Quasiclassical approach and high energy QED processes in the field of a heavy atom.

P. A. Krachkov

Budker Institute of Nuclear Physics

23 august, 2016

Outline

Introduction

QED processes in the field of a heavy atom

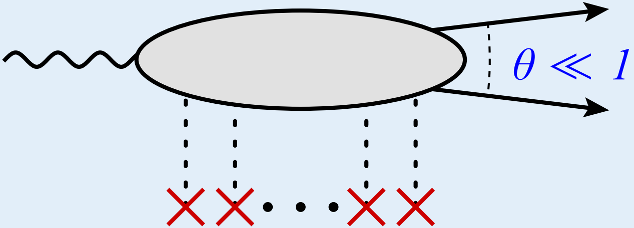
- Elastic scattering $e^{\pm}+Z \to e^{\pm}+Z$
- Bremsstrahlung $e^{\pm}+Z \to e^{\pm}+\gamma+Z$
- Pair production (PP) $\gamma+Z \to e^{+}+e^{-}+Z$
- Double bremsstrahlung $e^{\pm}+Z \to e^{\pm}+\gamma_1+\gamma_2+Z$
- Electroproduction $e^{\pm}+Z \to e^{\pm}+e^{+}+e^{-}+Z$
- Delbrück scattering $\gamma+Z \to (e^{+}+e^{-})+Z \to \gamma'+Z$
- Photon splitting $\gamma+Z \to (e^{+}+e^{-})+Z \to \gamma_1+\gamma_2+Z$

- Cross section
- Energy spectrum
- Differential cross section
- Polarization effects

Introduction

Typical experimental conditions

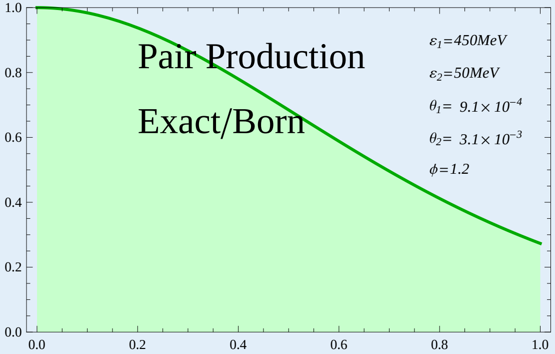
- High energy $E \gg m$
- Small characteristic angle $\theta \ll 1$
- High nuclear charge $Z \gg 1 \implies Z\alpha \sim 1$



Introduction

Coulomb corrections

- The Coulomb corrections in the differential cross section are 100% important except of small momentum transfer area $\Delta \ll m$.



- Total cross section: Born term contains large logarithm $\Longrightarrow CC \sim 10 \div 20\%$.

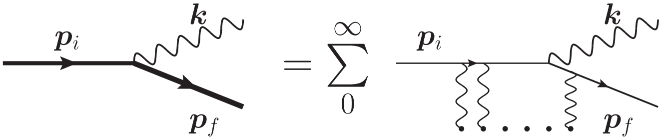
Furry representation

Diagram technique(Furry, 1934)

Furry representation allows to take into account exactly external field

$\rightarrow \quad G(\mathbf{r_1}, \mathbf{r_2}|\varepsilon) = \langle \boldsymbol{r}_2 | \frac{1}{\hat{\mathscr{P}} - m + i0} | \boldsymbol{r}_1 \rangle$

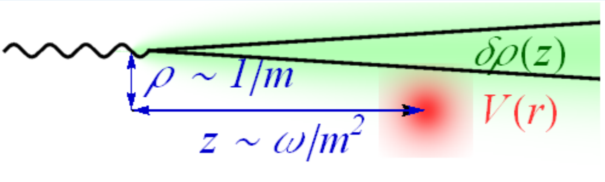
$\rightarrow \quad u^{in}_{\mathbf{p}}(\mathbf{r})$



Quasiclassical approximation

The characteristic parameters of the high energy processes

- $\Delta\tau \sim 1/m$ — The virtual pair life time in the comoving frame
- $\Delta t = \Delta\tau\gamma \sim \omega/m^2$ — The virtual pair life time in the LFR
- $\rho \sim 1/m$ — The loop transverse size in the LFR
- $z = \Delta t \sim \omega/m^2$ — The loop longitudinal size in the LFR
- $l \sim E\rho \sim \omega/m \gg 1$ — The angular momentum in the LFR
- $\theta \sim 1/l \sim m/\omega \ll 1$ — The characteristic angle in the LFR



