



# 4th Workshop on QCD Structure of the Nucleon

*11-15 July, 2016 Getxo, Spain*

## Spin-Dependent Fragmentation Functions in $e^+e^-$ annihilation

.....from an experimental point of view

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# OUTLINE

## INTRODUCTION

- Fragmentation Functions
- Spin-Dependent fragmentation functions in  $e^+e^-$  annihilation processes
- The Belle, BaBar, and BESIII experiments

## COLLINS FRAGMENTATION FUNCTION

- Reference frames and analysis strategy
- Results, comparisons and discussions
  - Collins effect vs. hadron fractional energies and transverse momenta
  - Extraction of Collins function

## DI-HADRON FRAGMENTATION FUNCTION

- Reference frames and analysis strategy
- Results
  - $H_1^\perp$  and  $G_1^\perp$

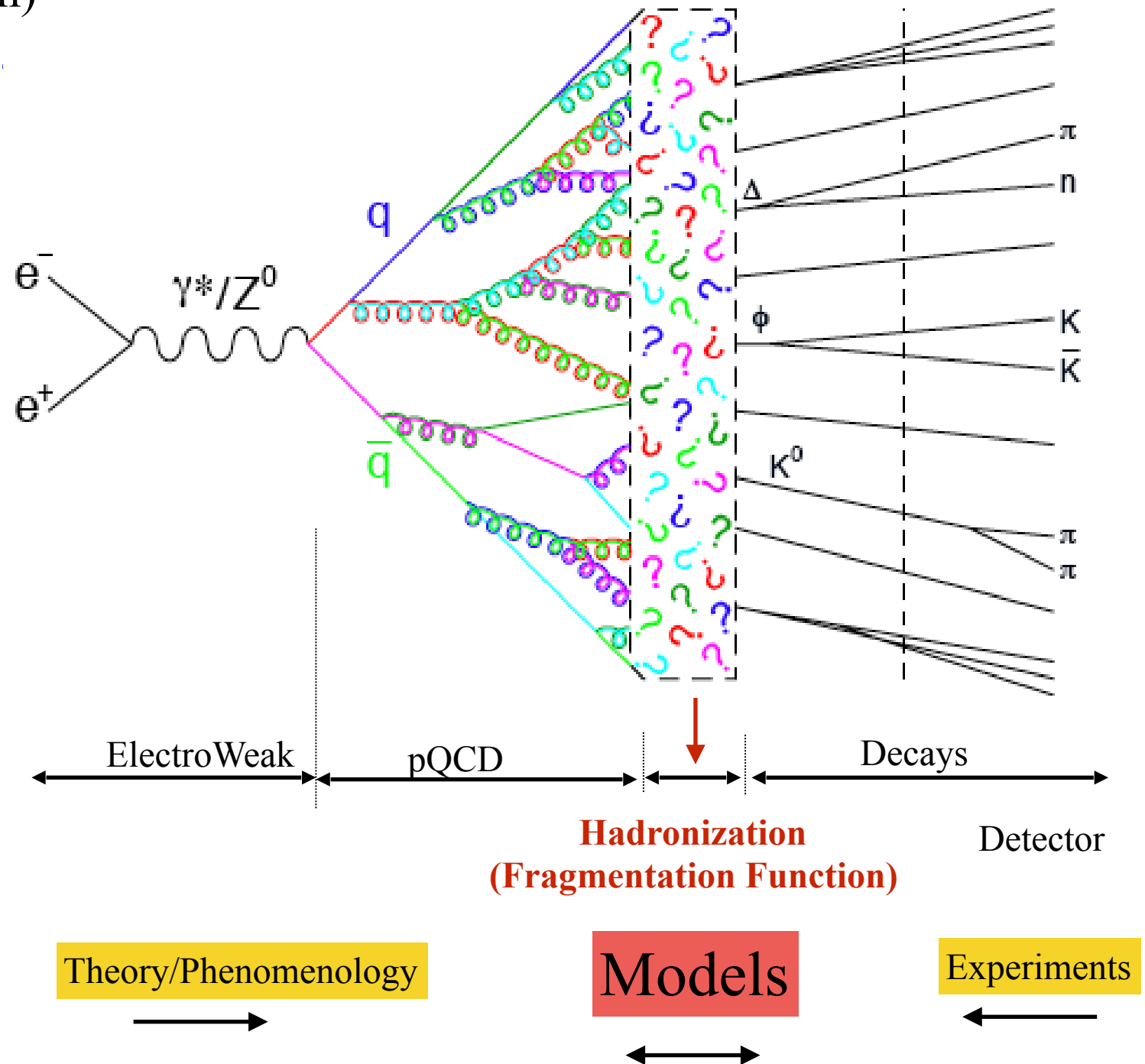
## SUMMARY and CONCLUSIONS

# Introduction: Fragmentation Functions in $e^+e^-$ annihilation processes

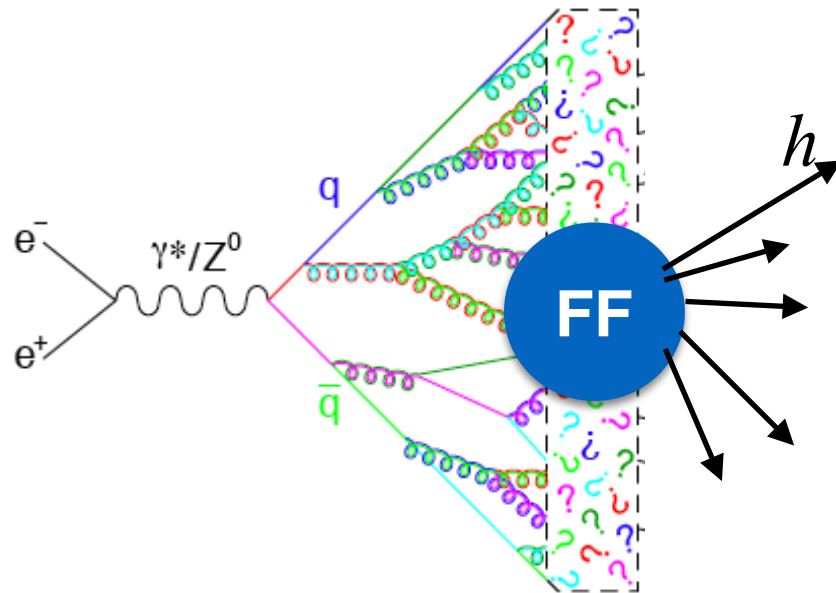
The process that transform quarks and gluons into colorless hadrons is referred as **FRAGMENTATION** (or hadronization)

$$e^+e^- \rightarrow q\bar{q}$$

1. Quarks and antiquarks fragment via radiation of gluons, each of which can radiate more gluons or split into a  $q\bar{q}$  pair (pQCD)
2. **Hadronization**: the partons transform into primary hadrons
3. Unstable primary hadrons decay into more stable particles that reach detector elements



# Introduction: Fragmentation Functions in $e^+e^-$ annihilation processes



- Fragmentation Functions (FFs) are used to describe the hadronization processes of partons
  - non-perturbative
  - universal functions
  - depend on the scaled energy of the hadron  $h$ :  

$$x = 2E_h/\sqrt{s}$$
- $e^+e^-$  annihilation environment offers the ideal conditions to access FFs (but limited sensitive to gluon FF and flavor dependence)

From the experimental point of view, the FFs describe final-state single particle energy distribution

- first observable: cross section in semi inclusive  $e^+e^-$  annihilation at center-of-mass energy  $\sqrt{s}$

$$\frac{1}{\sigma_0} \frac{d\sigma^{e^+e^- \rightarrow hX}}{dx} = F^h(x, s) = \sum_{i=q, \bar{q}} \int_x^1 \frac{dz}{z} C_i \left( z, \alpha_s(\mu), \frac{s}{\mu^2} \right) D_i^h \left( \frac{x}{z}, \mu^2 \right) + \mathcal{O} \left( \frac{1}{\sqrt{s}} \right)$$

$$\sigma_0 = \sum_q \frac{4\pi\alpha^2}{s} \left( 1 + \frac{\alpha_s}{\pi} + \dots \right)$$

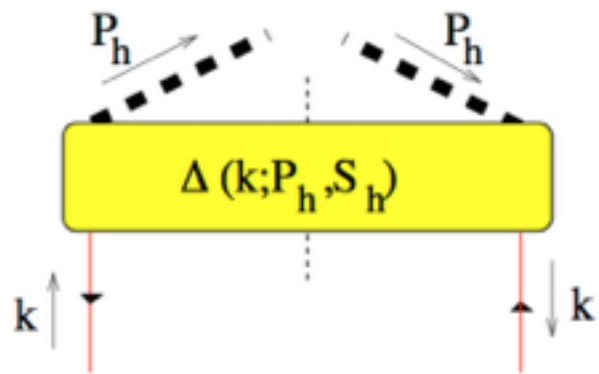
$\mu =$  factorization scale

process dependent  
short distance interaction

non-perturbative part:  
**parton fragmentation functions**

- $D_i^h(z, \mu^2)$ : probability that a parton  $i$  ( $i=u/\bar{u}, d/\bar{d}, s/\bar{s}, c/\bar{c}, b/\bar{b}, g$ ) fragments into a hadron  $h$  carrying a fraction  $z$  of the parton's momentum
- Collinear function: perturbative QCD corrections lead to logarithmic scaling violations via the evolution equations (DGLAP)

# Correlation functions in a nutshell



$$q \rightarrow h_1 X$$

Assuming the factorization into hard and soft physics

- hard part: partonic cross sections are calculated using pQCD
- the soft part describes the hadronization. The information on how quark fragments into a hadron with momentum  $P_h$  and spin  $S_h$  is encoded in the **correlation function**:

$$\Delta_{ij}(k, P_h, S_h) = \sum_X \int \frac{d^4x}{(2\pi)^4} e^{ik \cdot x} \langle 0 | \psi_i(x) | P_h, S_h; X \rangle \langle P_h, S_h; X | \bar{\psi}_j(0) | 0 \rangle$$

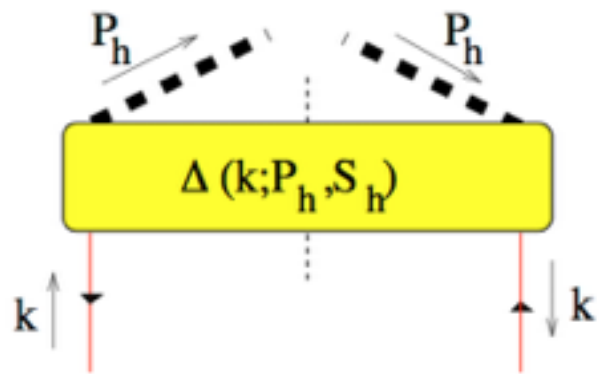
Collins, Soper, NPB 194, 445

General expression for **spin 1/2** hadron in the final state:

$$\begin{aligned} \Delta(k, P, S) = & MA_1 + A_2 \not{P} + A_3 \not{k} + (A_4/M) \sigma^{\mu\nu} P_\mu k_\nu + iA_5 (k \cdot S) \gamma_5 \\ & + MA_6 \not{S} \gamma_5 + (A_7/M) (k \cdot S) \not{P} \gamma_5 + (A_8/M) (k \cdot S) \not{k} \gamma_5 + \\ & + iA_9 \sigma^{\mu\nu} \gamma_5 \delta_\mu P_\nu + iA_{10} \sigma^{\mu\nu} \gamma_5 S_\mu k_\nu + i(A_{11}/M^2) (k \cdot S) \sigma^{\mu\nu} \gamma_5 k_\mu P_\nu \\ & + (A_{12}/M) \epsilon_{\mu\nu\rho\sigma} \gamma^\mu P^\nu k^\rho S^\sigma \end{aligned}$$

- Only terms which satisfy the condition of **hermiticity** and **P-invariance**
- Note that  $A_4$ ,  $A_5$ , and  $A_{12}$  are T-odd

# Correlation functions in a nutshell



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Collins, Soper, NPB 194, 445

General expression for **spin 0** hadron in the final state:

$$\Delta(k, P, S) = MA_1 + A_2 \not{P} + A_3 \not{k} + (A_4/M) \sigma^{\mu\nu} P_\mu k_\nu$$

FFs are obtained from the correlation function  $\Delta$  by projection with Dirac matrices  $\Gamma$ , and integration over components of the quark momentum

$$\Delta^{[\Gamma]}(z) \equiv \frac{1}{4z} \int dk^+ d^2\mathbf{k}_T \text{Tr}(\Delta \Gamma) \Big|_{k^- = P_h^- / z}$$

- LC quantization
- $\Gamma \in \{\gamma^-, \gamma^-\gamma_5, i\sigma^{\alpha-}\gamma_5, \dots\}$  determines quark spin states

$[\gamma^-]$ :

$[\gamma^-\gamma_5]$ :

$[i\sigma^{\alpha-}\gamma_5]$ :

# Fragmentation Functions

FF for one unpolarized hadron  
observed in a quark jet  
(no  $k_T$ )

$$D_1(z) = \left[ \bullet \rightarrow \text{red circle} \right]$$

$$G_1(z) = \left[ \bullet \rightarrow \text{red circle with blue arrow} \right] - \left[ \bullet \rightarrow \text{red circle with red arrow} \right]$$

$$H_1(z) = \left[ \uparrow \bullet \rightarrow \text{red circle with blue arrow} \right] - \left[ \downarrow \bullet \rightarrow \text{red circle with blue arrow} \right]$$

$$D_1(z, k_T^2) = \left[ \bullet \rightarrow \text{red circle} \right]$$

$$D_{1T}^\perp(z, k_T^2) = \left[ \bullet \rightarrow \text{blue circle with green arrow} \right]$$

$$G_{1L}(z, k_T^2) = \left[ \bullet \rightarrow \text{red circle with blue arrow} \right] - \left[ \bullet \rightarrow \text{red circle with red arrow} \right]$$

$$G_{1T}(z, k_T^2) = \left[ \bullet \rightarrow \text{green circle with green arrow} \right] - \left[ \bullet \rightarrow \text{green circle with red arrow} \right]$$

$$H_{1T}(z, k_T^2) = \left[ \uparrow \bullet \rightarrow \text{red circle with green arrow} \right] - \left[ \downarrow \bullet \rightarrow \text{red circle with green arrow} \right]$$

$$H_{1L}^\perp(z, k_T^2) = \left[ \uparrow \bullet \rightarrow \text{green circle with green arrow} \right] - \left[ \downarrow \bullet \rightarrow \text{green circle with green arrow} \right]$$

$$H_{1T}^\perp(z, k_T^2) = \left[ \uparrow \bullet \rightarrow \text{green circle with red arrow} \right] - \left[ \downarrow \bullet \rightarrow \text{green circle with red arrow} \right]$$

$$H_{1\perp}^\perp(z, k_T^2) = \left[ \uparrow \bullet \rightarrow \text{blue circle with green arrow} \right] - \left[ \downarrow \bullet \rightarrow \text{blue circle with green arrow} \right]$$

|                 | Dirac projection with $\Gamma$                                     | PDF | PDF |
|-----------------|--|-----|-----|
| vector          | $\gamma^+, \gamma^i, \gamma^-$                                     | D   | f   |
| axial vector    | $\gamma^+ \gamma_5, \gamma^i \gamma_5, \gamma^- \gamma_5$          | G   | g   |
| tensor          | $\sigma^{+i} \gamma_5, \sigma^{ij} \gamma_5, \sigma^{-i} \gamma_5$ | H   | h   |
| (pseudo) scalar | $1, \gamma_5$  | E   | e   |



# Fragmentation Functions

FF for one unpolarized hadron observed in a quark jet (no  $k_T$ )

$$D_1(z) = \left[ \bullet \rightarrow \text{red circle} \right]$$

$$D_1(z, k_T^2) = \left[ \bullet \rightarrow \text{red circle} \right]$$

$$D_{1\perp}(z, k_T^2) = 0 \left[ \bullet \rightarrow \text{blue circle with spin} \right]$$

FFs for one unpolarized hadron observed in a quark jet (with  $k_T$ )

$$G_1(z) = \left[ \bullet \rightarrow \text{red circle} \right] - \left[ \bullet \rightarrow \text{red circle} \right]$$

$$G_{1L}(z, k_T^2) = \left[ \bullet \rightarrow \text{red circle} \right] - \left[ \bullet \rightarrow \text{red circle} \right]$$

$$G_{1T}(z, k_T^2) = \left[ \bullet \rightarrow \text{green circle} \right] - \left[ \bullet \rightarrow \text{green circle} \right]$$

## T-odd FF: $H_1^\perp$ (Collins FF):

- chiral-odd function
- how a transversely polarized quark (anti-quark) fragments into unpolarized hadron
- quark transverse momentum dependence

$$H_1(z) = \left[ \uparrow \bullet \rightarrow \text{red circle} \right] - \left[ \downarrow \bullet \rightarrow \text{red circle} \right]$$

$$H_{1T}(z, k_T^2) = \left[ \uparrow \bullet \rightarrow \text{red circle} \right] - \left[ \downarrow \bullet \rightarrow \text{red circle} \right]$$

$$H_{1L}^\perp(z, k_T^2) = \left[ \uparrow \bullet \rightarrow \text{green circle} \right] - \left[ \downarrow \bullet \rightarrow \text{green circle} \right]$$

