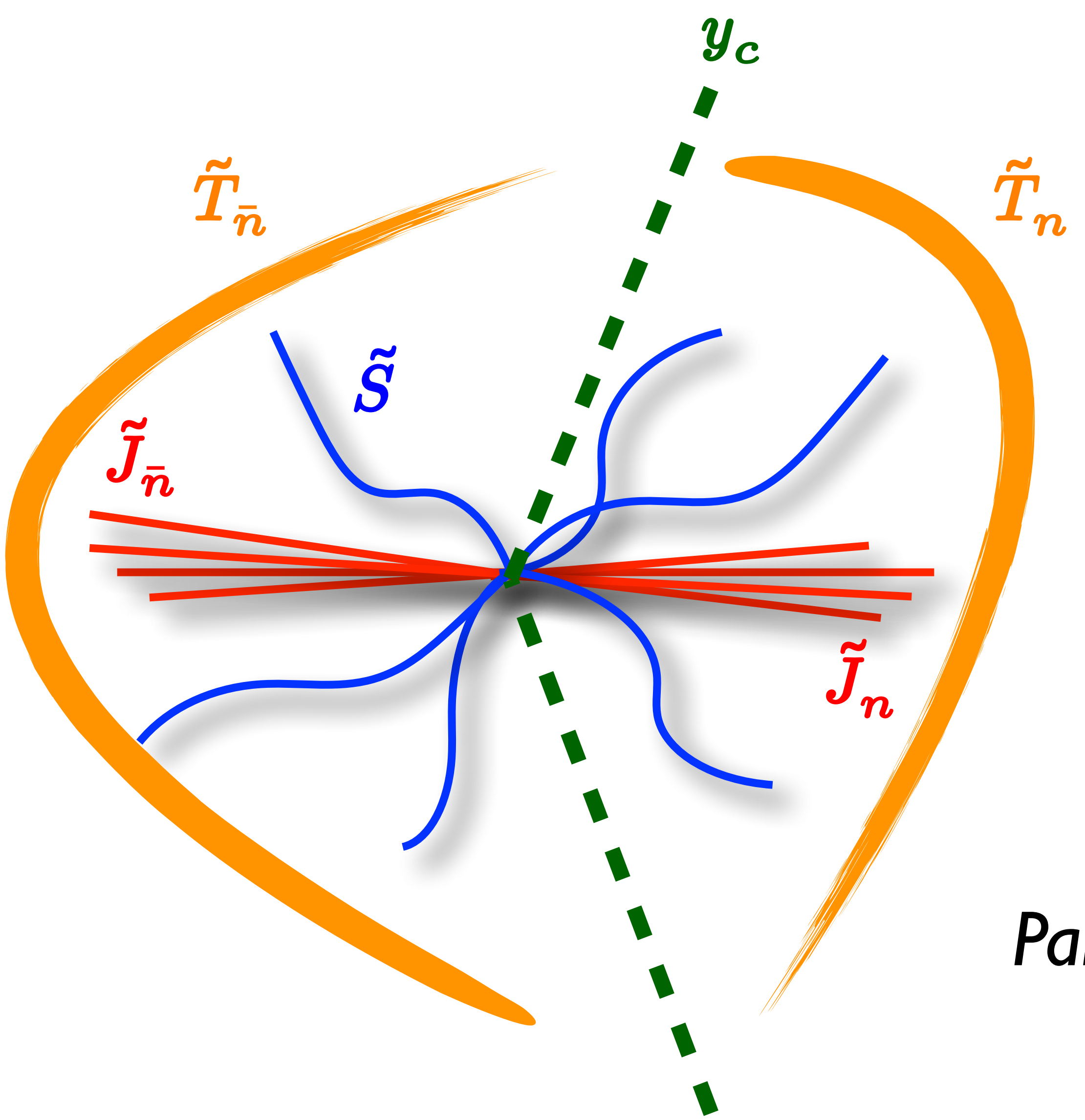
TMD Factorization & Evolution



Miguel G. Echevarría

Universitat de Barcelona

ICCUB

QCD Structure of the Nucleon (QCD-N'16)

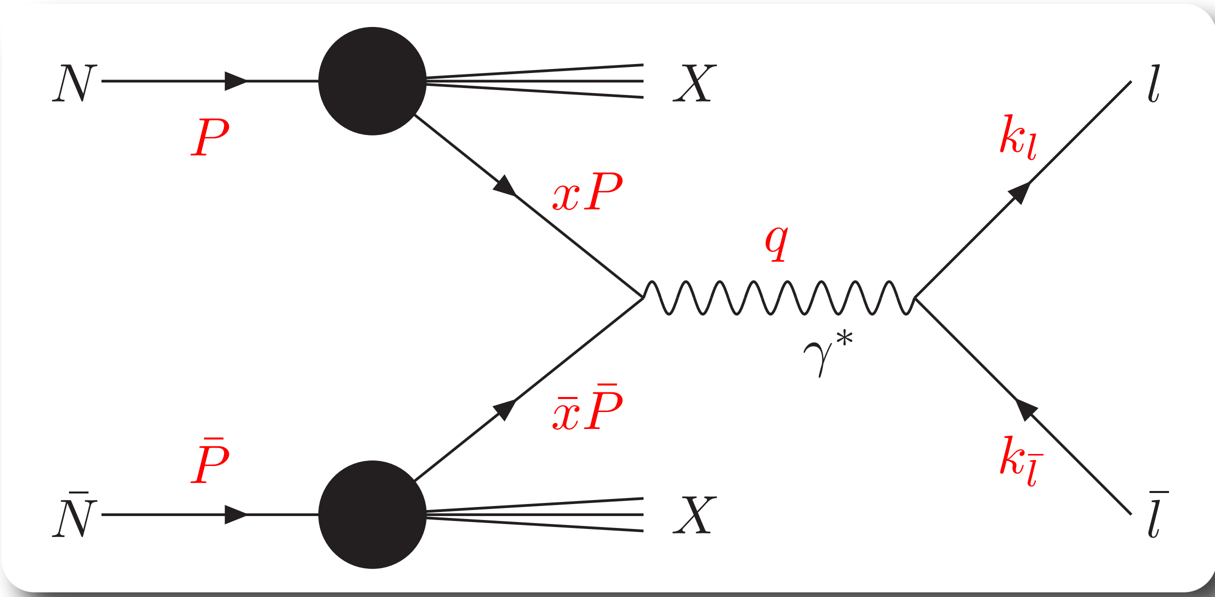
Palacio San Joseren (Getxo), Bilbao, Spain. July 11-15

Outline

1. *TMD factorization*
2. *TMD evolution*
3. *TMD refactorization*
4. *Example: gluon helicity TMDPDF*
5. *Recovering a complete TMD spectrum (matching TMD & Collinear frameworks)*
6. *Conclusions & Outlook*

TMD factorization

- *Take Drell-Yan as a benchmark process:*



$$q^2 = Q^2 \gg q_T^2$$

q_T large: perturbative origin
q_T small: non-perturbative origin

- *Same story applies to all processes with "at most two hadrons":*

$$H_1 + H_2 \to h + X$$
$$H_1 + H_2 \to [Q\bar{Q}] + X$$
$$e^- + H_1 \to e^- + H_2 + X$$
$$e^- + H_1 \to e^- + Q + \bar{Q} + X$$
$$e^+ + e^- \to H_1 + H_2 + X$$
$$e^+ + e^- \to [Q\bar{Q}] + H_1 + X$$
etc...

TMD factorization: EFT point of view

- We want to factorize a process which has different scales: $Q \gg q_T \geq \Lambda_{QCD}$

$$h_A(P, S_A) + h_B(\bar{P}, S_B) \to [l + \bar{l}](q_T) + X$$

$d\sigma$

QCD

=

$H(Q^2/\mu^2)$

$\tilde{F}_{q/A}^{[pol]}(x_A, b_T; Q, \mu)$

$SCET_{q_T}$

=

$H(Q^2/\mu^2)$

$\tilde{C}_{q/j}^{[pol]}(x_A, b_T^2\mu^2, Q^2/\mu^2)$

$f_{j/A}^{[pol]}(x_A; \mu)$

$SCET_{II}$

Same IR physics

Factorization Theorem = Multi-step Matching Procedure

TMD factorization: soft and collinear

- *Applying the SCET machinery, the cross-section is given in terms of collinear and soft:*

$$d\sigma = \sigma_0(\mu)\, H(Q^2,\mu)\, dy\, \frac{d^2 q_\perp}{(2\pi)^2} \int d^2 y_\perp\, e^{-i q_\perp \cdot y_\perp}\, J_n(x_A, y_\perp, \mu)\, J_{\bar{n}}(x_B, y_\perp, \mu)\, S(y_\perp, \mu)$$

$$J_n(0^+, y^-, \vec{y}_\perp) = \frac{1}{2}\sum_{\sigma_1} \langle N_1(P,\sigma_1)|\, \bar{\chi}_n(0^+, y^-, \vec{y}_\perp)\, \frac{\bar{\not{n}}}{2}\, \chi_n(0)\, |N_1(P,\sigma_1)\rangle|_{\text{zb subtracted}}$$

$$J_{\bar{n}}(y^+, 0^-, \vec{y}_\perp) = \frac{1}{2}\sum_{\sigma_2} \langle N_2(\bar{P},\sigma_2)|\, \bar{\chi}_{\bar{n}}(0)\, \frac{\not{n}}{2}\, \chi_{\bar{n}}(y^+, 0^-, \vec{y}_\perp)\, |N_2(\bar{P},\sigma_2)\rangle|_{\text{zb subtracted}}$$

$$S(0^+, 0^-, \vec{y}_\perp) = \langle 0|\, Tr\, \bar{T}[S_n^{T\dagger} S_{\bar{n}}^T](0^+, 0^-, \vec{y}_\perp) T[S_{\bar{n}}^{T\dagger} S_n^T](0)\, |0\rangle$$

But these matrix elements individually are ill-defined.
They contain mixed UV/Rapidity divergences...

