Physics at tau-charm-beauty facilities

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CERN-BINP workshop for young scientists in e+e- colliders 22-24 August, 2016

OUTLINE

- 1. e⁺e⁻ physics introduction
- 2. e⁺e⁻ facilities
- 3. New data in tau physics
- 4. Charm news
- 5. Beaty physics
- 6. Future projects
- 7. Conclusions

Fundamental particles



Interactions



Masses of fermions



Flavour conservation/violation



Time when quarks and leptons appeared



Now quarks are seen as jets in e⁺e⁻ \rightarrow q qbar process. Similar to e⁺e⁻ $\rightarrow \mu^{+}\mu^{-}$.

e⁺e⁻ collisions single photon channel





Space-time picture of e⁺e⁻ – annihilation to hadrons



Polarization in e⁺e⁻ single photon channel



Effects of the beam longitudinal polarization







e⁺e⁻ collisions two photon channel





C = +1 C even quantum numbers $J^{PC} = 0^{-+}, 0^{++}, 1^{++}, 2^{++}, 2^{-+}, ...$

(P)Scalar helicity suppressed ~ $(m_e/M)^2$. Vector and Tensor states $\chi_{c},\,\chi_{b}$ can be produced ;

 $e^+e^- \rightarrow \chi_c, \chi_b, \sigma \sim 0.1 \text{ pb}$ Electron width is measured, Γ_{ee} depends on form factors (QCD)

e⁺e⁻ collisions two (quasi real) photon - channel - 2



Photon-photon collider

C = +1

C even quantum numbers Spin 1 - excluded $J^{PC} = 0^{-+}, 0^{++}, 1^{++}, 1^{+-}, 2^{++}, 2^{-+}, ...$ $L_{\gamma\gamma} \sim \alpha^2 ln\gamma^3$. L ee

Scalar and tensor states χ_c , χ_b can be produced No helicity suppression of scalar states Spin=1 excluded Two photon width is measured and $\gamma\gamma$ to hadron cross section.



Quantum numbers: (the same as in single photon channel) $J=L+S=J_{\gamma}=1$ S=1 (spin) L=0,2, S,D-waves $P=(-1)^{L}=-1$ $C=(-1)^{L+S}=-1$

$${}^{3}S_{1}, {}^{3}D_{1} = {}^{2S+1}L_{J}$$

 $J^{PC} = 1^{--}$

σ (ISR)~ (10⁻² -10⁻³) σ_{ee}

e+e→hadrons in ISR



ISR – Initial State Radiation or Radiative Return $\frac{d\sigma(s,x)}{dxd(\cos\theta)} = H(s,x,\theta) \cdot \sigma_0(s(1-x))$ ons H- radiation function $H(s,x,\theta) = \frac{\alpha}{\pi x} \left(\frac{2-2x+x^2}{\sin^2\theta} - \frac{x^2}{2}\right), \quad x = \frac{2E_{\gamma}}{\sqrt{s}}$

Advantages of ISR

- 1. Full energy range from $2m_{\pi}$ up to \sqrt{s} is available
- 2. Detection efficiency is flat over reaction mechanism
- 3. No large radiative corrections But:
- 1. Low luminosity < 1 % L₀

$\begin{array}{l} L_{ISR} \sim 0.3\% \ L_0 \ , \\ with \ L_0 \sim 0.5 \ ab^{\text{-1}} \ \ \text{--->} \ L_{ISR} \sim 1.5 \ fb^{\text{-1}} \ ! \end{array}$

Advantages of e+e-

- 1. Good energy resolution
- 2. Effective study of narrow resonances

But:

1. No efficiency at small angles $\theta < 0.2$



e⁺e⁻ facilities

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BEPCII storage rings



Beam energy: 1.0 - 2.3 GeV Peak Luminosity: Design: 1×10³³ cm⁻²s⁻¹ Achieved: 0.65 × 10³³ cm⁻²s⁻¹ Optimum energy: 1.89 GeV Energy spread: 5.16 ×10⁻⁴ Circumference: 237 m

Beam energy measurement: Using Compton backscattering technique. Accuracy up to 5×10⁻⁵