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| (pseudo) scalar | $1, \gamma_5$  | E   | e   |



# Fragmentation Functions

FF for one unpolarized hadron  
observed in a quark jet  
(no  $k_T$ )

$$D_1(z) = \left[ \bullet \rightarrow \text{orange circle} \right]$$

$$D_1(z, k_T^2) = \left[ \bullet \rightarrow \text{orange circle} \right]$$

$$D_{1\perp}(z, k_T^2) = \left[ \bullet \rightarrow \text{blue circle with } \uparrow \right]$$

FFs for one unpolarized  
hadron observed in a quark jet  
(with  $k_T$ )

$$G_1(z) = \left[ \bullet \rightarrow \text{orange circle} \right] - \left[ \bullet \rightarrow \text{orange circle} \right]$$

$$G_{1L}(z, k_T^2) = \left[ \bullet \rightarrow \text{orange circle} \right] - \left[ \bullet \rightarrow \text{orange circle} \right]$$

$$G_{1T}(z, k_T^2) = \left[ \bullet \rightarrow \text{green circle with } \uparrow \right] - \left[ \bullet \rightarrow \text{green circle with } \uparrow \right]$$

FFs for spin 1/2 hadron  
observed in a quark jet  
(with  $k_T$ )

$$H_1(z) = \left[ \uparrow \bullet \rightarrow \text{orange circle with } \uparrow \right] - \left[ \downarrow \bullet \rightarrow \text{orange circle with } \uparrow \right]$$

$$H_{1T}(z, k_T^2) = \left[ \uparrow \bullet \rightarrow \text{orange circle with } \uparrow \right] - \left[ \downarrow \bullet \rightarrow \text{orange circle with } \uparrow \right]$$

$$H_{1L}^\perp(z, k_T^2) = \left[ \uparrow \bullet \rightarrow \text{green circle with } \uparrow \right] - \left[ \downarrow \bullet \rightarrow \text{green circle with } \uparrow \right]$$

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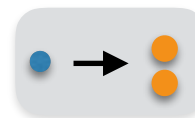
| Dirac projection with $\Gamma$ |  | PDF | PDF |
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| (pseudo) scalar                | $1, \gamma_5$  | E   | e   |

# Fragmentation Functions

$$\mathbf{R} \equiv \mathbf{P}_1 - \mathbf{P}_2$$

FF for one unpolarized hadron  
observed in a quark jet  
(no  $k_T$ )

$$\Delta^{[\gamma^-]}(z_1, z_2, \mathbf{k}_T, \mathbf{R}_T) = D_1$$

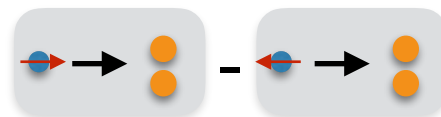


FFs for one unpolarized  
hadron observed in a quark jet  
(with  $k_T$ )

$$\Delta^{[\gamma^- \gamma^5]}(z_1, z_2, \mathbf{k}_T, \mathbf{R}_T) = \frac{\epsilon_T^{ij} R_{Ti} k_{Tj}}{M_1 M_2} G_1^\perp$$

$$\Delta^{[i\sigma^{i-} \gamma^5]}(z_1, z_2, \mathbf{k}_T, \mathbf{R}_T) = \frac{\epsilon_T^{ij} R_{Tj}}{M_1 + M_2} H_1^{\triangleleft} + \frac{\epsilon_T^{ij} k_{Tj}}{M_1 + M_2} H_1^\perp$$

FFs for spin 1/2 hadron  
observed in a quark jet  
(with  $k_T$ )



$G_1^\perp$ : T-odd FF

- chiral-even function
- log. polarized  $q \rightarrow$  two unpol. hadrons

FFs for two unpolarized  
hadrons observed in the same  
quark jet (with  $k_T$ )

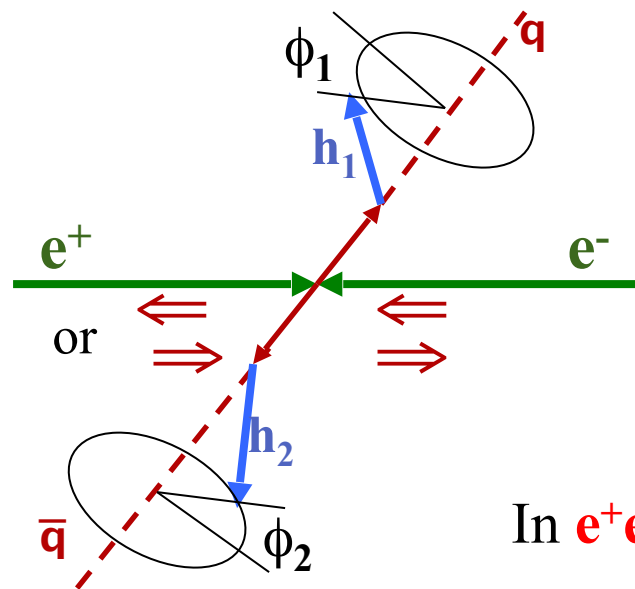


$H_1^*$ : T-odd FF

- Chiral-odd function
- Transv. polarized  $q \rightarrow$  two unpol. hadrons

The relative momentum of the hadron pair is an additional degree of freedom: the orientation of the two hadrons w.r.t. each other and the jet direction is an indicator of the quark transverse spin

# Collins Effect in $e^+e^-$ annihilation



Probability density for finding unpolarized hadron from a transversely polarized  $q$ :

$$D_1^{q\uparrow}(z, \mathbf{P}_\perp; s_q) = D_1^q(z, P_\perp) + \frac{P_\perp}{zM_h} H_1^{\perp q}(z, P_\perp) \mathbf{s}_q \cdot (\mathbf{k}_q \times \mathbf{P}_\perp)$$

In  $e^+e^-$  annihilation,  $\gamma^*$  (spin-1)  $\rightarrow$  spin-1/2  $q$  and  $\bar{q}$

- in a given event, the spin directions are unknown, but they must be parallel
- they have a polarization component transverse to the  $q$  direction  $\sim \sin^2\theta$  ( $\theta$  wrt the  $e^+e^-$ )
- exploit this correlation by using hadrons in opposite jets

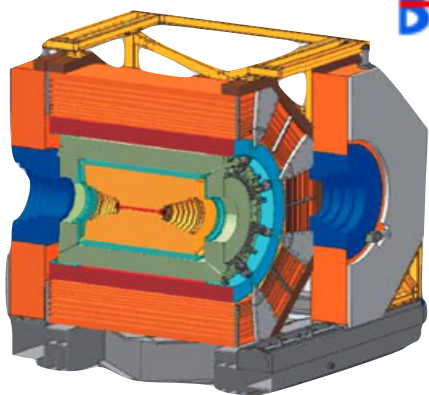
- $H_1^\perp \neq 0 \Rightarrow$  leads to an asymmetry in the angular distribution of final state particles (Collins effect)
- First non-zero Collins effect observed in SIDIS [PRL 94,012002\(2005\)](#), [NPB 765, 31\(2007\)](#)
- First non-zero independent measurement of the Collins effect for pion pairs in  $e^+e^-$  annihilation by Belle Collaboration @  $\sqrt{s} \sim 10.6$  GeV ([PRL 111,062002\(2008\)](#), [PRD 88,032011\(2013\)](#))
  - Confirmed by BaBar @  $\sqrt{s} \sim 10.6$  GeV ([PRD 90,052003 \(2014\)](#); [PRD 92,111101\(R\)\(2015\)](#) for  $KK$  and  $K\pi$ )
  - Measured at BESIII @  $\sqrt{s} = 3.65$  GeV ([PRL 116,42001\(2016\)](#))

$$e^+e^- \rightarrow q\bar{q} \rightarrow h_1 h_2 X \quad (q=u, d, s) \Rightarrow \sigma \propto \cos(\varphi_i) H_1^\perp(z_1) \otimes H_1^\perp(z_2)$$

# The BaBar, Belle and BESIII experiments

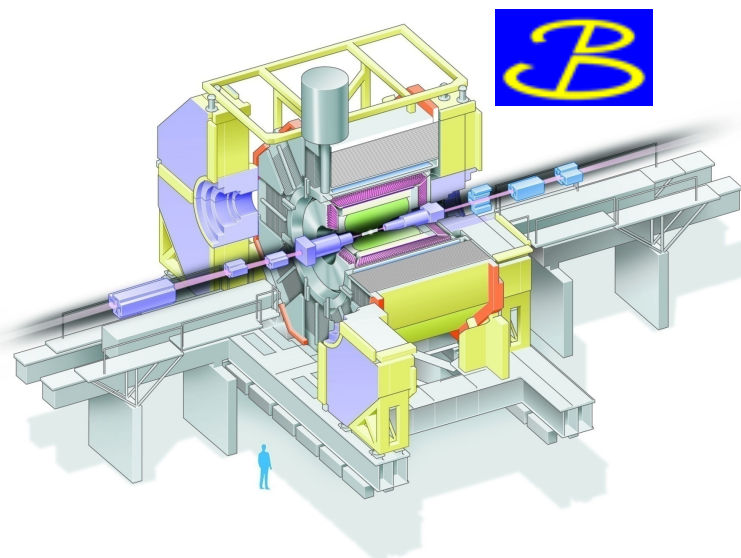
NIMA614,345(2010)

BESIII



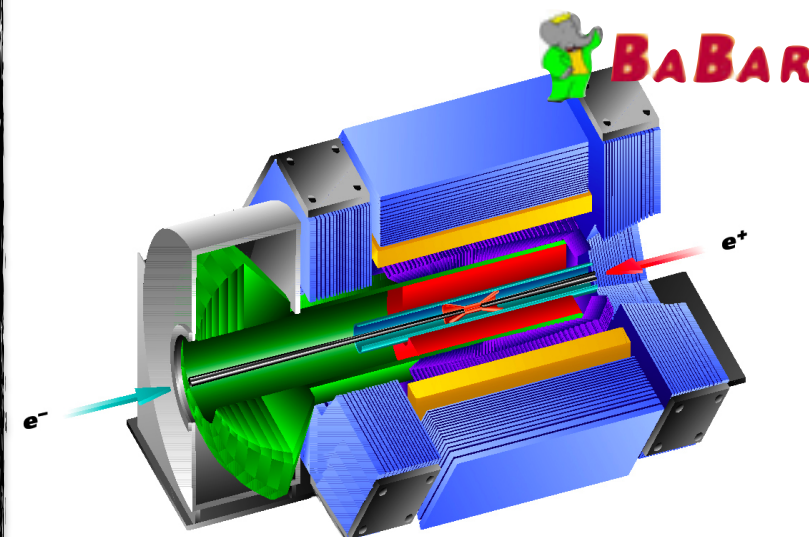
- Symmetric  $e^+e^-$  collider
- $\sqrt{s} = [2 - 4.6]$  GeV
- $62 \text{ pb}^{-1}$  @ 3.65 GeV used for Collins studies
- Below open-charm threshold

- Asymmetric-energy  $e^+e^-$  collider
- $\sqrt{s} \sim 10.6$  GeV ( $\Upsilon(4S)$ )
- $\beta\gamma=0.425$
- $L \sim 1 \text{ ab}^{-1}$



NIMA479,117(2002)

NIMA729,615(2013)

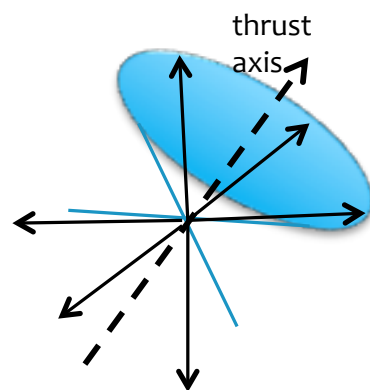


- Asymmetric-energy  $e^+e^-$  collider
- $\sqrt{s} \sim 10.6$  GeV ( $\Upsilon(4S)$ )
- $\beta\gamma=0.65$
- $L \sim 500 \text{ fb}^{-1}$

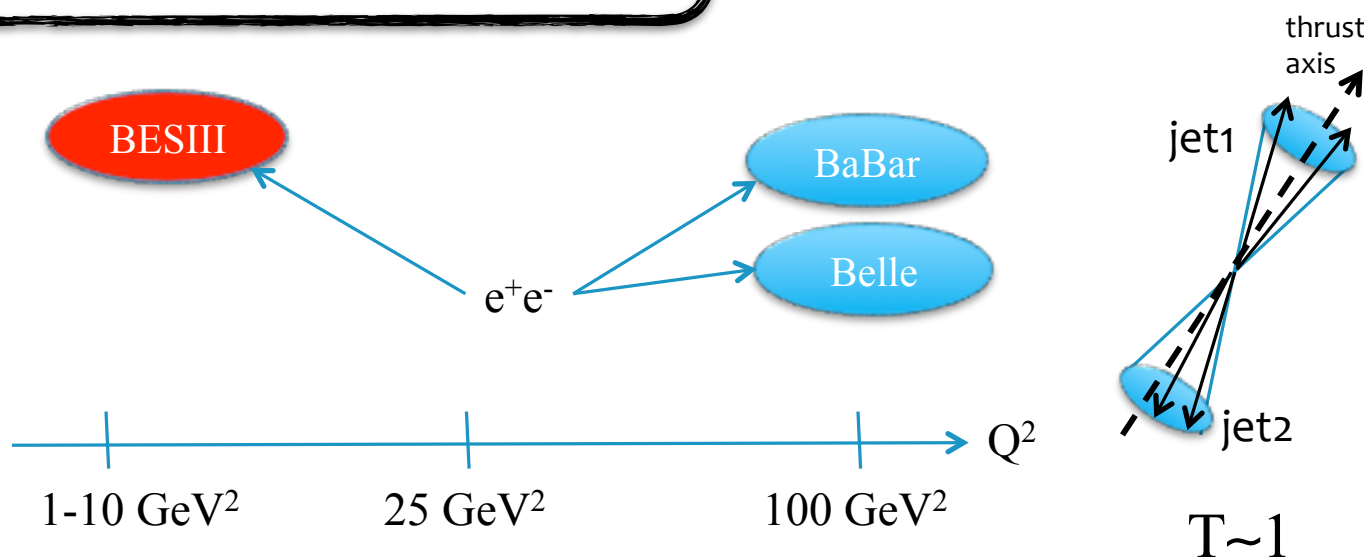
$$T = \sum_i \frac{|\mathbf{P} \cdot \hat{\mathbf{n}}|}{|P|}$$

thrust axis  $\equiv \hat{\mathbf{n}}$

$$0.5 \leq T \leq 1$$



$T \sim 0.5$

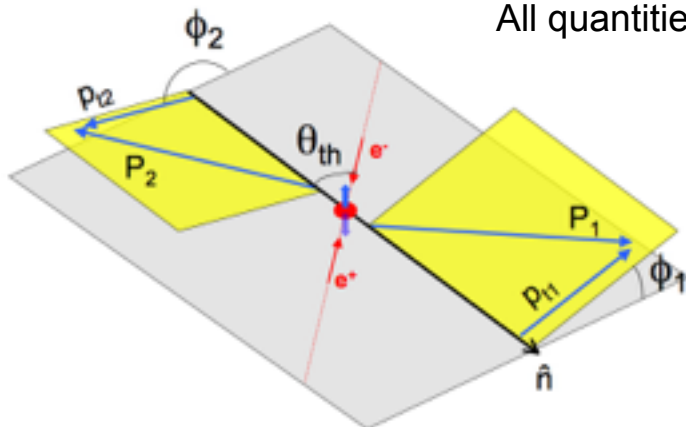


# Analysis Strategies and Reference Frames

[See NPB 806, 23 (2009)]

## RF12 or Thrust RF

All quantities in  $e^+e^-$  c.m.



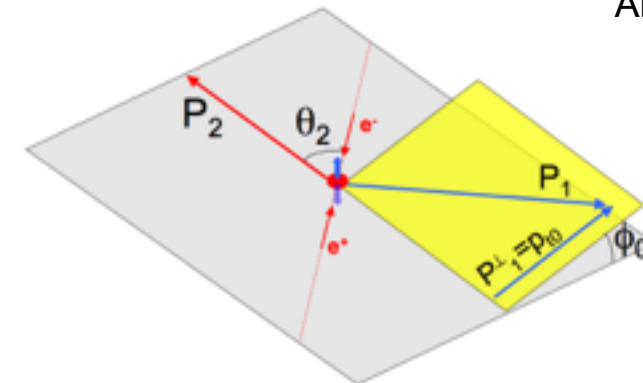
BaBar  
Belle  
~~BESIII~~

- **Thrust axis** to estimate the  $q\bar{q}$  direction
- $\phi_{1,2}$  defined using thrust-beam plane
- Modulation diluted by gluon radiation, detector acceptance,...

$$\sigma \sim 1 + \frac{\sin^2 \theta_{th}}{1 + \cos^2 \theta_{th}} \cos(\phi_1 + \phi_2) \frac{H_1^\perp(z_1) \bar{H}_1^\perp(z_2)}{D_1(z_1) \bar{D}_1(z_2)}$$

## RF0 or Second hadron momentum RF

All quantities in  $e^+e^-$  c.m.



