EBERHARD KARLS UNIVERSITÄT TÜBINGEN

FACULTY OF SCIENCE
Institute for Theoretical Physics

QCD-N'16

# In-jet Fragmentation

in collaboration with Asmita Mukherjee and Werner Vogelsang

[Kaufmann, Mukherjee, Vogelsang: PRD92:054015 & PRD93:114021]

11.07.2016, Tom Kaufmann

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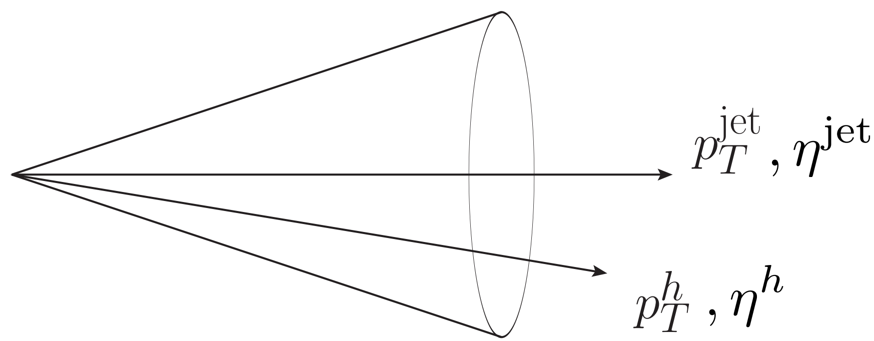
# Outline

- **Overview**
- Calculation of the cross section
- Numerical Results
- Photon-in-jet
- Summary & Outlook

# Process of interest

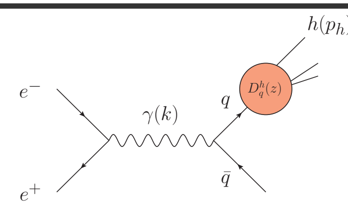
$$pp \to (\mathrm{jet}\, h)\, X$$

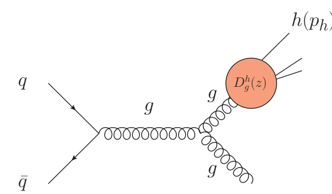
where the hadron is observed INSIDE a fully reconstructed jet and is part of the jet

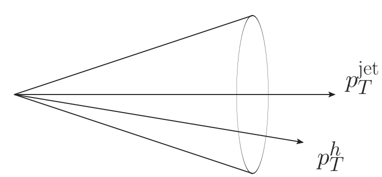



- Fragmentation functions
- Jet (sub-)structure
- Spin correlations (Collins effect)
- pA and AA scattering $\rightsquigarrow$ "jet quenching"

# Determination of FFs

<table>
<tr><th>Process</th><th>$D_g^h$ ?</th><th>Direct scan?</th></tr>
<tr><td>$e^+e^- \to hX$<br>$ep \to ehX$</td><td></td><td> 

$z = \frac{2p_h \cdot k}{k^2}$</td></tr>
<tr><td>$pp \to hX$</td><td></td><td> 

$\hat{p}_T = \frac{p_T^h}{z_c}$ $\quad z_c^{\min} = \frac{2p_T^h}{\sqrt{S}} \cosh\eta$<br>$d\sigma \propto \int_{z_c^{\min}}^1 \frac{dz_c}{z_c^2} \left[ f_a \otimes f_b \otimes d\hat{\sigma}_{ab}^c(\hat{p}_T, \dots) \right] D_c^h(z_c)$</td></tr>
<tr><td>$pp \to (\text{jet}\, h)\, X$</td><td></td><td> 

$z = z_h \equiv \frac{p_T^h}{p_T^{\text{jet}}}$</td></tr>
</table>

# A Warning…

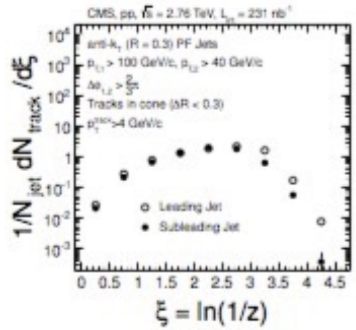
Description of hadron-jet momentum correlation is not unique! Some possible definitions:

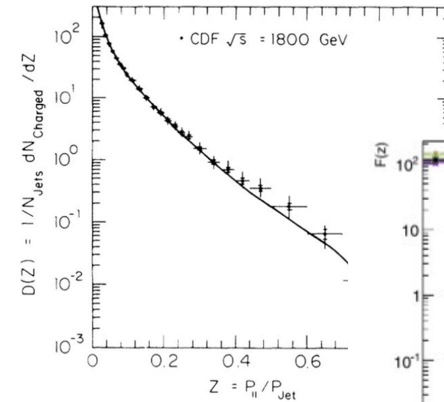
$$z_h = \ ?$$

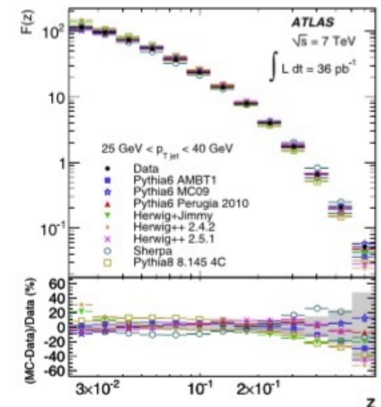
$$\frac{p_T}{p_T^{\mathrm{jet}}} \qquad \frac{\vec{p}_T \cdot \vec{p}_T^{\,\mathrm{jet}}}{|\vec{p}_T^{\,\mathrm{jet}}|^2} \qquad \frac{p_T}{p_T^{\mathrm{jet}}} \cos \Delta R$$

$$\Delta R \equiv \sqrt{\Delta\eta^2 + \Delta\phi^2}$$

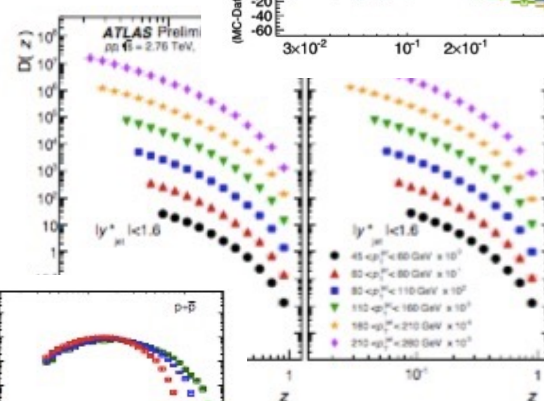
# What's available - Experiment



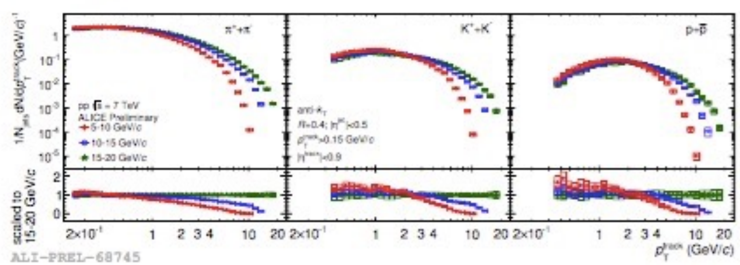



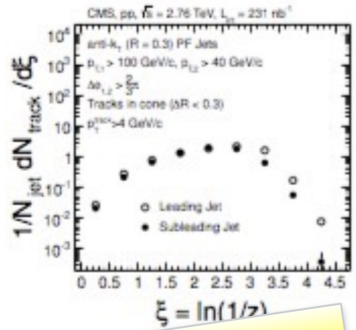

- Tevatron
  - CDF: PRL65(1990)
- LHC
  - ATLAS: EPJ C71(2011), Preliminary ATLAS-CONF-2015-022
  - CMS: JHEP10(2012)
  - ALICE: arXiv:1408.5723, Nucl.Phys. A931 (2014)

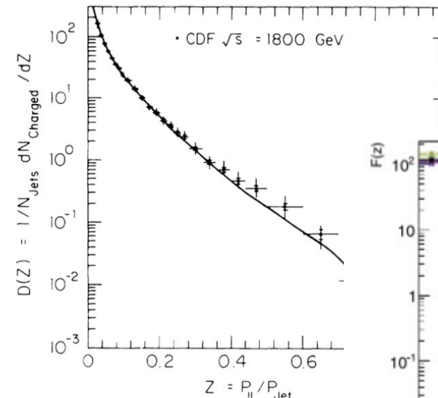


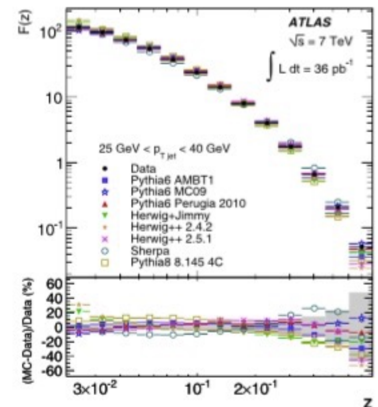

- RHIC
  - STAR: in progress…



# What's available - Experiment




- Tevatron
  - CDF ... 90)