Summary of BESIII experiments



MORE:

- 3554 MeV 24 pb⁻¹ τ mass; 4100-4400 MeV 0.5 fb⁻¹ coarse scan
- On-going data taking





KEDR at VEPP-4



- + laser energy calibration
- + γ –beam
 - $\sigma/E_{\gamma} \sim 2$ % at 1 GeV measured BGO ~ 0.5% project
- + e- beam,

 $\sigma/E~$ ~ 2 % at 1 GeV

Physical program at KEDR E = 2 - 12 GeV1. Measurements of Ψ, Y, τ masses 2. $\gamma\gamma \rightarrow$ hadrons 3. Rad.transitions cc,bb $\rightarrow \gamma + X$ 4. $\mathbf{R} = \frac{\sigma(e^+e^- \rightarrow hadr)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$

Physics at e+e- $(\tau - c - b)$ colliders

1. QED: $e^+e^- \rightarrow e^+e^-$, $\mu^+\mu^-$, $\gamma\gamma$, used for luminosity normalization ~0.1%

2. QCD: e⁺e⁻
$$\rightarrow$$
 MM, BB, multihadrons, X,Y, Z, R = $\frac{\sigma(e^+e^- \rightarrow hadr)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$

- 3. **EW** : flavor transitions of low lying b,c \rightarrow s,u,d, CPV, mixing ...
- 4. **BSM** physics : study of rare decays of τ , D, B; LFV, FCNC, ...

Tau lepton physics in e⁺e⁻ $\rightarrow \tau^+ \tau^-$ process

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1. Lepton universality 2. LFV 3. V_{us} 4. α_S 5. $(g-2)_{\mu}$ 6. CPV

Lepton universality, $g_{\tau} = g_e = g_{\mu}$, ~10⁻³ $g_{\tau}/g_e = 1.0031 \pm 0.0016,$ $g_{\rm T}/g_{\mu} = 1.0013 \pm 0.0016.$ $g_{\mu}/g_{e} = 1.0018 \pm 0.0014$ Checked at ~ 10⁻³ level (A.Lusiani, HFAG) **Basic relations** $\Gamma(L \to \nu_L \ell \overline{\nu}_\ell(\gamma)) = \frac{B(L \to \nu_L \ell \overline{\nu}_\ell)}{\tau_L} = \frac{G_L G_\ell m_L^5}{192\pi^3} f\left(\frac{m_\ell^2}{m^2}\right) r_W^L r_\gamma^L ,$ Benerjee(TAU2012) LEP: W-> $\mu\nu$ / W-> $e\nu$ -> g_{μ}/g_{e} = 0.997+-0.010 $f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$ KLOE: K-> $\pi\mu\nu$ /K-> $\pi e\nu$ -> g_{μ}/g_{e} = 1.0010+-0.0025 HFAG: $\tau \to Kv / \pi \to \mu v \to g_{\tau}/g_{\mu} = 0.9857 + 0.0079$ $r_{\gamma}^{L} = 1 + \frac{\alpha(m_{L})}{2\pi} \left(\frac{25}{4} - \pi^{2}\right)$

Lepton universality in K,D,B decays



Different sign of deviation in D and B decays !

τ Michel parameters – SM test

$e^+e^- \rightarrow \tau^+\tau^-$, $\tau_1 \rightarrow Ivv$, $I = e, \mu$; 1409.4969

Lepton spectrum in τ decay ,1409.4969 (depends on Michel parameters)

$$\frac{d\Gamma(\tau^{\mp})}{d\Omega dx} = \frac{4G_F^2 M_\tau E_{\text{max}}^4}{(2\pi)^4} \sqrt{x^2 - x_0^2} \left(x(1-x) + \frac{2}{9}\rho(4x^2 - 3x - x_0^2) + \eta x_0(1-x) \right)$$
$$\mp \frac{1}{3} P_\tau \cos\theta_\ell \xi \sqrt{x^2 - x_0^2} \left[1 - x + \frac{2}{3}\delta(4x - 4 + \sqrt{1 - x_0^2}) \right] \right), \ x = \frac{E_\ell}{E_{\text{max}}}, \ x_0 = \frac{m_\ell}{E_{\text{max}}}$$
$$\text{In the SM: } \rho = \frac{3}{4}, \ \eta = 0, \ \xi = 1, \ \delta = \frac{3}{4}$$