BaBar  
Belle  
BESIII

- Use **one track** in a pair
- Very clean experimentally (no thrust axis), less so theoretically
- Gives quark direction for higher pion momentum

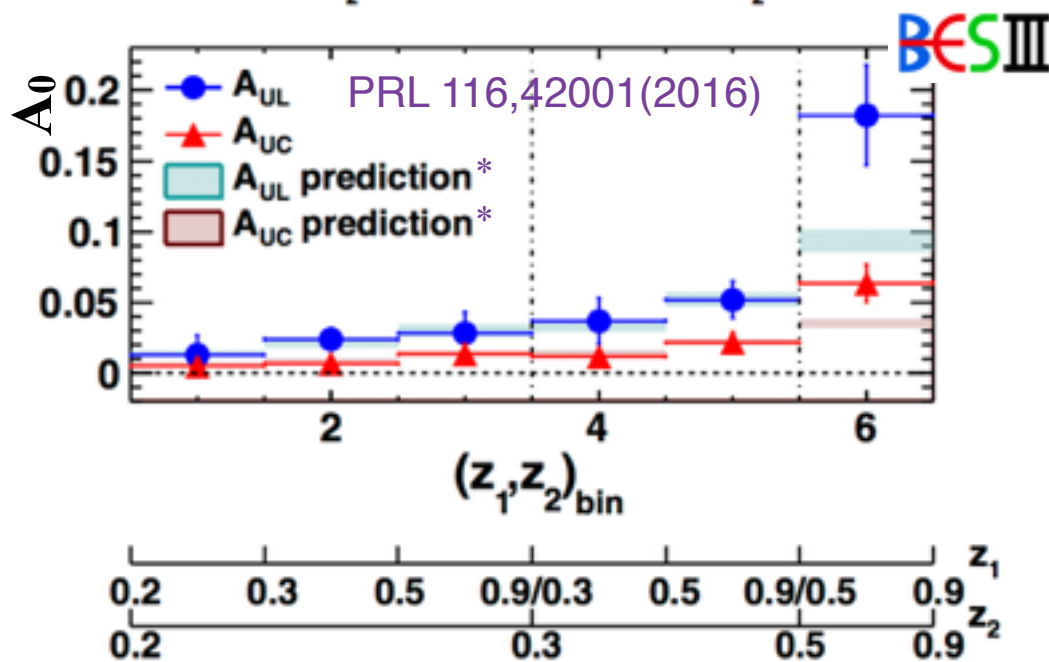
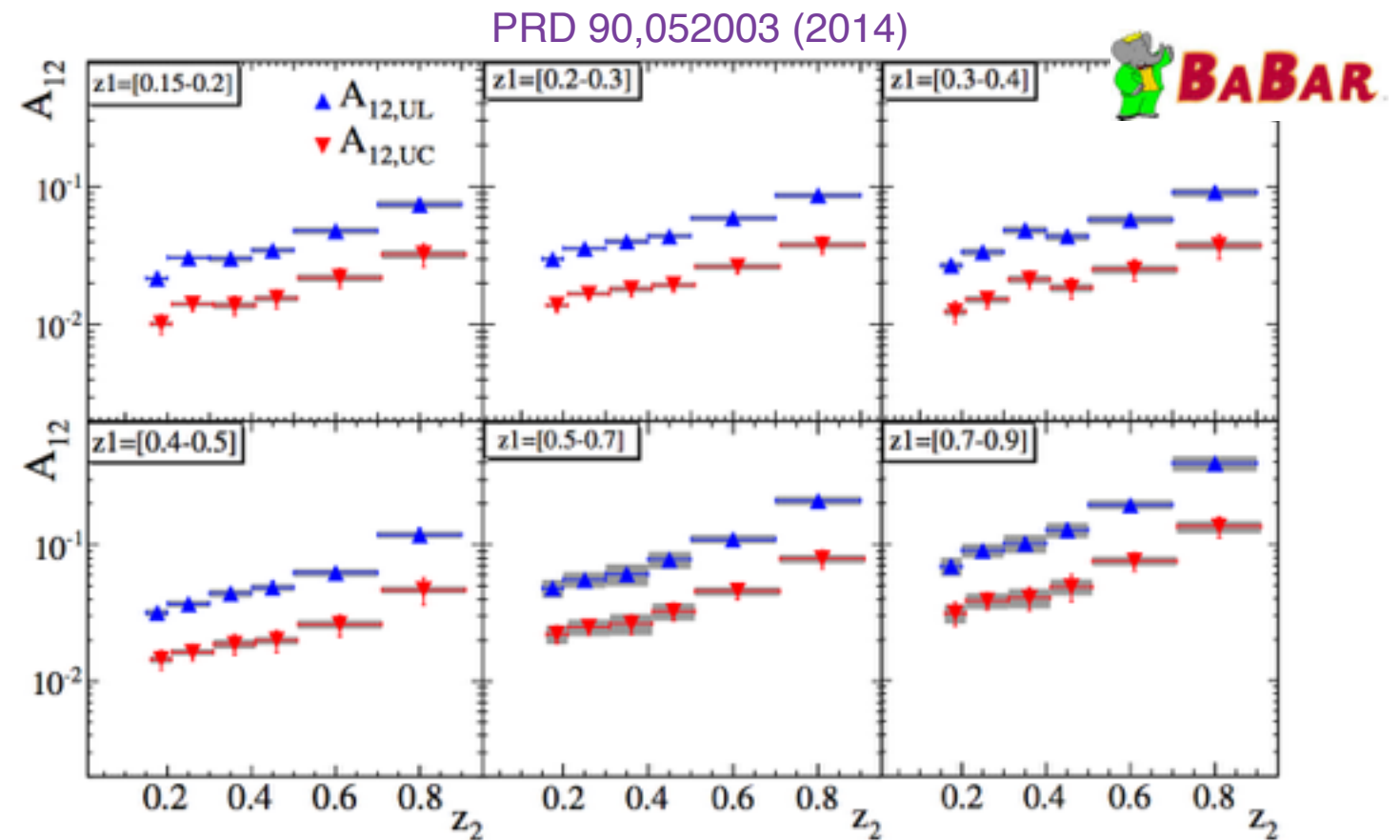
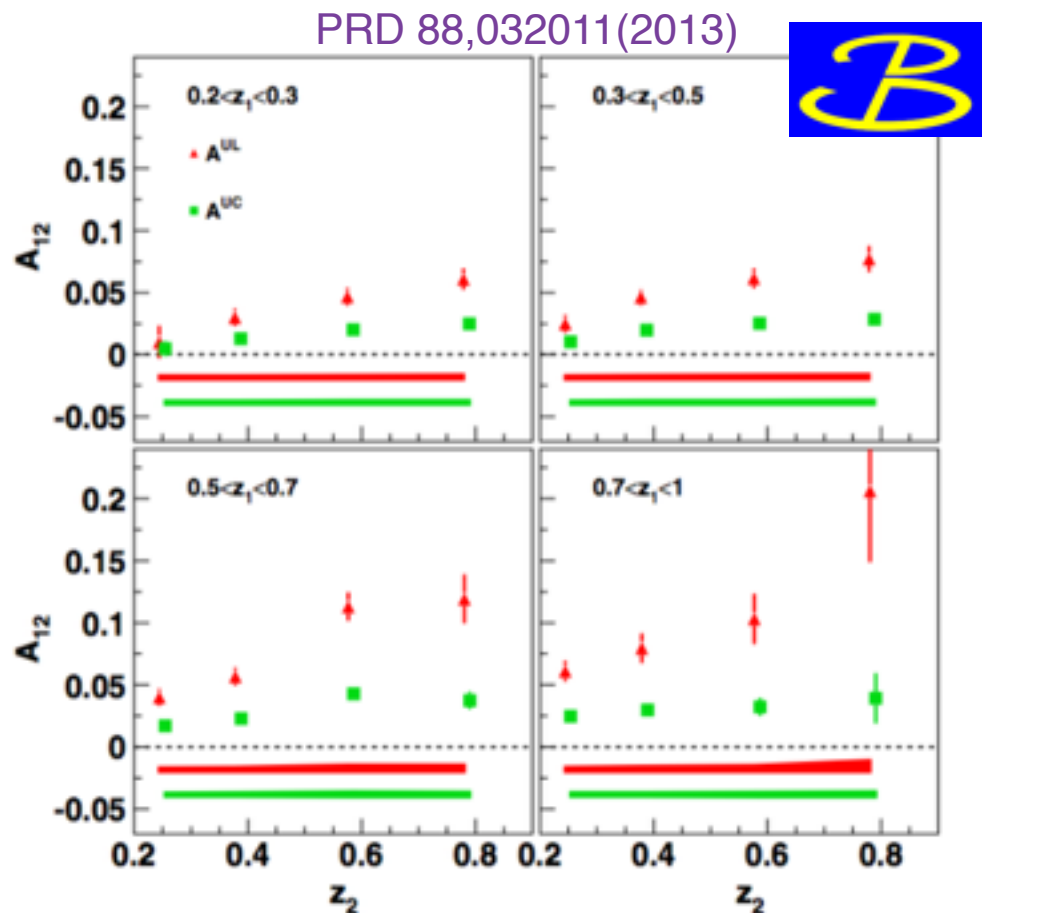
$$\sigma \sim 1 + \frac{\sin^2 \theta_2}{1 + \cos^2 \theta_2} \cos(2\phi_0) \mathcal{F} \left[ \frac{H_1^\perp(z_1) \bar{H}_1^\perp(z_2)}{D_1(z_1) \bar{D}_1(z_2)} \right]$$

- Azimuthal modulations (Collins effect) may be measured as a function of the pions fractional energy ( $z_{1,2}=2E_\pi/\sqrt{s}$ ), pions transverse momentum ( $p_{t1}, p_{t2}, p_{t0}$ ), and as a function of the polar angle of the reference axis ( $\theta_{th}, \theta_2$ )
- Ratio of normalized distribution of pion pairs with opposite charge sign ( $R^U$ ) / same charge sign ( $R^L$ ): cancels detector effects (or  $(R^{C=U+L})/R^L$ )  $\Rightarrow$  **sensitivity to fav. and dis. fragmentation processes**

$$\frac{R_\alpha^U}{R_\alpha^{L(C)}} = \frac{N^U(\phi_\alpha) / \langle N^U(\phi_\alpha) \rangle}{N^{L(C)}(\phi_\alpha) / \langle N^{L(C)}(\phi_\alpha) \rangle} \rightarrow B_\alpha^{UL(UC)} + A_\alpha^{UL(UC)} \cdot \cos(\phi_\alpha)$$



# Collins Effect vs $(z_1, z_2)$

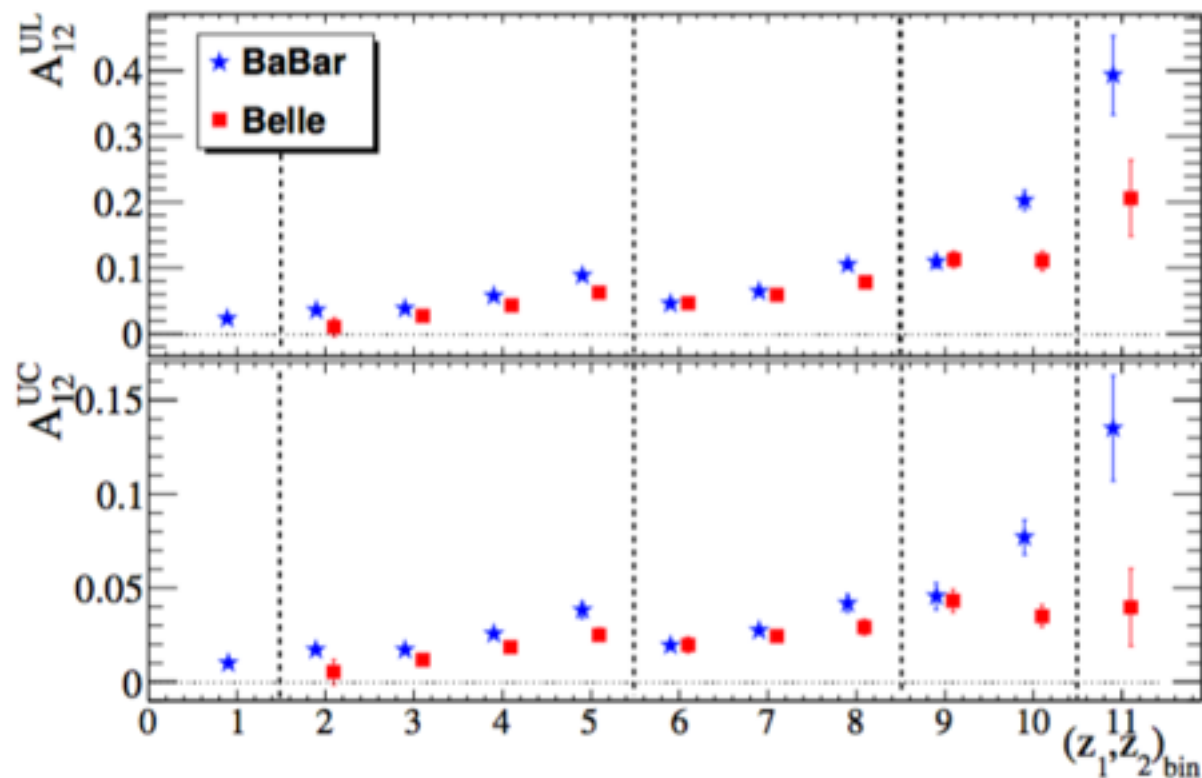
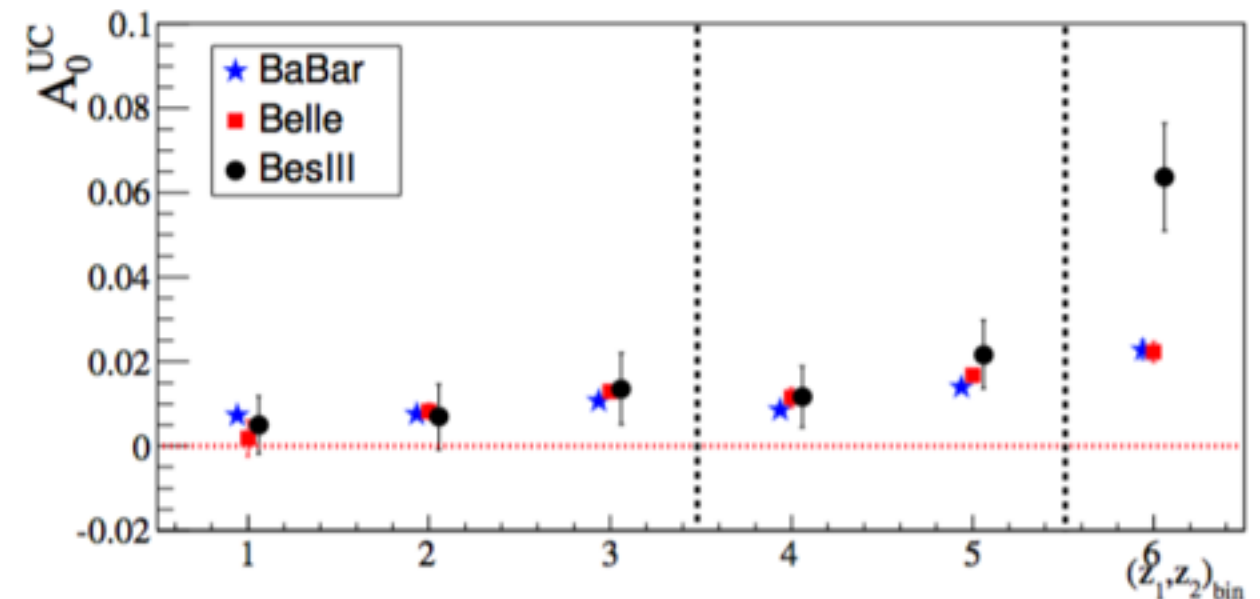
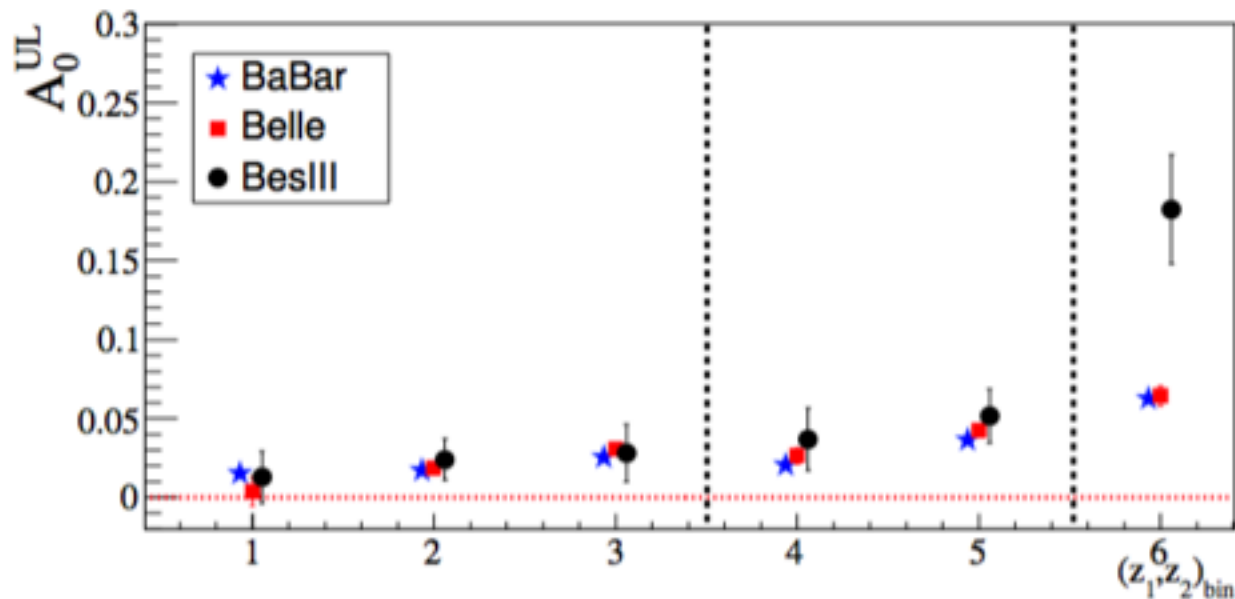


