Institute for Theoretical Physics

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Small-z Resummation in NNLO FF Analysis

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Speaker:
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Outline

- Goal
- Time-Like Evolution
- Our E+E- NNLO Fit
- Improving: Small-z Resummation
- Conclusions & Outlook

Towards a Global NNLO FF Fit

Anderle, Ringer, Stratmann

Ingredients needed to achieve the goal:

DATA SETS:

SI-e^+e^- → old: TPC(Phys. Rev. Lett 61, 1263 (1998)), SLD(Phys. Rev. D59,052001 (1999)),
ALEPH(Phys. Lett. B357, 487 (1995)),
DELPHI(Eur. Phys. J. C5, 585 (1998),Eur. Phys. J.C6, 19 (1999))
OPAL(Eur. Phys. J. C16, 407 (2000),Eur. Phys. J.C7, 369 (1999)),
TASSO(Z. Phys.C42, 189 (1989))

SIDIS → old: EMC(Z. Phys. C52, 361 (1991)), JLAB(Phys. Rev. Lett. 98, 022001)

SI- p(anti-)p → old: CDF(Phys. Rev. Lett. 61,1819 (1988)), UA1(Nucl. Phys. B335,261 (1990)),
UA2(Z. Phys. C27, 329 (1985))

Towards a Global NNLO FF Fit

Anderle, Ringer, Stratmann

Ingredients needed to achieve the goal:

DATA SETS:

SI-e^+e^- → new: BaBar(Phys. Rev. D 88, 032011 (2013)), Belle(Phys. Rev. Lett. 111, 062002 (2013))

SIDIS → new: HERMES(Ph.D. thesis, Erlangen Univ., Germany, September 2005), Compass(PoS DIS 2013, 202 (2013)), JLAB@12GeV

SI- p(anti-)p → new: Phenix(Phys. Rev. D 76,051106 (2007)), Alice(Phys. Lett. B 717, 162 (2012).), Brahms(Phys. Rev. Lett. 98, 252001 (2007)), Star(Phys. Rev. Lett. 97, 152302 (2006))

pp→ (Jet h)X → future: Star, CMS(JHEP 1210, 087 (2012)), Alice(arXiv:1408.5723), Atlas(Eur. Phys. J. C 71, 1795 (2011))

Towards a Global NNLO FF Fit

Anderle, Ringer, Stratmann

Ingredients needed to achieve the goal:

NNLO EVOLUTION KERNELS:

Splitting functions →

NNLO-Non Singlet: Mitov, Moch, Vogt (Phys.Lett. B638 (2006) 61-67)

NNLO-Singlet: Moch, Vogt (Phys.Lett.B659 (2008) 290-296)

NNLO-Singlet: Almasy, Mitov, Moch, Vogt (Nucl.Phys. B854 (2012)) 133-152)

Both computed in x-Space and in Mellin Space

TOWARDS A GLOBAL NNLO FF FIT

Anderle, Ringer, Stratmann

Ingredients needed to achieve the goal:

NNLO COEFFICINT FUNCTIONS:

SI-e^+e^- → **x-Space** Rijken, van Neerven (Phys.Lett.B386(1996)422, Nucl.Phys.B488(1997)233,Phys.Lett.B392(1997)207)

Mellin-Space Mitov, Moch (Nucl.Phys.B751 (2006) 18-52)
Blümlein, Ravindran (Nucl.Phys.B749 (2006) 1-24)

SIDIS → NOT COMPUTED YET but work in progress

$$\gamma q' \to q\bar{q}q', \quad \gamma g \to q\bar{q}q'$$

Anderle, de Florian, Rotstein, Vogelsang

SI- p(anti-)p → NOT COMPUTED YET

pp→ (Jet h)X → NOT COMPUTED YET

TOWARDS A GLOBAL NNLO FF FIT

Anderle, Ringer, Stratmann

Ingredients needed to achieve the goal:

NNLO COEFFICINT FUNCTIONS:

SIDIS → Soft gluon Resummed results (can be expanded @ NNLO)

Anderle, Ringer, Vogelsang (Phys.Rev. D87 (2013) 094021, Phys.Rev. D87 (2013) 3, 034014)

SI- p(anti-)p → Soft gluon Resummed results (can be expanded @ NNLO)

Work in progress for $\frac{d\sigma}{dp_T d\eta}^{(NNLL)}$ Hinderer, Ringer, Sterman, Vogelsang

pp→ (Jet h)X → Resummed results (can be expanded @ NNLO)

Work in progress from T. Kaufmann, Vogelsang

Towards a Global NNLO FF Fit

Anderle, Ringer, Stratmann

Ingredients needed to achieve the goal:

NNLO COEFFICINT FUNCTIONS:

To include the last processes we need a

NNLO Mellin Space Fitting Program

The NNLO Evolution Code "Pegasus_FF"

Existing NNLO Evolution CODES:

X-SPACE APFEL(time-like version C/C++, Fortran77, Python)
Bertone1,Carrazza, Rojo (CERN-PH-TH/2013-209)

Mellin SPACE MELA(Fortran77)
Bertone1,Carrazza, Nocera (CERN-PH-TH-2014-265)

Newly born:

Mellin SPACE Pegasus_FF (Fortran77) → based on Pegasus(Fortran77)
Anderle, Ringer, Stratmann — Vogt (Comput.Phys.Commun.170:65-92,2005)

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THE TIME-LIKE EVOLUTION

In the factorisation procedure, the absorption of *collinear singularities* by fragmentation functions (FF)(in case of massless partons) leads to scaling violation and the appearance of a factorisation scale μ_F

The scale dependance of FF is governed by the Time-Like DGLAP

$$\frac{\partial}{\partial \ln \mu_F^2} D_i^h(x, \mu_F^2) = \sum_j \int_x^1 \frac{dy}{y} P_{ji}\left(y, \alpha_s(\mu_F^2)\right) D_j^h\left(\frac{x}{y}, \mu_F^2\right)$$

Time-Like Splitting function perturbatively calculable $\quad P_{ji}(y, \alpha_s) = \sum_{k=0} a_s^{k+1} P_{ji}^{(k)}(y)$

Usually rewritten into $2n_f - 1$ equations (charge conjugation and flavour symmetry)

NON-SINGLET

$$D^h_{\mathrm{NS};v} = \sum_{i=1}^{n_f} (D^h_{q_i} - D^h_{\bar{q}_i})$$

$$D^h_{\mathrm{NS};\pm} = (D^h_{q_i} \pm D^h_{\bar{q}_i}) - (D^h_{q_j} \pm D^h_{\bar{q}_j})$$

$$\frac{\partial}{\partial \ln \mu_F^2} D^h_{\mathrm{NS};\pm,v}(x,\mu_F^2) = P^{\pm,\mathrm{v}}(x,\mu_F^2) \otimes D^h_{\mathrm{NS};\pm,v}(x,\mu_F^2)$$

and two coupled

SINGLET

$$D^h_{\Sigma} = \sum_{i=1}^{n_f} (D^h_{q_i} + D^h_{\bar{q}_i})$$

$$D^h_g$$

$$\frac{\partial}{\partial \ln \mu_F^2} \begin{pmatrix} D^h_{\Sigma}(x,\mu_F^2) \\ D^h_g(x,\mu_F^2) \end{pmatrix} = \begin{pmatrix} P^{\mathrm{qq}} & 2n_f P^{\mathrm{gq}} \\ \frac{1}{2n_f} P^{\mathrm{qg}} & P^{\mathrm{gg}} \end{pmatrix} \otimes \begin{pmatrix} D^h_{\Sigma}(x,\mu_F^2) \\ D^h_g(x,\mu_F^2) \end{pmatrix}$$