τ Michel parameters

Michel parameters (MP) - parameters in lepton spectrum $\tau_1 \rightarrow Ivv$, sensitive to EW modification, NP

$$e^+e^- \rightarrow \tau^+\tau^-$$
, $\Delta L = 485$ ifb, arXiv1409.4969 (2014)

$$\tau_1 \rightarrow |\nu\nu\rangle$$
, $| = e, \mu$; $\tau_2 \rightarrow \rho\nu \rightarrow \pi\pi^0\nu$,

spin-spin correlation,

precisely measured 9 dim phase space kinematics, fit gives four MP uncertainties







Lepton flavor violation (LFV) in tau decays

 $\tau \rightarrow \mu \gamma$ $B_{\tau \mu \gamma} < 5.10^{-8}$ (Belle) LF conserved when $m_v = 0$ V oscillations lead to $m_v \neq 0$ SM extended to $m_v \neq 0$ SM estimate 10^{-54} is many orders below experiment LFV will be a sign of new physics (W---H charged)



Summary of Tau LFV decay limits



Limits on Br($\tau \rightarrow |\gamma, 3|$) ~ 2.10⁻⁸ at Babar and Belle.

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CVC

 $\label{eq:cvc} \text{CVC} - \text{establishes a relation between } \tau - \text{lepton decays to vector hadronic state} \\ \text{and corresponding isovector channel of e+e- annihilation cross section} \\$









 V_{us} calculated from π , K decay constant and τ decays. V_{us} from τ agrees with V_{us} from CKM triangle.





0.

G.Lafferti

E (GeV)

Charm energy region



Quarkonia are similar to atom


A comparison of charmonium and positronium structure



e+e- workshop



KEDR recent results

$$\mathsf{R} = \frac{\sigma(e^+e^- \to hadr)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



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X,Y,Z states in charmonium

X,Y,Z are new objects in quarkonium systems. Most of them – unexpected. They have quantum numbers of cc and bb states They may have electric charge ! Their nature – molecule, hybrid, 4-quark ? No X,Y,Z found in Y(1S) decays

X

- charmonium-like states with $J^{PC} \neq 1^{--}$ Observed in B decays, e⁺e⁻, pp and pp collisions

Υ

- charmonium-like states with $J^{PC}=1^{--}$. Observed in direct e⁺e⁻ annihilation or in ISR

Ζ

- charmonium -like complex states . Must contain at least a cc(bb) and a light qq pair

Exotic states



4q-mesons

States of tightly bound quarks. Decay to ordinary hadrons via the position reordering.



Molecular mesons

Weakly bound through pion exchange pairs of mesons. Decay to constituent mesons or J/Psi + X.



Hybrid mesons

Bound states of gluon and colored diquark.







 Y_c - charmonium-like states with $J^{PC}=1^{--}$. Observed in direct e^+e^- annihilation or in ISR



Observation of Z_c in Y(4360) $\rightarrow \pi \pi \Psi(2S) \rightarrow \pi Z_c$ (4054)



Z states:

- charmonium -like states . Must contain at least a cc and a light qq pair

Table of Z states

$e^+e^- \rightarrow \pi {}^{\pm}Z_c(3900)^{\pm} \rightarrow \pi^+\pi^- J/\psi$



State	Z _c decay	Reaction
Z _C (3900) ^{±0}	π ^{±0} J/Ψ	$e^+e^- \rightarrow \pi^{\pm 0} Z_C$
Z _C (3885) ^{±0}	(D D*) ^{±0}	$e^+e^- \rightarrow \pi^{\pm 0} Z_C$
Z _C (4020) ^{±0}	$\pi^{\pm 0}h_{C}$	$e^+e^- \rightarrow \pi^{\pm 0} Z_C$
Z _C (4025) ^{±0}	(D* D*)±0	$e^+e^- \rightarrow \pi^{\pm 0} Z_C$

Nature of Z states: -

$$c\bar{c}g, cq\bar{q} \ \bar{c}, (\bar{c}q)(c\bar{q}), c\bar{c}\pi\pi, \dots$$

All Z_c found unpredicted

tetraquark?