- Significant non-zero asymmetries  $A_{12}$ ,  $A_0$  in all bins
- Strong dependence on  $(z_1, z_2)$  observed in all the experiments
- $A_{UC} < A_{UL}$  as expected; complementary informations about favored and disfavored fragmentation processes

\*PRD 93,014009(2016)

# Collins Effect vs $(z_1, z_2)$ : comparisons

EPJA, 52, 1-15 (2016)

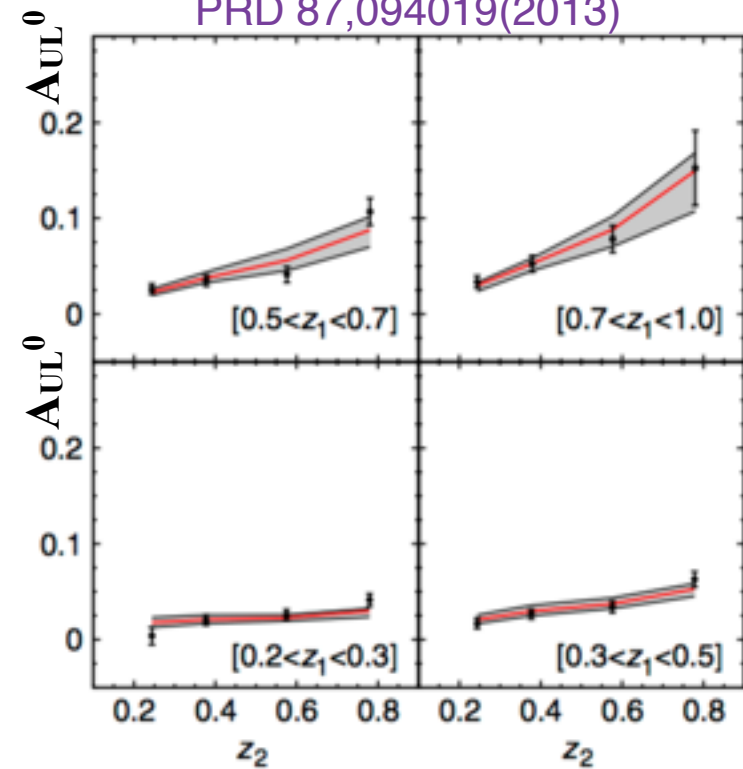


- Symmetric  $(z_1, z_2)$  bins are used for the comparisons: results falling in the same large  $(z_1, z_2)$  interval are averaged taking into account statistical and systematic uncertainties
- $A_0$ : very good agreement between BaBar and Belle data; larger asymmetry for BESIII in the last bin ( $z_{1,2} > 0.5$ )
- $A_{12}$ : some tensions between BaBar and Belle, which can be attributed to experimental features:
  - thrust axis corrections
  - background corrections
  - $z < 0.9$  for BaBar,  $z < 1$  for Belle



# Extracted Collins Functions

PRD 87,094019(2013)



- Polynomial parameterization:

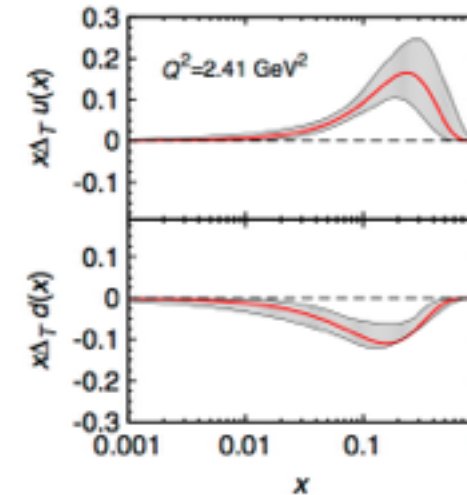
$$\frac{\Delta^N D_{\pi^+/u^{\uparrow}, \bar{d}^{\uparrow}}(z, p_{\perp})}{D_{\pi^+/u, \bar{d}}} = 2\mathcal{N}_{fav}^C(z)h(p_{\perp})\frac{e^{-p_{\perp}/\langle p_{\perp}^2 \rangle}}{\pi\langle p_{\perp}^2 \rangle}$$

$$h(p_{\perp}) = \sqrt{2}e\frac{p_{\perp}}{M_h}e^{-p_{\perp}^2/M_h^2}$$

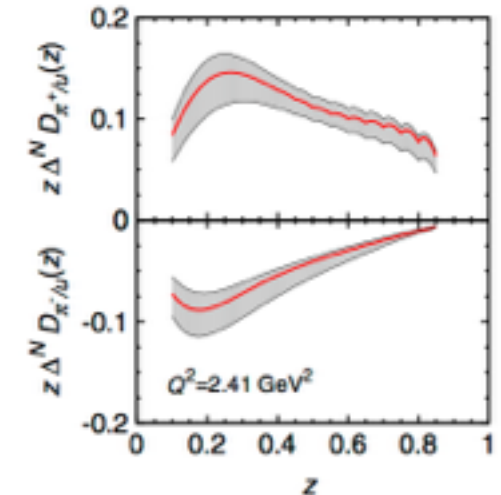
$$\mathcal{N}_{fav}^C = \mathcal{N}_{fav}^C z[(1-a-b) + az + bz^2]$$

- global analysis: SIDIS+BELLE data (arXiv:1303.2076; PLB693,11; PRD86,032011)
- $z\Delta^N D_{h/q^{\uparrow}}(z) = 4zH_1^{\perp(1/2)q}(z)$
- tension between fav Collins function from  $A^0$  and  $A^{12}$  (PRD87)

Transversity PDF

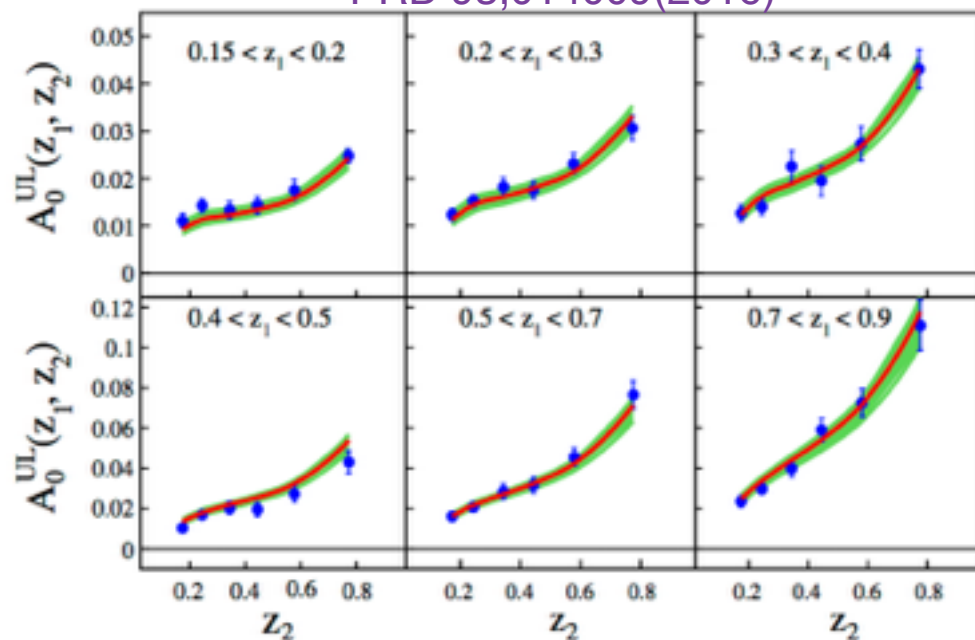


Fav. and dis. Collins FFs

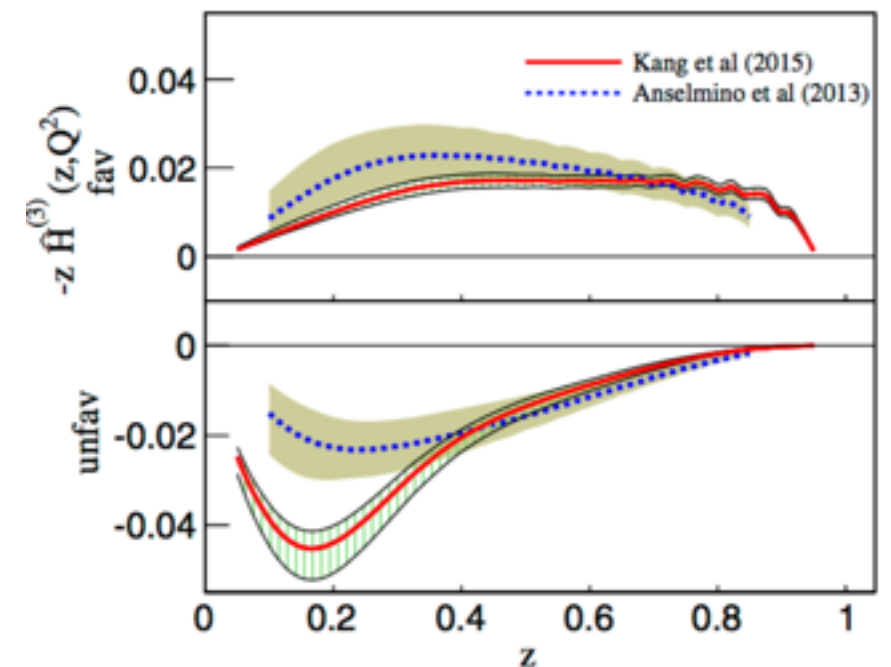


Note: no TMD evolution taken into account

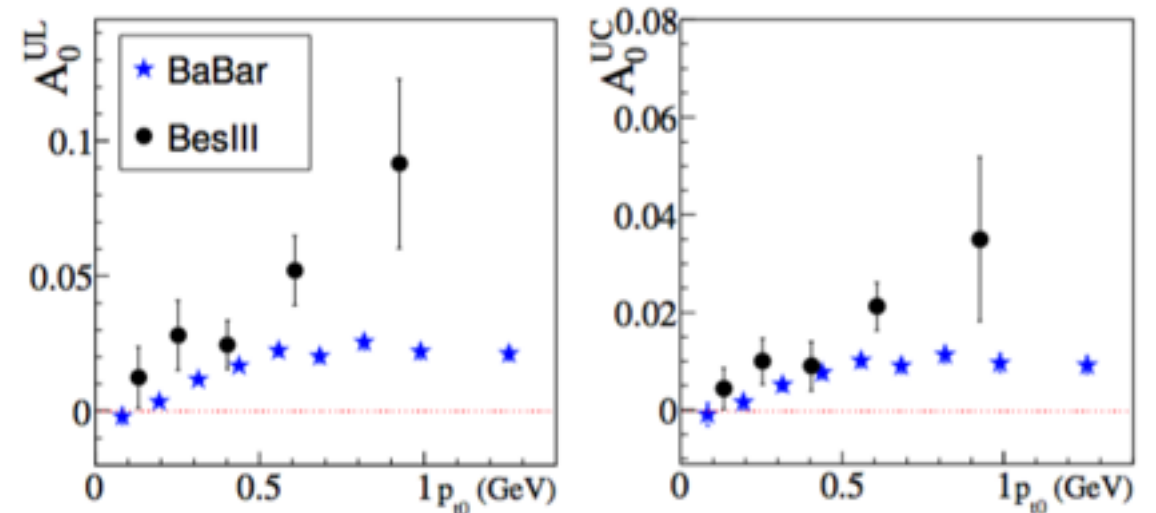
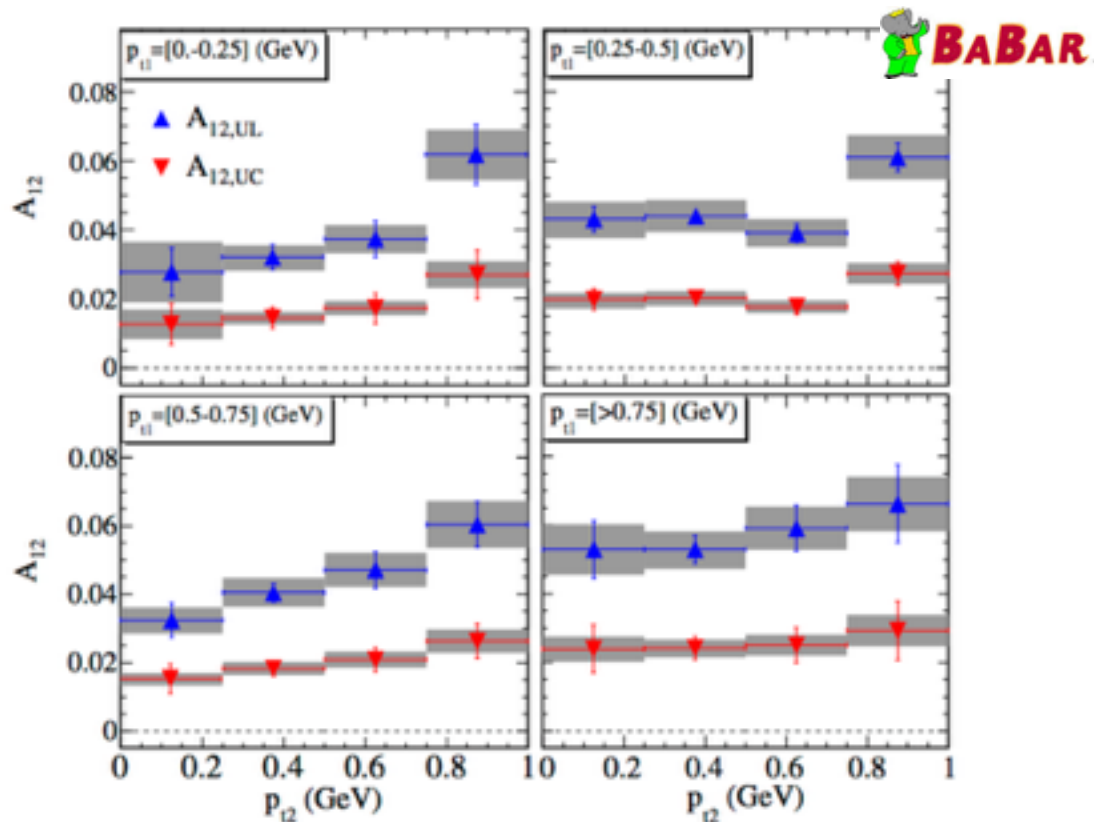
PRD 93,014009(2016)



- Global fit with TMD evolution (PRL103,152002; PLB744,250; PLB673,127; PRL 107,072003, PRD90,052003; PRD78,032011)
- Good description of both Belle and BaBar data: magnitude and shape of the data are very well reproduced: fav. and dis. parameters allowed to be different and independent each other

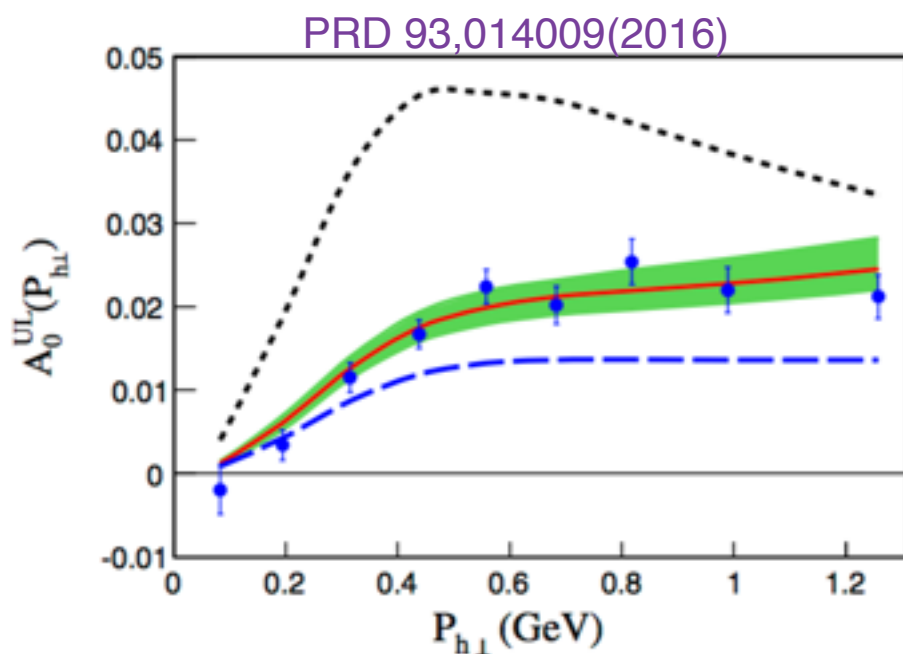


# Collins Effect vs $p_t$ and $\theta_{(th,2)}$



The asymmetries increase for increasing  $p_t$ :