The splitting functions are accordingly separated in the singlet and non-singlet sectors

NON-SINGLET

$$P_{\mathrm{ns}}^{\pm} = P_{\mathrm{qq}}^{\mathrm{v}} \pm P_{\mathrm{q\bar{q}}}^{\mathrm{v}}$$

$$P_{\mathrm{ns}}^{\mathrm{v}} = P_{\mathrm{qq}}^{\mathrm{v}} - P_{\mathrm{q\bar{q}}}^{\mathrm{v}} + n_f (P_{\mathrm{qq}}^{\mathrm{s}} - P_{\mathrm{q\bar{q}}}^{\mathrm{s}}) \equiv P_{\mathrm{ns}}^{-} + P_{\mathrm{ns}}^{\mathrm{s}}$$

SINGLET

$$P_{\mathrm{qq}} = P_{\mathrm{ns}}^{+} + n_f (P_{\mathrm{qq}}^{\mathrm{s}} + P_{\mathrm{\bar{q}q}}^{\mathrm{s}}) \equiv P_{\mathrm{ns}}^{+} + P_{\mathrm{ps}}$$

$$P_{\mathrm{gq}} \equiv P_{\mathrm{gq}_i} = P_{\mathrm{g\bar{q}}_i}$$

$$P_{\mathrm{qg}} \equiv n_f\, P_{\mathrm{q}_i\mathrm{g}} = n_f\, P_{\mathrm{\bar{q}}_i\mathrm{g}}$$

The splitting functions are accordingly separated in the singlet and non-singlet sectors

NON-SINGLET

$$P_{\mathrm{ns}}^{\pm} = P_{\mathrm{qq}}^{\mathrm{v}} \pm \cancel{P_{\mathrm{q\bar{q}}}^{\mathrm{v}}}$$

$$P_{\mathrm{ns}}^{\mathrm{v}} = P_{\mathrm{qq}}^{\mathrm{v}} - \cancel{P_{\mathrm{q\bar{q}}}^{\mathrm{v}}} + n_f\left(\cancel{P_{\mathrm{qq}}^{\mathrm{s}}} - \cancel{P_{\mathrm{q\bar{q}}}^{\mathrm{s}}}\right) \equiv P_{\mathrm{ns}}^{-} + \cancel{P_{\mathrm{ns}}^{\mathrm{s}}}$$

@LO $$P_{\mathrm{ns}}^{\mathrm{v}} = P_{\mathrm{ns}}^{\pm}$$

SINGLET

$$P_{\mathrm{qq}} = P_{\mathrm{ns}}^{+} + n_f\left(\cancel{P_{\mathrm{qq}}^{\mathrm{s}}} + \cancel{P_{\mathrm{\bar{q}q}}^{\mathrm{s}}}\right) \equiv P_{\mathrm{ns}}^{+} + \cancel{P_{\mathrm{ps}}}$$

$$P_{\mathrm{gq}} \equiv P_{\mathrm{gq}_i} = P_{\mathrm{g\bar{q}}_i}$$

$$P_{\mathrm{qg}} \equiv n_f\, P_{\mathrm{q}_i\mathrm{g}} = n_f\, P_{\mathrm{\bar{q}}_i\mathrm{g}}$$

The splitting functions are accordingly separated in the singlet and non-singlet sectors

NON-SINGLET

$$P_{\rm ns}^{\pm} = P_{\rm qq}^{\rm v} \pm P_{\rm q\bar{q}}^{\rm v}$$

$$P_{\rm ns}^{\rm v} = P_{\rm qq}^{\rm v} - P_{\rm q\bar{q}}^{\rm v} + n_f (P_{\rm qq}^{\rm s} - P_{\rm q\bar{q}}^{\rm s}) \equiv P_{\rm ns}^{-} + P_{\rm ns}^{\rm s}$$

@NLO

$$P_{\rm qq}^{\rm S} = P_{\rm q\bar{q}}^{\rm S}$$

$$P_{\rm ns}^{\rm v} = P_{\rm ns}^{-}$$

SINGLET

$$P_{\rm qq} = P_{\rm ns}^{+} + n_f (P_{\rm qq}^{\rm s} + P_{\rm \bar{q}q}^{\rm s}) \equiv P_{\rm ns}^{+} + P_{\rm ps}$$

$$P_{\rm gq} \equiv P_{{\rm gq}_i} = P_{{\rm g\bar{q}}_i}$$

$$P_{\rm qg} \equiv n_f \, P_{{\rm q}_i{\rm g}} = n_f \, P_{\bar{\rm q}_i{\rm g}}$$

The splitting functions are accordingly separated in the singlet and non-singlet sectors

NON-SINGLET

$$P_{\rm ns}^{\pm} = P_{\rm qq}^{\rm v} \pm P_{\rm q\bar{q}}^{\rm v}$$

$$P_{\rm ns}^{\rm v} = P_{\rm qq}^{\rm v} - P_{\rm q\bar{q}}^{\rm v} + n_f (P_{\rm qq}^{\rm s} - P_{\rm q\bar{q}}^{\rm s}) \equiv P_{\rm ns}^{-} + P_{\rm ns}^{\rm s}$$

@NNLO

Responsable for $s,\ \bar{s}$ asymmetry

$$[s-\bar{s}](x,Q^2) \neq 0$$

Rodrigo, Catani, de Florian, Vogelsang (arXiv:hep-ph/0406338)

SINGLET

$$P_{\rm qq} = P_{\rm ns}^{+} + n_f (P_{\rm qq}^{\rm s} + P_{\rm \bar{q}q}^{\rm s}) \equiv P_{\rm ns}^{+} + P_{\rm ps}$$

$$P_{\rm gq} \equiv P_{{\rm gq}_i} = P_{{\rm g\bar{q}}_i}$$

$$P_{\rm qg} \equiv n_f \, P_{{\rm q}_i{\rm g}} = n_f \, P_{{\rm \bar{q}}_i{\rm g}}$$