Furry-Sommerfeld-Maue wave functions

(Furry, 1934; Sommerfeld and Maue, 1935)

Using partial wave expansion and neglecting terms of order $(Z\alpha)^2/l$ Furry obtained the wave function of Dirac equation in the Coulomb field $V(r) = -Z\alpha/r$:

$$\psi^{(+)} = e^{\pi Z\alpha/2}\Gamma(1-iZ\alpha)\,e^{i\mathbf{kr}}\left(1-i\frac{\alpha\nabla}{2\varepsilon}\right){}_1F_1\left(iZ\alpha,1,i\left[kr-\mathbf{kr}\right]\right)u$$

Furry-Sommerfeld-Maue wave functions

(Furry, 1934; Sommerfeld and Maue, 1935)

Using partial wave expansion and neglecting terms of order $(Z\alpha)^2/l$ Furry obtained the wave function of Dirac equation in the Coulomb field $V(r) = -Z\alpha/r$:

$$\psi^{(+)} = e^{\pi Z\alpha/2}\Gamma(1-iZ\alpha)\, e^{i\mathbf{kr}}\left(1 - i\frac{\alpha\nabla}{2\varepsilon}\right){}_1F_1(iZ\alpha, 1, i[kr - \mathbf{kr}])\, u$$

Useful tool

Pair production, Bremsstrahlung— (Bethe and Maximon, 1954; Davies et al., 1954)

Furry-Sommerfeld-Maue wave functions

(Furry, 1934; Sommerfeld and Maue, 1935)

Using partial wave expansion and neglecting terms of order $(Z\alpha)^2/l$ Furry obtained the wave function of Dirac equation in the Coulomb field $V(r) = -Z\alpha/r$:

$$\psi^{(+)} = e^{\pi Z\alpha/2}\Gamma(1 - iZ\alpha)\, e^{i\mathbf{kr}} \left(1 - i\frac{\alpha\nabla}{2\varepsilon}\right) {}_1F_1(iZ\alpha, 1, i[kr - \mathbf{kr}])\, u$$

Questions:

- How to derive analog of the FSM functions for arbitrary localized potential?
- How to derive quasiclassical Green's functions?
- How to go beyond leading quasiclassical approximation?

Quasiclassical approximation

Quasiclassical Green's function(Lee et al., 2000; Krachkov and Milstein, 2015)

$$D(\boldsymbol{r}_2,\boldsymbol{r}_1|\varepsilon) = d_0(\boldsymbol{r}_2,\boldsymbol{r}_1|\varepsilon) + \boldsymbol{\alpha}\cdot\boldsymbol{d}_1(\boldsymbol{r}_2,\boldsymbol{r}_1|\varepsilon) + \boldsymbol{\Sigma}\cdot\boldsymbol{d}_2(\boldsymbol{r}_2,\boldsymbol{r}_1|\varepsilon)$$

$$d_0 \sim l_c d_1 \sim l_c^2 d_2 \qquad l_c \sim \varepsilon/\Delta \gg 1$$

$$d_0(\boldsymbol{r}_2,\boldsymbol{r}_1|\varepsilon) = \frac{ie^{i\kappa r}}{4\pi^2 r}\int d\boldsymbol{Q}\exp\left[iQ^2 - ir\int_0^1 dx V(\boldsymbol{R}_x)\right]$$
$$\times\left\{1 + \frac{ir^3}{2\kappa}\int\limits_0^1 dx\int\limits_0^x dy(x-y)\boldsymbol{\nabla}_\perp V(\boldsymbol{R}_x)\cdot\boldsymbol{\nabla}_\perp V(\boldsymbol{R}_y)\right\}$$

$$\mathbf{R}_x = \mathbf{r}_1 + x\mathbf{r} + \mathbf{q}\sqrt{2x\bar{x}r/k} \Longleftarrow \text{quantum fluctuations}$$