- less pronounced for  $A_{12}$ , but large uncertainties due to the  $p_t$  resolution
- steeper  $p_t$  dependence for BESIII
- different kinematic regions:  $\langle Z \rangle_{\text{BESIII}} > \langle Z \rangle_{\text{BaBar}}$



— NLL': next-to-leading-logarithm approximation

- - - LL: leading logarithmic calculation

..... No TMD evolution

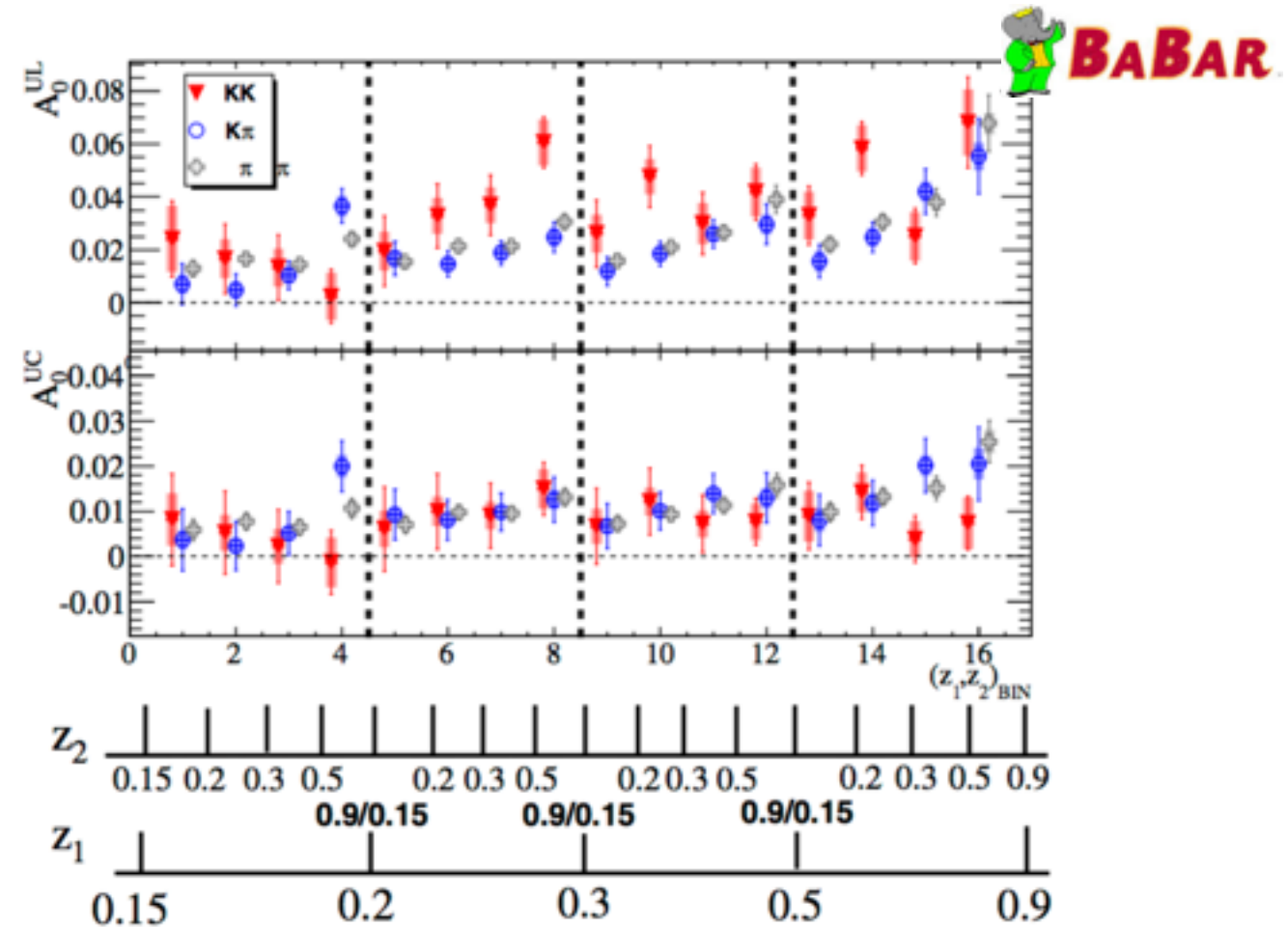
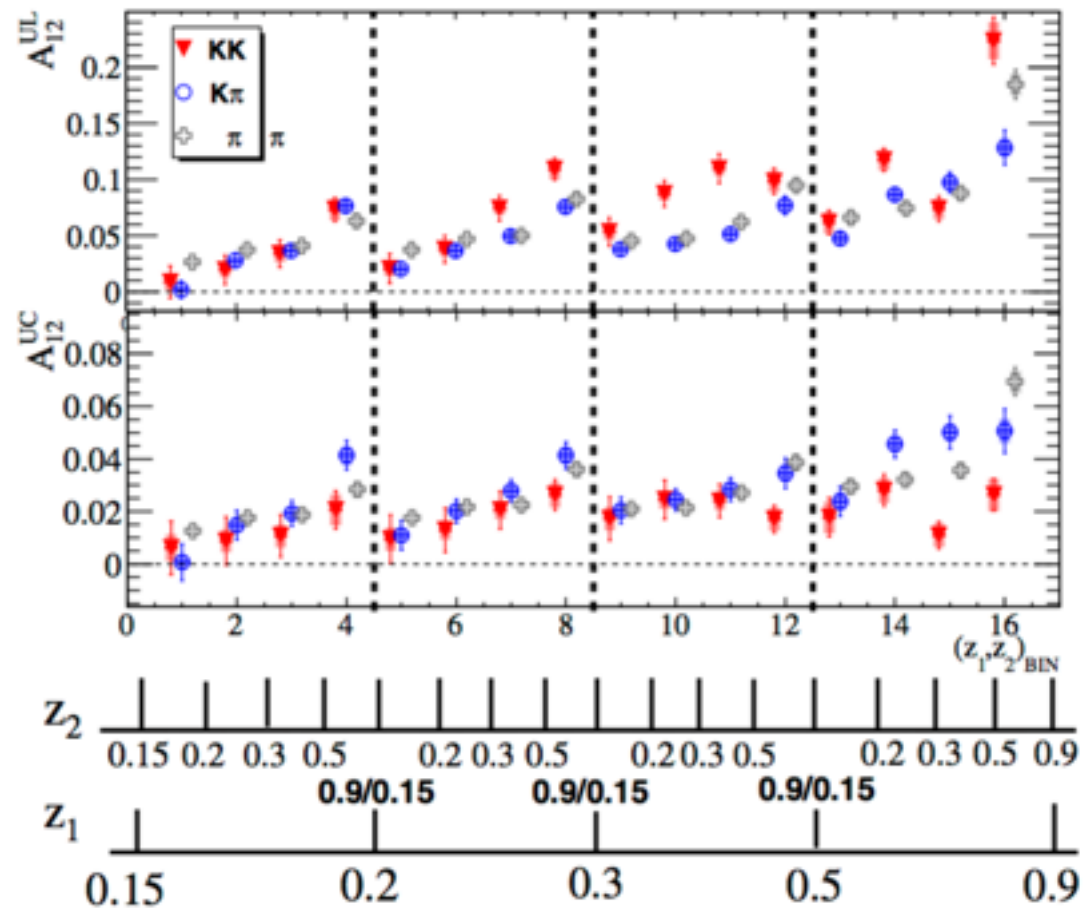
Calculation performed with fixed parameters from Table I in PRD93,014009

- $A^{\text{UL}}$  and  $A^{\text{UC}}$  asymmetries are described very well
- TMD evolution at NLL' describes  $e^+e^-$  and SIDIS data adequately well
- better description including higher orders: improvement of the theoretical uncertainties

# Collins Effect For $KK$ and $K\pi$

Simultaneous measurement of  $KK$ ,  $K\pi$  and  $\pi\pi$  Collins asymmetries from BaBar data

PRD 92,111101(R)(2015)

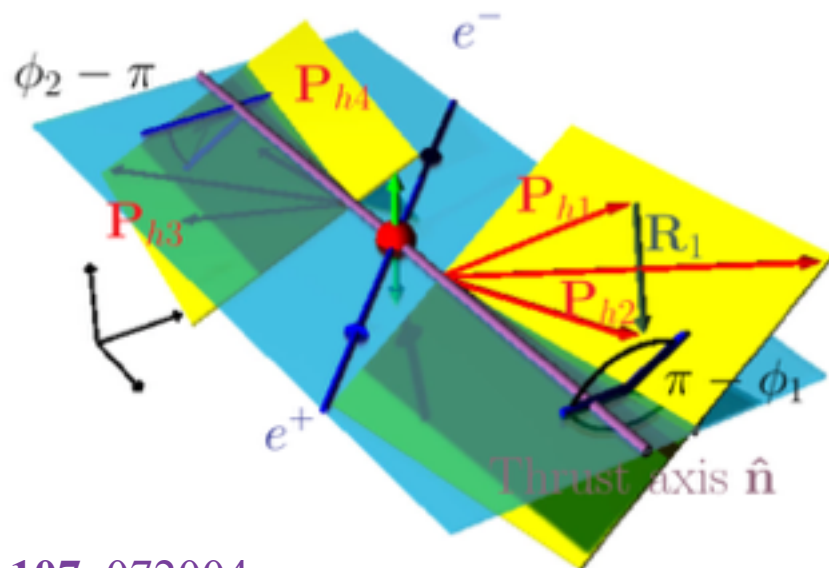


- Rising of the asymmetry as a function of  $z$ :
  - more pronounced for U/L
- $A^{UL}$   $KK$  asymmetry slightly higher than pion asymmetry for high  $z$
- $KK$  asymmetry consistent with zero at lower  $z$
- $\pi\pi$  asymmetries consistent with previous measurements (PRD90, 052003)



# Di-hadron Fragmentation Functions

$$e^+e^- \rightarrow (h_1 h_2)(h_3 h_4) X$$



- The Collins effect is a challenging observable (both theoretically and experimentally) due to its  $\mathbf{k}_T$  dependence
- Di-hadron or Interference FFs (IFFs) represent an alternative way to access spin information
  - complementary access to transversity
  - two hadrons orientation as indication the quark transverse spin
  - collinear model can be used for factorization

PRL107, 072004

$$a_{12R}(z_1, z_2, m_1^2, m_2^2) \propto \frac{1}{2} \frac{\sin^2 \theta}{1 + \cos^2 \theta} \cdot \frac{\sum_{q, \bar{q}} e_q^2 z_1^2 z_2^2 H_1^{\chi, q}(z_1, m_1^2) H_1^{\chi, \bar{q}}(z_2, m_2^2)}{\sum_{q, \bar{q}} e_q^2 z_1^2 z_2^2 D_1^q(z_1, m_1^2) D_1^{\bar{q}}(z_2, m_2^2)}$$

PRD67, 094003

$$R_{12} = \frac{N(\phi_1 + \phi_2)}{\langle N \rangle} = a_{12} \cos(\phi_1 + \phi_2) + b_{12} + c_{12} \sin(\phi_1 + \phi_2) + d_{12} \cos(2(\phi_1 + \phi_2))$$

$$\propto H^{\chi q_1} \cdot H^{\chi q_1}$$

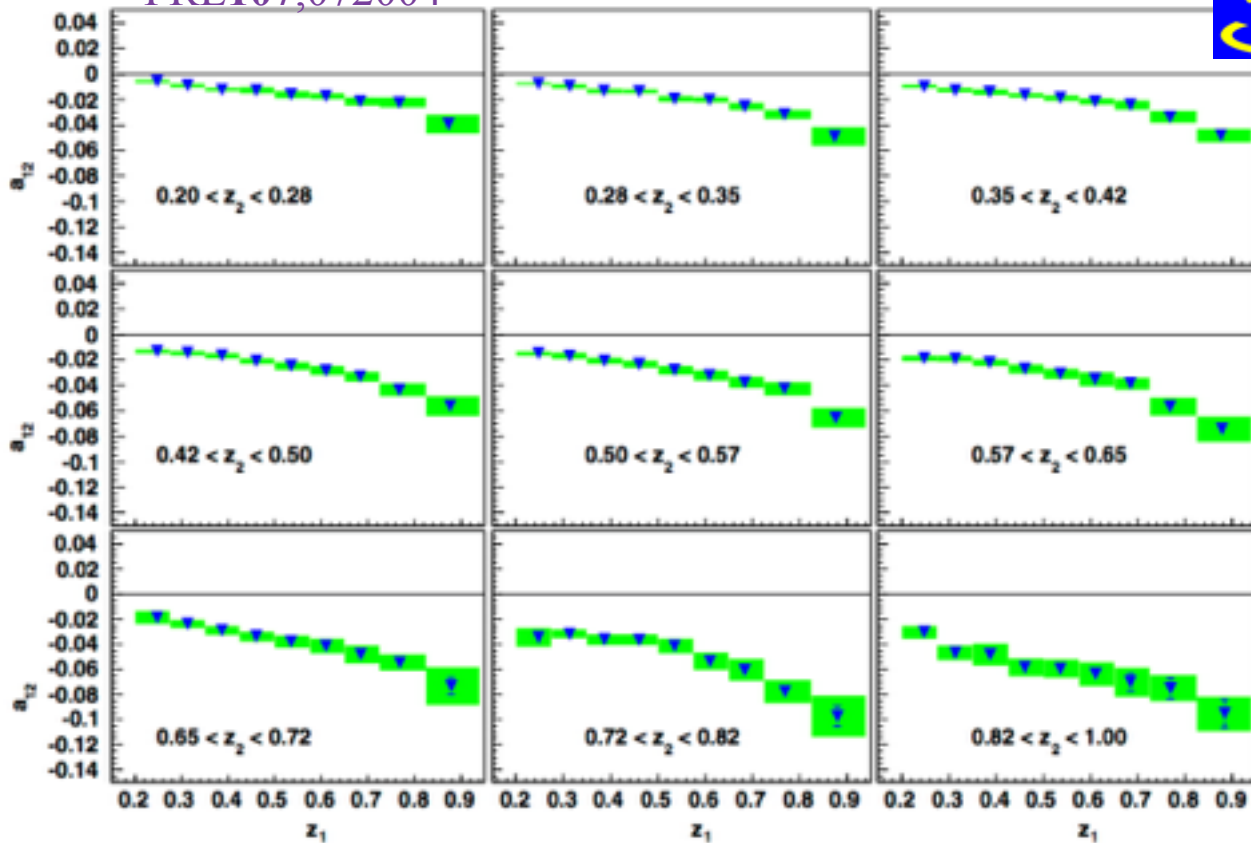
should be consistent with unity

higher-order terms (orthogonal terms, they should not interfere each other)

- $9 \times 9$   $z_1, z_2$  bins
- $8 \times 8$   $m_1, m_2$  bins

# $H_1^*$ and transversity extraction

PRL107,072004

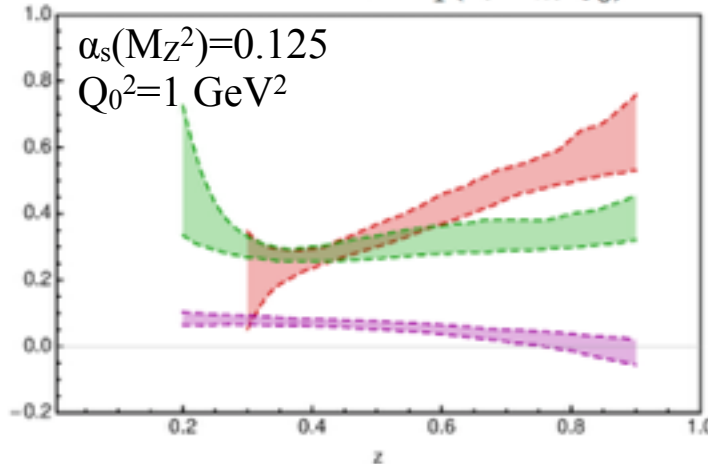


- Large azimuthal asymmetries for two  $\pi^+\pi^-$  pairs in opposite hemispheres measured by Belle using  $672 \text{ pb}^{-1}$
- Monotonically decreasing with fractional energy and invariant mass (see back-up)
- no sign change observed in contrast to what predicted in PRL80, 1166

- Global fit analysis, JHEP05(2015)123, in order to extract  $H_1^*$  and transversity (JHEP06,017; PRL107,072004, EPJ Web Conf. 85, 02018) based on “replica method”

- collinear factorization framework
- transversity consistent with extraction from PRD91, 014034\* (different approach used)
- disagreement for  $x \geq 0.1$  w.r.t. the outcome of the Collins effect

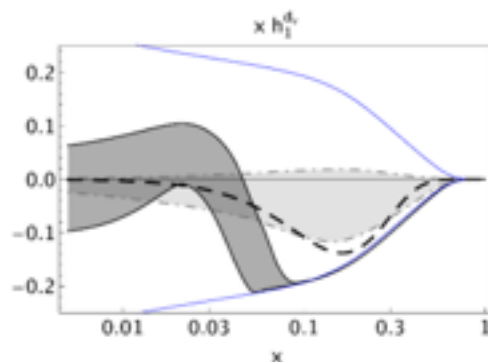
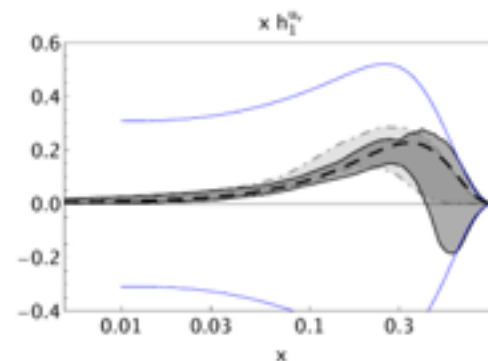
$$R(z, M_h) = \frac{|R|}{M_h} \frac{H_1^{*u}(z, M_h; Q_0^2)}{D_1^u(z, M_h; Q_0^2)}$$



JHEP05(2015)123

- $M_h=0.4 \text{ GeV}$
- $M_h=0.8 \text{ GeV}$
- $M_h=1 \text{ GeV}$

bands:  $1\sigma$  unc.