Integrated Parton Distribution Function (1/2)

- The integrated PDF:

$$f_n(x) = \frac{1}{2}\int \frac{dy^-}{2\pi} e^{-i\frac{1}{2}y^- x P^+} \frac{1}{2}\sum_S \langle PS| \left[\bar{\xi}_n W_n\right](0^+, y^-, \vec{0}_\perp) \frac{\bar{\not{n}}}{2} \left[W_n^\dagger \xi_n\right](0) |PS\rangle$$

$$W_n(x) = \bar{P}\exp\left[ig\int_{-\infty}^{0} ds\, \bar{n}\cdot A_n(x+s\bar{n})\right]$$

- I will use the following regulator: [MGE, Idilbi, Scimemi '11]

$$\frac{i(\not{p}+\not{k})}{(p+k)^2 + i\Delta^-} \longrightarrow \frac{1}{k^- + i\delta^-}, \; \delta^- = \frac{\Delta^-}{p^+}$$

$$\frac{i(\bar{\not{p}}-\not{k})}{(\bar{p}-k)^2 + i\Delta^+} \longrightarrow \frac{1}{-k^+ + i\delta^+}, \; \delta^+ = \frac{\Delta^+}{\bar{p}^-}$$

Dimensional regularization for UV

- This regulator consists just in keeping finite the "epsilons" of the propagators.
- We send them to zero unless they regulate some divergence.

Of course the physics is (should be!) independent of the regulator!!

Integrated Parton Distribution Function (1/2)

- *The integrated PDF:*

$$f_n(x) = \frac{1}{2}\int \frac{dy^-}{2\pi} e^{-i\frac{1}{2}y^- x P^+} \frac{1}{2}\sum_S \langle PS| \left[\bar{\xi}_n W_n\right](0^+, y^-, \vec{0}_\perp) \frac{\bar{\not{n}}}{2} \left[W_n^\dagger \xi_n\right](0)\,|PS\rangle$$

$$W_n(x) = \bar{P}\exp\left[ig\int_{-\infty}^{0} ds\,\bar{n}\cdot A_n(x+s\bar{n})\right]$$

- *I will use the following regulator:* [MGE, Idilbi, Scimemi '11]

$$\frac{i(\not{p}+\not{k})}{(p+k)^2+i\Delta^-} \longrightarrow \frac{1}{k^-+i\delta^-}\,,\ \delta^- = \frac{\Delta^-}{p^+}$$

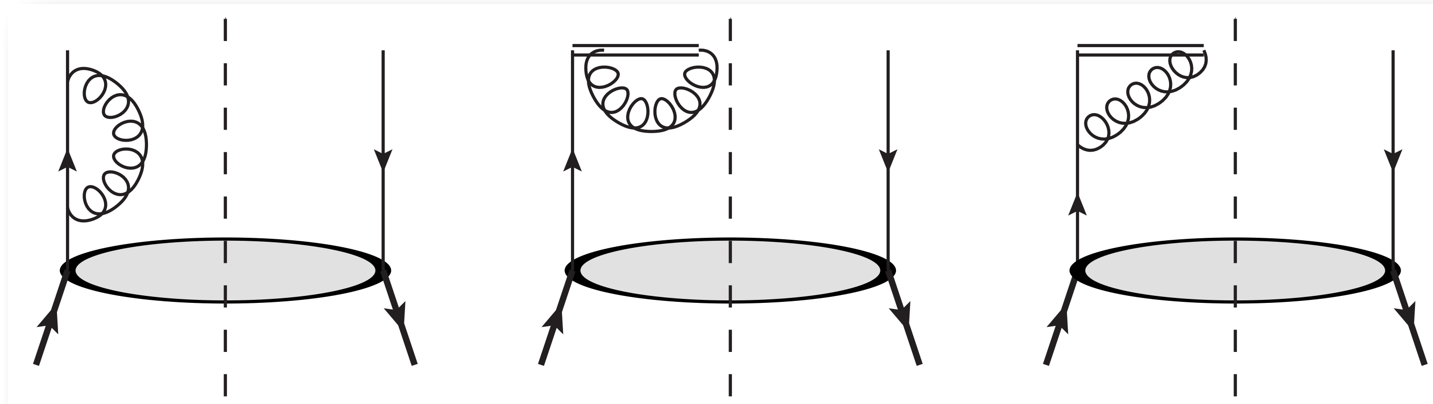
$$\frac{i(\bar{\not{p}}-\not{k})}{(\bar{p}-k)^2+i\Delta^+} \longrightarrow \frac{1}{-k^++i\delta^+}\,,\ \delta^+ = \frac{\Delta^+}{\bar{p}^-}$$

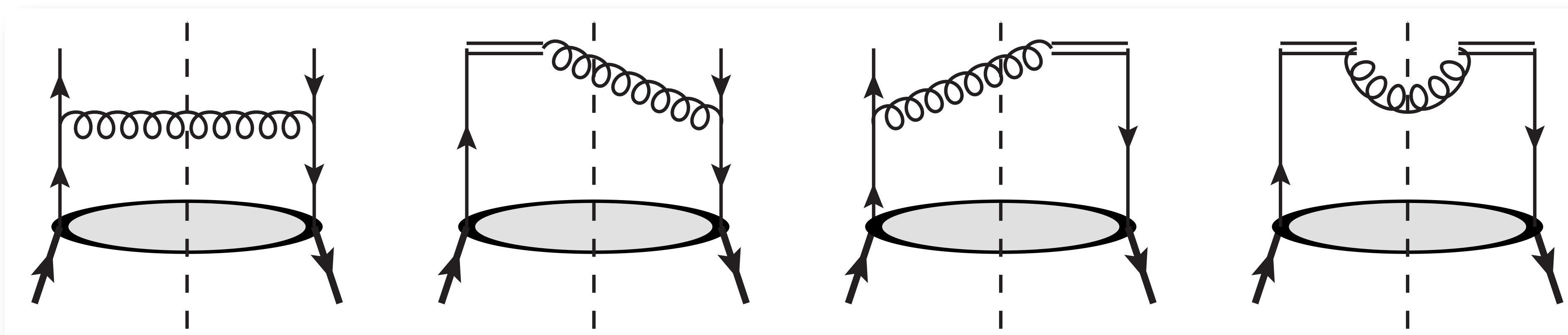
Dimensional regularization for UV

- *This regulator consists just in keeping finite the "epsilons" of the propagators.*
- *We send them to zero unless they regulate some divergence.*

Of course the physics is (should be!) independent of the regulator!!

Integrated Parton Distribution Function (2/2)





$$f_n = \delta(1-x) + \frac{\alpha_s C_F}{2\pi}\left\{P_{q/q}\left(\frac{1}{\varepsilon_{UV}} - \ln\frac{\Delta^-}{\mu^2}\right) - \frac{1}{4}\delta(1-x) - (1-x)[1+\ln(1-x)]\right\}$$

$$P_{q/q} = \frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)$$

- *The UV pole is cancelled by **renormalization***
- *The IR pole (logarithm) is washed out by **confinement***

Naive TMD

- *One could think of defining the TMDPDF by "extending" the PDF:*

[Collins, Soper '81, '82]

$$F_n^{naive}(0^+, y^-, \vec{y}_\perp) = \frac{1}{2}\sum_\sigma \langle P,\sigma|\,\left[\bar{\xi}_n W_n\right](0^+, y^-, \vec{y}_\perp)\,\frac{\bar{n}\!\!\!/}{2}\,\left[W_n^\dagger \xi_n\right](0)\,|P,\sigma\rangle$$

Need ***transverse*** *gauge links to maintain gauge invariance*

[Belitsky, Ji, Yuan 0208038]
[Idilbi, Scimemi 1009.2776]
[MGE, Idilbi, Scimemi 1104.0686]

- *If we calculate this matrix element we get:*

$$\tilde{F}_n^{naive} = \delta(1-x) + \frac{\alpha_s C_F}{2\pi}\Big\{\delta(1-x)\Big[\frac{2}{\varepsilon_{UV}}\ln\frac{\delta^+}{p^+} + \frac{3}{2\varepsilon_{UV}} + \frac{3}{2}L_T + 2L_T\ln\frac{\delta^+}{p^+}\Big] + (1-x) - L_T P_{q/q} - P_{q/q}\ln\frac{\Delta^-}{\mu^2} - \frac{1}{4}\delta(1-x) - (1-x)[1+\ln(1-x)]\Big\}$$

$$L_T = \ln\frac{\mu^2 b_T^2}{4e^{-2\gamma_E}}$$

It is ill-defined!! Mixed UV/Rapidity divergences...
Cannot be renormalized, nor OPEd onto collinear PDF

Naive TMD

- *One could think of defining the TMDPDF by "extending" the PDF:*

[Collins, Soper '81, '82]

$$F_n^{naive}(0^+, y^-, \vec{y}_\perp) = \frac{1}{2}\sum_\sigma \langle P,\sigma|\ \left[\bar{\xi}_n W_n\right](0^+, y^-, \vec{y}_\perp)\ \frac{\bar{\not{n}}}{2}\ \left[W_n^\dagger \xi_n\right](0)\,|P,\sigma\rangle$$

Need ***transverse*** *gauge links to maintain gauge invariance*

[Belitsky, Ji, Yuan 0208038]
[Idilbi, Scimemi 1009.2776]
[MGE, Idilbi, Scimemi 1104.0686]

- *If we calculate this matrix element we get:*

$$\begin{aligned}\tilde{F}_n^{naive} = \delta(1-x) + \frac{\alpha_s C_F}{2\pi}\Big\{\delta(1-x)\Big[\frac{2}{\varepsilon_{UV}}\ln\frac{\delta^+}{p^+} + \frac{3}{2\varepsilon_{UV}} \\ + \frac{3}{2}L_T + 2L_T\ln\frac{\delta^+}{p^+}\Big] + (1-x) - L_T P_{q/q} \\ - P_{q/q}\ln\frac{\Delta^-}{\mu^2} - \frac{1}{4}\delta(1-x) - (1-x)[1+\ln(1-x)]\Big\}\end{aligned}$$

$$L_T = \ln\frac{\mu^2 b_T^2}{4e^{-2\gamma_E}}$$

It is ill-defined!! Mixed UV/Rapidity divergences...
Cannot be renormalized, nor OPEd onto collinear PDF

