Expansion by Region and LHC Scalar Boson Production at N3LO

Theory Seminar at DESY Hamburg

Franz Herzog (Nikhef)

Collaborators:
C Anastasiou, F Duhr, C Dulat, E Furlan, T Gehrmann, A Lazopoulos, B Mistlberger

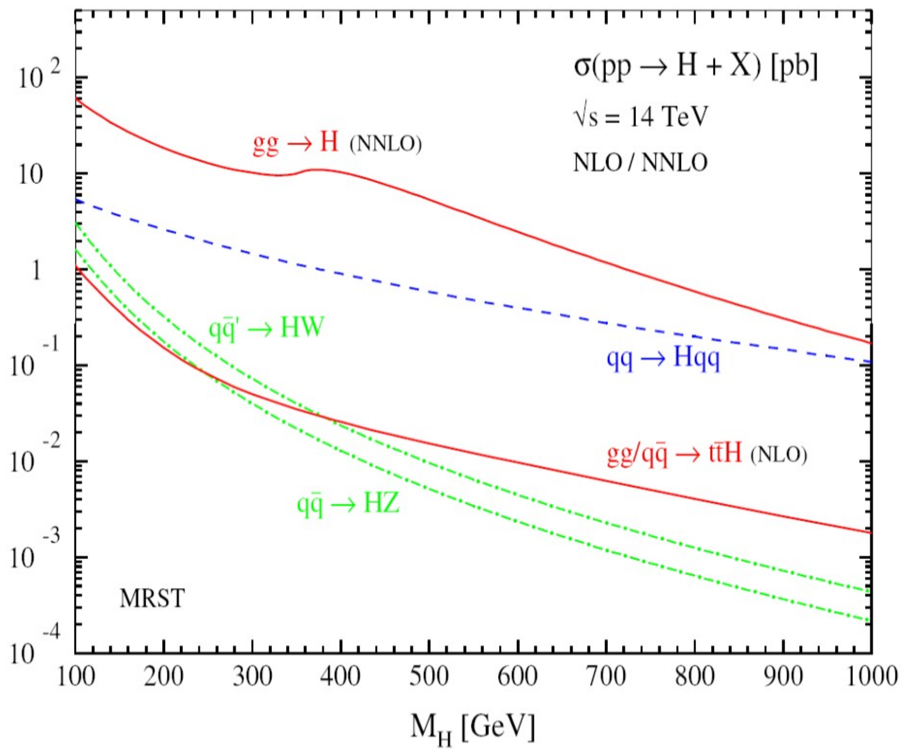
Motivation

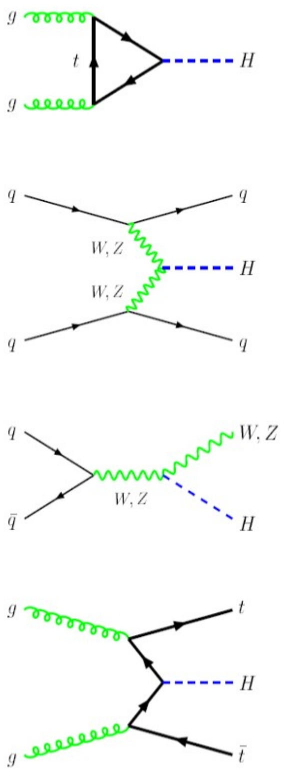
- How many Higgs bosons at the LHC?
  - Total Higgs boson cross-section is an important number for LHC Higgs coupling extractions
  - used for normalisations in Parton shower, fully differential NNLO, resummed differential predictions
  - N3LO QCD corrections are required due to poorly converging perturbation theory
- N3LO correction demand evaluation of new difficult multi-loop and phase space integrals with several scales.
  - Such integrals are often difficult to evaluate in closed form.
  - Sometimes a well posed Taylor Expansion can allow for sufficiently accurate evaluation.

Overview

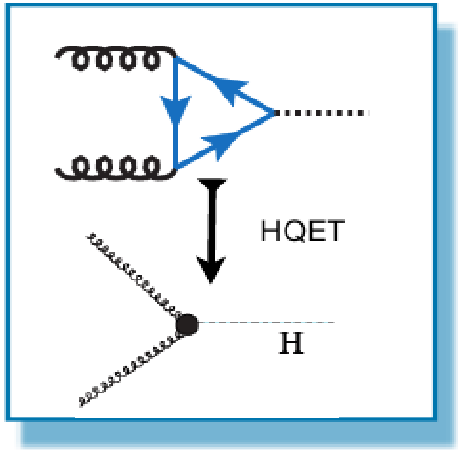
- Higgs boson production at the LHC in gluon fusion
- Soft limit, Expansion by Region and N3LO QCD Corrections
- Results including N3LO and study of remaining sources of uncertainty
- 750 GeV Scalar production

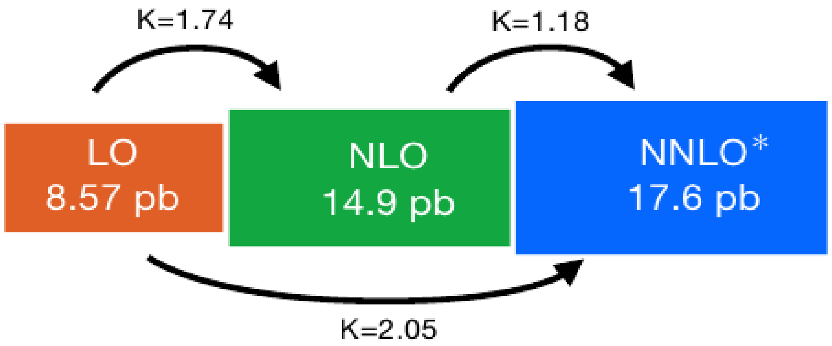
LHC Higgs Production in the Standard Model





Higgs Production in gluon fusion





Higgs Production in Gluon Fusion

Total Cross section is mostly gluon induced:

$$\sigma_{PP\to H}(\tau) \sim \int_{\tau}^{1} \frac{dz}{z}\, \mathcal{L}_{gg}(\tau/z)\, \sigma_{gg\to H}(z)$$

Behaviour of gluon luminosity and cross section is enhanced at threshold or z=1.

Expansion by Region

[V.A. Smirnov, M Beneke]

Analytic structure around z=1 :

$$\sigma^{(L)}_{ij\to H}(z;\epsilon) = \delta(1-z)F^{(L)}_0(\epsilon)+\sum_{n=2}^{2L}(1-z)^{-n\epsilon-1}F^{(L)}_n(z;\epsilon)$$

- L = #loops, $\epsilon = (4-D)/2$, $z = \frac{m_H^2}{\hat{s}}$
- $F_n(z)$ are analytic in the strip: z in (0,1]
- Discontinuities at z=1 captured by powers $(1-z)^{-n\epsilon-1}$
- Terms of different n=2,3,4.. can be identified to correspond to different Regions

Feynman Diagrammatic Expansion

Cross Section is expanded perturbatively:

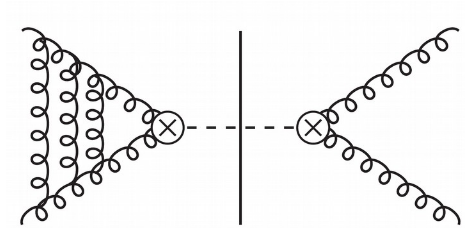
$$\sigma_{ij\to H}(\alpha_s; z; \epsilon) = \sum_L \left(\frac{\alpha_s}{\pi}\right)^L \sigma^{(L)}_{ij\to H}(z; \epsilon)$$

At each order L, need to consider diagrams with l loops and r cuts

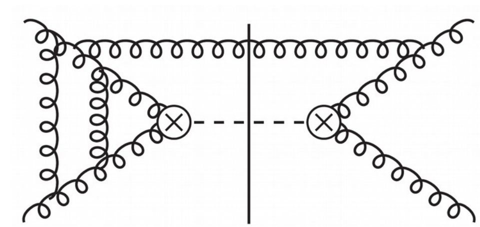
$$\sigma^{(L)}_{ij\to H} = \sum_{l+r=L} \sigma^{(l,r)}_{ij\to H}$$