Quasiclassical approximation

Quasiclassical Green's function for Coulomb field

$$D(\boldsymbol{r}_2, \boldsymbol{r}_1|\varepsilon) = \frac{ie^{i\kappa(r_1+r_2)}}{8\pi\kappa r_1 r_2} \int d\boldsymbol{q} \left(\frac{2\kappa\sqrt{r_1 r_2}}{q}\right)^{2iZ\alpha} \exp\left[\frac{i(r_1+r_2)q^2}{2pr_1r_2} + i\boldsymbol{q}\cdot\boldsymbol{\theta}\right]$$
$$\left\{\left[1+\frac{i\pi(Z\alpha)^2}{2q}\right]\left[1+Z\alpha\frac{\boldsymbol{\alpha}\cdot\boldsymbol{q}}{q^2}\right] - \frac{i\pi(Z\alpha)^2}{4q^3}\frac{\boldsymbol{r}}{r}\cdot[\boldsymbol{q}\times\boldsymbol{\Sigma}]\right\}$$

Applications

e^+e^- pair production.

Total cross section

Bethe-Maximon for Coulomb corrections

$$\sigma_C^{(0)} = -\frac{28\alpha\,(Z\alpha)^2}{9m^2} f\,(Z\alpha)$$

is valid formaly for $\omega \gg m$.

e^+e^- pair production.

Total cross section

Bethe-Maximon for
Coulomb corrections

$$\sigma_C^{(0)} = -\frac{28\alpha\left(Z\alpha\right)^2}{9m^2} f\left(Z\alpha\right)$$

is valid formaly for $\omega \gg m$.

(Lee et al., 2004)

Using derived quasiclassical Green's function with the first QC correction we have obtained

$$\sigma_C^{(1)} = \frac{\alpha\left(Z\alpha\right)^2 \pi^4}{2m\omega} \operatorname{Im} g(Z\alpha).$$

$$g(Z\alpha) = Z\alpha \frac{\Gamma(1-iZ\alpha)\Gamma(1/2+iZ\alpha)}{\Gamma(1+iZ\alpha)\Gamma(1/2-iZ\alpha)}$$

Huge coefficient π^4 !

e^+e^- pair production.

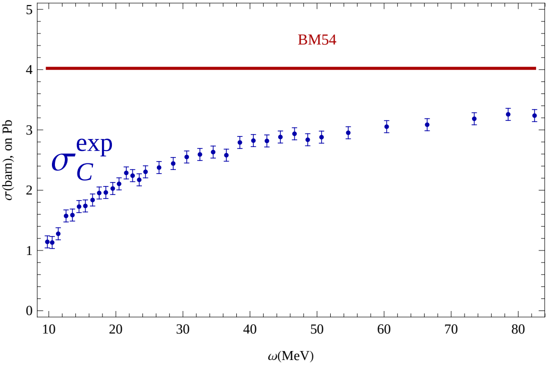
Total cross section

Bethe-Maximon for Coulomb corrections

$$\sigma_C^{(0)} = -\frac{28\alpha (Z\alpha)^2}{9m^2} f(Z\alpha)$$

is valid formaly for $\omega \gg m$.

Experimental data



e^+e^- pair production.

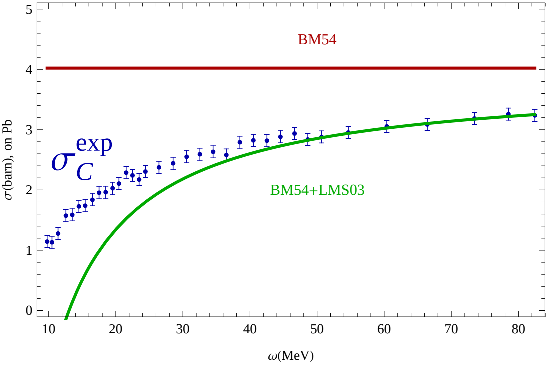
Total cross section

Bethe-Maximon for Coulomb corrections

$$\sigma_C^{(0)} = -\frac{28\alpha\,(Z\alpha)^2}{9m^2} f\,(Z\alpha)$$

is valid formaly for $\omega \gg m$.

Experimental data



QC correction taken into account.