THE SOLUTION

We can solve the integro-differential DGLAP equation analytically in Mellin space at each fixed order since it becomes an Ordinary Differential Equation

$$\frac{\partial \boldsymbol{q}(N,a_s)}{\partial a_s} = \{\beta_{\mathrm{N^mLO}}(a_s)\}^{-1}\, \boldsymbol{P}_{\mathrm{N^mLO}}(N,a_s)\, \boldsymbol{q}(N,a_s)$$

$$= -\frac{1}{\beta_0 a_s}\Big[\boldsymbol{P}^{(0)}(N) + a_s\big(\boldsymbol{P}^{(1)}(N) - b_1\boldsymbol{P}^{(0)}(N)\big) + a_s^2\big(\boldsymbol{P}^{(2)}(N) - b_1\boldsymbol{P}^{(1)}(N) + (b_1^2 - b_2)\boldsymbol{P}^{(0)}(N)\big) + \ldots\Big]\, \boldsymbol{q}(N,a_s)$$

$$f(N,\alpha_s) = \int_0^1 dy\, y^{N-1} f(y,\alpha_s) \qquad N \in \mathbb{C}$$

where here $\mathbf{P}(N,\alpha_S)$ and $\mathbf{q}(N,\alpha_S)$ are the Mellin-Transform of either singlet or non-singlet splitting function and FF respectively the general solution can be expressed in terms of the evolution matrices ***U*** (constructed from the splitting functions) as a simple multiplication

$$\begin{aligned} \boldsymbol{q}(N, a_s) &= \boldsymbol{U}(N, a_s)\, \boldsymbol{L}(N, a_s, a_0)\, \boldsymbol{U}^{-1}(N, a_0)\, \boldsymbol{q}(N, a_0) \\ &= \left[1 + \sum_{k=1}^{\infty} a_s^k\, \boldsymbol{U}_k(N)\right] \boldsymbol{L}(a_s, a_0, N) \left[1 + \sum_{k=1}^{\infty} a_0^k\, \boldsymbol{U}_k(N)\right]^{-1} \boldsymbol{q}(a_0, N) \end{aligned}$$

where ***L*** is defined by the LO solution

$$\boldsymbol{q}_{\mathrm{LO}}(N, a_s, N) = \left(\frac{a_s}{a_0}\right)^{-\boldsymbol{R}_0(N)} \boldsymbol{q}(N, a_0) \equiv \boldsymbol{L}(N, a_s, a_0)\, \boldsymbol{q}(N, a_0)$$

$$\boldsymbol{R}_0 \equiv \frac{1}{\beta_0} \boldsymbol{P}^{(0)}$$

TRUNCATED AND ITERATED SOLUTION

Since both $\beta_{\mathrm{N^mLO}}$ and $\boldsymbol{P}_{\mathrm{N^mLO}}$ have an expansion in powers of α_s

there are different ways of defining the N$^{\mathrm{m}}$LO solution

$$\begin{aligned}\boldsymbol{q}_{\mathrm{N^3LO}}(a_{\mathrm{s}}) \;=\; \Big[&\boldsymbol{L} + a_{\mathrm{s}}\,\boldsymbol{U}_1\,\boldsymbol{L} - a_0\,\boldsymbol{L}\,\boldsymbol{U}_1 \\ &+ a_{\mathrm{s}}^2\,\boldsymbol{U}_2\,\boldsymbol{L} - a_{\mathrm{s}}a_0\,\boldsymbol{U}_1\,\boldsymbol{L}\,\boldsymbol{U}_1 + a_0^2\,\boldsymbol{L}\left(\boldsymbol{U}_1^2 - \boldsymbol{U}_2\right) \\ &+ a_{\mathrm{s}}^3\,\boldsymbol{U}_3\,\boldsymbol{L} - a_{\mathrm{s}}^2 a_0\,\boldsymbol{U}_2\,\boldsymbol{L}\,\boldsymbol{U}_1 + a_{\mathrm{s}}a_0^2\,\boldsymbol{U}_1\,\boldsymbol{L}\left(\boldsymbol{U}_1^2 - \boldsymbol{U}_2\right) \\ &- a_0^3\,\boldsymbol{L}\left(\boldsymbol{U}_1^3 - \boldsymbol{U}_1\boldsymbol{U}_2 - \boldsymbol{U}_1\boldsymbol{U}_2 + \boldsymbol{U}_3\right)\Big]\,\boldsymbol{q}(a_0)\end{aligned}$$

$$\boldsymbol{R}_0 \equiv \frac{1}{\beta_0}\boldsymbol{P}^{T,(0)}\,,\quad \boldsymbol{R}_k \equiv \frac{1}{\beta_0}\boldsymbol{P}^{T,(k)} - \sum_{i=1}^{k} b_i \boldsymbol{R}_{k-i}\,,$$

$$[\boldsymbol{U}_k, \boldsymbol{R}_0] = \boldsymbol{R}_k + \sum_{i=1}^{k-1} \boldsymbol{R}_{k-1}\boldsymbol{U}_i + k\boldsymbol{U}_k\,.$$

TRUNCATED AND ITERATED SOLUTION

TRUNCATED: Keep only terms up to α_s^m in the solution

$$\boldsymbol{q}_{\mathrm{N^3LO}}(a_\mathrm{s}) = \Big[\boldsymbol{L} + a_\mathrm{s}\,\boldsymbol{U}_1\,\boldsymbol{L} - a_0\,\boldsymbol{L}\,\boldsymbol{U}_1 + a_\mathrm{s}^2\,\boldsymbol{U}_2\,\boldsymbol{L} - a_\mathrm{s}a_0\,\boldsymbol{U}_1\,\boldsymbol{L}\,\boldsymbol{U}_1 + a_0^2\,\boldsymbol{L}\left(\boldsymbol{U}_1^2 - \boldsymbol{U}_2\right) + a_\mathrm{s}^3\,\boldsymbol{U}_3\,\boldsymbol{L} - a_\mathrm{s}^2 a_0\,\boldsymbol{U}_2\,\boldsymbol{L}\,\boldsymbol{U}_1 + a_\mathrm{s}a_0^2\,\boldsymbol{U}_1\,\boldsymbol{L}\left(\boldsymbol{U}_1^2 - \boldsymbol{U}_2\right) - a_0^3\,\boldsymbol{L}\left(\boldsymbol{U}_1^3 - \boldsymbol{U}_1\boldsymbol{U}_2 - \boldsymbol{U}_1\boldsymbol{U}_2 + \boldsymbol{U}_3\right)\Big]\,\boldsymbol{q}(a_0)$$

$$\boldsymbol{R}_0 \equiv \frac{1}{\beta_0}\boldsymbol{P}^{T,(0)}\,, \quad \boldsymbol{R}_k \equiv \frac{1}{\beta_0}\boldsymbol{P}^{T,(k)} - \sum_{i=1}^{k} b_i \boldsymbol{R}_{k-i}\,, \quad [\boldsymbol{U}_k, \boldsymbol{R}_0] = \boldsymbol{R}_k + \sum_{i=1}^{k-1}\boldsymbol{R}_{k-1}\boldsymbol{U}_i + k\boldsymbol{U}_k\,.$$