Feynman Diagrams for Higgs production at N3LO in HQET

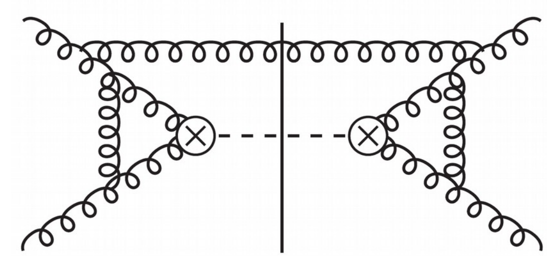
$$\sigma^{(3,0)}_{ij\to H} = \sum$$



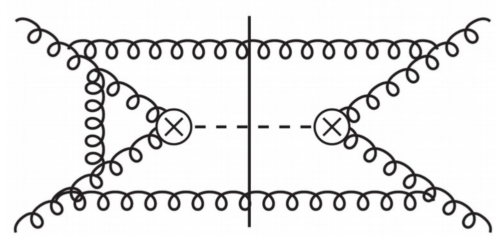
$$\sigma^{(2,1)}_{ij\to H} = \sum$$



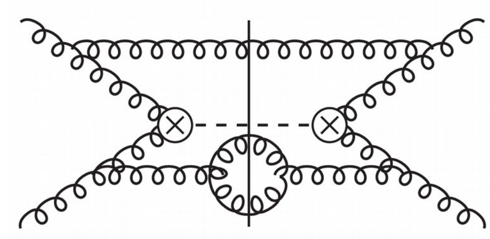
$$+\sum$$



$$\sigma^{(1,2)}_{ij\to H} = \sum$$



$$\sigma^{(0,3)}_{ij\to H} = \sum$$

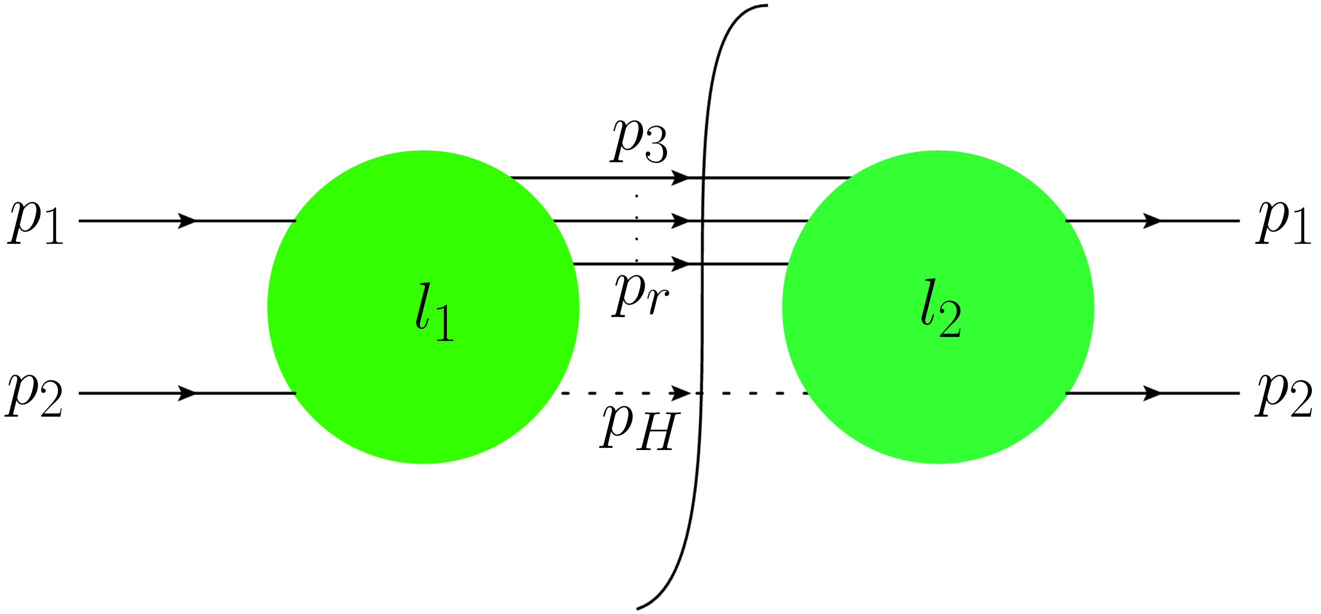


~100.000 diagrams

Soft Expansion

- Expand cut momenta around Threshold:

$\sigma^{(l_1+l_2,r)}_{ij\to H} =$



$$p_1 + p_2 = p_H + \sum_{k=0}^{r} p_k$$

Soft Expansion of cut momenta

Define the following kinematical variables:

$$p_H^2 = m_H^2 \qquad\qquad s_{12} = (p_1 + p_2)^2$$

The soft limit is parametrized by

$$z = \frac{m_H^2}{s_{12}} \to 1 \qquad \text{such that} \qquad p_1 + p_2 = p_H + \mathcal{O}(1-z)$$

The cut momenta must have the following scaling :

$$p_{k=3..r} \sim (1-z)$$

Reverse Unitarity Crash Course

- A trick to use conventional IBP identities for

$$\delta^+(q^2) \equiv \frac{1}{(q^2)_c}$$

- Differentiate using

$$\frac{d}{dx}\frac{1}{(f(x))_c} = \frac{-1}{(f(x))_c^2}\frac{df}{dx}$$

- Satisfy extra constraint

$$\frac{1}{(q^2)_c^{-\nu}} = 0 \quad \text{for} \quad \nu \geq 0$$

Phase Space Example

$$\Phi_3(z;\epsilon) = \int d^D p_3 \delta^+(p_3^2)\ d^D p_4 \delta^+(p_4^2)\ d^D p_H \delta^+(p_H^2 - m_H^2)\ \delta^D(p_1 + p_2 - p_3 - p_4 - p_H)$$

has the following cut propagator representation

$$= \int d^D p_3 d^D p_4 \left(\frac{1}{p_3^2}\right)_c \left(\frac{1}{p_4^2}\right)_c \left(\frac{1}{s_{12}\bar{z} - 2p_{12}.p_{34} + 2p_3.p_4}\right)_c$$

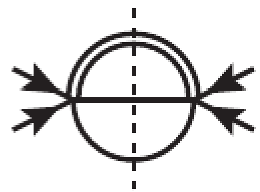
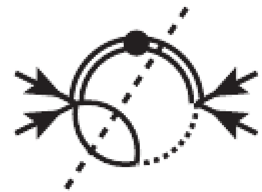
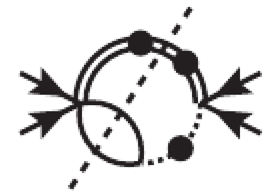
$$\sim \bar{z} \qquad \sim \bar{z}^2$$

Expanding the Cut propagators around Threshold we obtain

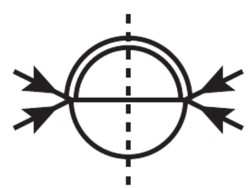
$$= \sum_{n=0}^{\infty} \int d^D p_3 d^D p_4 \left(\frac{1}{p_3^2}\right)_c \left(\frac{1}{p_4^2}\right)_c \left(\frac{1}{\bar{z}s_{12} - 2p_{12}.p_{34}}\right)_c^{1+n} (-2p_3.p_4)^n$$

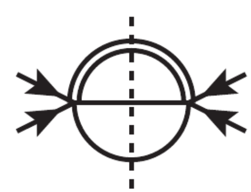
Phase Space Example

Diagrammatically this can be written as

$\Phi_3(\bar{z};\epsilon) = \bar{z}^{3-4\epsilon}\Big[$  $-\,\bar{z}$  $+\,\bar{z}^2$  $+\,\mathcal{O}(\bar{z}^3)\Big]$

Since the soft expansion just raises or lowers powers of denominators, soft IBPS can be used to reduce everything in terms of the first term in the expansion:

 $= -\dfrac{1-\epsilon}{2}$  ,

 $= \dfrac{(1-\epsilon)(2-\epsilon)(3-2\epsilon)}{4(5-4\epsilon)}$ 

Soft Expansion of loop momenta

- The scaling of loop momenta is not fixed by momentum conservation in the soft limit. Use

$$k^\mu = \alpha p_1^\mu + \beta p_2^\mu + k_\perp^\mu \qquad d^D k = (p_1.p_2) d\alpha \, d\beta \, d^{D-2} k_\perp$$

	Momentum	Measure
Hard:	$k^\mu \sim 1$	$d^D k \sim 1$
Soft:	$k^\mu \sim (1-z)$	$d^D k \sim (1-z)^{4-2\epsilon}$
Collinear 1:	$\beta \sim (1-z), \; k_\perp \sim \sqrt{1-z}$	$d^D k \sim (1-z)^{2-\epsilon}$
Collinear 2:	$\alpha \sim (1-z), \; k_\perp \sim \sqrt{1-z}$	$d^D k \sim (1-z)^{2-\epsilon}$

Exp. by Region = Exp. by Subgraph

$$F_\Gamma = \sum_\gamma F_{\Gamma/\gamma} \circ \mathcal{T}^{(n)}_{r(\Gamma/\gamma)} F_\gamma + \mathcal{O}(\lambda^{n+1})$$

- Let Γ be a graph, γ a subgraph, F_γ its integrand, Γ/γ a reduced graph
- $\mathcal{T}^{(n)}_{r(\Gamma/\gamma)}$ is a Taylor Expansion (and substitution) operator of degree n
- γ are identified as the non-homogenously scaling subgraphs of a certain (here only soft or collinear) region
- lines connected to soft cut lines may become soft propagators
- Lines connected to both soft lines and non-soft external massless momenta may give collinear regions

Feynman diagrams vs regions

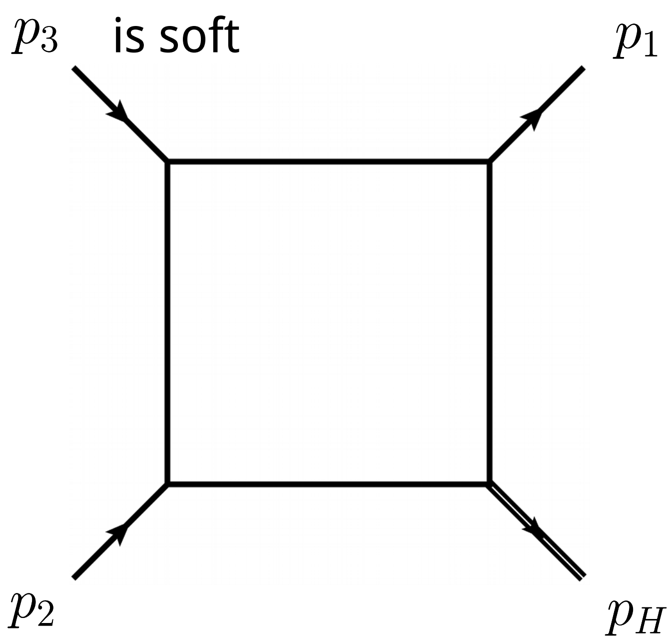
$$\sigma^{(L)}_{ij\to H}(z;\epsilon) = \delta(1-z)F_0^{(L)}(\epsilon)+\sum_{n=2}^{2L}(1-z)^{-n\epsilon-1}F_n^{(L)}(z;\epsilon)$$

vs $$\sigma^{(L)}_{ij\to H} = \sum_{l+r=L}\sigma^{(l,r)}_{ij\to H}$$

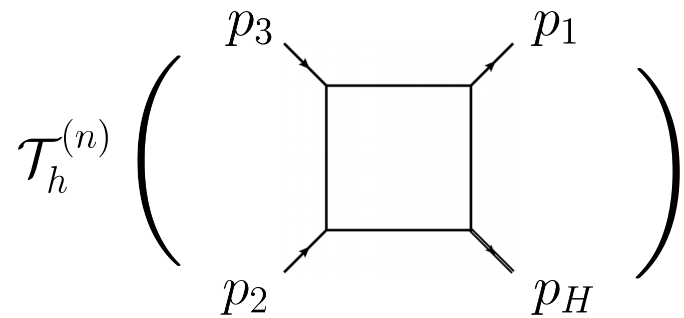
Loop diagrams contribute with different scalings, e.g.

$$\sigma^{(l,r)}_{ij\to H}(z;\epsilon) = \sum_{n=2r}^{2L}(1-z)^{-n\epsilon-1}F_n^{(l,r)}(z;\epsilon) \qquad l<L$$