- hadronic molecule
- hadro charmonium ?
- hybrid

e⁺e⁻ B factory

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The B factory is e⁺e⁻ collider, operating in energy region of production of particles with b quark. Energy interval max ~ 9 – 13 GeV in c.m.s. $(\eta_b - Y(6s) - B_c - region)$

e⁺e⁻ B factories summary

Cross sections at Y(4S)			
e + e> τ + τ -	0.919 nb		
e + e> μ + μ -	1.147 nb		
e + e> uds -	2.09 nb		
e + e> c cbar	1.3 nb		
e + e> b bar	1.05 nb		

Integrated luminosity

Babar ~ 500 ifb Belle ~ 1000 ifb

 Number of produced events

 $\tau\tau$ ~ 1.4 10 ⁹

 $\mu\mu$ ~ 2.2 10 ⁹

 bb
 ~ 1.6 10 ⁹

 cc
 ~ 2. 10 ⁹

 uds
 ~ 3.1 10 ⁹

Two b_s , τ_s in an event, self_tagging kinematics

e⁺e⁻ hadronic cross-section in Y region



BABAR and BELLE important results



Transitions between bb quarkonium



Observation of $h_b(1P,2P)$



Hyperfine splitting (1+- - 1++) is not seen

Observation of Z_b in Y(5S) $\rightarrow \pi \pi Y(nS) \rightarrow \pi Z_b$

(not predicted !)

 $Z_{b}(10610), Z_{b}(10650), J^{P} = 1^{+}$



 $Z_b \rightarrow \pi Y(nS)$, n=1,2,3

$$B_{g} \rightarrow \mu \mu$$
 , $B_{d} \rightarrow \mu \mu$

$$B_d = (b\bar{d}), B_s = (b\bar{s}),$$

Standard model :
$$B_s \rightarrow \mu\mu = 3.65 \pm 0.23 \ 10^{-9}$$

 $B_d \rightarrow \mu\mu = 1.06 \pm 0.09 \ 10^{-10}$



Experiment :
$$B_s \rightarrow \mu\mu = 2.8 \pm 0.65 \ 10^{-9}$$

LHCb, CMS $B_d \rightarrow \mu\mu = 3.9 \pm 1.5 \ 10^{-10}$
2.2 σ !

LHCb and CMS have advantages in statistics. BELLE2 can confirm their result.

New physics ?
$$\sum_{s}^{b} Z' \qquad \mu^{+}$$

Semileptonic decays



No suppression in SM

Data, HFAG: R(D*)=0.322±0.018±0.012 R(D) =0.391±0.041±0.028 Models: Leptoquarks Composite higgs Z' Hadronic effects

SM R(D*)=0.252±0.003 R(D) =0.300±0.010

Most serious deviation from SM

Cancellations:

- In theory calculations,
- In experimental systematics



Decays with ee or $\mu\mu$ pairs

$$R_k = BR(B^+ \rightarrow K^+ \mu^+ \mu^-) / BR(B^+ \rightarrow K^+ e^+ e^-)$$

PDG : BR(B⁺ \rightarrow K⁺I⁺I⁻) ~ 5.10⁻⁷

 $R_k(SM) = 1.00 \pm 0.01$ $R_k(LHCb)=0.745\pm0.08\pm0.036$ $\mu\mu$ deficit at M_{I+I-} <2.5 GeV



Other heavy particles can contribute instead of W, t, Z.

Decay $B^+ \rightarrow \tau^+ \nu_{\tau}$

2-3 neutrinos in an event !



Precise SM prediction: $Br(B \rightarrow l\nu) = \frac{G_F^2 m_B}{8\pi} m_l^2 (1 - \frac{m_l^2}{m_B^2}) f_B^2 |V_{ub}|^2 \tau_B$ Experiment: BR(B $\rightarrow \tau \nu$) = (11.4 +- 2.2) 10⁻³ Theory: BR(B $\rightarrow \tau \nu$) = 7.4 10⁻³ (exp.- theory) ~ 1.8 σ !