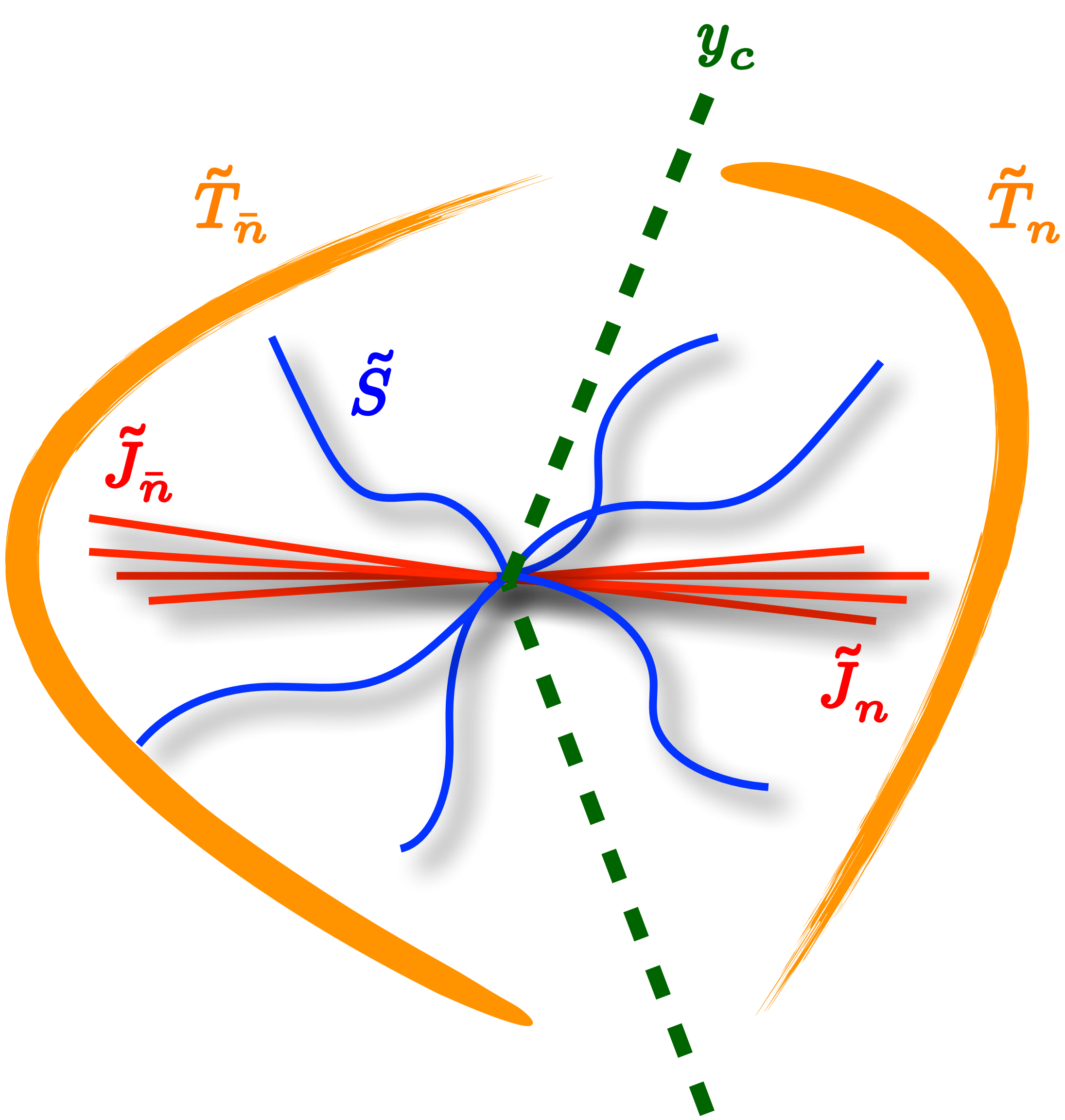
Definition of TMDs (1/2)

- *Proper definition is a bit tricky…*

$$k_n \sim Q(1,\lambda^2,\lambda)$$
$$k_{\bar n} \sim Q(\lambda^2,1,\lambda)$$
$$k_s \sim Q(\lambda,\lambda,\lambda)$$

$$y = \frac{1}{2}\ln\left|\frac{k^+}{k^-}\right|$$ *Different rapidities (mixed under boosts)*

$$k_n^2 \sim k_{\bar n}^2 \sim k_s^2 \sim Q^2\lambda^2$$ *Same invariant mass!*



Cancel spurious rapidity divergences

$$\zeta_A = (p^+)^2 e^{-2y_c}$$

$$\tilde{T}_n(x_A,\vec{k}_{n\perp},S_A;\zeta_A,\mu) = \tilde{J}_n\sqrt{\tilde{S}}$$

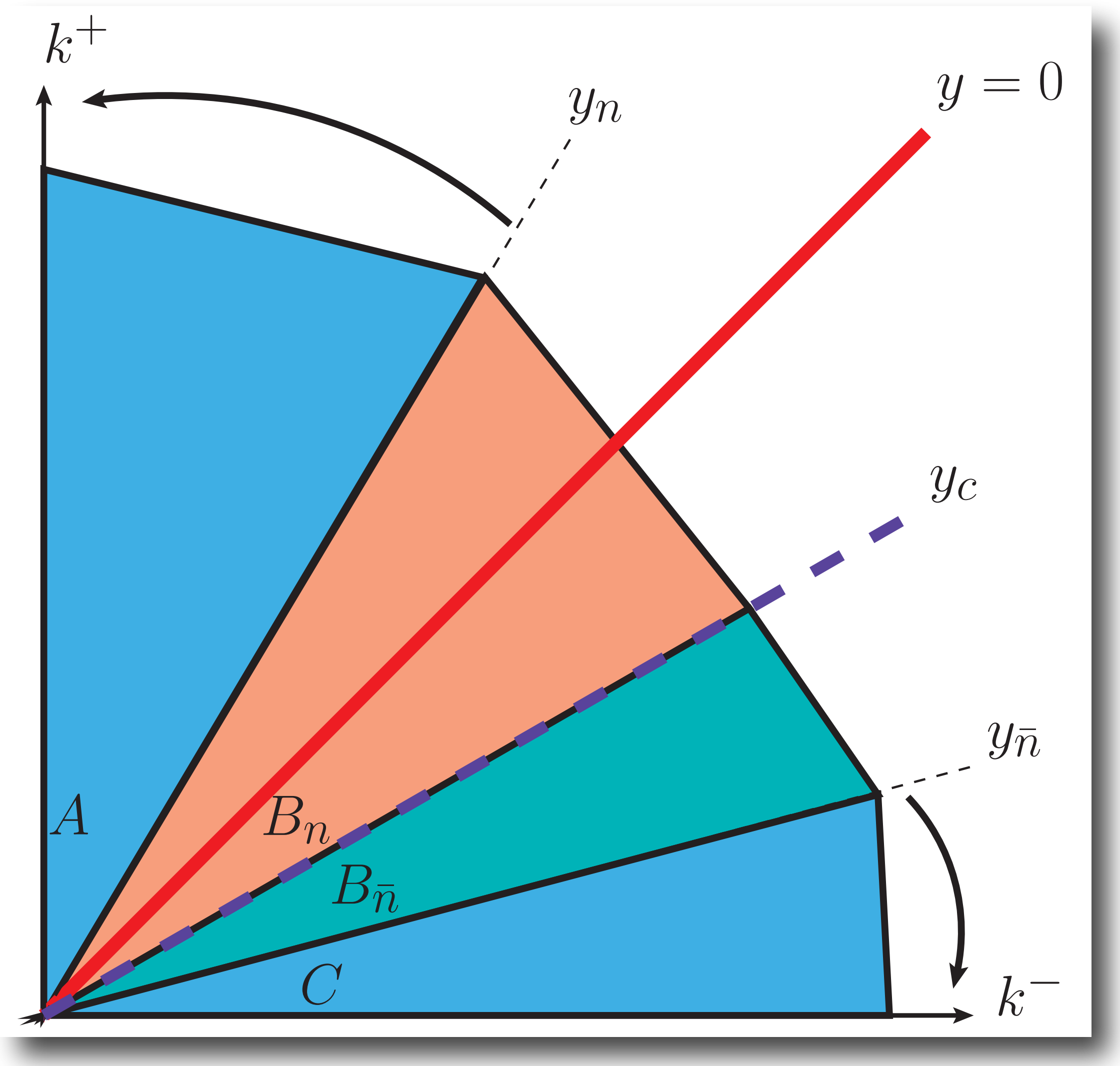
$$\tilde{T}_{\bar n}(x_B,\vec{k}_{\bar n\perp},S_B;\zeta_B,\mu) = \tilde{J}_{\bar n}\sqrt{\tilde{S}}$$

$$\zeta_B = (\bar{p}^-)^2 e^{+2y_c}$$

[MGE, Idilbi, Scimemi 1111.4996, 1211.1947, 1402.0869]
[MGE, Kasemets, Mulders, Pisano 1502.05354]
[Collins' book '11]

Definition of TMDs (2/2)

[MGE, Idilbi, Scimemi 1211.1947]
[MGE, Scimemi, Vladimirov, 1604.07869]



- *Problem: inclusion of the wrong region of rapidity space in the collinear*

$$\tilde{S}(b_T;\mu;\delta^+,\delta^-) = \tilde{S}_+(b_T;\mu;\nu\delta^+)\,\tilde{S}_-(b_T;\mu;\delta^-/\nu)$$

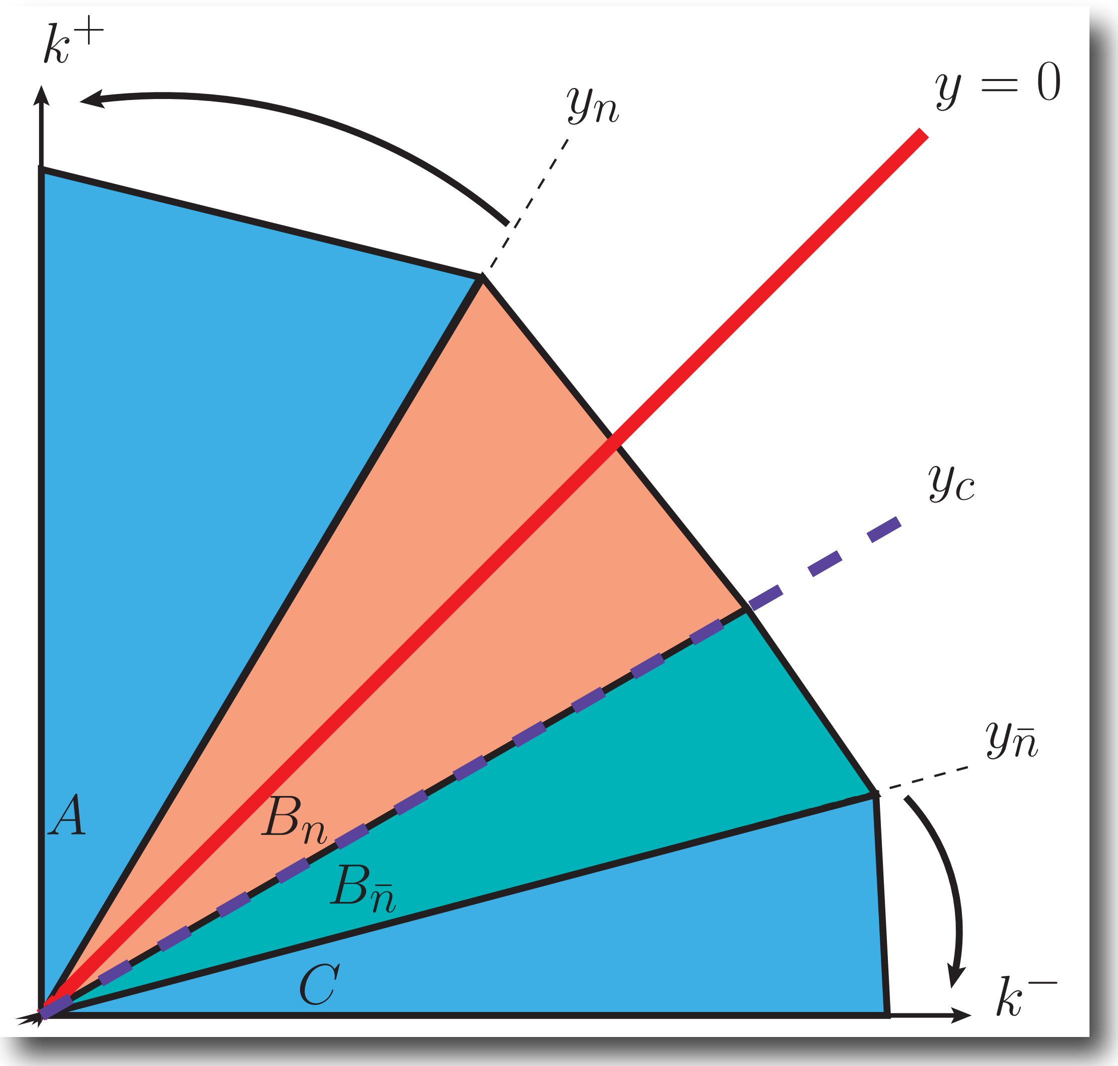
- *The used regulator is not important:*