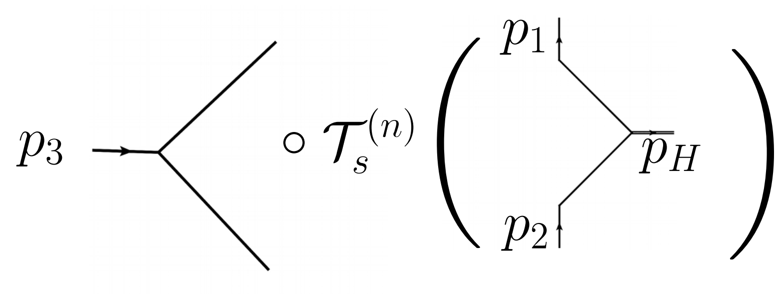
1Loop Example



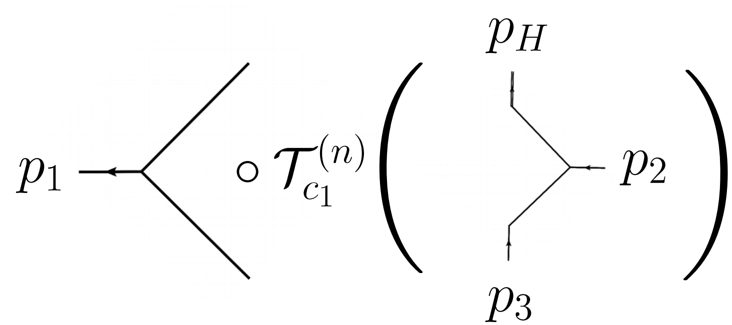
- Hard:



- Soft:



- Collinear 1:
- Collinear 2 similar



All order resummed

$$u, t \sim (1-z) \quad s \sim 1$$

Hard

Collinear1

$$\mathrm{Box}(-t,-u,s) = \frac{2c_\Gamma}{\epsilon^2}\Gamma(1+\epsilon)\Gamma(1-\epsilon)\, e^{-i\pi\epsilon}\, \frac{\left(\frac{tu}{s}\right)^{-\epsilon}}{tu} - \frac{2c_\Gamma}{\epsilon(1+\epsilon)}\frac{t^{-\epsilon-1}}{s}\,{}_2F_1\left(1, 1+\epsilon; 2+\epsilon; \frac{u}{s}\right)$$

$$- \frac{2c_\Gamma}{\epsilon(1+\epsilon)}\frac{u^{-\epsilon-1}}{s}\,{}_2F_1\left(1, 1+\epsilon; 2+\epsilon; \frac{t}{s}\right)$$

Collinear2

$$- \frac{2c_\Gamma}{\epsilon(1+\epsilon)}\, e^{i\pi\epsilon}\, s^{-2-\epsilon}\, F_2\left(2+\epsilon; 1+\epsilon, 1+\epsilon; 2+\epsilon, 2+\epsilon; \frac{u}{s}, \frac{t}{s}\right) ,$$

Soft

$$F_2(a; b_1, b_2; c_1, c_2; x_1, x_2) = \sum_{n,m=0}^{\infty} \frac{(a)_{m+n}\,(b_1)_m\,(b_2)_n}{(c_1)_m\,(c_2)_n}\,\frac{x_1^m\, x_2^n}{m!\, n!}$$

Conventional Result

$$s_{12} = s,\ s_{23} = t,\ s_{31} = u$$

$$\mathrm{Box}(s_{12}, s_{23}, s_{31}) = \frac{2c_\Gamma}{\epsilon^2}\frac{1}{s_{12}s_{23}}\left\{ (-s_{23})^{-\epsilon}\, {}_2F_1\left(1, -\epsilon; 1-\epsilon; -\frac{s_{31}}{s_{12}}\right)\right.$$
$$\left. + (-s_{12})^{-\epsilon}\, {}_2F_1\left(1, -\epsilon; 1-\epsilon; -\frac{s_{31}}{s_{23}}\right) - (-M_h^2)^{-\epsilon}\, {}_2F_1\left(1, -\epsilon; 1-\epsilon; -\frac{M_h^2 s_{31}}{s_{12}s_{23}}\right) \right\}$$

Functions simpler, but more difficult to expand around soft limit

Soft Phase Space & Loop Integrals via IBPs

- Integrals in soft and hard regions can be reduced over combined Phasespace and loop momentum space

$$\mathcal{M}_{\mathcal{S}_i}^{(l,r)} = \int d\Phi^{(r,l)} F(\{k_{1..l}, p_{3..r}\})$$

- Problems occur in the collinear regions, due to appearance of non-linear propagators

Subtleties in the Collinear Region

First term in collinear region of 1L box:

$$D_1 = k^2$$
$$D_2 = (k - p_1)^2$$
$$D_3 = (k + \tfrac{s_{13}}{s_{12}} p_2)^2$$
$$D_4 = 2k.p_2$$

$$\int \frac{d^D k}{D_1 D_2 D_3 D_4}$$

Denominators are not independent: $1 = \dfrac{D_1 + \frac{s_{13}}{s_{12}} D_4}{D_3}$

Collinear Box can be partial fractioned into trivial triangles

$$\int \frac{1}{D_1 D_2 D_3 D_4} = \int \frac{1}{D_2 D_3^2 D_4} + \frac{s_{13}}{s_{12}} \int \frac{1}{D_1 D_2 D_3^2}$$

Subtleties in the Collinear Region

Consider IBP for combined Phase space and loop:

$$\int d^Dp_3 d^Dk \; p_2 \cdot \frac{\partial}{\partial k} \left(\frac{1}{(p_3)_c^2 s_{23}} \frac{1}{D_1 D_2 D_3^2} \right) = 0$$

Eventually produces term:

$$p_2 \cdot \frac{\partial}{\partial k} \frac{1}{D_3} = \frac{D_1 - D_3}{D_3^2} \frac{s_{12}}{s_{13}}$$

$\frac{1}{s_{13}}$ takes integral out of topology.

IBPs do not close for collinear region.

Subtleties in collinear region

Workaround: Replace trivial triangles with tadpoles, e.g.

$$\int \frac{1}{D_1 D_2 D_3} \sim \int \frac{d^D k}{(k^2 + s_{13})^3}$$

Allows for efficient IBP reduction of combined phase space and loop integration all collinear regions, also with more legs, loops to all orders in expansion..

Integral Statistics

	NNLO	N3LO
#diagrams	~1.000	~100.000
#integrals	~50.000	517.531.178
#masters	27	1.028
#soft masters	5	78

- The number of soft masters is much smaller than the number of full masters.
- Soft Master Integrals can be used to express the Higgs Cross section and/or the full kinematic Master Integrals to all orders in the soft expansion by direct Integrand expansion.
- Soft Master Integrals can be used as a boundary condition to solve the differential equations.
- Soft Master Integrals can be used to construct an Ansatz for the full Master Integrals, which can be fixed by the differential equations.

Truncated Series Solution via Differential Equation

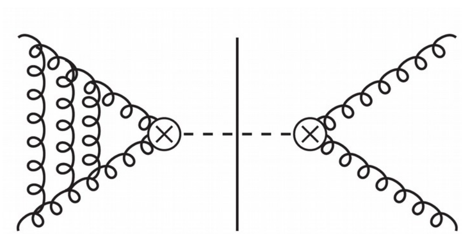
Ansatz: $$\mathcal{M}_i(z,\epsilon) = \sum_{k=0}^{6} \bar{z}^{-k\epsilon} \sum_{l=l_0}^{n} \mathcal{M}_i^{(k,l)}\, \bar{z}^l + \mathcal{O}(\bar{z}^{n+1})$$

↓

$$\frac{\partial}{\partial z}\mathcal{M}_i(z,\epsilon) = \sum_j C_{ij}(z,\epsilon)\mathcal{M}_j(z,\epsilon)$$

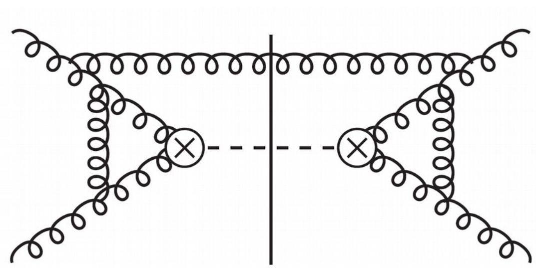
- Substituting the Ansatz into the differential equations yields a linear system which can be solved order by order in zbar.
- Solved for the first 38 coefficients of the full Masters in terms of Soft Masters from knowledge of boundary.
- First few terms checked by explicit computation via Integrand Expansion.

Status of Higgs Production at N3LO



Triple Virtual

Known from QCD Form Factor
[Baikov, Chetyrkin, Smirnov, Smirnov, Steinhauser; Gehrmann, Glover, Huber, Ikizlerli, Studerus]

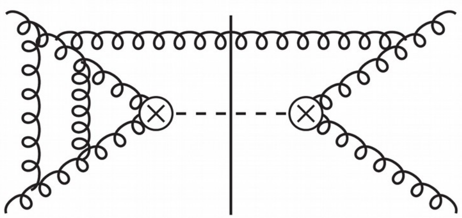


Real-Virtual Squared

Known [Anastasiou, Duhr, Dulat, FH, Mistlberger; Kilgore]

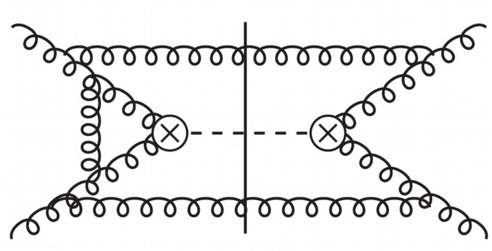
+UV and IR counter terms

Known[Pak, Rogal, Steinhauser; Anastasiou, Buehler, Duhr, FH; Höschele, Hoff, Pak, Steinhauser, Ueda; Buehler, Lazopoulos]

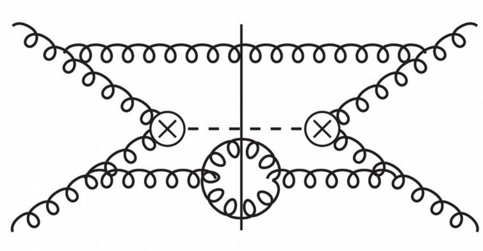


Double Virtual- Real

Known [Dulat, Mistlberger; Duhr, Gehrmann]