- It solves the equation exactly only up to terms of order n > m

Truncated and Iterated Solution

ITERATED: Keep the all the m-terms generated from $\beta_{\mathrm{N^mLO}}$ and $\boldsymbol{P}_{\mathrm{N^mLO}}$

$$\boldsymbol{q}_{\mathrm{N^3LO}}(a_s) = \Big[\boldsymbol{L} + a_s\boldsymbol{U}_1\boldsymbol{L} - a_0\boldsymbol{L}\boldsymbol{U}_1 + a_s^2\boldsymbol{U}_2\boldsymbol{L} - a_s a_0\boldsymbol{U}_1\boldsymbol{L}\boldsymbol{U}_1 + a_0^2\boldsymbol{L}\left(\boldsymbol{U}_1^2 - \boldsymbol{U}_2\right) + a_s^3\boldsymbol{U}_3\boldsymbol{L} - a_s^2 a_0\boldsymbol{U}_2\boldsymbol{L}\boldsymbol{U}_1 + a_s a_0^2\boldsymbol{U}_1\boldsymbol{L}\left(\boldsymbol{U}_1^2 - \boldsymbol{U}_2\right) - a_0^3\boldsymbol{L}\left(\boldsymbol{U}_1^3 - \boldsymbol{U}_1\boldsymbol{U}_2 - \boldsymbol{U}_1\boldsymbol{U}_2 + \boldsymbol{U}_3\right)\Big]\boldsymbol{q}(a_0)$$

$$\boldsymbol{R}_0 \equiv \frac{1}{\beta_0}\boldsymbol{P}^{T,(0)}\,,\quad \boldsymbol{R}_k \equiv \frac{1}{\beta_0}\boldsymbol{P}^{T,(k)} - \sum_{i=1}^{k} b_i\boldsymbol{R}_{k-i}\,,\qquad [\boldsymbol{U}_k, \boldsymbol{R}_0] = \boldsymbol{R}_k + \sum_{i=1}^{k-1}\boldsymbol{R}_{k-1}\boldsymbol{U}_i + k\boldsymbol{U}_k\,.$$

- It corresponds to the solution done in x-Space

- It introduces more higher order scheme-dependent terms

Truncated and Iterated Solution

ITERATED-TRUNCATED = theoretical uncertainty of order $\mathcal{O}({\alpha_s}^{m+1})$

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OUR SIA FIT

Anderle, Ringer, Stratmann Phys.Rev. D92 (2015) no.11, 114017

Parametrization of light patrons FF @ μ_0

$$D_i^h(z, Q_0) = \frac{N_i z^{\alpha_i}(1-z)^{\beta_i}[1+\gamma_i(1-z)^{\delta_i}]}{B[2+\alpha_i, \beta_i+1] + \gamma_i B[2+\alpha_i, \beta_i+\delta_i+1]}$$

So that $N_i = \int_0^1 z D_i^h dz$

Heavy Quark Treatment:

NON PERTURBATIVE INPUT: at $\mu > m_q$ the evolution is set to evolve with $n_f + 1$ for flavours and for the q-heavy quark FF the same functional form as for the light quark is set at $\mu = m_q$

Data sets:

15 Data Set: from Sld, Aleph, Delphi, Opal, Tpc, BaBar, Belle either inclusive, uds tagged, b tagged or c tagged (relevant scales 10.5,29,91.2 GeV). We use a GLOBAL CUT 0.075<z<0.95

PARAMETERS FOR PI+

With only SIA one needs some extra constrains:

parameter	LO	NLO	NNLO
$N_{u+\bar{u}}$	0.735	0.572	0.579
$\alpha_{u+\bar{u}}$	-0.371	-0.705	-0.913
$\beta_{u+\bar{u}}$	0.953	0.816	0.865
$\gamma_{u+\bar{u}}$	8.123	5.553	4.062
$\delta_{u+\bar{u}}$	3.854	1.968	1.775
$N_{s+\bar{s}}$	0.243	0.135	0.271
$\alpha_{s+\bar{s}}$	*-0.371*	*-0.705*	*-0.913*
$\beta_{s+\bar{s}}$	*4.807*	*2.784*	*2.640*
N_g	0.273	0.211	0.174
α_g	2.414	2.210	1.595
β_g	*8.000*	*8.000*	*8.000*
$N_{c+\bar{c}}$	0.405	0.302	0.338
$\alpha_{c+\bar{c}}$	-0.164	-0.026	-0.233
$\beta_{c+\bar{c}}$	5.114	6.862	6.564
$N_{b+\bar{b}}$	0.462	0.405	0.445
$\alpha_{b+\bar{b}}$	-0.090	-0.411	-0.695
$\beta_{b+\bar{b}}$	4.301	4.039	3.681
$\gamma_{b+\bar{b}}$	24.85	15.80	11.22
$\delta_{b+\bar{b}}$	12.25	11.27	9.908

5 free param needed

charge conjugation and isospin symmetry $D^{\pi^\pm}_{u+\bar{u}} = D^{\pi^\pm}_{d+\bar{d}}$,

1 free param, 2 fixed by

$\alpha_{s+\bar{s}} = \alpha_{u+\bar{u}}, \quad \beta_{s+\bar{s}} = \beta_{u+\bar{u}} + \delta_{u+\bar{u}}$