Bremsstrahlung

Differential cross section

Momentum transfer distribution (Lee et al., 2005)

$$\frac{d\sigma_C^{BS}}{d^2\Delta_\perp} \propto \Delta_\perp^2 \left[|A_0(\Delta_\perp)|^2 - |A_{0B}(\Delta_\perp)|^2 \right],$$

$$A_0 = \frac{-i}{\Delta_\perp^2} \int d\boldsymbol{r} \exp\left[-i\boldsymbol{\Delta}\cdot\boldsymbol{r} - i\chi(\rho)\right] \boldsymbol{\Delta}_\perp \cdot \boldsymbol{\nabla}_\perp V(\boldsymbol{r})$$

Bremsstrahlung

Differential cross section

Momentum transfer distribution (Lee et al., 2005)

$$\frac{d\sigma_C^{BS}}{d^2\Delta_\perp} \propto \Delta_\perp^2 \left[|A_0(\Delta_\perp)|^2 - |A_{0B}(\Delta_\perp)|^2 \right],$$

$$A_0 = \frac{-i}{\Delta_\perp^2} \int d\boldsymbol{r} \exp\left[-i\boldsymbol{\Delta}\cdot\boldsymbol{r} - i\chi(\rho)\right] \boldsymbol{\Delta}_\perp \cdot \boldsymbol{\nabla}_\perp V(\boldsymbol{r})$$

For Coulomb potential at $\Delta_\perp \gg \Delta_{min} \sim m^2/\varepsilon$

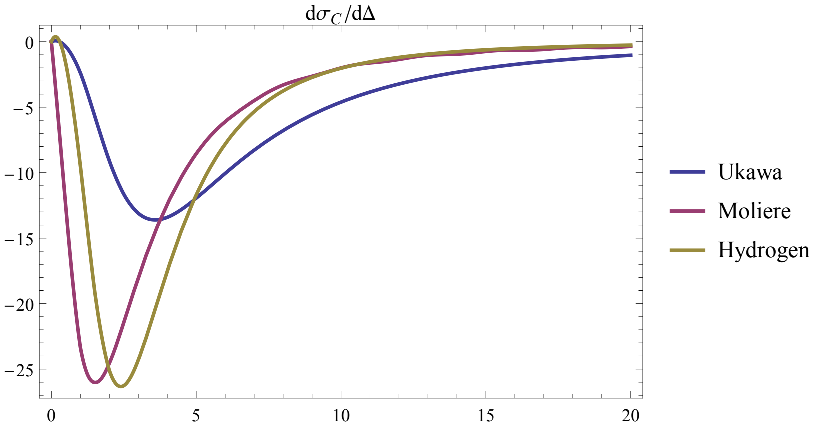
$$A(\Delta_\perp) = A_B(\Delta_\perp) \frac{\Gamma(1-iZ\alpha)}{\Gamma(1+iZ\alpha)} \left(\frac{4}{\Delta_\perp^2}\right)^{-iZ\alpha} \implies |A(\Delta_\perp)| = |A_B(\Delta_\perp)|$$

Coulomb corrections come from the region $\Delta_\perp \sim \Delta_{\min}$

Bremsstrahlung

Differential cross section

(Lee et al., 2005)



$d\sigma_C^{BS}/d^2\Delta_\perp$ essentially depends on screening, $\sigma_C^{BS} \propto (Z\alpha)^2 f(Z\alpha)$ is universal!

Recent results

Exact in $Z\alpha$!

- $\gamma + Z \to e^+ + e^- + Z$: Differential cross section, spectrum, total cross section (Lee et al., 2012, 2004) with the account of the first QC correction and screening.

Recent results

Exact in $Z\alpha$!

- $\gamma + Z \to e^+ + e^- + Z$: Differential cross section, spectrum, total cross section (Lee et al., 2012, 2004) with the account of the first QC correction and screening.

- $e^\pm + Z \to e^\pm + \gamma + Z$: Differential cross section, spectrum (Lee et al., 2005; Krachkov and Milstein, 2015), with the account of the first QC correction and screening.

Recent results

Exact in $Z\alpha$!

- $\gamma+Z\to e^{+}+e^{-}+Z$: Differential cross section, spectrum, total cross section (Lee et al., 2012, 2004) with the account of the first QC correction and screening.
- $e^{\pm}+Z\to e^{\pm}+\gamma+Z$: Differential cross section, spectrum (Lee et al., 2005; Krachkov and Milstein, 2015), with the account of the first QC correction and screening.
- $\gamma+Z\to e^{+}+e^{-}+\gamma'+Z$: Differential cross section (Krachkov et al., 2014)

Recent results

Exact in $Z\alpha$!