Double Real - Virtual



Triple Real

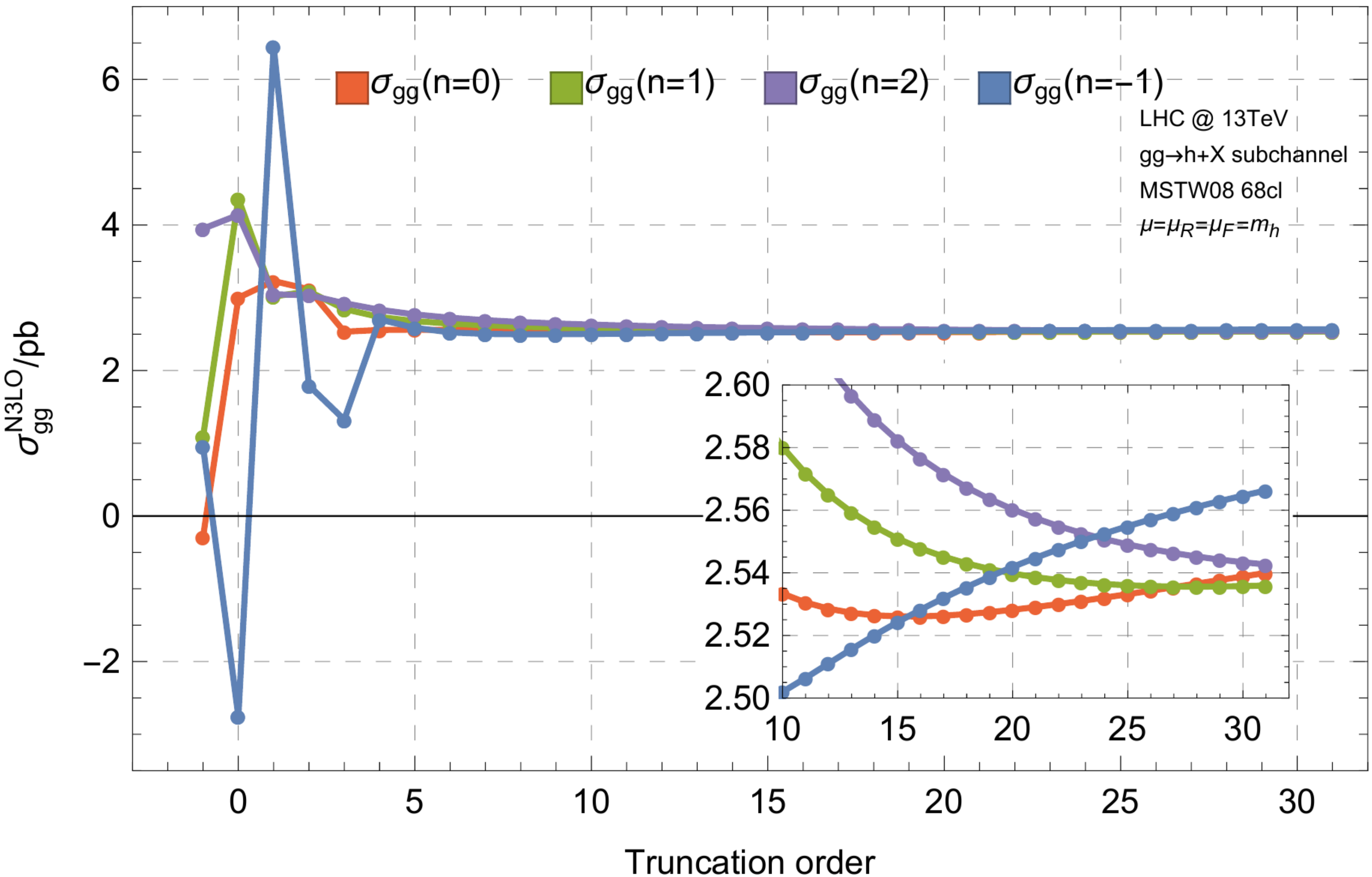
qq` channel known [Chihaya Anzai, Alexander Hasselhuhn, Maik Hoschele, Jens Hoff, William Kilgore, Matthias Steinhauser, Takahiro Ueda]

2 terms in soft expansion [Anastasiou, Duhr, Dulat, FH, Mistlberger,Furlan; Li, Mantueffel, Schabinger, Zhu]
37 terms [Anastasiou, Duhr, Dulat, FH, Mistlberger]
Known [to be published]

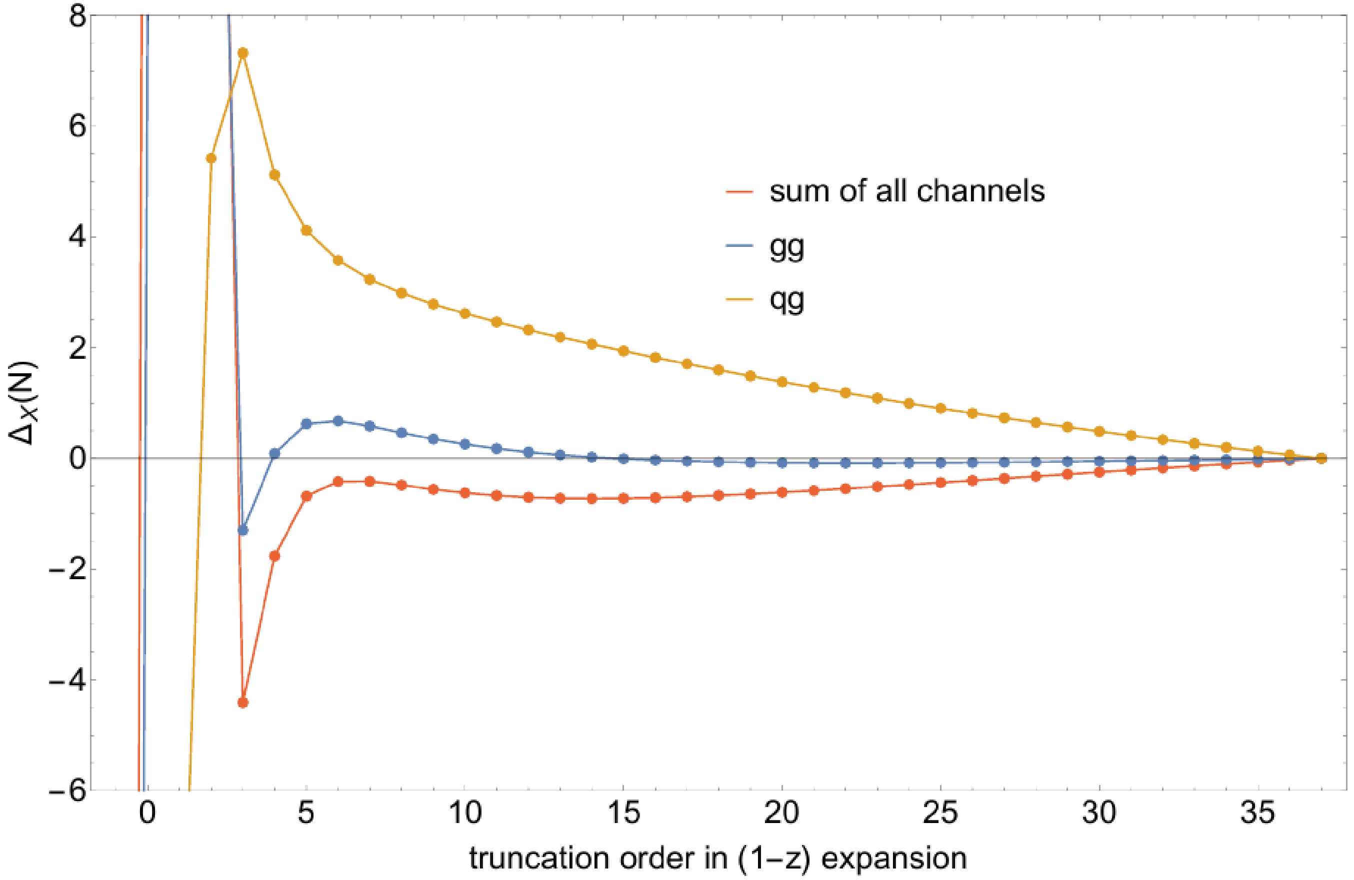
2 terms in soft expansion [Anastasiou, Duhr, Dulat, Mistlberger; Zhu]
37 terms [Anastasiou, Duhr, Dulat, FH, Mistlberger]