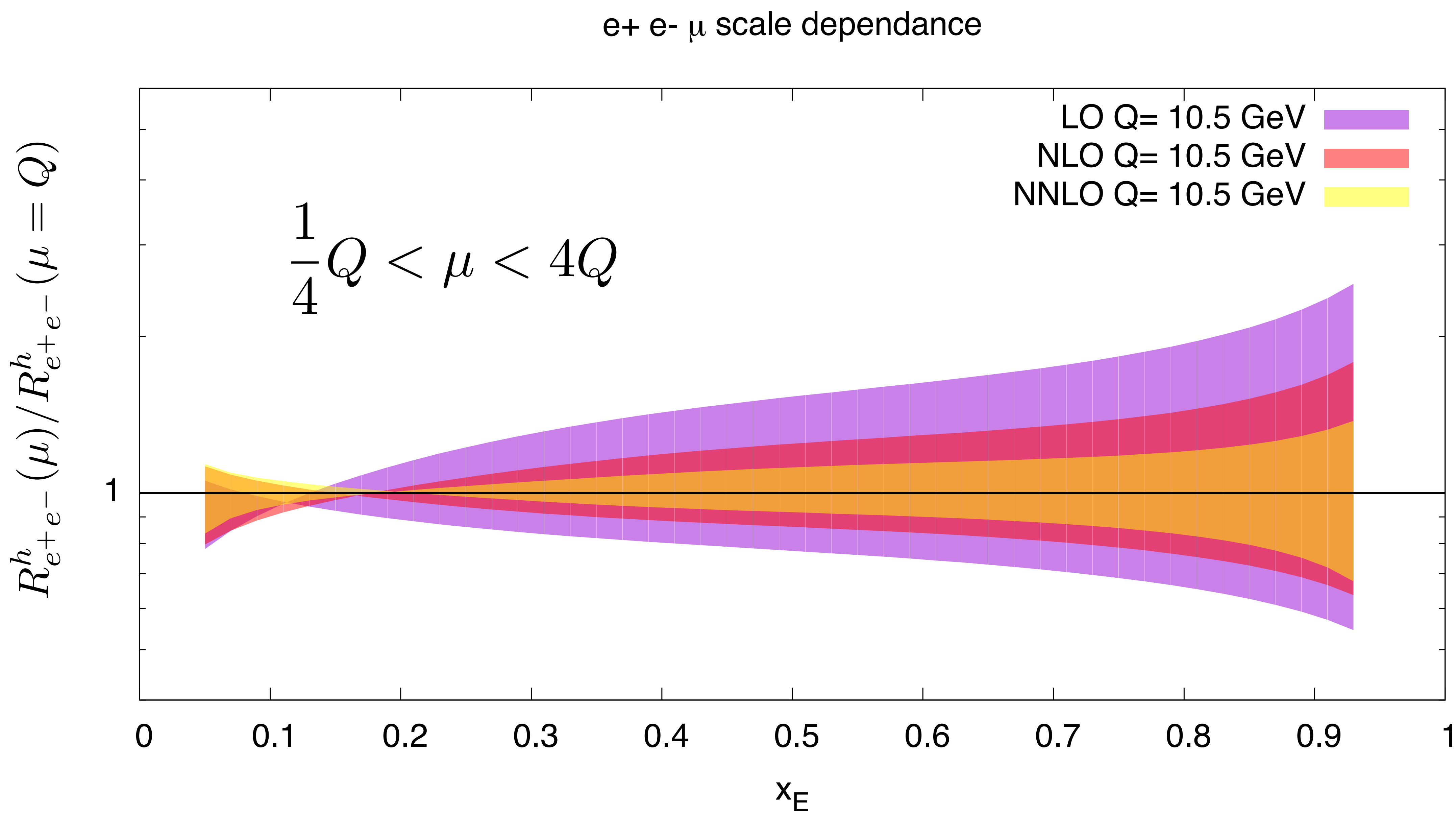
2 free param, 1 fixed

3 free param

5 free param

TOT = 16 free param

Scale Dependence



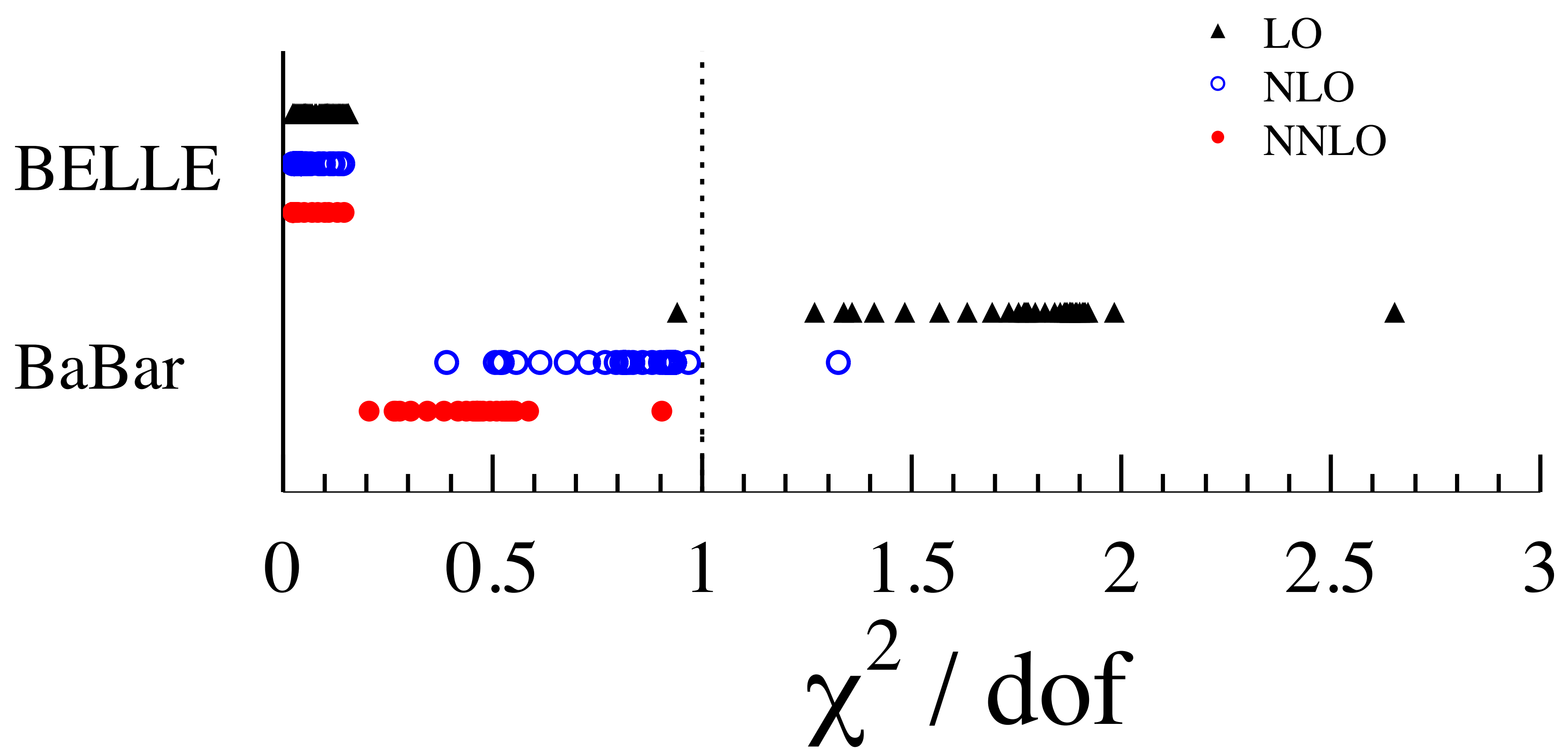
Multiplicity $R^h_{e^+e^-} \equiv \frac{1}{\sigma^{\text{tot}}} \frac{d^2\sigma^h}{dx_E \, d\cos\theta}$

using input parameter for FF of Kretzer (Phys.Rev. D62 (2000) 054001) and truncated-solution