$$\tilde{T}_n(x_A,b_T;\zeta_A,\mu) = \tilde{J}_n(x_A,b_T;\mu;\delta^+)\,\tilde{S}_+^{-1}(b_T;\mu;\nu\delta^+)$$ [EIS formalism]

$$\tilde{T}_n(x_A,b_T;\zeta_A,\mu) = \lim_{\substack{y_n\to+\infty \\ y_{\bar{n}}\to-\infty}} \tilde{J}_n(x_A,b_T;\mu;y_{\bar{n}})\sqrt{\frac{\tilde{S}(y_n,y_c)}{\tilde{S}(y_c,y_{\bar{n}})\,\tilde{S}(y_n,y_{\bar{n}})}}$$ [Collins formalism]

Definition of TMDs (2/2)

[MGE, Idilbi, Scimemi 1211.1947]
[MGE, Scimemi, Vladimirov, 1604.07869]



- *Problem: inclusion of the wrong region of rapidity space in the collinear*

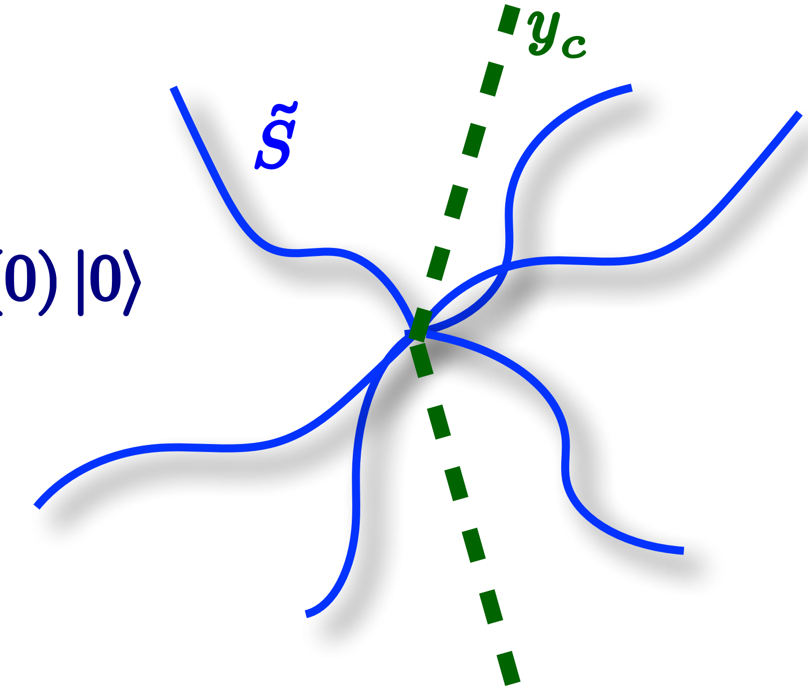
$$\tilde{S}(b_T;\mu;\delta^+,\delta^-) = \tilde{S}_+(b_T;\mu;\nu\delta^+)\,\tilde{S}_-(b_T;\mu;\delta^-/\nu)$$

- *The used regulator is not important:*

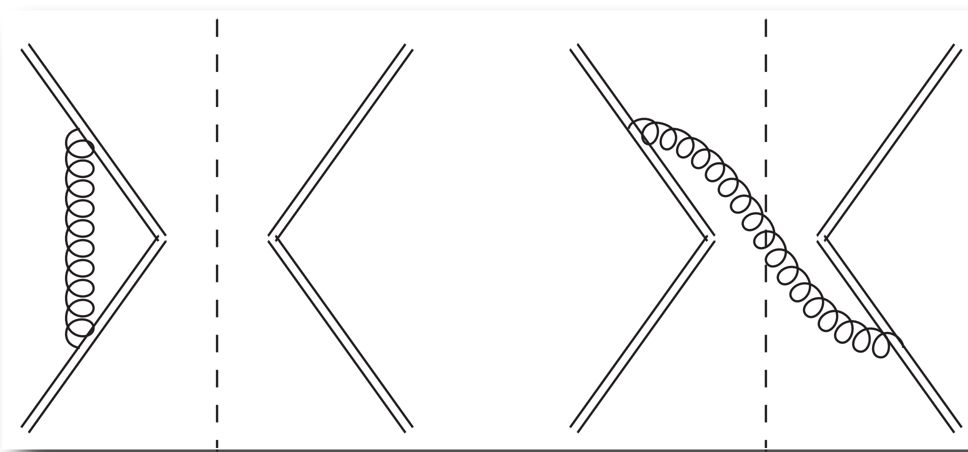
$$\tilde{T}_n(x_A,b_T;\zeta_A,\mu) = \tilde{J}_n(x_A,b_T;\mu;\delta^+)\,\tilde{S}_+^{-1}(b_T;\mu;\nu\delta^+)$$ [EIS formalism]

$$\tilde{T}_n(x_A,b_T;\zeta_A,\mu) = \lim_{\substack{y_n\to+\infty \\ y_{\bar{n}}\to-\infty}} \tilde{J}_n(x_A,b_T;\mu;y_{\bar{n}})\sqrt{\frac{\tilde{S}(y_n,y_c)}{\tilde{S}(y_c,y_{\bar{n}})\,\tilde{S}(y_n,y_{\bar{n}})}}$$ [Collins formalism]

Splitting of the soft function at NLO



$$\tilde{S}(b_T;\mu) = \frac{1}{N_c^2-1}\sum_{X_s}\langle 0|\left(S_n^\dagger S_{\bar{n}}\right)^{ab}(b_\perp)|X_s\rangle\langle X_s|\left(S_{\bar{n}}^\dagger S_n\right)^{ba}(0)|0\rangle$$



$$\tilde{S}\left(b_T;\mu;\delta^+,\delta^-\right) = 1+\frac{\alpha_s C_A}{2\pi}\left[-\frac{2}{\varepsilon_{UV}^2}+\frac{2}{\varepsilon_{UV}}\ln\frac{\delta^+\delta^-}{\mu^2}+L_T^2+2L_T\ln\frac{\delta^+\delta^-}{\mu^2}+\frac{\pi^2}{6}\right]$$

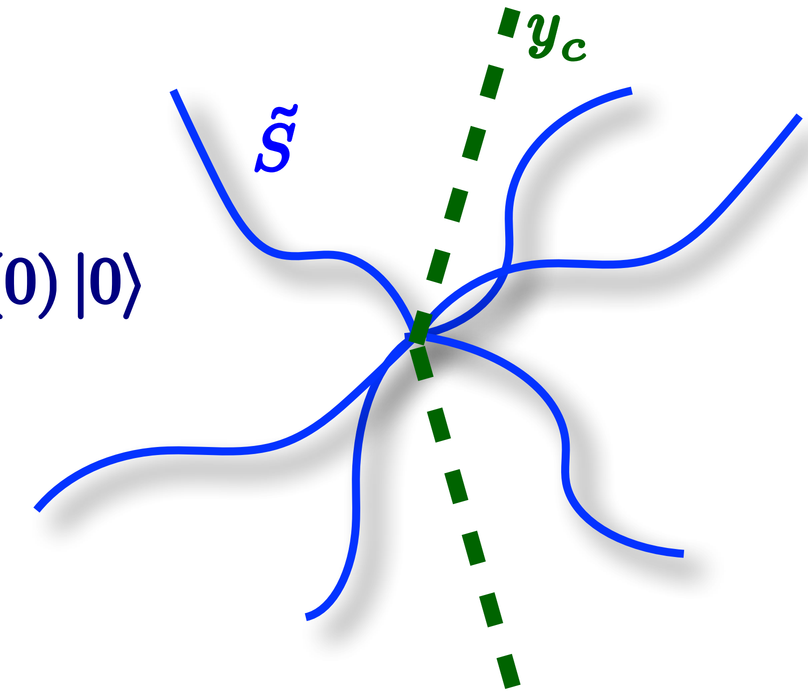
$$L_T = \ln\frac{\mu^2 b_T^2}{4e^{-2\gamma_E}}$$