$\alpha_s(M_Z^2)=0.125$   
 $Q^2=2.4 \text{ GeV}^2$

- JHEP05 flexible scenario
- $H_1^\perp$  PRD87 (DGLAP for collinear part)
- $H_1^\perp$  PRD91 (TMD framework)
- Soffer bound

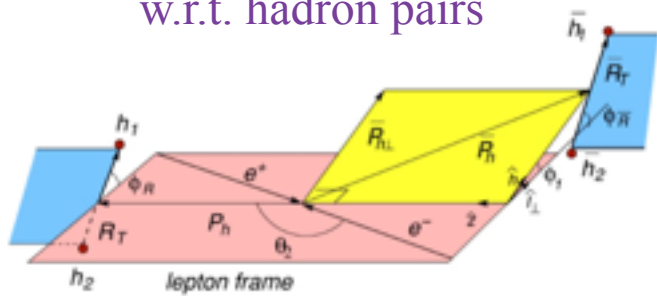
\* see back-up for more details

# Di-hadron Fragmentation Functions

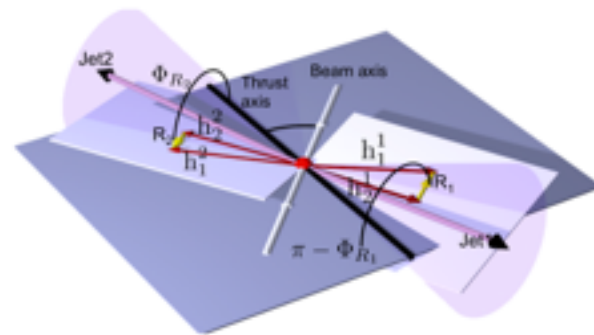
Starting from the fully integrated  $e^+e^-$  cross section into four unpolarized hadrons with two leading hadrons in each jet, authors of ref. [PRD67, 094003](#) explicitly derive the asymmetry:

$$A(y, z, \bar{z}, M_h^2 \bar{M}_h^2) = \frac{\langle \cos(2(\phi_R - \phi_{\bar{R}})) \rangle}{\langle 1 \rangle} = \frac{\sum_{a, \bar{a}} e_a^2 \frac{3\alpha^2}{2Q^2} z^2 \bar{z}^2 A(y) \frac{1}{M_1 M_2 \bar{M}_1 \bar{M}_2} \boxed{G_1^{\perp a}(z, M_h^2) \bar{G}_1^{\perp a}(\bar{z}, \bar{M}_h^2)}}{\sum_{a, \bar{a}} e_a^2 \frac{6\alpha^2}{Q^2} A(y) z^2 \bar{z}^2 D_1^a(z, M_h^2) \bar{D}_1^a(\bar{z}, \bar{M}_h^2)}$$

[PRD67,094003](#)  
asymmetric frame  
w.r.t. hadron pairs



[arXiv:1505.08020](#)  
symmetric frame



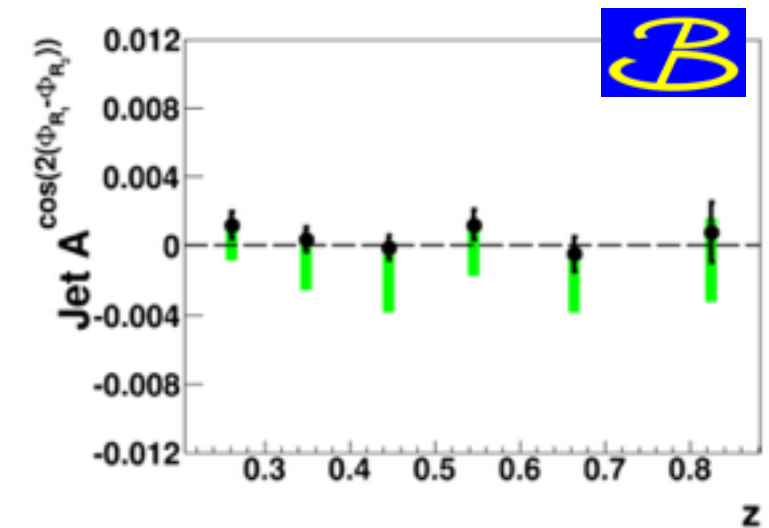
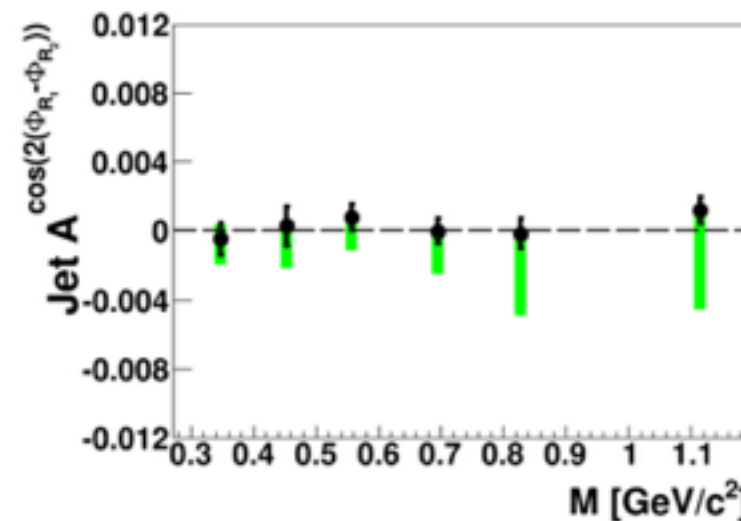
- longitudinally polarized quark IFF  $G_1^\perp$
  - chiral-even function related to the jet handedness
  - asymmetric reference frame
  - experimentally: switch to a symmetric frame
    - Belle preliminary: [arXiv:1505.08020](#)
    - angles are computed using the jet axis of di-jet event
    - jet axes reconstructed using anti-kT jet algorithm
- [JHEP0804, 063](#)

Two-dimensional  $\chi^2$  fit is performed to the normalized di-pion pairs:

$$1 + A^{\cos(\phi_{R1} + \phi_{R2})} \cos(\phi_{R1} + \phi_{R2}) + \boxed{A^{\cos(2(\phi_{R1} - \phi_{R2}))} \cos(2(\phi_{R1} - \phi_{R2}))}$$

**NO SIGNAL observed at Belle**

BUT more investigations about the thrust axis method and jet-axes reconstruction are needed



# Summary and Conclusions










- Spin-dependent fragmentation functions provide key informations for understanding the hadronic structure and can also be used as a tool for the extraction of parton distribution functions
- $e^+e^-$  annihilation experiments offer the ideal conditions to access FFs

| one hadron FF  | without $k_T$                     | with $k_T$  |
|----------------|-----------------------------------|---|
| spin-0         | $D_1$                             | $H_1^\perp$   |
| spin-1/2       | $D_1, G_1, H_1$                   | $D_{1T}^\perp, H_1^\perp, G_{1T}, H_{1L}^\perp, H_{1L}^\perp$   |
| spin-1         | $D_1, D_{1LL}, G_1, H_1, H_{1LT}$ | $D_{1T}^\perp, H_1^\perp, G_{1T}, H_{1L}^\perp, H_{1L}^\perp, D_{1T}^\perp, D_{1LT}, D_{1TT}, G_{1LT}, G_{1TT}, H_{1LL}^\perp, H'_{1LT}, H_{1LT}^\perp$ |
| two hadrons FF | without $k_T$                     | with $k_T$  |
| spin-0         | $D_1, H_1^\triangleright$         | $G_1^\perp, H_1^\perp$  |

higher twist  
T-odd



# Summary and Conclusions

- Many attempts from Belle, BaBar and BESIII in order to study spin-dependent FF
  - Collins effects investigated as a function of several kinematic variables at two different center-of-mass energies
  - Di-hadron FFs: only Belle results available, but they could be studied also at BaBar and BESIII
  - lack of knowledge for the corresponding  $k_T$  dependence for the unpolarized functions. These informations are required to have a more reliable extraction from global fit analyses
- Prospects
  - Continue to measure precise spin-dependent FFs at Belle, BaBar and BESIII
  - Ongoing works:
    - $\pi^0, \eta$  (Hairong Li from Indiana) **Collins asymmetry** (preliminary results will be ready soon) 
    - **K Collins asymmetries** (F. Giordano, R. Seidl), and  $k_T$  Collins dependence (BaBar)  
    - $\pi\rho^0$  **Collins asymmetry** (Belle) 
    - **Unpolarized IFFs for pions and kaons** (preliminary results will be ready for SPIN-2016)  (A. Vossen)
  - More works planned   
  - Measure other interesting QCD-related quantities:
    - $\Lambda$  FFs  $D_1^\perp(z, k_T)$ , which is the fragmentation counterpart to the Sivers function (Yinghui Guan, KEK/Indiana; preliminary results will be ready for SPIN-2016) 

BK slides

# Transversity distributions from single- and di-hadron production

A. Martin, F. Bradamante, V. Barone, [PRD91, 014034](#)

- Extraction of the transversity point by point in  $x$  both from signal-hadron and di-hadron data
- no data parameterization used
- analyzing power determined from  $e^+e^-$  measurement (PRL107,072004):

$$|\tilde{a}_P^{hh}(Q^2)| = \left| \frac{\tilde{H}_{1u}^{\leftarrow}(Q^2)}{\tilde{D}_{1u}^{hh}(Q^2)} \right|$$

$$= \sqrt{-\frac{1}{5}(1+\mu^2)(5+\lambda^2) \frac{\langle 1+\cos^2\theta_2 \rangle}{\langle \sin^2\theta_2 \rangle} A_{e^+e^-}^{hh}(Q^2)},$$

$$\frac{\langle \sin^2\theta_2 \rangle}{\langle 1+\cos^2\theta_2 \rangle} = 0.7636, \quad \mu=0.5 \text{ (charm contr.)}, \lambda=0.5 \text{ (strange contr.)}$$

$$A_{e^+e^-}^{hh} = -0.0196 \pm 0.0002 \pm 0.0022$$

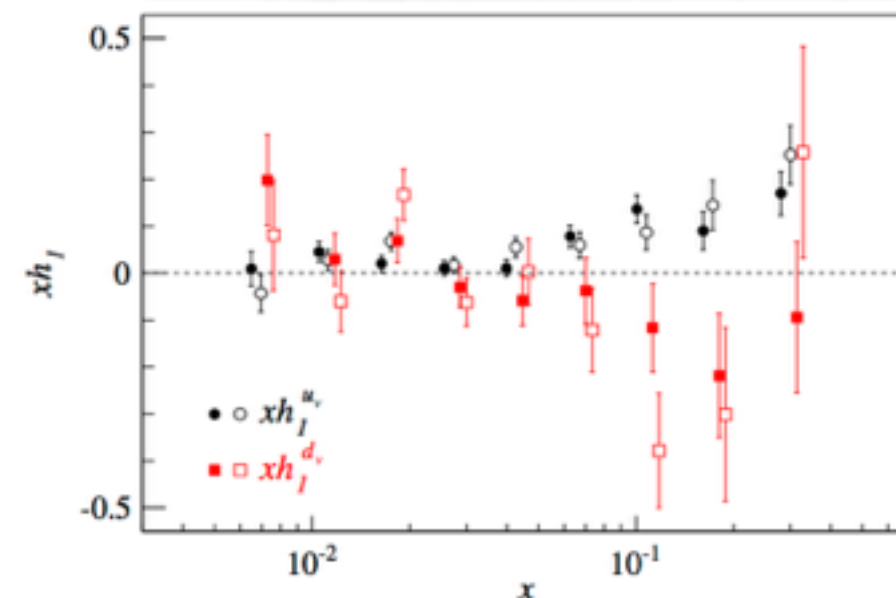
$$|\tilde{a}_P^{hh}(Q_B^2)| = 0.201 \quad \text{at } Q_B^2 \approx 110 \text{ GeV}^2/c^2$$

- use infos from  $e^+e^-$  data to get the transversity distributions from the SIDIS data (COMPASS data only, without any corrections and neglecting  $Q^2$  evolution of the analyzing power):

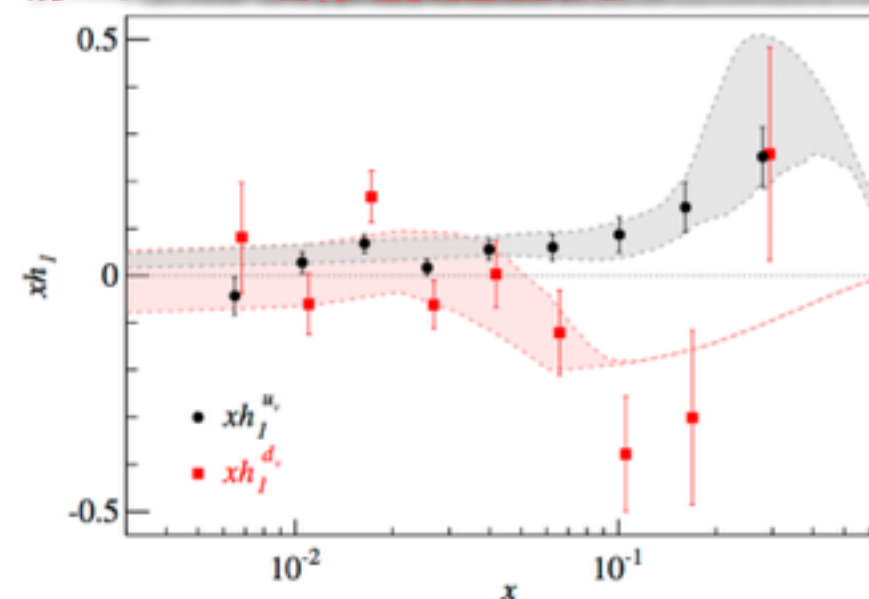
$$4xh_1^{u_v} - xh_1^{d_v} = \frac{1}{\tilde{a}_P^{hh}} (4xf_1^{u+\bar{u}} + xf_1^{d+\bar{d}} + \lambda xf_1^{s+\bar{s}}) A_P^{hh},$$

$$xh_1^{u_v} + xh_1^{d_v} = \frac{1}{3} \frac{1}{\tilde{a}_P^{hh}} (5xf_1^{u+\bar{u}} + 5xf_1^{d+\bar{d}} + 2\lambda xf_1^{s+\bar{s}}) A_d^{hh}$$

**The transversity values obtained from the di-hadron asymmetries and from the Collins asymmetries are very well compatible: supports the fact that the same distributions are measured in the two processes**