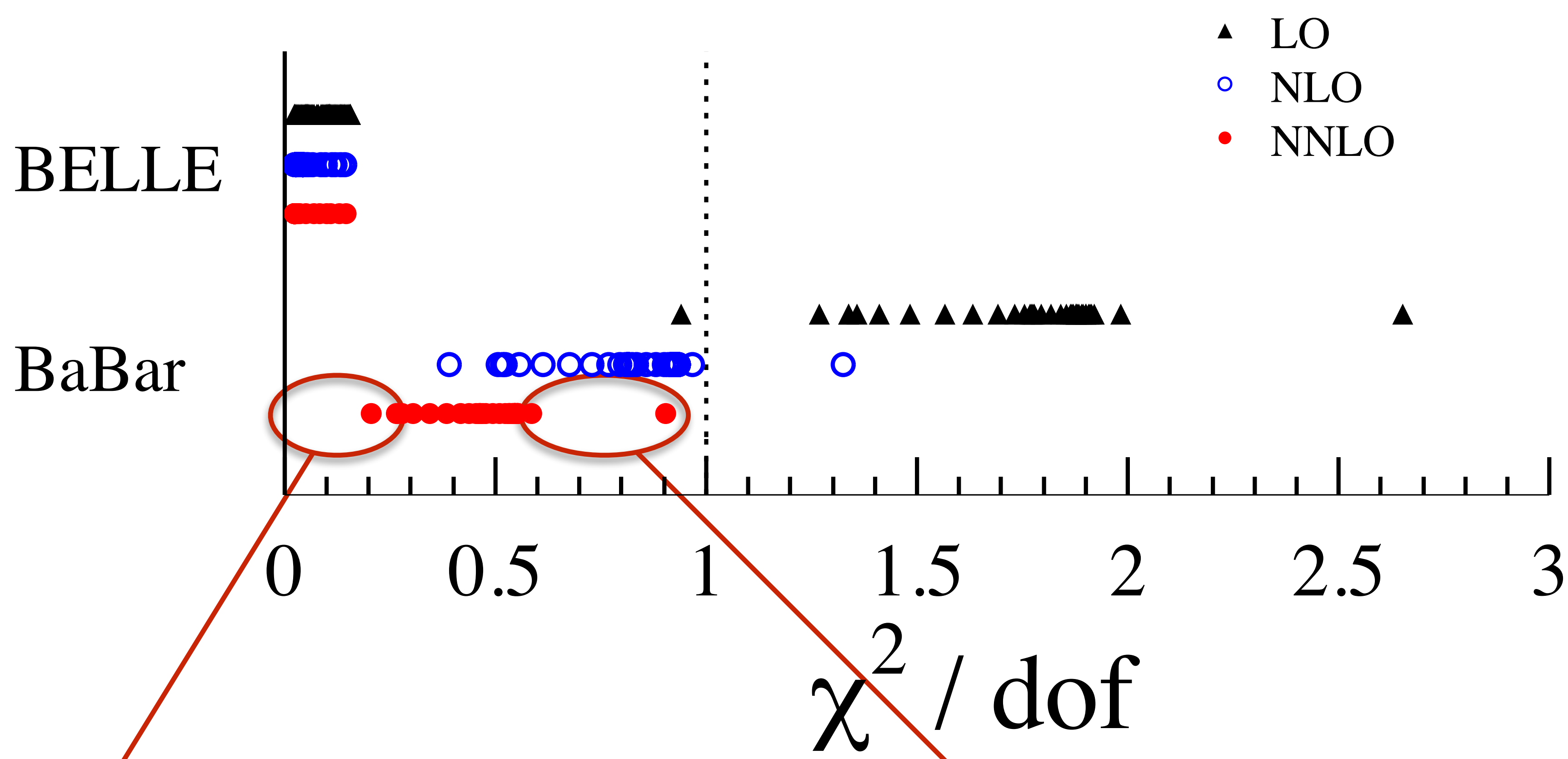
χ^2 Comparison

experiment	data type	# data in fit	χ^2 LO	NLO	NNLO
SLD [40]	incl.	23	15.0	14.8	15.5
	uds tag	14	9.7	18.7	18.8
	c tag	14	10.4	21.0	20.4
	b tag	14	5.9	7.1	8.4
ALEPH [41]	incl.	17	19.2	12.8	12.6
DELPHI [42]	incl.	15	7.4	9.0	9.9
	uds tag	15	8.3	3.8	4.3
	b tag	15	8.5	4.5	4.0
OPAL [43]	incl.	13	8.9	4.9	4.8
TPC [44]	incl.	13	5.3	6.0	6.9
	uds tag	6	1.9	2.1	1.7
	c tag	6	4.0	4.5	4.1
	b tag	6	8.6	8.8	8.6
BABAR [10]	incl.	41	108.7	54.3	37.1
BELLE [9]	incl.	76	11.8	10.9	11.0
TOTAL:		288	241.0	190.0	175.2

χ^2 Comparison



χ^2 COMPARISON



Small z Logs

$$\alpha_s^k \frac{\ln^{2k}(z)}{z}$$

Threshold Logs

$$\alpha_s^k \left(\frac{\ln^{2k-1}(1-x)}{1-x}\right)_+$$

Anderle, Ringer, Vogelsang (Phys.Rev. D87 (2013) no.3, 034014)

+Hadron Mass Cor.

Accardi, Anderle, Ringer (Phys.Rev. D91 (2015) 3, 034008)

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Small-z Logarithms (SIA)

N^{k}LO Small-z Logarithms in Splitting Functions and Singlet Coefficient Functions

$$P_{gi}^{(k-1)} \propto a_s^k \frac{\ln^{2k-1-a}(z)}{z}$$

$$C_{T,g}^{(k)} \propto a_s^k \frac{\ln^{2k-1-a}(z)}{z}$$

$$C_{L,g}^{(k)} \propto a_s^k \frac{\ln^{2k-2-a}(z)}{z}$$

$$a = 0, 1, 2 \qquad i \in \{q, g\}$$

Double Log Enhancement

spoils perturbative convergence even for $\alpha_s \ll 1$

In Mellin Space they correspond to N =1 Poles

$$\mathcal{M}\left[\frac{\ln^{2k-1}(z)}{z}\right] \equiv \int_0^1 dx\, x^{N-1} \frac{\ln^{2k-1}(z)}{z} = \frac{(-1)^k(2k-1)!}{(N-1)^{2k}}$$

Resummation Accuracy

For example P_{gg} with $N-1=\bar{N}$

Fixed Order →

Resummation ↓

LO	$\alpha_s/\bar{N}$	α_s				
NLO	$\alpha_s/\bar{N}^3$	$\alpha_s/\bar{N}^2$	$\alpha_s/\bar{N}$	α_s		
NNLO	$\alpha_s/\bar{N}^5$	$\alpha_s/\bar{N}^4$	$\alpha_s/\bar{N}^3$	$\alpha_s/\bar{N}^2$	$\alpha_s/\bar{N}$	α_s
...	...	...	...	...	...	
N^{k-1}LO	$\alpha_s/\bar{N}^{2k-1}$	$\alpha_s/\bar{N}^{2k-2}$	$\alpha_s/\bar{N}^{2k-3}$	$\alpha_s/\bar{N}^{2k-4}$	$\alpha_s/\bar{N}^{2k-5}$	...

NNLL : *Vogt (2011), Kom ,Vogt, Yeats (2012)*

NLL : *Mueller (83), Albino, Bolzoni, Kniehl, Kotikov (11)*

LL : *Mueller (81); Bassetto, Ciafaloni, Marchesini, Mueller (82).*

RESUMMATION VIA UNFACTORIZED SIA

van Neerven, Rijken (1996)
Vogt (2011), Kom, Vogt, Yeats(2012)

One can proceed by using "all-order" mass factorization: e.g.

A) starting from the *unfactorized gluon singlet transversal parton structure function in dimensional regularisation (IR-singularities not yet factorized out and "re-absorbed" in FF)*

$$\hat{\mathcal{F}}_g^T(N, a_s, \epsilon) = \sum_{i=q,g} \bar{C}_i^T(N, a_s, \epsilon)\, \Gamma_{ig}^N(N, a_s, \epsilon)$$

D-Dimensional coef. function:
only positive powers of ϵ

$$\bar{C}_i^T(N, a_s, \epsilon) = \delta_{iq} + \sum_{l=1}^{\infty} a_s^l \sum_{k=0}^{\infty} \epsilon^k \bar{c}_{T,i}^{(l,k)}(N)$$

Transition function:
incorporates all IR $1/\epsilon$ poles, calculable order by order as a combination of splitting functions

$$\beta_D(a_s)\, \frac{\partial \Gamma_{ik}}{\partial a_s}\, \Gamma_{kj}^{-1} = P_{ij}$$

B) "Plug-in" the small $\bar{N} = N - 1$ limit for known fixed order coefficient functions and splitting functions (e.g. NNLO)

C) Impose equality order by order in a_s with the small $\bar{N} = N - 1$ limit for the unfactorized structure function which reads