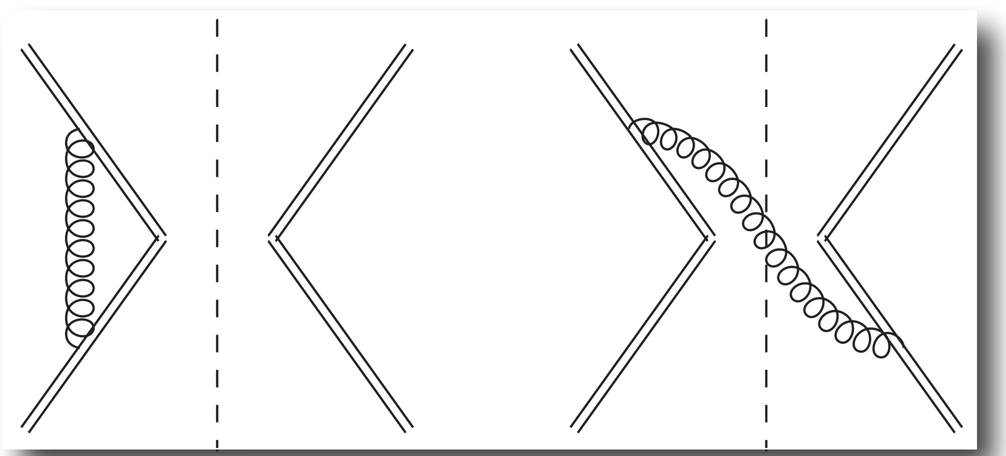
$$\tilde{S}\left(b_T;\mu;\delta^+,\delta^-\right) = \tilde{S}_-\left(b_T;\nu,\mu;\delta^-\right)\tilde{S}_+\left(b_T;1/\nu,\mu;\delta^+\right)$$

$$\tilde{S}_- = 1+\frac{\alpha_s C_A}{2\pi}\left[-\frac{1}{\varepsilon_{UV}^2}+\frac{1}{\varepsilon_{UV}}\ln\frac{\nu(\delta^-)^2}{\mu^2}+\frac{1}{2}L_T^2+\frac{1}{\varepsilon_{UV}}L_T\ln\frac{\nu(\delta^-)^2}{\mu^2}+\frac{\pi^2}{12}\right]$$

Splitting of the soft function at NLO

$$\tilde{S}(b_T;\mu) = \frac{1}{N_c^2-1}\sum_{X_s}\langle 0|\left(S_n^\dagger S_{\bar{n}}\right)^{ab}(b_\perp)|X_s\rangle\langle X_s|\left(S_{\bar{n}}^\dagger S_n\right)^{ba}(0)|0\rangle$$





$$\tilde{S}\left(b_T;\mu;\delta^+,\delta^-\right) = 1 + \frac{\alpha_s C_A}{2\pi}\left[-\frac{2}{\varepsilon_{UV}^2} + \frac{2}{\varepsilon_{UV}}\ln\frac{\delta^+\delta^-}{\mu^2} + L_T^2 + 2L_T\ln\frac{\delta^+\delta^-}{\mu^2} + \frac{\pi^2}{6}\right]$$

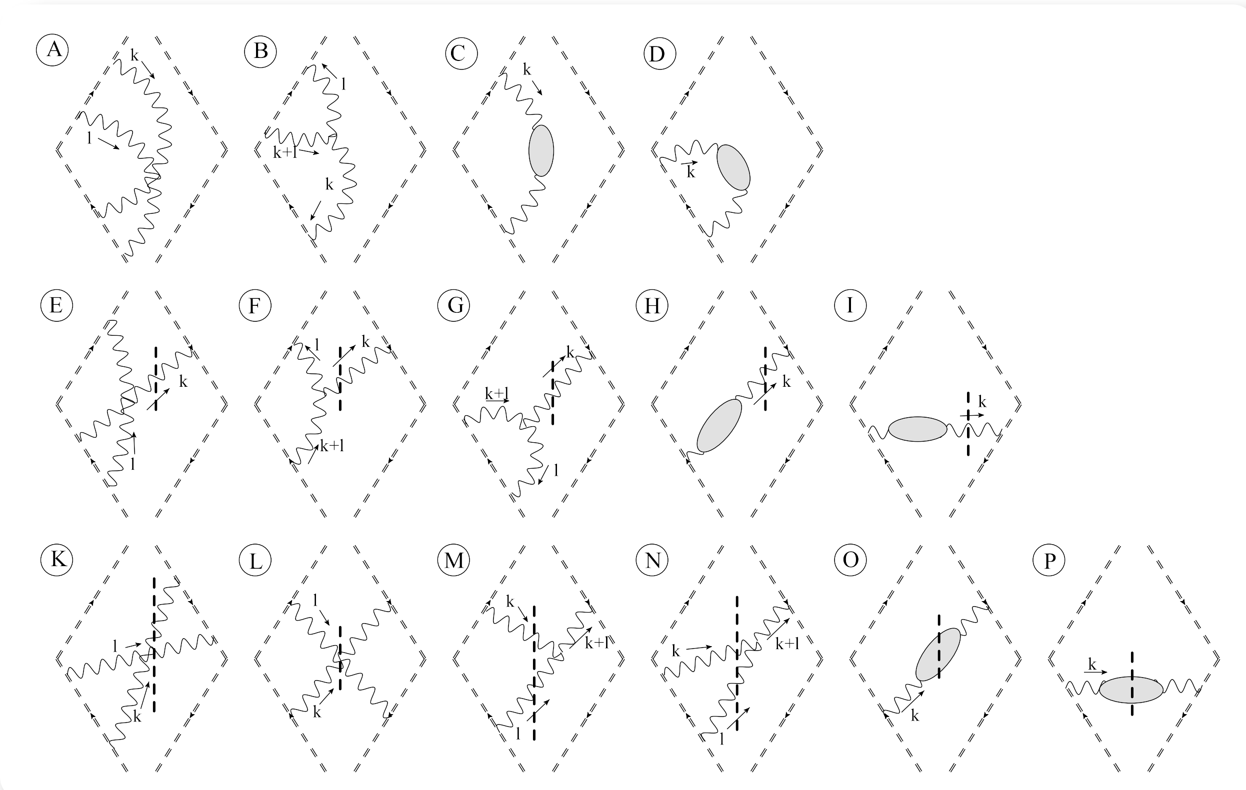
$$L_T = \ln\frac{\mu^2 b_T^2}{4e^{-2\gamma_E}}$$

$$\tilde{S}\left(b_T;\mu;\delta^+,\delta^-\right) = \tilde{S}_-\left(b_T;\nu,\mu;\delta^-\right)\,\tilde{S}_+\left(b_T;1/\nu,\mu;\delta^+\right)$$

$$\tilde{S}_- = 1 + \frac{\alpha_s C_A}{2\pi}\left[-\frac{1}{\varepsilon_{UV}^2} + \frac{1}{\varepsilon_{UV}}\ln\frac{\nu(\delta^-)^2}{\mu^2} + \frac{1}{2}L_T^2 + \frac{1}{\varepsilon_{UV}}L_T\ln\frac{\nu(\delta^-)^2}{\mu^2} + \frac{\pi^2}{12}\right]$$

Splitting of the soft function at NNLO

- *We have shown the splitting at NNLO:* [MGE, Scimemi, Vladimirov 1511.05590]



$$\tilde{S}(\mathbf{L}_\mu, \mathbf{L}_{\sqrt{\delta^+\delta^-}}) = \tilde{S}^{\frac{1}{2}}(\mathbf{L}_\mu, \mathbf{L}_{\delta^+/\nu})\,\tilde{S}^{\frac{1}{2}}(\mathbf{L}_\mu, \mathbf{L}_{\nu\delta^-})$$

$$\tilde{S}(b_T) = e^{a_s C_F (S^{[1]} + a_s S^{[2]} + \dots)}$$

$$\text{Diagram} = \mu^{4\epsilon}\left(A_0\delta^{-2\epsilon} + A_1\delta^{-\epsilon}B^{\epsilon} + A_2 B^{2\epsilon}\right) + \mathcal{O}(\delta)$$

$$S^{[2]} = \frac{d^{(2,2)}}{C_F}\left(\frac{3}{\epsilon^3} + \frac{2\mathbf{l}_\delta}{\epsilon^2} + \frac{\pi^2}{6\epsilon} + \frac{4}{3}\mathbf{L}_\mu^3 - 2\mathbf{L}_\mu^2\mathbf{l}_\delta + \frac{2\pi^2}{3}\mathbf{L}_\mu + \frac{14}{3}\zeta_3\right) - \frac{d^{(2,1)}}{C_F}\left(\frac{1}{2\epsilon^2} + \frac{\mathbf{l}_\delta}{\epsilon} - \mathbf{L}_\mu^2 + 2\mathbf{L}_\mu\mathbf{l}_\delta - \frac{\pi^2}{4}\right)$$
$$- \frac{d^{(2,0)}}{C_F}\left(\frac{1}{\epsilon} + 2\mathbf{l}_\delta\right) + C_A\left(\frac{\pi^2}{3} + 4\ln 2\right)\left(\frac{1}{\epsilon^2} + \frac{2\mathbf{L}_\mu}{\epsilon} + 2\mathbf{L}_\mu^2 + \frac{\pi^2}{6}\right) + C_A\left(8\ln 2 - 9\zeta_3\right)\left(\frac{1}{\epsilon} + 2\mathbf{L}_\mu\right)$$
$$+ \frac{656}{81}T_R N_f + C_A\left(-\frac{2428}{81} + 16\ln 2 - \frac{7\pi^4}{18} - 28\ln 2\,\zeta_3 + \frac{4}{3}\pi^2\ln^2 2 - \frac{4}{3}\ln^4 2 - 32\,\mathrm{Li}_4\left(\frac{1}{2}\right)\right) + \mathcal{O}(\epsilon)$$

TMD factorization: final formula

- The cross-section is finally given in terms of two TMDs:

$$d\sigma = \sigma_0(\mu)\, H(Q^2,\mu)\, dy\, \frac{d^2 q_\perp}{(2\pi)^2} \int d^2 y_\perp\, e^{-i q_\perp \cdot y_\perp}\, \tilde{F}_n(x_A, y_\perp, \mu)\, \tilde{F}_{\bar{n}}(x_B, y_\perp, \mu) + O(q_T/Q)$$

- There is no soft factor in the factorization theorem
- These are the hadronic quantities that we extract from experiment

Evolution of TMDs (1/3)

- *TMDs depend on **two scales**: **renormalization** and **rapidity** scales*
- *We know the evolution of **all** (un)polarized TMDs (**universal** evolution kernel):*

$$\tilde{T}^{[pol]}_{j\leftrightarrow A}(x, b_\perp, S_A; \zeta_{A,f}, \mu_f) = \tilde{T}^{[pol]}_{j\leftrightarrow A}(x, b_\perp, S_A; \zeta_{A,i}, \mu_i)\, \tilde{R}^j\left(b_T; \zeta_{A,i}, \mu_i, \zeta_{A,f}, \mu_f\right)$$

- *The dependence on the renormalization scale is:*

$$\frac{d}{d\ln\mu}\ln\tilde{T}^{[pol]}_{j\leftrightarrow A}(x, b_\perp, S_A; \zeta_A, \mu) = \gamma_j\left(\alpha_s(\mu), \ln\frac{\zeta_A}{\mu^2}\right)$$