Future projects in τ -c-b physicS

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- 1. SuperKEKB B factory
- 2. Super C-Tau-Charm factory
 - Novosibirsk
 - Bejing

SuperKEKB Project



KEKB upgrade → SuperKEKB(nano-beam)



Super KEK-B

- increased beam current,
- reduced beam size,
- higher beam background,
- larger crossing angle,
- $L_{int} \rightarrow 50 \text{ ab}^{-1}$

2017 – collisions tuningpartial BELLE22018 – full comissioning

 $L_{max} = 8 \ 10^{35} \ cm^{-2} s^{-1}$



BELLE II prospects in physics

$$R(D^{(*)}) = \frac{\Gamma(B^0 \rightarrow D^{(*)} \tau \nu)}{\Gamma(B^0 \rightarrow D^{(*)} l \nu)_{l=u,e}}$$



new Belle measurement $R(D^*) = 0.302 \pm 0.030(stat) \pm 0.011(syst)$

BELLE-II goal $\rightarrow \pm 0.01$ (12 σ instead of 3.9 σ ?)

LFV

$$\begin{split} \tau &\rightarrow \mu\gamma, \text{BR limit 4 } 10^{-8} \rightarrow 3 \; 10^{-9} \\ \tau &\rightarrow \mu\mu\mu, \text{BR limit2 } 10^{-8} \rightarrow 8 \; 10^{-10} \\ \text{NP models} \rightarrow 10^{-7} \rightarrow 10^{-10} \\ \text{Belle II compared to CLEO, Babar,} \\ \text{Belle, LHCb} \end{split}$$



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BELLE II prospects in physics



BR(B→ $\tau\nu$)=1.14±0.22)x10⁻⁴ - current measurement SM = 0.75x10⁻⁴ Suppressed by V_{ub} in SM, but could be enhanced by NP process, e.g., charged Higgs Belle II could reduce the error in BR(B→ $\tau\nu$) to 5% For BR(B→ $\mu\nu$) →10 % Instead of 1.8 σ → 7 σ ?



Super C-Tau project in Novosibirsk

Physics at tau-charm facility



Super C-Tau factory project in China

China: HIEPA



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Production rate / year

SCTF, 1
$$ab^{-1}$$

J/ Ψ - 1 10¹¹
DD - 3 10⁹
D_s D_s - 3 10⁹
 $\tau^{+}\tau^{-}$ - 3 10⁹

Compare with BELLE-II, at Y(4S), L= 10 ab⁻¹ Charm - 10^{10} $\tau^+\tau^-$ - 10^{10}

Conclusions

- 1. First generations B-factories in KEK and SLAC completed data taking with overall luminosity of 1.5 inv.ab.
- 2. CP-violation in b-sector is confirmed, many new results in
 - τ –c–b physics are obtained. Some results are in tension with SM.
- 3. BES-III continues data taking with max luminosity of 10³³ cm⁻²s⁻¹.
- 4. BELLE-2 detector in KEK is close to start data taking at upgraded collider with the max luminosity of 8 10³⁵ cm⁻²s⁻¹.
- 5. A project of super-charm-tau collider is developing in Novosibirsk and Bejing
- 6. e^+e^- facilities in τ -c-b physics are in strong competitions with LHC detectors LHCb, ATLAS, CMS.

Thank you for listening !

Backup slides

Total electric charge of quarks and leptons = 0



Quarks and leptons could be relatives . Hint of transition between quarks and leptons? Total electron and proton charge = 0q(p)+q(e⁻)=0

Why?

Hint of mutual transitions between them ?