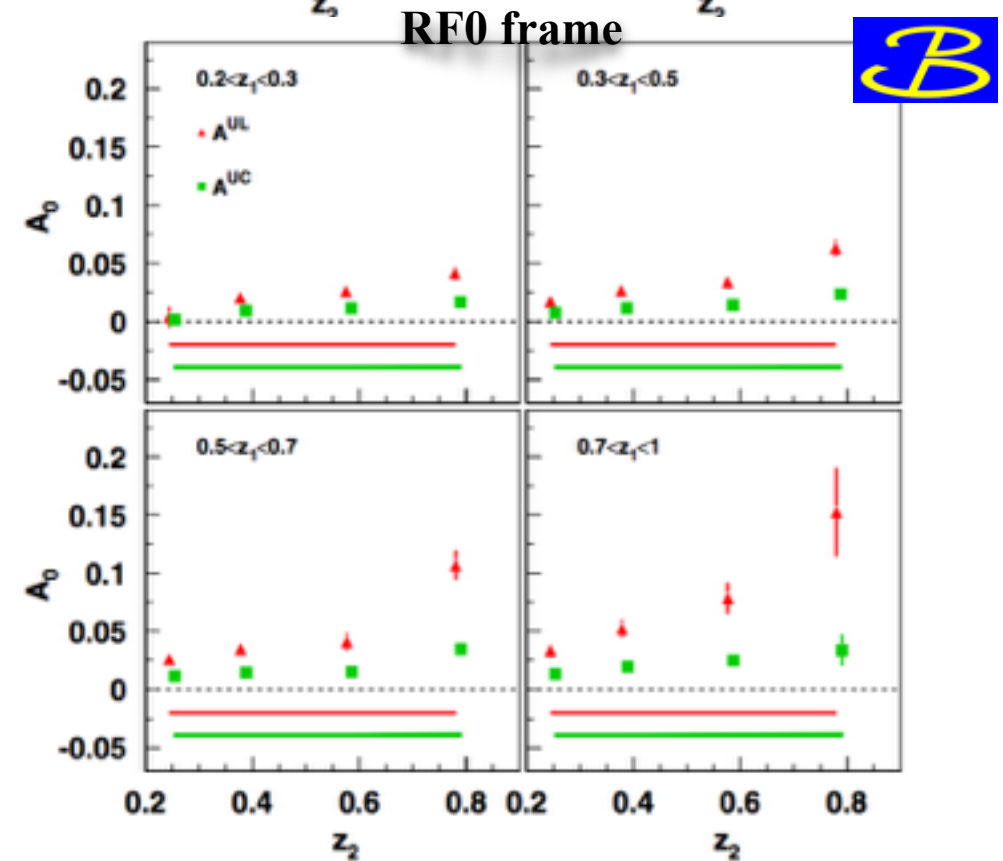
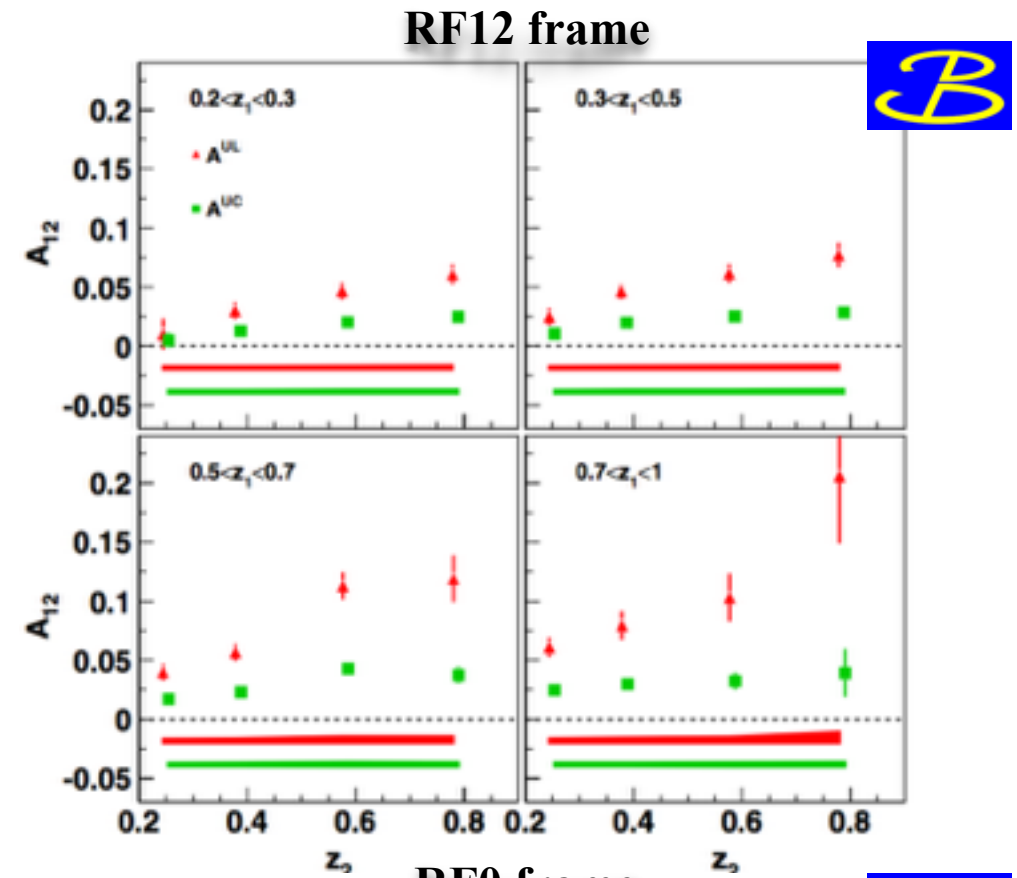
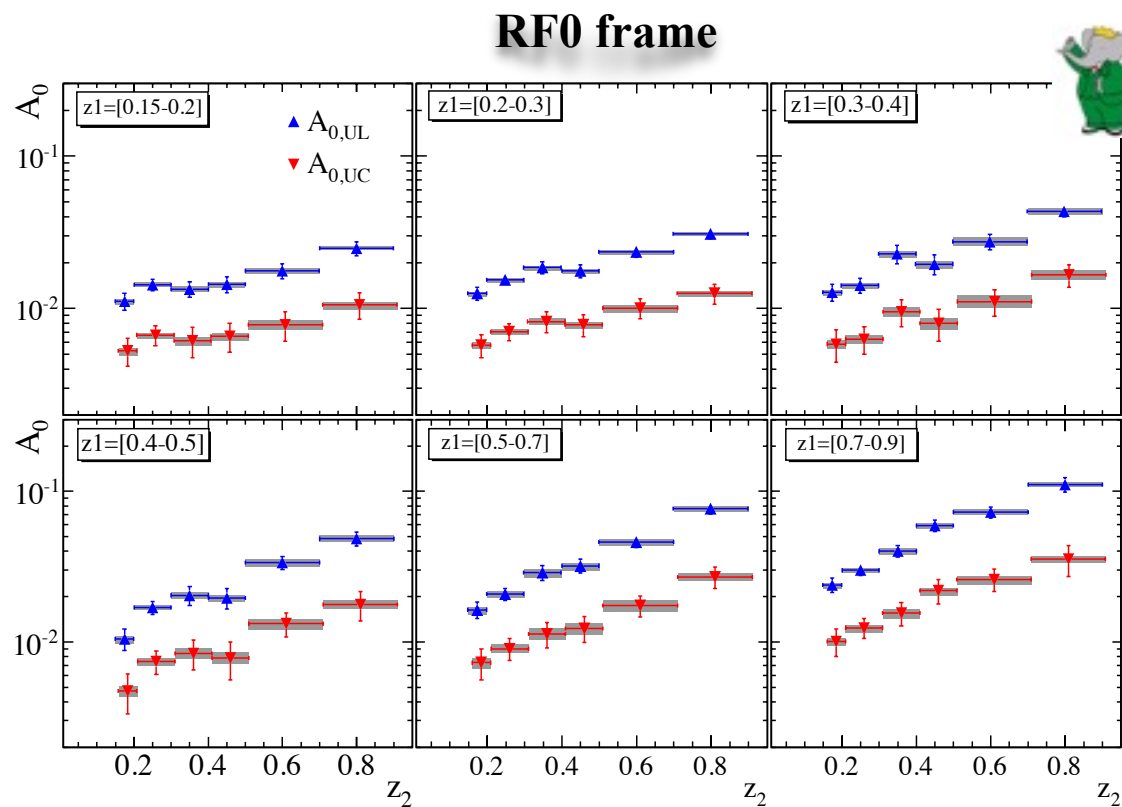
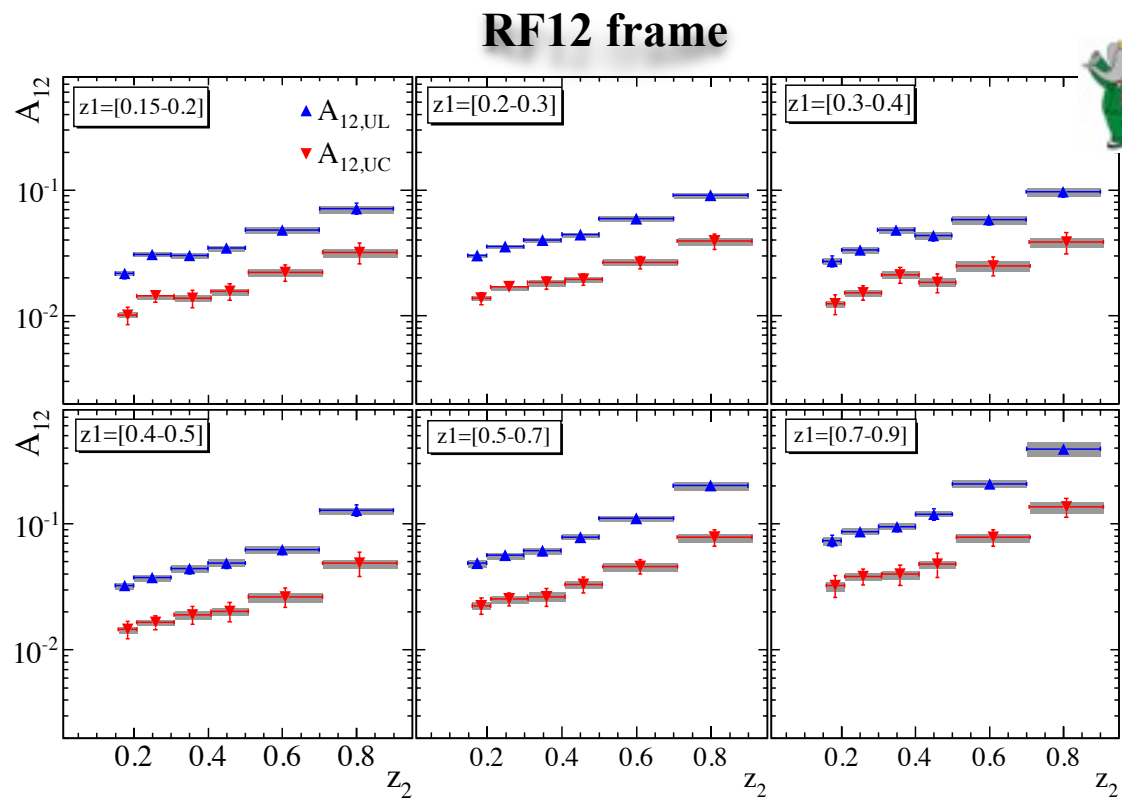


- □ Transversity from di-hadron data
- ■ Transversity from Collins asymmetries

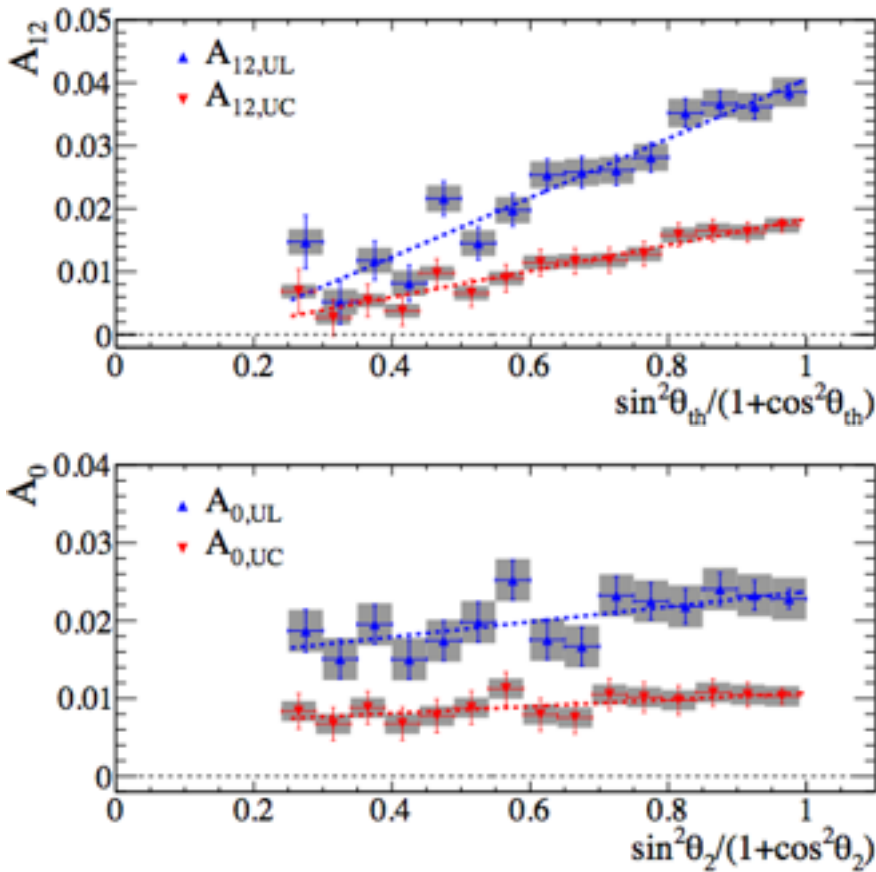


- ■ Transversity distribution extracted from di-hadron data
- ■ Bacchetta, Courtoy, Radici, JHEP03, 119 ( $Q^2=2.4 \text{ GeV}^2$ )

# Collins asymmetries vs $z$



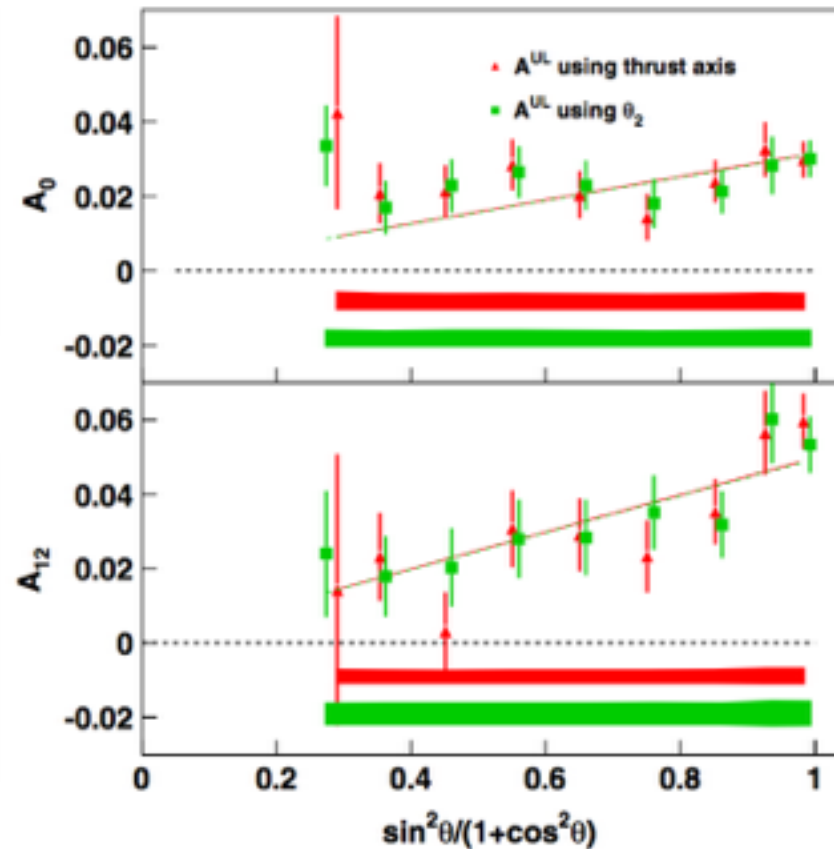
# Collins asymmetries vs $\sin^2\theta/(1+\cos^2\theta)$



RF12: thrust polar angle  $\theta_{th}$

$$\frac{\sin^2 \theta_{th}}{1 + \cos^2 \theta_{th}} \cos(\phi_1 + \phi_2) \frac{H_1^\perp(z_1) \bar{H}_1^\perp(z_2)}{D_1(z_1) \bar{D}_1(z_2)}$$

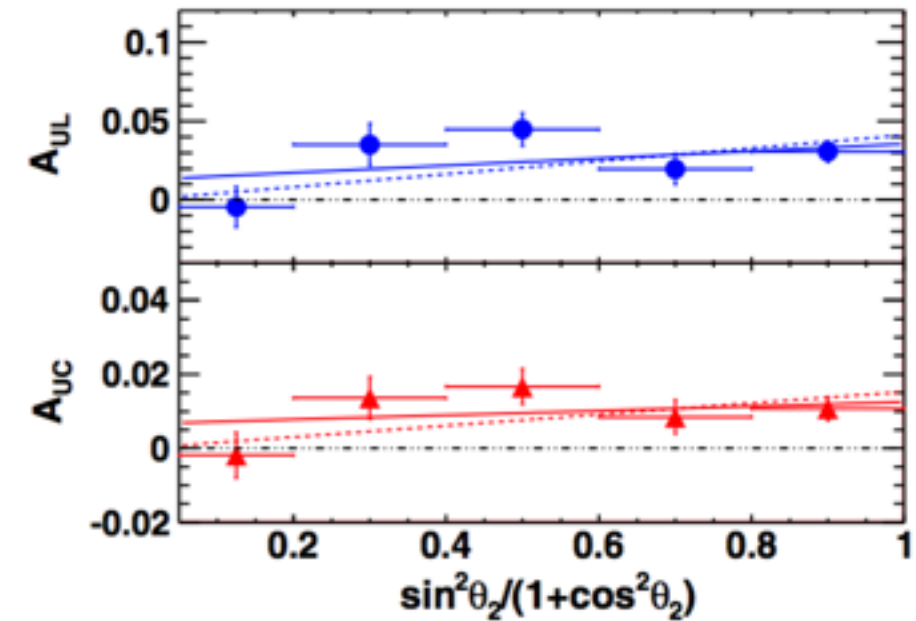
==> Intercept consistent with zero, as expected



RF0: thrust polar angle  $\theta_2$

$$\frac{\sin^2 \theta_2}{1 + \cos^2 \theta_2} \cos(2\phi_0) \mathcal{F} \left[ \frac{H_1^\perp(z_1) \bar{H}_1^\perp(z_2)}{D_1(z_1) \bar{D}_1(z_2)} \right]$$

==> The linear fit gives a non-zero constant parameter  $\rightarrow$  the second hadron momentum provides a worse estimation of the  $q\bar{q}$  direction?