Known at 3-loops

$$\gamma_j\left(\alpha_s(\mu), \ln\frac{\zeta_A}{\mu^2}\right) = -\Gamma^j_{cusp}(\alpha_s(\mu))\ln\frac{\zeta_A}{\mu^2} - \gamma^j_{nc}(\alpha_s(\mu))$$

Evolution of TMDs (1/3)

- *TMDs depend on* ***two scales****:* ***renormalization*** *and* ***rapidity*** *scales*
- *We know the evolution of* ***all*** *(un)polarized TMDs (****universal*** *evolution kernel):*

$$\tilde{T}^{[pol]}_{j\leftrightarrow A}(x, b_\perp, S_A; \zeta_{A,f}, \mu_f) = \tilde{T}^{[pol]}_{j\leftrightarrow A}(x, b_\perp, S_A; \zeta_{A,i}, \mu_i)\, \tilde{R}^j\left(b_T; \zeta_{A,i}, \mu_i, \zeta_{A,f}, \mu_f\right)$$

- *The dependence on the* ***renormalization scale*** *is:*

$$\frac{d}{d\ln\mu} \ln \tilde{T}^{[pol]}_{j\leftrightarrow A}(x, b_\perp, S_A; \zeta_A, \mu) = \gamma_j\left(\alpha_s(\mu), \ln\frac{\zeta_A}{\mu^2}\right)$$

Known at 3-loops

$$\gamma_j\left(\alpha_s(\mu), \ln\frac{\zeta_A}{\mu^2}\right) = -\Gamma^j_{cusp}(\alpha_s(\mu))\ln\frac{\zeta_A}{\mu^2} - \gamma^j_{nc}(\alpha_s(\mu))$$

Evolution of TMDs (2/3)

- *The dependence on the **rapidity scale** is:*

$$\frac{d}{d\ln\zeta_A}\ln\tilde{T}^{[pol]}_{j\leftrightarrow A}(x,b_\perp,S_A;\zeta_A,\mu)=-D_j(b_T;\mu)$$

Known at NLO. Recently at NNLO.

Indirect: [Becher, Neubert 1007.4005]
Direct: [MGE, Scimemi, Vladimirov 1511.05590]
[Li, Zhu 1604.01404]

$$\frac{dD_j}{d\ln\mu}=\Gamma^j_{cusp}(\alpha_s(\mu))$$

Cusp does __not__ completely determine D_j

- *Combining the evolution in both scales:*

$$\tilde{R}^j(b_T;\zeta_{A,i},\mu_i,\zeta_{A,f},\mu_f)=\exp\left[\int_{\mu_i}^{\mu_f}\frac{d\hat{\mu}}{\hat{\mu}}\gamma_j\left(\alpha_s(\hat{\mu}),\ln\frac{\zeta_{A,f}}{\hat{\mu}^2}\right)\right]\left(\frac{\zeta_{A,f}}{\zeta_{A,i}}\right)^{-D_j(b_T;\mu_i)}$$

The evolution itself contains some
__non-perturbative input__
(in the D_j term at large b_T)

Evolution of TMDs (2/3)

- *The dependence on the **rapidity scale** is:*

$$\frac{d}{d\ln\zeta_A}\ln\tilde{T}^{[pol]}_{j\leftrightarrow A}(x, b_\perp, S_A; \zeta_A, \mu) = -D_j(b_T; \mu)$$

Known at NLO. Recently at NNLO.

Indirect: [Becher, Neubert 1007.4005]
Direct: [MGE, Scimemi, Vladimirov 1511.05590]

[Li, Zhu 1604.01404]

$$\frac{dD_j}{d\ln\mu} = \Gamma^j_{cusp}(\alpha_s(\mu))$$

Cusp does __not__ completely determine D_j

- *Combining the evolution in both scales:*

$$\tilde{R}^j(b_T; \zeta_{A,i}, \mu_i, \zeta_{A,f}, \mu_f) = \exp\left[\int_{\mu_i}^{\mu_f}\frac{d\hat{\mu}}{\hat{\mu}}\gamma_j\left(\alpha_s(\hat{\mu}), \ln\frac{\zeta_{A,f}}{\hat{\mu}^2}\right)\right]\left(\frac{\zeta_{A,f}}{\zeta_{A,i}}\right)^{-D_j(b_T;\mu_i)}$$

*The evolution itself contains some __**non-perturbative input**__ (in the D_j term at large b_T)*

Evolution of TMDs (3/3)

- *Currently known perturbative ingredients allow NNLL' evolution:*

$$D = C_F \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{4\pi}\right)^n \sum_{k=0}^{n} \mathbf{L}_\mu^k d^{(n,k)}$$

Order	Γ^j_{cusp}	γ^j_{nc}	D_j
LL	α_s	α_s^0	α_s^0
NLL	α_s^2	α_s	α_s
NNLL	α_s^3	α_s^2	α_s^2
NNNLL	α_s^4	α_s^3	α_s^3

??

[Li, Zhu 1604.01404]

$$d^{(3,0)} = \frac{-1}{2} C_A^2 \left(-\frac{176}{3}\zeta_3\zeta_2 + \frac{6392\zeta_2}{81} + \frac{12328\zeta_3}{27} + \frac{154\zeta_4}{3} - 192\zeta_5 - \frac{297029}{729}\right)$$
$$- C_A T_r N_f \left(-\frac{824\zeta_2}{81} - \frac{904\zeta_3}{27} + \frac{20\zeta_4}{3} + \frac{62626}{729}\right) - 2T_r^2 N_f^2 \left(-\frac{32\zeta_3}{9} - \frac{1856}{729}\right)$$
$$- C_F T_r N_f \left(\frac{-304\zeta_3}{9} - 16\zeta_4 + \frac{1711}{27}\right)$$

Refactorization of TMDs (1/2)

- TMDs contain perturbative information when transverse momentum is large:

$$\tilde{T}_{i\leftrightarrow A}(x,b_T;\zeta,\mu) = \sum_{j=q,\bar{q},g} \tilde{C}^T_{i\leftrightarrow j}(x,b_T;\zeta,\mu) \otimes t_{j\leftrightarrow A}(x;\mu) + O(b_T\Lambda_{QCD})$$

- For each TMD we have a different OPE. For example:

$$\tilde{f}_1^{q/A}(x,b_T;\zeta,\mu) = \sum_{j=q,\bar{q},g} \int_x^1 \frac{d\bar{x}}{\bar{x}} \tilde{C}^f_{q/j}(\bar{x},b_T;\zeta,\mu) f_{j/A}(x/\bar{x};\mu)$$

$$\tilde{h}_1^{\perp g/A\,(2)}(x,b_T;\zeta,\mu) = \sum_{j=q,\bar{q},g} \int_x^1 \frac{d\bar{x}}{\bar{x}} \tilde{C}^h_{g/j}(\bar{x},b_T;\zeta,\mu) f_{j/A}(x/\bar{x};\mu)$$

$$\tilde{g}_{1L}^{g/A}(x,b_T;\zeta,\mu) = \sum_{j=q,\bar{q},g} \int_x^1 \frac{d\bar{x}}{\bar{x}} \tilde{C}^g_{g/j}(\bar{x},b_T;\zeta,\mu) g_{j/A}(x/\bar{x};\mu)$$

$$\tilde{f}_{1T}^{\perp g/A\,(1)}(x,b_T;\zeta,\mu) = \sum_{j=q,\bar{q},g} \int_x^1 \frac{d\bar{x}_1}{\bar{x}_1}\frac{d\bar{x}_2}{\bar{x}_2} \tilde{C}^{sivers}_{g/j}(\bar{x}_1,\bar{x}_2,b_T;\zeta,\mu) T_{F\,j/A}(x_1/\bar{x}_1,x_2/\bar{x}_2;\mu)$$

Refactorization of TMDs (1/2)

- *TMDs contain perturbative information when transverse momentum is large:*

$$\tilde{T}_{i\leftrightarrow A}(x, b_T; \zeta, \mu) = \sum_{j=q,\bar{q},g} \tilde{C}^T_{i\leftrightarrow j}(x, b_T; \zeta, \mu) \otimes t_{j\leftrightarrow A}(x; \mu) + O(b_T \Lambda_{QCD})$$

- *For each TMD we have a different OPE. For example:*

$$\tilde{f}_1^{q/A}(x, b_T; \zeta, \mu) = \sum_{j=q,\bar{q},g} \int_x^1 \frac{d\bar{x}}{\bar{x}} \tilde{C}^f_{q/j}(\bar{x}, b_T; \zeta, \mu) f_{j/A}(x/\bar{x}; \mu)$$

$$\tilde{h}_1^{\perp g/A\,(2)}(x, b_T; \zeta, \mu) = \sum_{j=q,\bar{q},g} \int_x^1 \frac{d\bar{x}}{\bar{x}} \tilde{C}^h_{g/j}(\bar{x}, b_T; \zeta, \mu) f_{j/A}(x/\bar{x}; \mu)$$

$$\tilde{g}_{1L}^{g/A}(x, b_T; \zeta, \mu) = \sum_{j=q,\bar{q},g} \int_x^1 \frac{d\bar{x}}{\bar{x}} \tilde{C}^g_{g/j}(\bar{x}, b_T; \zeta, \mu) g_{j/A}(x/\bar{x}; \mu)$$

$$\tilde{f}_{1T}^{\perp g/A\,(1)}(x, b_T; \zeta, \mu) = \sum_{j=q,\bar{q},g} \int_x^1 \frac{d\bar{x}_1}{\bar{x}_1} \frac{d\bar{x}_2}{\bar{x}_2} \tilde{C}^{sivers}_{g/j}(\bar{x}_1, \bar{x}_2, b_T; \zeta, \mu) T_{F\,j/A}(x_1/\bar{x}_1, x_2/\bar{x}_2; \mu)$$

Refactorization of TMDs (2/2)

- *All matchings at NNLO for unpolarized quark/gluon TMD distribution and fragmentation functions in:* [MGE, Scimemi, Vladimirov, 1604.07869]