Results

Convergence of Soft Expansion

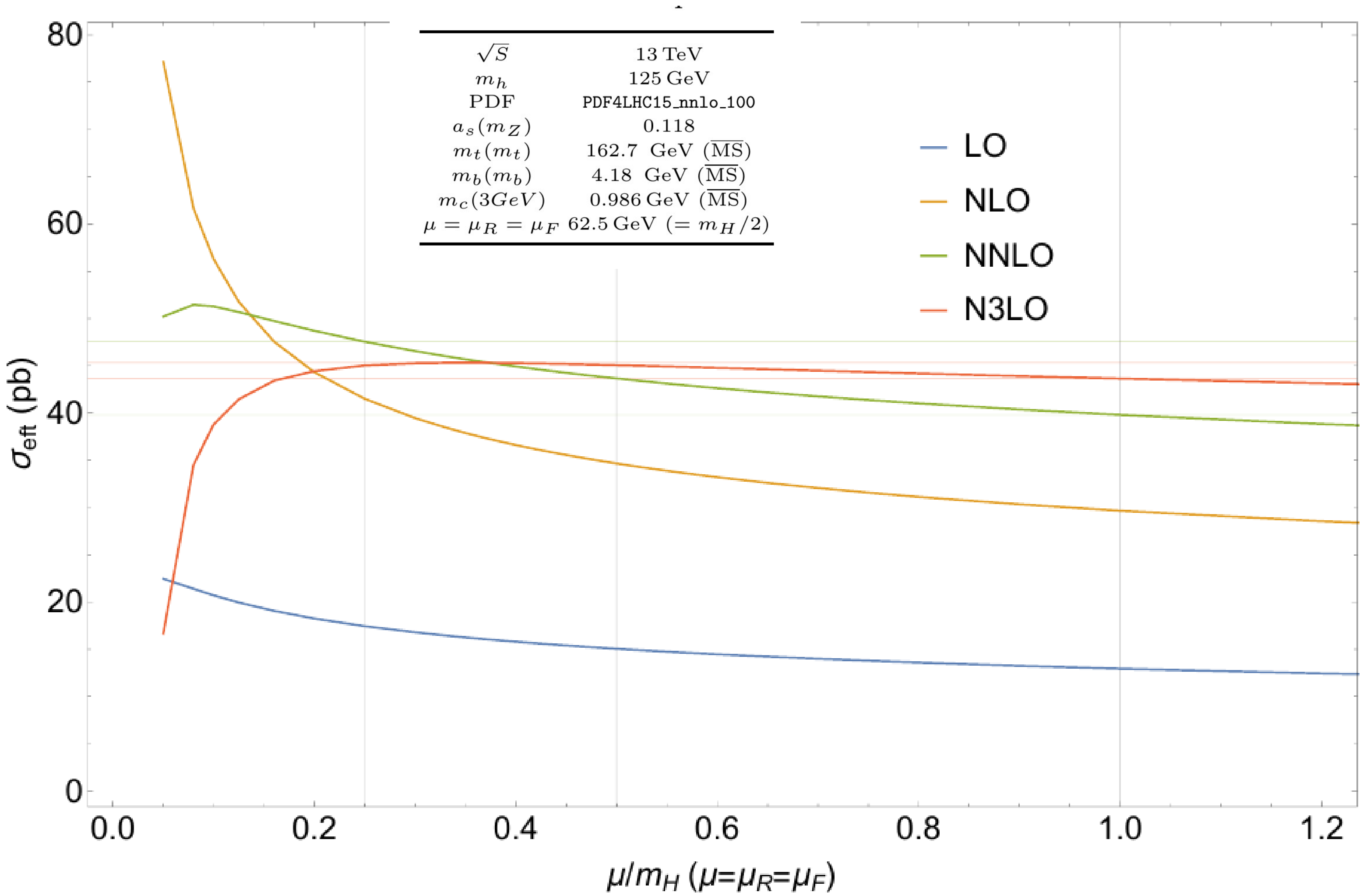


Convergence in Different Channels

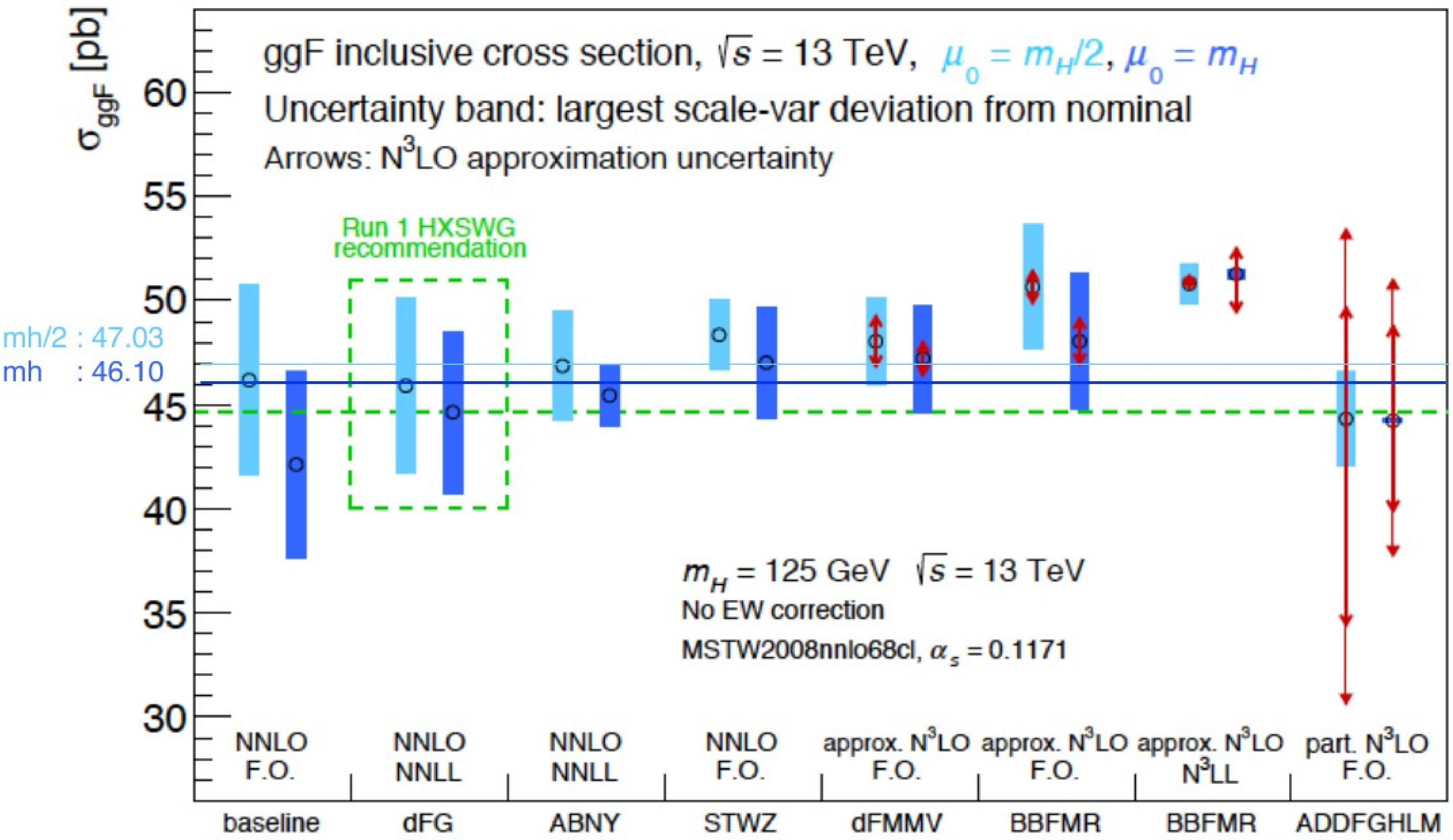


$$\Delta_X(N) \equiv \frac{\sigma^{(3)}_{X,EFT}(N) - \sigma^{(3)}_{X,EFT}(N_{\text{last}})}{\sigma^{(3)}_{X,EFT}(N_{\text{last}})}\, 100\%\,.$$