- $\gamma + Z \to e^+ + e^- + Z$: Differential cross section, spectrum, total cross section (Lee et al., 2012, 2004) with the account of the first QC correction and screening.
- $e^\pm + Z \to e^\pm + \gamma + Z$: Differential cross section, spectrum (Lee et al., 2005; Krachkov and Milstein, 2015), with the account of the first QC correction and screening.
- $\gamma + Z \to e^+ + e^- + \gamma' + Z$: Differential cross section (Krachkov et al., 2014)
- $e^\pm + Z \to e^\pm + \gamma_1 + \gamma_2 + Z$: Differential cross section (Krachkov et al., 2015)

Recent results

Exact in $Z\alpha$!

- $\gamma + Z \to e^+ + e^- + Z$: Differential cross section, spectrum, total cross section (Lee et al., 2012, 2004) with the account of the first QC correction and screening.
- $e^\pm + Z \to e^\pm + \gamma + Z$: Differential cross section, spectrum (Lee et al., 2005; Krachkov and Milstein, 2015), with the account of the first QC correction and screening.
- $\gamma + Z \to e^+ + e^- + \gamma' + Z$: Differential cross section (Krachkov et al., 2014)
- $e^\pm + Z \to e^\pm + \gamma_1 + \gamma_2 + Z$: Differential cross section (Krachkov et al., 2015)
- $e^\pm + Z \to e^\pm + e^+ + e^- + Z$: Dierential cross section

Summary

- Quasiclassical approach provides effective reliable framework for investigation of the high-energy QED processes in the field of heavy atom.
- Quasiclassical Green's functions and wave functions in arbitrary localized potential are derived with the account of the first QC correction (with typical relative magnitude θ or m/ε).
- Applications include all basic high-energy QED processes in the field of heavy atom.

Thank you for attention!

References I

Bethe, H. A. and L. C. Maximon: 1954, 'Theory of Bremsstrahlung and Pair Production. 1. Differential Cross Section'. Phys. Rev. 93, 768–784.

Davies, H., H. A. Bethe, and L. C. Maximon: 1954, 'Theory of Bremsstrahlung and Pair Production. 2. Integral Cross Section for Pair Production'. Phys. Rev. 93, 788–795.

Furry, W. H.: 1934, 'Approximate Wave Functions for High Energy Electrons in Coulomb Fields'. Phys. Rev. 46, 391–396.

Krachkov, P. A., R. N. Lee, and A. I. Milstein: 2014, 'High-energy e^+e^- photoproduction in the field of a heavy atom accompanied by bremsstrahlung'. Phys. Rev. A90(6), 062112.

Krachkov, P. A., R. N. Lee, and A. I. Milstein: 2015, 'Double bremsstrahlung from high-energy electrons in an atomic field'. Phys. Rev. A91(6), 062109.

Krachkov, P. A. and A. I. Milstein: 2015, 'Charge asymmetry in the differential cross section of high-energy bremsstrahlung in the field of a heavy atom'. Phys. Rev. A91(3), 032106.

Lee, R. N., A. I. Milstein, and V. M. Strakhovenko: 2000, 'Quasiclassical Green function in an external field and small-angle scattering'. JETP 117, 75.

Lee, R. N., A. I. Milstein, and V. M. Strakhovenko: 2004, 'High-energy expansion of Coulomb corrections to the e+ e- photoproduction cross-section'. Phys. Rev. A69, 022708.

Lee, R. N., A. I. Milstein, and V. M. Strakhovenko: 2012, 'Charge Asymmetry in the Differential Cross Section of High-Energy e+e- Photoproduction in the Field of a Heavy Atom'. Phys. Rev. A85, 042104.

Lee, R. N., A. I. Milstein, V. M. Strakhovenko, and O. Y. Schwarz: 2005, 'Coulomb Corrections to Bremsstrahlung in the Electric Field of a Heavy Atom at High Energies'. JETP 127, 5–17.

Sommerfeld, A. and A. W. Maue: 1935, 'Verfahren zur näherungsweisen Anpassung einer Lösung der Schrödinger-an die Diracgleichung'. Annalen der Physik 414(7), 629–642.