- *In particular, we got for the first time all the coefficients for TMDFFs at NNLO:*

$$\tilde{D}_{i\to A}(z,b_T;\zeta,\mu)=\sum_{j=q,\bar{q},g}\int_z^1\frac{dt}{t^{3-2\varepsilon}}\tilde{C}_{i\to j}(z/t,b_T;\zeta,\mu)\,d_{j\to A}(t;\mu)$$

$$\tilde{C}_{i\to j}(z,b_T;\zeta,\mu)\quad\forall i,j$$

- *And checked consistency of previous calculations regarding TMDPDFs*

- *Waiting for e+e- data…*

TMDs: Non-perturbative ingredients

- *This is how a resummed TMD looks like:*

$$\tilde{T}_{i\leftrightarrow A}(x, b_T; \zeta, \mu) = \sum_{j=q,\bar{q},g} \tilde{C}^T_{i\leftrightarrow j}(x, \hat{b}_T; \mu_b^2, \mu_b) \otimes t_{j\leftrightarrow A}(x; \mu_b)$$
$$\times \exp\left[\int_{\mu_b}^{\mu} \frac{d\hat{\mu}}{\hat{\mu}} \gamma_j\left(\alpha_s(\hat{\mu}), \ln\frac{\zeta}{\hat{\mu}^2}\right)\right] \left(\frac{\zeta}{\mu_b^2}\right)^{-D_j(\hat{b}_T;\mu_b)}$$
$$\times \tilde{T}^{NP}_{i\leftrightarrow A}(x, b_T; \zeta)$$

- *General philosophy: only parametrize what* ***cannot*** *be calculated*
- *The non-perturbative part of D_j is* ***universal***
- *The non-perturbative part of D_j seems not well-constrained by current data*

[MGE, D'Alesio, Melis, Scimemi 1407.3311]

- *Higher-order calculations allow better determination of non-perturbative ingredients*
- *At low b_T the TMDs are neither supposed to be correct (q_T>Q region)*

Gluon helicity TMDPDF (1/2)

[MGE, Kasemets, Mulders, Pisano 1502.05354]

- *The resummed/evolved expression is:*

$$\tilde{g}_{1L}^{g}(x_A, b_T; Q^2, Q) = \exp\left\{\int_{Q_i}^{Q} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_G\left(\alpha_s(\bar{\mu}), \ln\frac{Q^2}{\bar{\mu}^2}\right)\right\} \left(\frac{Q^2}{Q_i^2}\right)^{-D_g(\hat{b}_T; Q_i)}$$
$$\times \sum_{j=q,\bar{q},g} \int_{x_A}^{1} \frac{d\bar{x}}{\bar{x}}\, \tilde{C}_{g/j}^{g}(\bar{x}, \hat{b}_T; Q_i^2, Q_i)\, g_{j/A}(x_A/\bar{x}; Q_i)\, \tilde{g}_{1L}^{g,NP}(x_A, b_T; Q)$$

- *The resummation scale is:* $Q_i = 2e^{-\gamma_E}/\hat{b}_T$

- *The non-perturbative model and the prescription to avoid the Landau pole:*

$$\hat{b}_T(b_T) = b_c\left(1 - e^{-(b_T/b_c)^2}\right)^{1/2}, \quad b_c = 3\, GeV^{-1}$$

$$\tilde{g}_{1L}^{g,NP}(x_A, b_T; Q) = \exp\left[-b_T^2(\lambda_g + \lambda_Q \ln(Q^2/Q_0^2))\right], \quad Q_0 = 1\, GeV$$

Gluon helicity TMDPDF (1/2)

[MGE, Kasemets, Mulders, Pisano 1502.05354]

- *The resummed/evolved expression is:*

$$\tilde{g}^g_{1L}(x_A, b_T; Q^2, Q) = \exp\left\{\int_{Q_i}^{Q} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_G\left(\alpha_s(\bar{\mu}), \ln\frac{Q^2}{\bar{\mu}^2}\right)\right\} \left(\frac{Q^2}{Q_i^2}\right)^{-D_g(\hat{b}_T; Q_i)}$$
$$\times \sum_{j=q,\bar{q},g} \int_{x_A}^{1} \frac{d\bar{x}}{\bar{x}}\, \tilde{C}^g_{g/j}(\bar{x}, \hat{b}_T; Q_i^2, Q_i)\, g_{j/A}(x_A/\bar{x}; Q_i)\, \tilde{g}^{g,NP}_{1L}(x_A, b_T; Q)$$

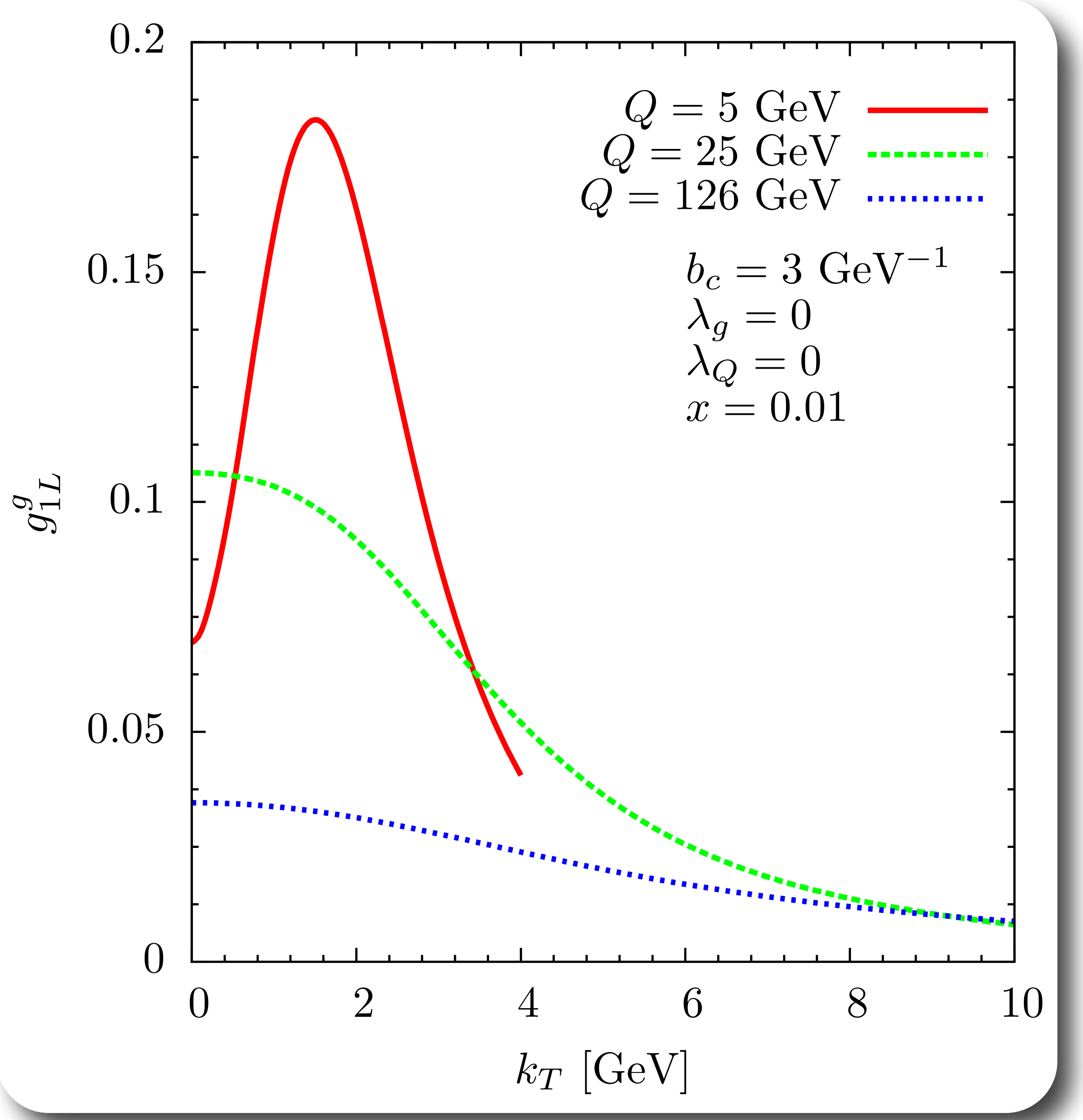
- *The resummation scale is:* $Q_i = 2e^{-\gamma_E}/\hat{b}_T$

- *The non-perturbative model and the prescription to avoid the Landau pole:*

$$\hat{b}_T(b_T) = b_c\left(1 - e^{-(b_T/b_c)^2}\right)^{1/2}, \quad b_c = 3\; GeV^{-1}$$

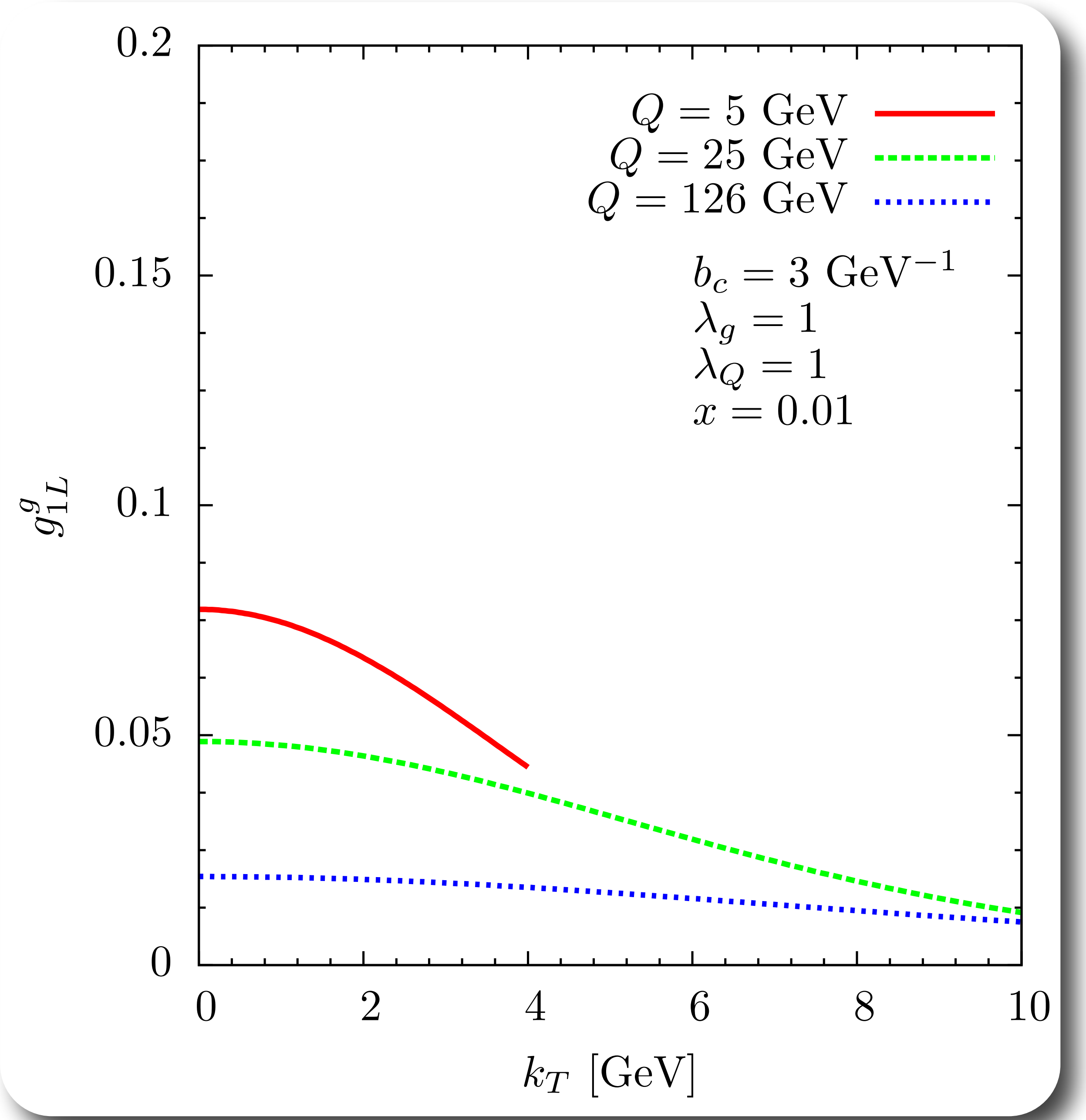
$$\tilde{g}^{g,NP}_{1L}(x_A, b_T; Q) = \exp\left[-b_T^2(\lambda_g + \lambda_Q \ln(Q^2/Q_0^2))\right], \quad Q_0 = 1\; GeV$$