A.Vogt JHEP10 (2011) 025

$$\hat{\mathcal{F}}_g^{T,(n)}(N,\epsilon) = a_s^n \frac{1}{\epsilon^{2n-1}} \sum_{l=0}^{n-1} \frac{1}{N-1-2(n-l)\epsilon} \left(A_{T,g}^{(l,n)} + \epsilon B_{T,g}^{(l,n)} + \epsilon^2 C_{T,g}^{(l,n)} + \dots\right)$$

LL NLL NNLL

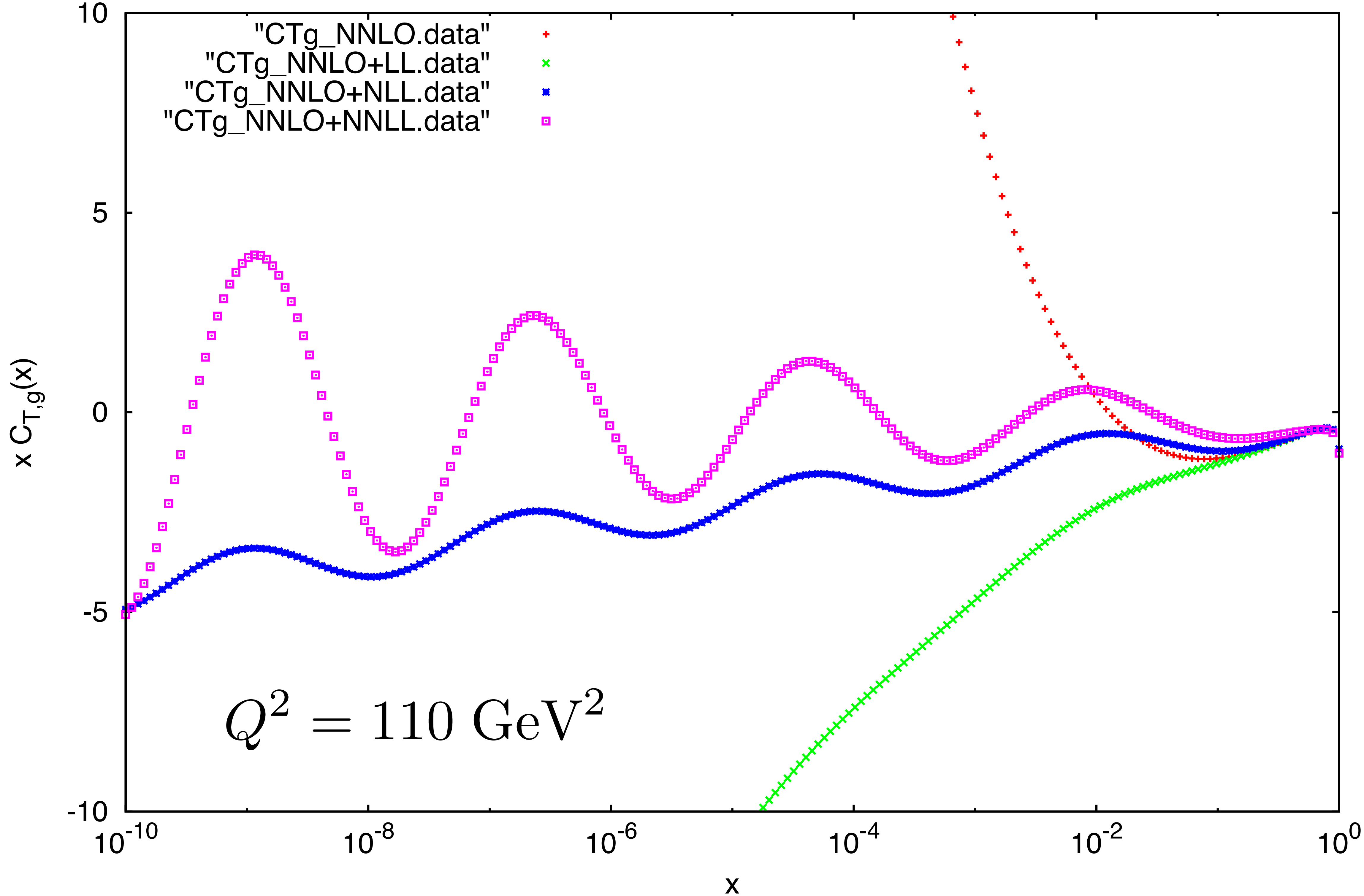
D) solve recursively order by order for $\bar{c}_{T,i}^{(n,k)}$ $P_{ij}^{(n-1)}$ $A_{T,g}^{(l,n)}$ $B_{T,g}^{(l,n)}$ $C_{T,g}^{(l,n)}$:

- **KLN - Cancellations**
- **fixed order calculation constrains**

→ System of equation solvable

E) From the coefficient of the small N expansion deduct closed form

C_g^T



"CTg_NNLO.data"
"CTg_NNLO+LL.data"
"CTg_NNLO+NLL.data"
"CTg_NNLO+NNLL.data"
10
5
0
-5
-10
x C_{T,g}(x)
10^{-10}
10^{-8}
10^{-6}
10^{-4}
10^{-2}
10^{0}
x
$Q^2 = 110\ \mathrm{GeV}^2$