Scale Variation at N3LO



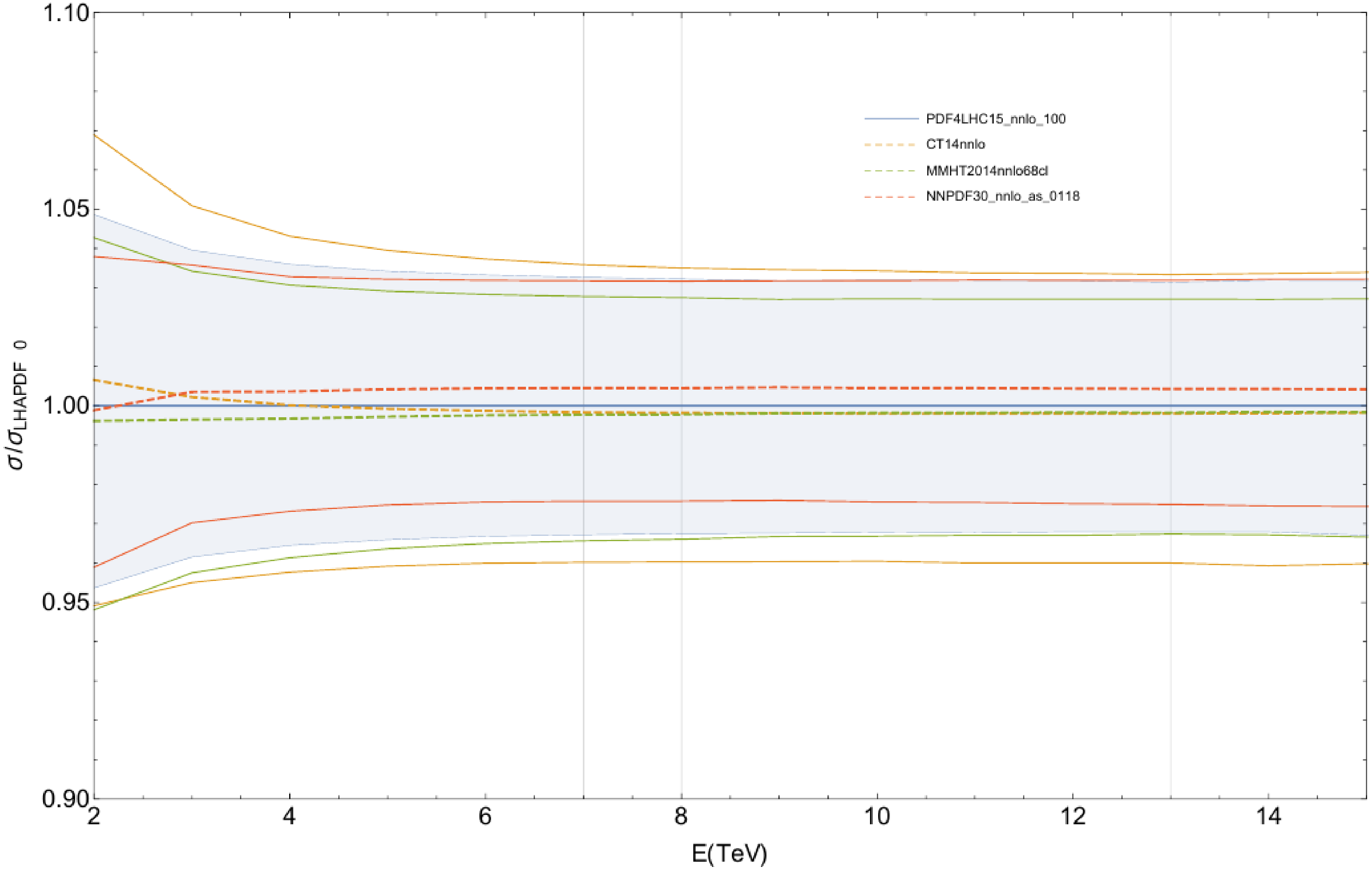
Comparison with Approximate Results

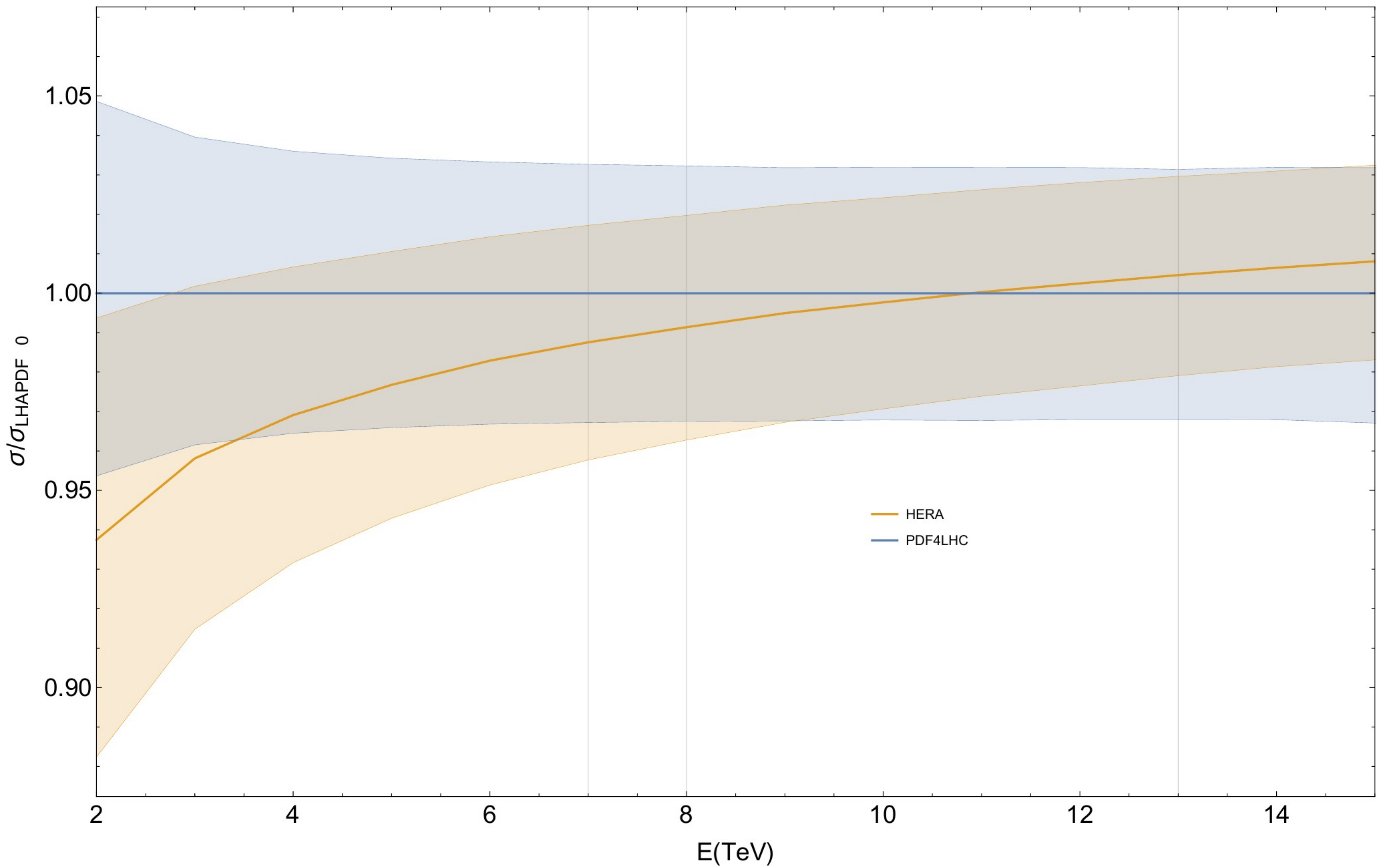


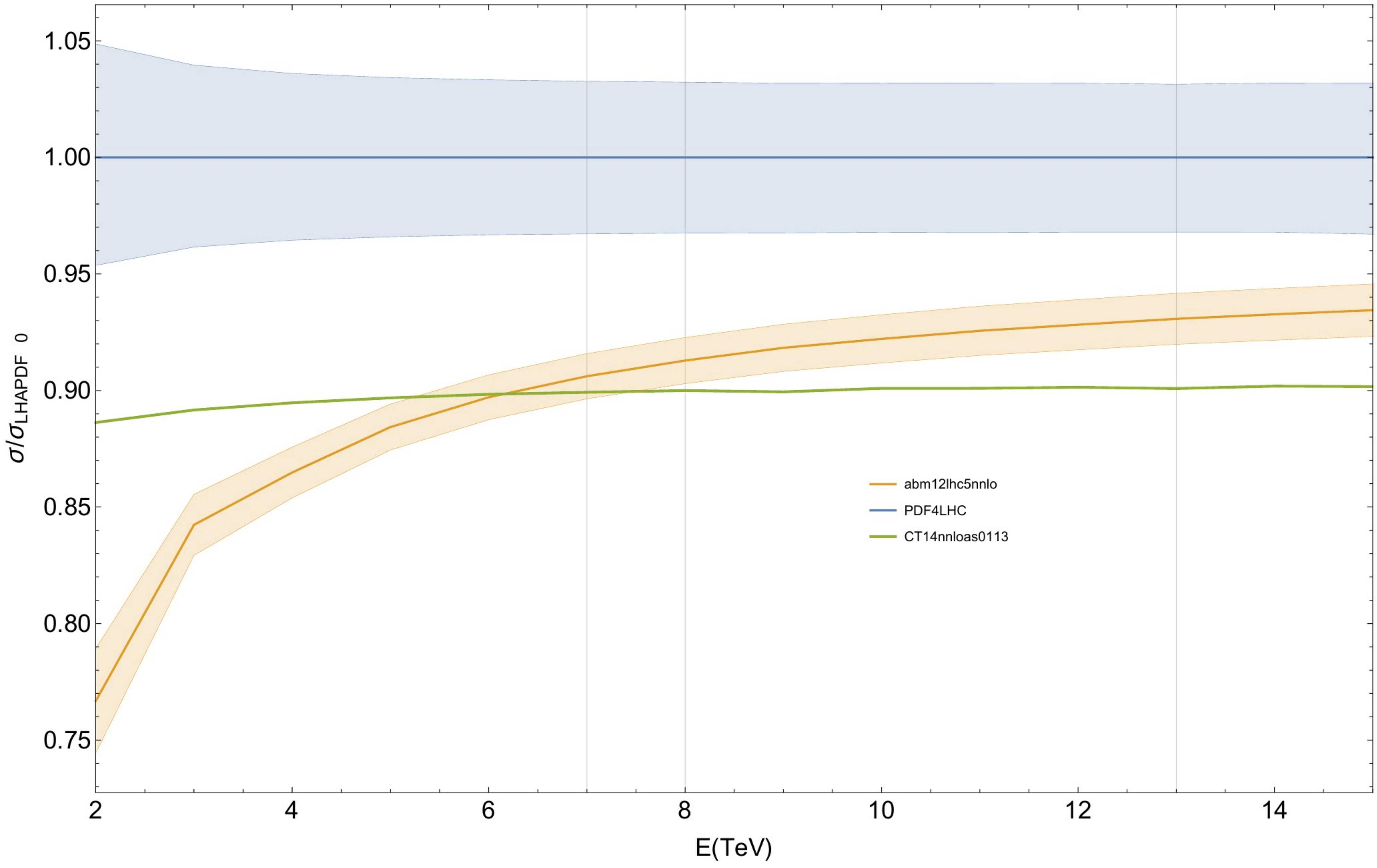
Sources of Uncertainities in SM

- Truncation
- QCD perturbative & non-perturbative?
- EW, mixed QCD-EW
- Exact mass effects c,b,t and parameteric uncertainties
- PDFs+ α_s

PDF Uncertainties




1.05
1.00
0.95
0.90
σ/σ LHAPDF 0
2
4
6
8
10
12
14
E(TeV)
HERA
PDF4LHC


1.05
1.00
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$\sigma/\sigma_{\mathrm{LHAPDF}\ 0}$
2
4
6
8
10
12
14
E(TeV)
abm12lhc5nnlo
PDF4LHC
CT14nnloas0113

Uncertainty for SM Higgs at 13TeV LHC

$\delta(\text{scale})$	$\delta(\text{trunc})$	$\delta(\text{PDF-TH})$	$\delta(\text{EW})$	$\delta(t, b, c)$	$\delta(1/m_t)$
$^{+0.10\text{ pb}}_{-1.15\text{ pb}}$	± 0.18 pb	± 0.56 pb	± 0.49 pb	± 0.40 pb	± 0.49 pb
$^{+0.21\%}_{-2.37\%}$	$\pm 0.37\%$	$\pm 1.16\%$	$\pm 1\%$	$\pm 0.83\%$	$\pm 1\%$

Size of different contributions

$$\sigma = 48.58\,\text{pb}^{+2.22\,\text{pb}\,(+4.56\%)}_{-3.27\,\text{pb}\,(-6.72\%)}\ (\text{theory}) \pm 1.56\,\text{pb}\,(3.20\%)\ (\text{PDF}+\alpha_s)\,.$$

$$\begin{aligned}
48.58\,\text{pb} = \quad & 16.00\,\text{pb} & (+32.9\%) & \quad (\text{LO, rEFT})\\
& +20.84\,\text{pb} & (+42.9\%) & \quad (\text{NLO, rEFT})\\
& -\ \ 2.05\,\text{pb} & (-4.2\%) & \quad ((t,b,c),\ \text{exact NLO})\\
& +\ \ 9.56\,\text{pb} & (+19.7\%) & \quad (\text{NNLO, rEFT})\\
& +\ \ 0.34\,\text{pb} & (+0.2\%) & \quad (\text{NNLO},\ 1/m_t)\\
& +\ \ 2.40\,\text{pb} & (+4.9\%) & \quad (\text{EW, QCD-EW})\\
& +\ \ 1.49\,\text{pb} & (+3.1\%) & \quad (\text{N}^3\text{LO, rEFT})
\end{aligned}$$

750 GeV CP-even Scalar at the LHC

A generic scalar produced in gluon fusion can be described model independently by the effective Lagrangian

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \mathcal{L}_S - \frac{1}{4v} C_S \, S \, G^a_{\mu\nu} G^{\mu\nu}_a$$

leads to the cross section

$$\sigma_S(m_S, \Gamma_S, \Lambda_{\rm UV}) = |C_S(\mu, \Lambda_{\rm UV})|^2 \, \eta(\mu, m_S, \Gamma_S)$$

Model independent calculation from SM calculation

$$\sigma_S(m_S, \Gamma_S, \Lambda_{\rm UV}) = \left| \frac{C_S(\mu_0, \Lambda_{\rm UV})}{C(\mu_0, m_t)} \right|^2 \sigma_H(m_S, \Gamma_S, m_t)$$

In a recent paper we have published numbers & uncertainties, allowing to extract the cross section in any BSM model provided the Wilson Coefficient is known in that model.