Gluon helicity TMDPDF (2/2)



Quark/gluon helicity PDFs from

[de Florian, Sassot, Stratmann, Vogelsang '14]



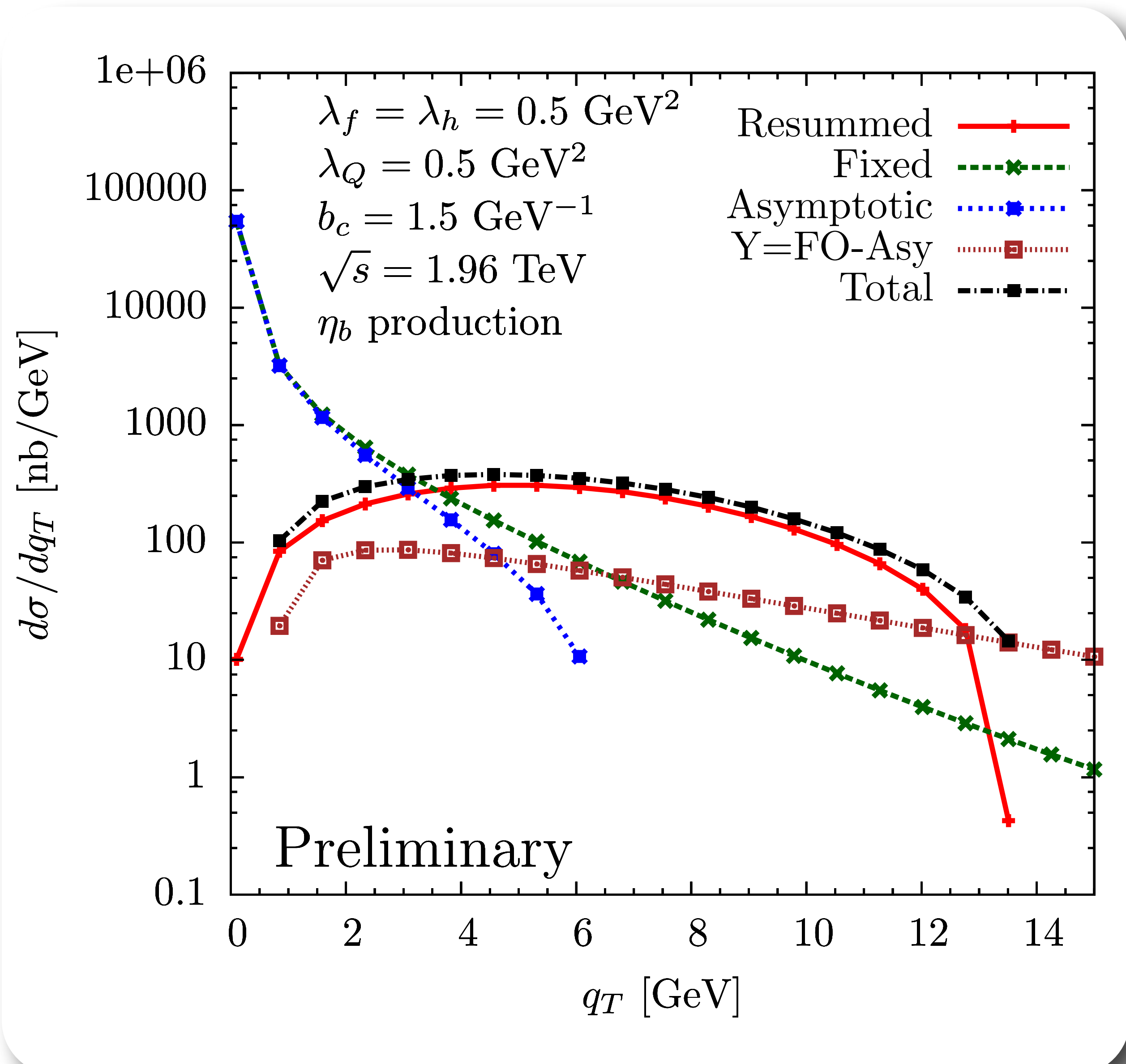
The larger the scale,
the wider and lower the distribution

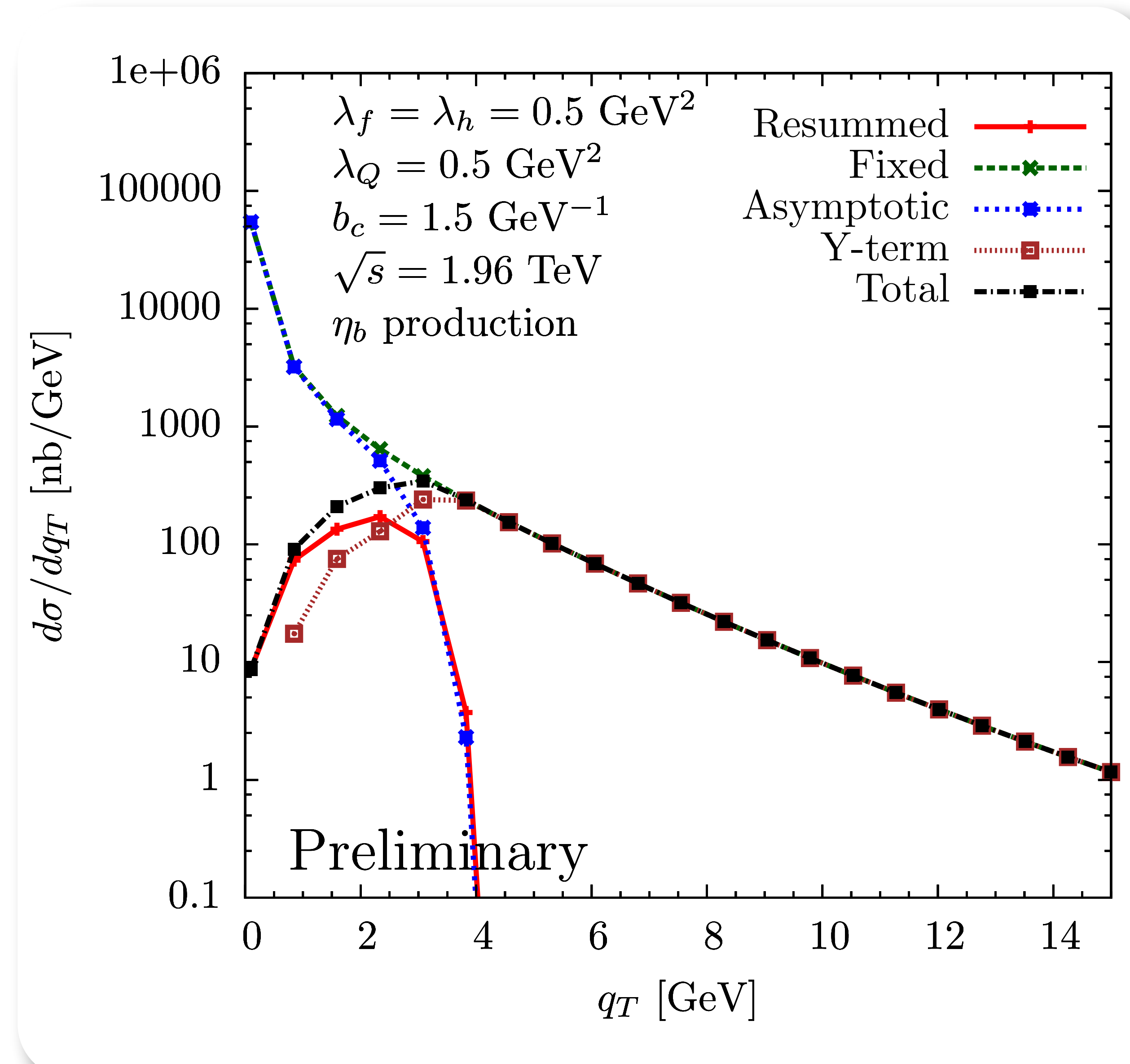
Recovering a complete TMD spectrum

- *In practice we need to properly match TMD and collinear regions*
- *Example:* $p + p \to \eta_b(q_T) + X$

[MGE, Kasemets, Lansberg, Pisano, Signori 16XX.XXXX]

[Collins, Gamberg, Prokudin, Rogers, Sato, Wang 1605.00671]





→ *See B. Wang's talk*

Conclusions & Outlook

- *After ~ 3 decades we finally know how to properly define TMDs*
- *TMD evolution is universal, and currently known at NNLL'*
- *Matching coefficients for all unpolarized TMDs currently known at NNLO; for some polarized TMDs at NLO*
- *TMD pheno is a mess: non-perturbative ingredients, different regions mixed under Fourier transform, need to match TMD and collinear regions,…*

★ *We need new experimental data: unpolarized e+e-, more unpolarized Drell-Yan, etc*

★ *Push the pheno: perform global fits exploiting all available perturbative information*

★ *Better constrain collinear twist-3 functions: basis for spin asymmetries*

★ …