RESUMMED SOLUTION FOR EVOLUTION

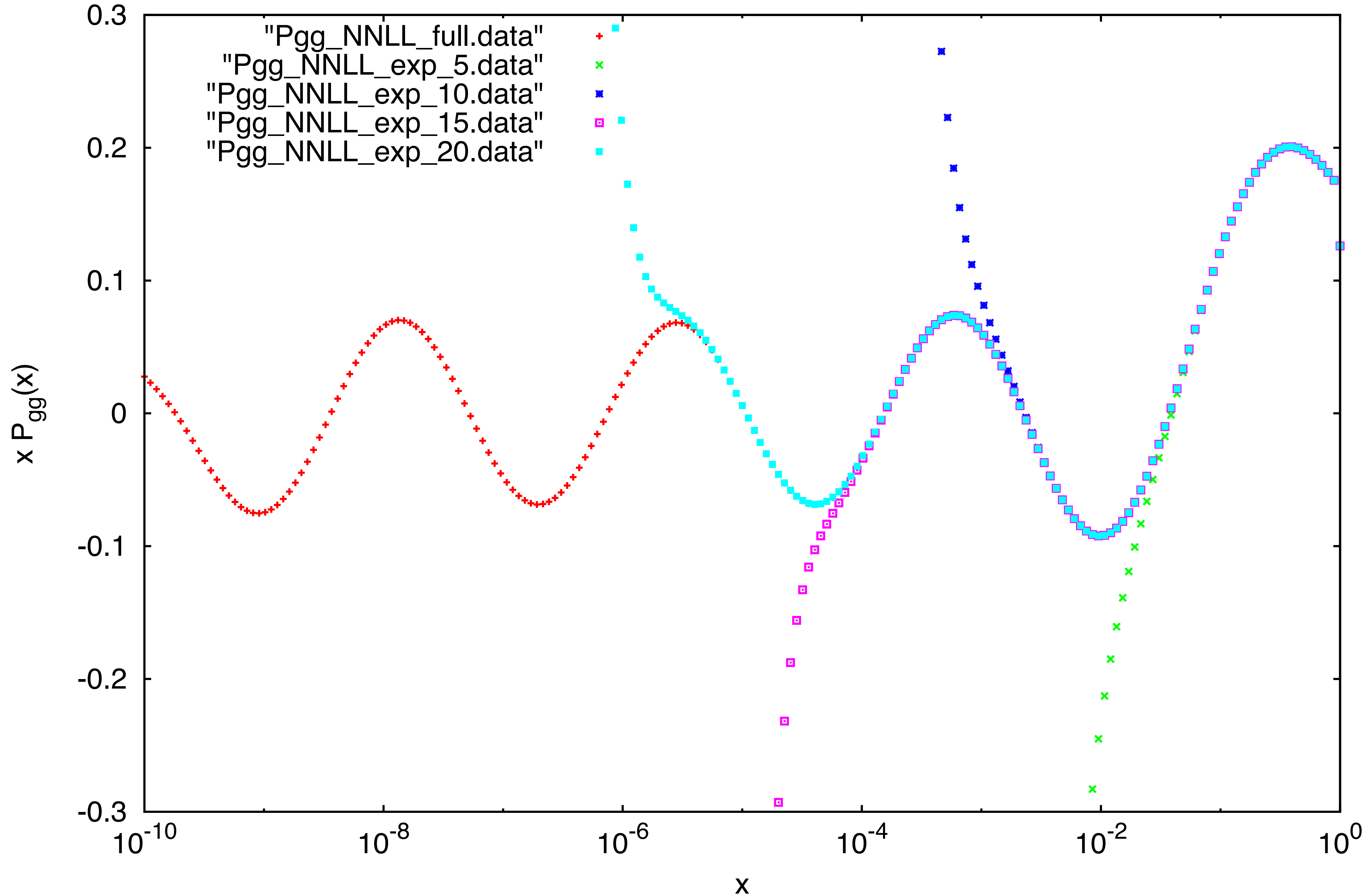
If NmLL accuracy then for k > m $\boldsymbol{P}^{T,(k)} = \boldsymbol{P}^{T,\mathrm{resum}}|_{a_s^k}$

$$
\begin{aligned}
\boldsymbol{q}(N,a_{\mathrm{s}}) &= \boldsymbol{U}(N,a_{\mathrm{s}})\,\boldsymbol{L}(N,a_{\mathrm{s}},a_0)\,\boldsymbol{U}^{-1}(N,a_0)\,\boldsymbol{q}(N,a_0) \\
&= \left[1+\sum_{k=1}^{\infty} a_{\mathrm{s}}^k\,\boldsymbol{U}_k(N)\right]\boldsymbol{L}(a_{\mathrm{s}},a_0,N)\left[1+\sum_{k=1}^{\infty} a_0^k\,\boldsymbol{U}_k(N)\right]^{-1}\boldsymbol{q}(a_0,N)
\end{aligned}
$$

$$
\boldsymbol{R}_0 \equiv \frac{1}{\beta_0}\boldsymbol{P}^{T,(0)}\,, \quad \boldsymbol{R}_k \equiv \frac{1}{\beta_0}\boldsymbol{P}^{T,(k)} - \sum_{i=1}^{k} b_i \boldsymbol{R}_{k-i}\,,
$$

$$
[\boldsymbol{U}_k,\boldsymbol{R}_0] = \boldsymbol{R}_k + \sum_{i=1}^{k-1}\boldsymbol{R}_{k-1}\boldsymbol{U}_i + k\boldsymbol{U}_k\,.
$$

How many terms?



RESUMMED SCALE DEPENDANCE

In SIA the dependance of the coefficient functions on the factorization scale μ_F can be expressed through the coefficients $c_{k,i}^{(l,m)}$

$$C_{k,i}(N, \alpha_s, \log(Q^2/\mu_F^2)) = c_{k,i}^{(0)}(N) + \sum_{l=1}^{\infty} \left(\frac{\alpha_s}{4\pi}\right)^l \left(c_{k,i}^{(l)}(N) + \sum_{m=1}^{l} c_{k,i}^{(l,m)}(N) \log^m \left(\frac{Q^2}{\mu_F^2}\right)\right)$$

which can be calculated order by order solving the renormalization group equation:

$$\left[\left\{ \frac{\partial}{\partial \log \mu_F^2} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s} \right\} \delta_{ij} - P_{ij}^T \right] C_{k,i}(N, \alpha_s, \log(Q^2/\mu_F^2)) = 0$$

$$k = L, T \qquad i = q, g$$

This leads to the following recursive formula for the coefficients $c_{k,i}^{(l,m)}$

$$c_{k,j}^{(l,m)} = \frac{1}{m} \sum_{w=m-1}^{l-1} c_{k,i}^{(w,m-1)} \left(P_{ij}^{T\;(l-w-1)} - w\beta_{l-w-1}\delta_{ij} \right)$$

Resummed Scale Dependance

Taking the small $\bar{N} = N - 1$ limit, one can write a closed form for the $\log^m\left(\frac{Q^2}{\mu_F^2}\right)$ dependance up to NNNLL

LL $$C_{k,i}^{LL}(N,\alpha_s) = c_{k,i}^{LL}(N,\alpha_s)$$

NLL $$C_{k,i}^{NLL}(N,\alpha_s,\log(Q^2/\mu_F^2)) = c_{k,i}^{NLL}(N,\alpha_s) + \log\left(\frac{Q^2}{\mu_F^2}\right)(c_{k,j}^{LL}\,P_{ji}^{T,\,LL})(N,\alpha_s)$$

NNLL $$C_{k,i}^{NNLL}(N,\alpha_s,\log(Q^2/\mu_F^2)) = c_{k,i}^{NNLL}(N,\alpha_s) + \log\left(\frac{Q^2}{\mu_F^2}\right)(c_{k,j}^{NLL}\,P_{ji}^{T,\,NLL})(N,\alpha_s)$$
$$+\log\left(\frac{Q^2}{\mu_F^2}\right)\left(\frac{\alpha_s}{4\pi}\right)^2\frac{\partial}{\partial(\alpha_s/4\pi)}c_{k,i}^{LL}(N,\alpha_s)$$
$$+\log^2\left(\frac{Q^2}{\mu_F^2}\right)(c_{k,j}^{LL}\,P_{jg}^{T,\,LL}\,P_{gi}^{T,\,LL})(N,\alpha_s)$$

NNNLL → Scale dependance given by NNLL quantities and 3 powers of $\log\left(\frac{Q^2}{\mu_F^2}\right)$

Outline

- Goal
- Time-Like Evolution
- Our E+E- NNLO Fit
- Improving: Small-z Resummation
- Conclusions & Outlook

Conclusions

- We have performed a NNLO fit for SIA exploring the new features appearing @ NNLO.
- At NNLO it is clear that the extreme phase space regions are better described and that more of the small z and large z logs are taken into account
- We are working on a resummed version of our fit and looking to extend our analysis to an approximate NNLO global fit using expanded NLL results
- We have presented our on-going work towards a resummed FF fit including small-z resummation including its extension to the scale dependance

Thanks for Your Attention

Any questions?