Light colored particles in the loop

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \mathcal{L}_S - \frac{\lambda_{\rm wc}}{4v} C\, S\, G^a_{\mu\nu} G_a^{\mu\nu} - \lambda_t \frac{m_t}{v}\, S\, \bar{t}t$$

General Parameterization for our best prediction in such a scenario:

$$\begin{aligned}\sigma_S[\lambda_{\rm wc}, \lambda_t] \; &= \lambda_{\rm wc}^2 \sigma_S^{\rm N^3LO}[1,0] - \lambda_{\rm wc}\lambda_t \sigma_S^{\rm NLO}[1,0] + \lambda_t(\lambda_t - \lambda_{\rm wc})\sigma_S^{\rm NLO}[0,1] \\ &+ \lambda_{\rm wc}\lambda_t\, \sigma_S^{\rm NLO}[1,1]\,.\end{aligned}$$

Light quarks in the loop

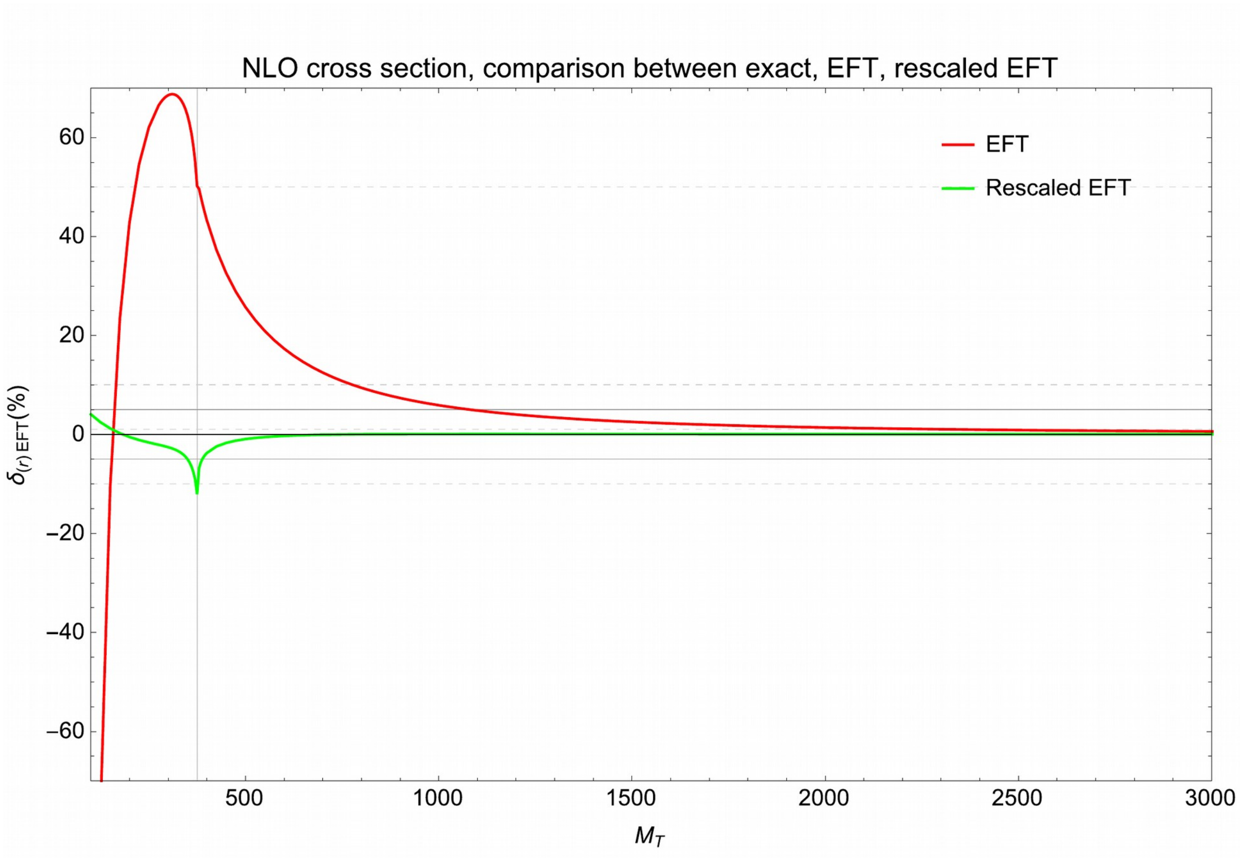


Figure 7: Percent difference (4.3) between the exact and rescaled EFT (rEFT) cross section at NLO (red line/green line) as a function of the quark mass for the production of a 750 GeV CP-even scalar. The vertical line corresponds to $m_S/2$.

750 GeV Uncertainties

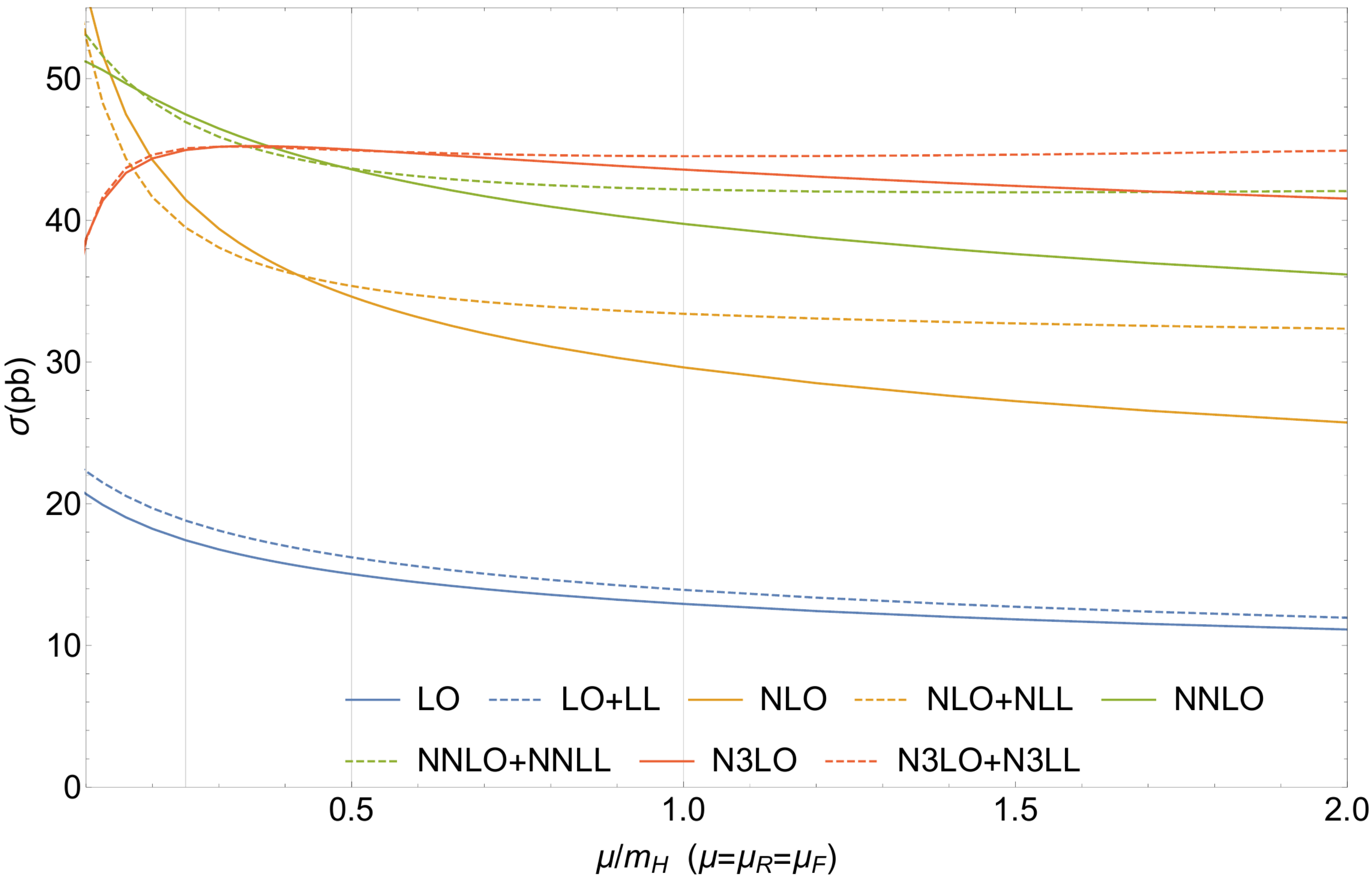
$\sqrt{s}$	Component	value[fb]	δ(theory) [%]	δ(pdf+α_S) [%]
7 TeV	$\sigma_S^{\mathrm{N^3LO}}[1,0]$	69.72	$^{+2.0}_{-4.2}$	7.0
	$\sigma_S^{\mathrm{NLO}}[1,0]$	55.59	19.52	6.95
	$\sigma_S^{\mathrm{NLO}}[0,1]$	61.71	22.69	6.94
	$\sigma_S^{\mathrm{NLO}}[1,1]$	152.6	22.1	6.92
8 TeV	$\sigma_S^{\mathrm{N^3LO}}[1,0]$	111.4	$^{+1.9}_{-4.0}$	6.1
	$\sigma_S^{\mathrm{NLO}}[1,0]$	89.37	19.18	6.23
	$\sigma_S^{\mathrm{NLO}}[0,1]$	98.92	22.3	6.22
	$\sigma_S^{\mathrm{NLO}}[1,1]$	245.3	21.71	6.2
13 TeV	$\sigma_S^{\mathrm{N^3LO}}[1,0]$	496.9	$^{+2.0}_{-3.7}$	4.0
	$\sigma_S^{\mathrm{NLO}}[1,0]$	404.6	18.3	4.5
	$\sigma_S^{\mathrm{NLO}}[0,1]$	442.7	21.3	4.4
	$\sigma_S^{\mathrm{NLO}}[1,1]$	1108	20.7	4.4
14 TeV	$\sigma_S^{\mathrm{N^3LO}}[1,0]$	609.7	$^{+1.9}_{-3.7}$	3.8
	$\sigma_S^{\mathrm{NLO}}[1,0]$	497.3	18.21	4.26
	$\sigma_S^{\mathrm{NLO}}[0,1]$	543.	21.14	4.2
	$\sigma_S^{\mathrm{NLO}}[1,1]$	1361	20.57	4.21