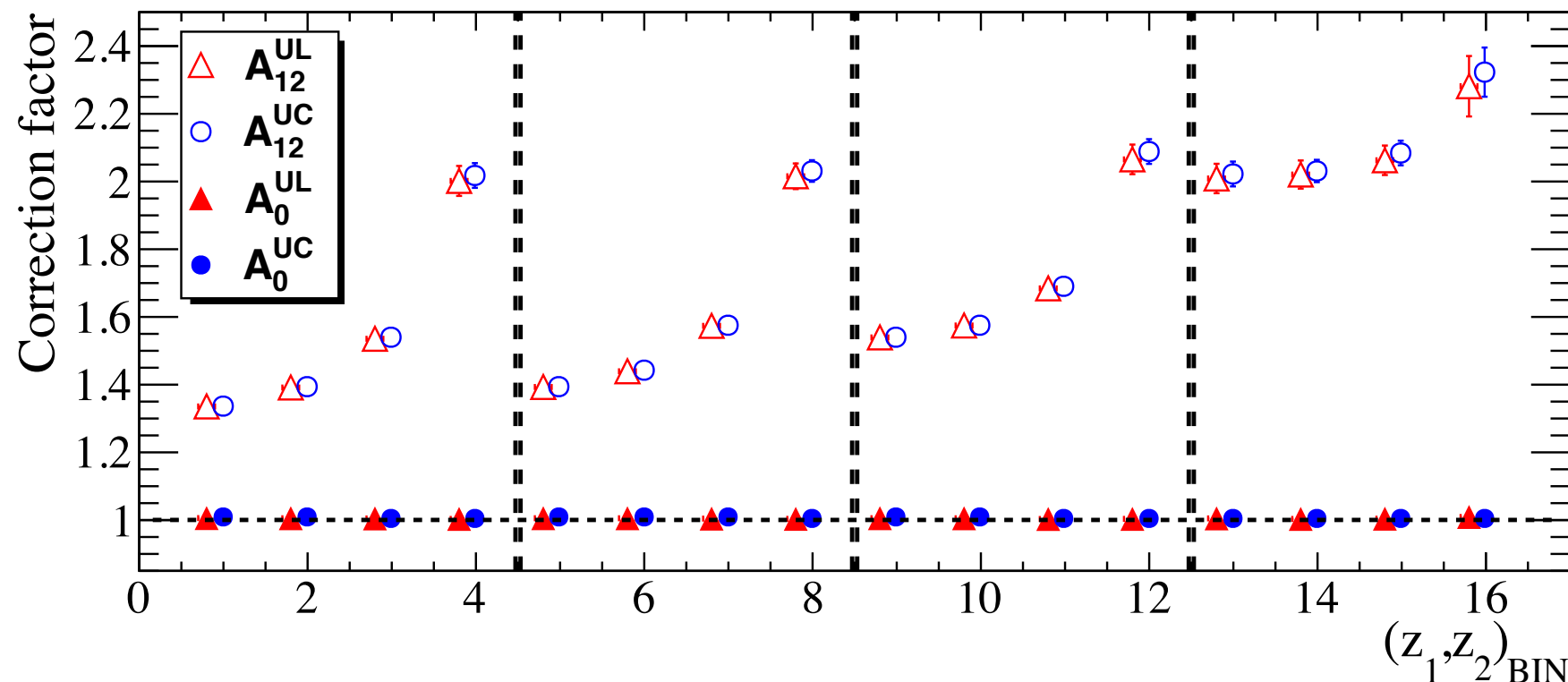
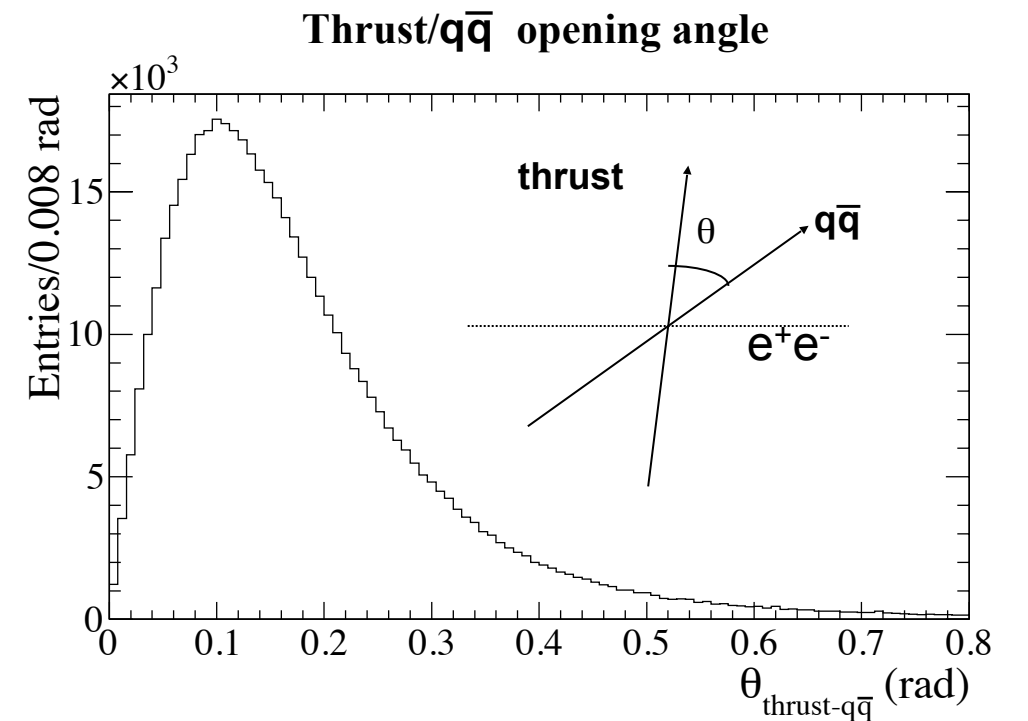


# Effect of the thrust axis reconstruction

The experimental method assumes the thrust axis as  $q\bar{q}$  direction, but this is only a rough approximation

- RF12: the azimuthal angles are calculated respect to the thrust axis  $\rightarrow$  large smearing;
- RF0: no thrust axis needed  $\rightarrow$  smearing due only to PID and tracking resolution.

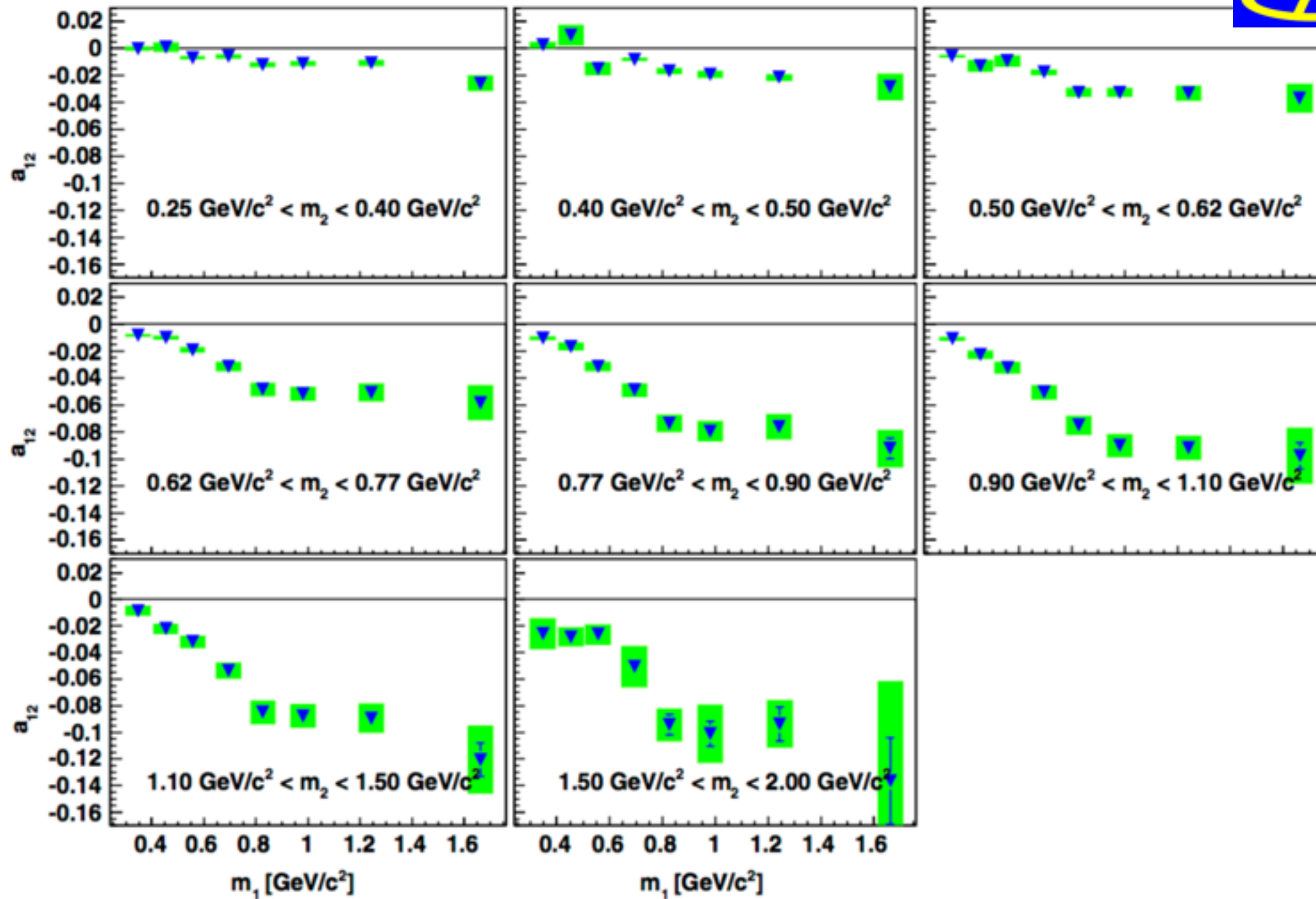
$\Rightarrow$  Using the MC sample, we introduce in the simulation several values of asymmetries, and we study the differences between the simulated and the reconstructed ones



- RF12: strong dilution observed
  - correction ranges between 1.3 to 2.3 for increasing  $z$
- RF0: no dilution observed
  - no correction needed

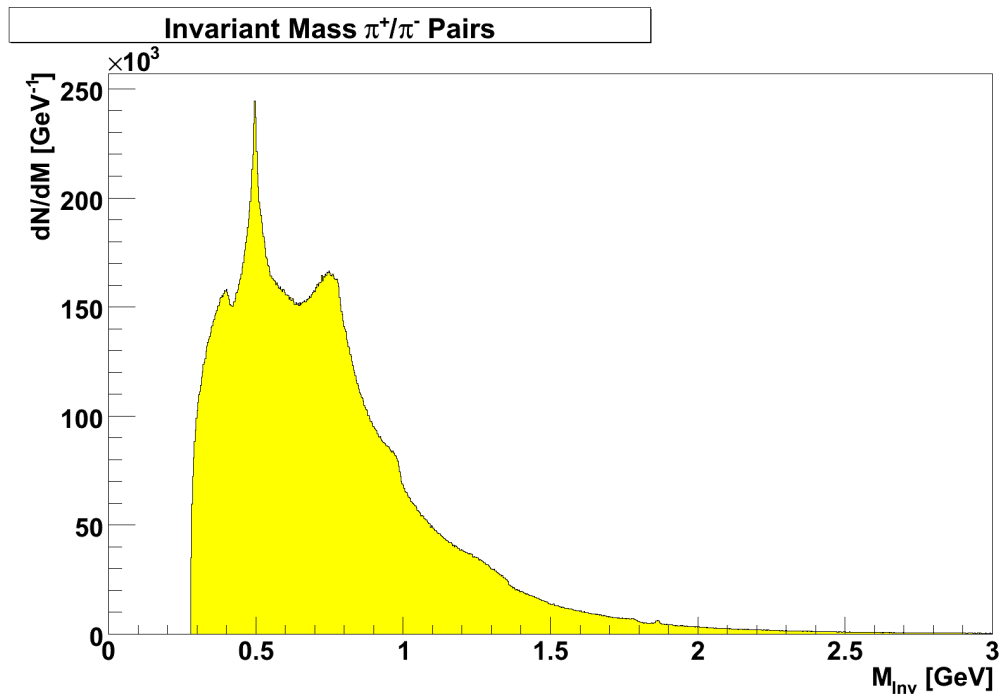
Same corrections applied for the three hadron pair combinations

# Di-hadron FFs vs. $m$





# Di-hadron FFs: asymmetry extraction



- Build normalized yields:

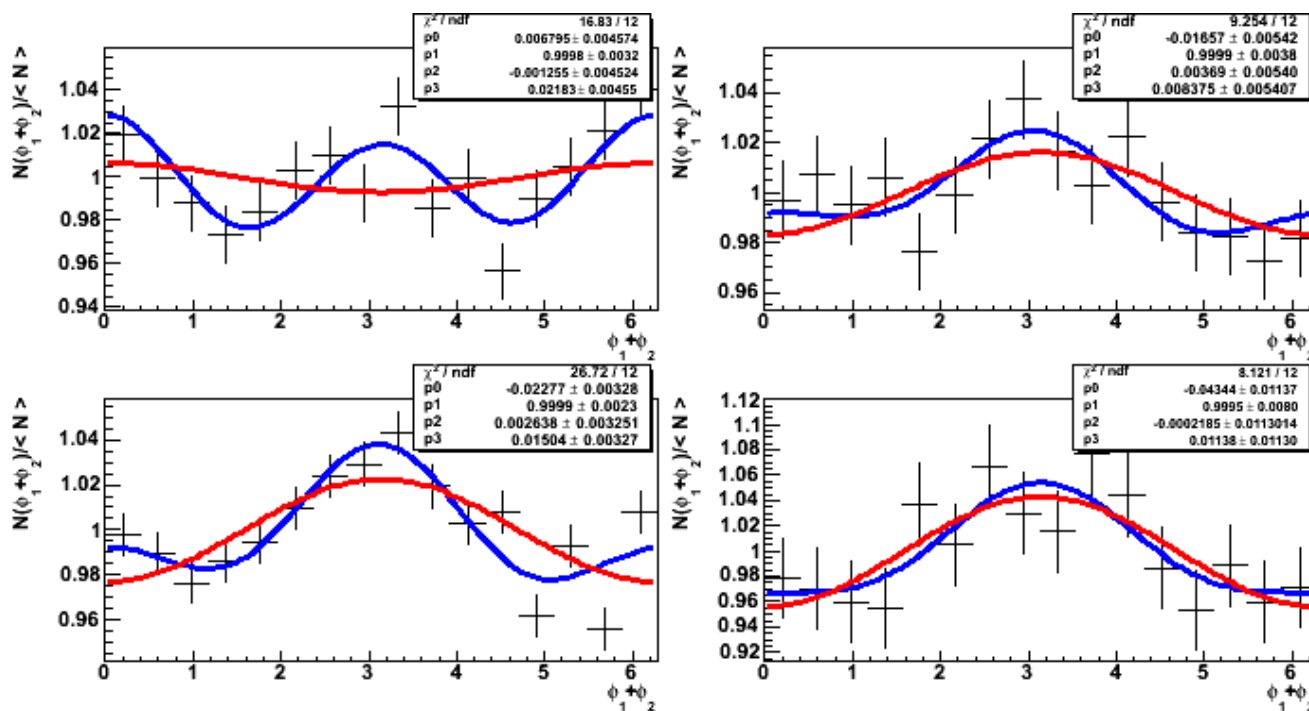
$$\frac{N(\phi_1 + \phi_2)}{\langle N \rangle},$$

- Fit with:

$$a_{12} \cos(\phi_1 + \phi_2) + b_{12}$$

OR

$$a_{12} \cos(\phi_1 + \phi_2) + b_{12} + c_{12} \cos 2(\phi_1 + \phi_2) + d_{12} \sin(\phi_1 + \phi_2)$$



Amplitude  $a_{12}$  directly measures ( IFF ) x ( -IFF ) (no double ratios)

# Global analysis

RHIC and SIDIS experiments measure:

Transversity  $\delta q(x)$  X

Collins Fragmentation function

or Interference Fragmentation function (IFF)



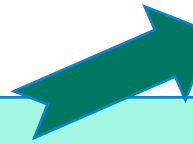
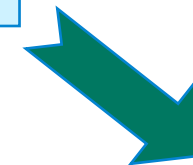
2 Unknown  
Functions measured  
together

- Universality understood
- Evolution ?

From  $e^+e^-$  experiments:

Collins X Collins - well studied for charged pion pairs; z dependence for KK and Kpi; other combinations can be studied

or IFF X IFF – charged pions



Transversity