Model of grand Unification

Search of proton decay $p \rightarrow \pi^0 + e^+$ $\tau_p > 10^{32} \text{ years}$

Fundamental constant and dimensions



Three constants

+

- **h** Plank constant (min. spin)
- **c** light speed (max speed)

$$G = hc/m_P^2 = 6.7 \ 10^{-8} \ cm^3 \ gr^{-1} \ s^{-1}$$

 $m_P = 1,2 \cdot 10^{19} \, GeV/c^2$;

Dimensions do not directly correspond to constant

Redefinitiom of constants:	
$l_P = \frac{\hbar c}{m_P} = 10^{-33} cm$	
$t_P = \frac{l_P}{c} = 10^{-43} \mathrm{sec}$	

Physical program for S-C-Tau facility

Physics below open charm

- Light hadron spectroscopy
- Rare and forbidden decays
- Physics with τ lepton
- Process of LFV and CPV

1. QCD and —

- 2. Light hadrons
- 3. Tau physics
- 4. Charmonium
- 5. Charm
- 6. X,Y,Z states

Above threshold of open charm

- Physics with D mesons
- Charm baryons
- f_D and f_{Ds}
- $D_0 D_0$ mixing
- X,Y,Z particles

Physics below J/ Ψ

- Search of multiquark states with s-quark
- Hadron form factors
- Study of Y(2160)
- ...

Levels of bottomonium



THE BOTTOMONIUM SYSTEM from the Particle Data Group, http://pdglive.lbl.gov/


STCF: Machine Parameters

	INFN	BINP	China
Beam Energy, <u>GeV</u>	1.0-2.3	1.0-2.5	1.0-3.5
Circumference, m	340.7	<mark>813</mark>	992.8
Number of bunches	530	390	540
Bunch Current, mA	3.29	4.4	5
Beam Current, A	1.0-1.745	1.7	2.7
Emittance Horiz, nm	56.11-4.89	8	10
Emittance Vert, nm	0.015-0.012	0.04	0.05
Bunch length, mm	10.1-6.9	16-10	<mark>1</mark> 0
beta x (IP), cm	7	4	100
beta y (IP), cm	0.06	0.08	0.1
RF frequency, MHz	476	508	500.06
Luminosity, cm ⁻² s ⁻¹	0.2-1x10 ³⁵	0.63-1x10 ³⁵	1.05x10 ³⁵
			Y.Sakai

Tau polarization in $e^+e^- \rightarrow \tau^+\tau^-$ process, electrons – longitudinally polarized (R.Li)

$$\begin{aligned} \boldsymbol{\xi}_{\mathrm{ph}} \cdot \boldsymbol{N} &= \pm \frac{(\boldsymbol{e}_{z}\boldsymbol{N})}{1 - \frac{\beta^{2}}{2}N_{\perp}^{2}} = \pm \frac{\cos\theta}{1 - \frac{\beta^{2}}{2}\sin^{2}\theta} \text{ - tau longitidunal polarization} \\ \boldsymbol{\xi}_{\mathrm{ph}} \cdot \frac{\boldsymbol{e}_{z} - \boldsymbol{N}(\boldsymbol{e}_{z}\boldsymbol{N})}{|\boldsymbol{e}_{z} - \boldsymbol{N}(\boldsymbol{e}_{z}\boldsymbol{N})|} &= \pm \frac{\frac{1}{\gamma}\sin\theta}{1 - \frac{\beta^{2}}{2}\sin^{2}\theta} \text{ - tau transverse polarization} \\ \boldsymbol{\xi}_{\mathrm{ph}} \cdot \boldsymbol{e}_{z} &= \pm \frac{\frac{1}{\gamma} + \left(1 - \frac{1}{\gamma}\right)(\boldsymbol{e}_{z}\boldsymbol{N})^{2}}{1 - \frac{\beta^{2}}{2}N_{\perp}^{2}} = \pm \frac{\frac{1}{\gamma} + \left(1 - \frac{1}{\gamma}\right)\cos^{2}\theta}{1 - \frac{\beta^{2}}{2}\sin^{2}\theta} \text{ - tau polarization along electrons} \\ \boldsymbol{\xi}_{\mathrm{ph}}^{2} = \frac{1 - \beta^{2}N_{\perp}^{2}}{\left(1 - \frac{\beta^{2}}{2}N_{\perp}^{2}\right)^{2}} = \frac{1 - \beta^{2}\sin^{2}\theta}{\left(1 - \frac{\beta^{2}}{2}\sin^{2}\theta\right)^{2}} \text{ - tau polarization} \end{aligned}$$