Conclusions & Outlook

- Expansion by region is a useful and interesting technique for the evaluation of Feynman Diagrams.
- Mathematics behind this exp by region is still not perfectly well understand – systematics still has to be worked out case by case
- Here we developed new methods for the efficient evaluation of the soft expansion using Expansion by Region and differential equations
- Successfully applied these techniques in Higgs production at N3LO in QCD.
- Completed first N3LO calculation for the LHC.
- Carefully analysed current theoretical uncertainties
- Experimental uncertainties for LHC Higgs (& other possible BSM scalars) boson production data can now be matched with sufficiently accurate predictions
- Follow up applications: W/Z production, bb $\rightarrow$ H
- To improve on Higgs production: exact t,b-mass effects, EW corrections, fully differential

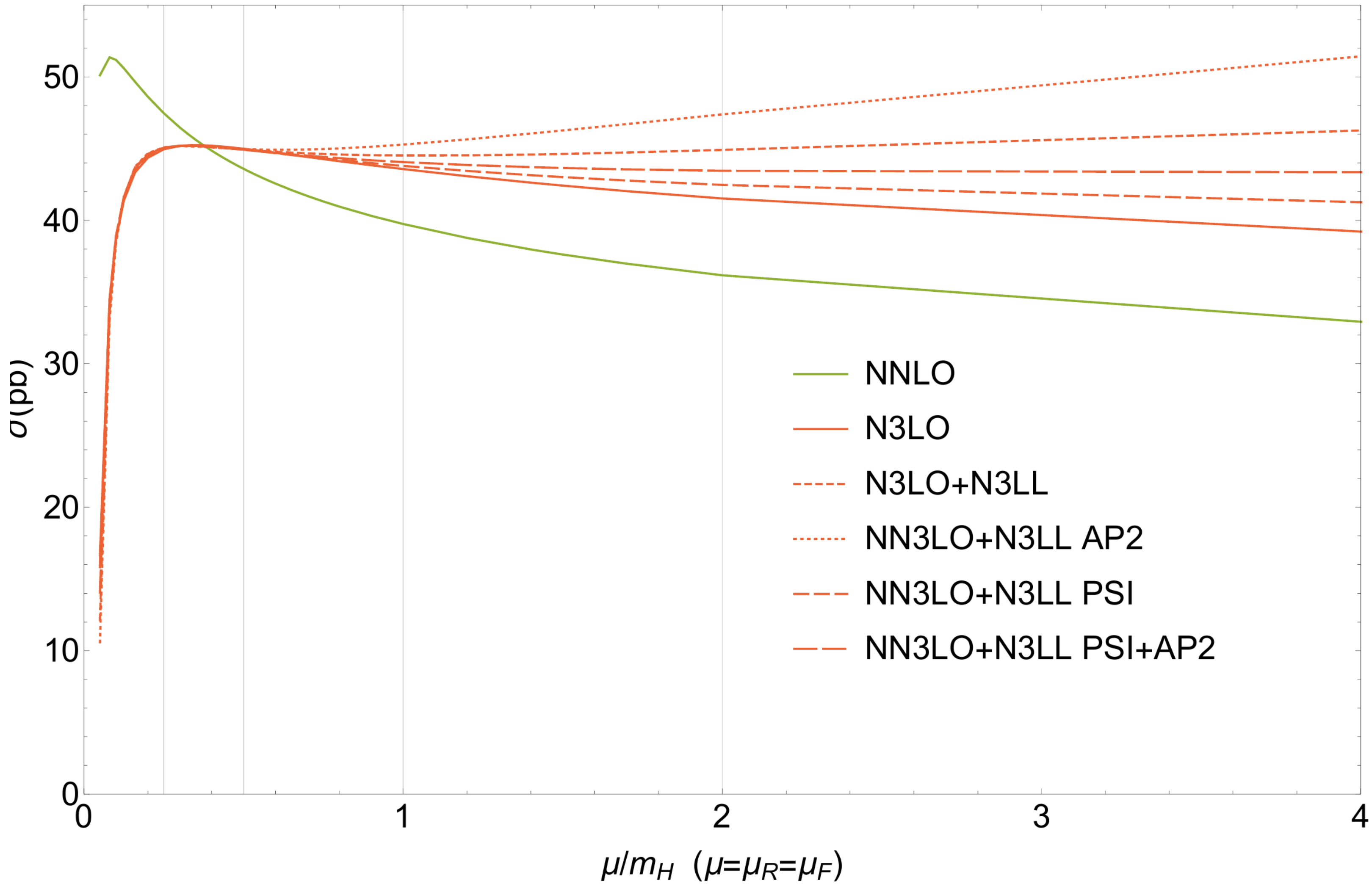
Threshold Resummation

a la Collins/Sterman/Catani



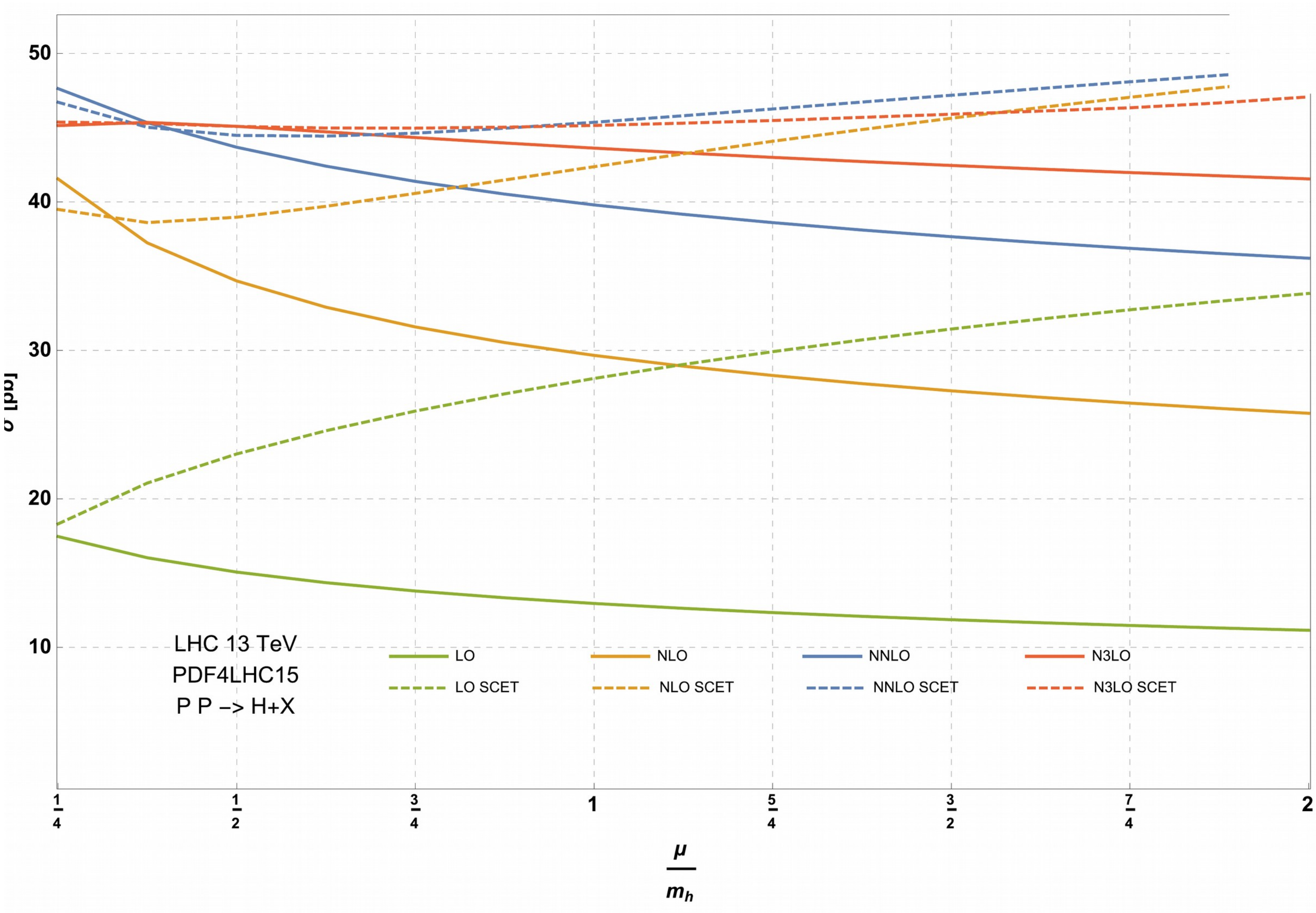
Threshold Resummation

Scheme Dependence



Threshold Resummation

a la SCET (Becher/Neubert)



t,b,c mass effects

$$\delta(tbc)^{\overline{\text{MS}}} = \pm \left| \frac{\delta\sigma_{ex;t}^{NLO} - \delta\sigma_{ex;t+b+c}^{NLO}}{\delta\sigma_{ex;t}^{NLO}} \right| (R_{LO}\delta\sigma_{EFT}^{NNLO} + \delta_t \hat{\sigma}_{gg+qg,EFT}^{NNLO}) \simeq \pm 0.31\,\text{pb}$$

$$\delta(t,b,c) = 1.3\,\delta(t,b,c)^{\overline{\text{MS}}}$$

1.3 motivated from 30% scheme dependence at NLO

Negligibilty of Parameteric Mass Uncertainties

Top quark			Bottom quark			Charm quark		
$\delta m_t = 1$ GeV	$\sigma^{NLO}_{ex;t+b+c}$	34.77	$\delta m_b = 0.03$ GeV	$\sigma^{NLO}_{ex;t+b+c}$	34.77	$\delta m_c = 0.026$ GeV	$\sigma^{NLO}_{ex;t+b+c}$	34.77
$m_t + \delta m_t$	$\sigma^{NLO}_{ex;t+b+c}$	34.74	$m_b + \delta m_b$	$\sigma^{NLO}_{ex;t+b+c}$	34.76	$m_c + \delta m_c$	$\sigma^{NLO}_{ex;t+b+c}$	34.76
$m_t - \delta m_t$	$\sigma^{NLO}_{ex;t+b+c}$	34.80	$m_b - \delta m_b$	$\sigma^{NLO}_{ex;t+b+c}$	34.79	$m_c - \delta m_c$	$\sigma^{NLO}_{ex;t+b+c}$	34.78

Truncation zoomed in