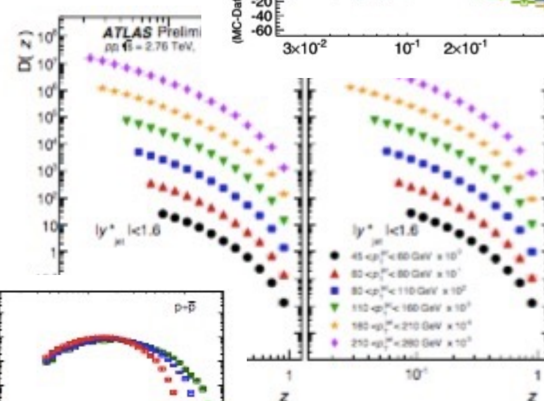
dijet sample

- LHC
  - ATLAS: EPJ C71(2011) ... ATLAS-CONF-2015-022
  - CMS ... 2012)
  - ALICE: arXiv:140... Phys. A931 (2014)

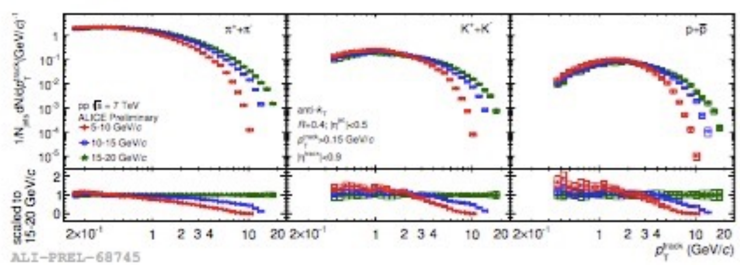
unidentified hadrons

dijet sample

charged jets




- RHIC
  - STAR: in progress…



# What's available - Theory

- $e^+e^-$ collisions:
  - Jain, Procura, Waalewijn; JHEP05(2011) — SCET
  - Procura, Waalewijn; PRD85(2012) — SCET
  - …
- $pp$ collisions:
  - Arleo, Fontannaz, Guillet, Nguyen; JHEP04(2014) — MC@NLO
  - Ritzmann, Waalewijn; PRD90(2014) — SCET@NNLO
  - Kaufmann, Mukherjee, Vogelsang; PRD92(2015) — NJA@NLO
  - Chien, Kang, Ringer, Vitev, Xing; JHEP05(2016) — SCET@LO+NLL,R
  - Kang, Ringer, Vitev; arXiv:1606.07063 — SCET@NLO+NLL,R
  - Dai, Kim, Leibovich; arXiv:1606.07411 — SCET@NLO+NLL,R
  - …

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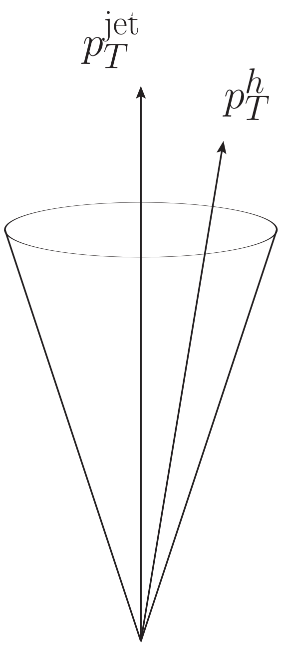
# Calculation of the cross section

[Kaufmann, Mukherjee, Vogelsang: PRD92:054015]

- NLO cross section for the process $pp \to (\text{jet}\, h)\, X$
- differential in jet's transverse momentum $p_T^{\text{jet}}$ and rapidity $\eta^{\text{jet}}$ , and the variable

$$z_h \equiv \frac{p_T^h}{p_T^{\text{jet}}}$$



# Narrow Jet Approximation (NJA)

[Jäger, Stratmann, Vogelsang: PRD70:034010; Mukherjee, Vogelsang: PRD86:094009; Kaufmann, Mukherjee, Vogelsang: PRD91:034001]

- parameter which characterizes size of a jet

cone

$$(\phi_J - \phi_i)^2 + (\eta_J - \eta_i)^2 \leq R^2$$

(anti-) kT

$$d_{ij} = \min\left((k_T^i)^{2p}, (k_T^j)^{2p}\right) \frac{(\phi_i - \phi_j)^2 + (\eta_i - \eta_j)^2}{R^2}$$

$$J_\beta(p) \equiv E - \beta \frac{m^2}{E} \,, \quad \beta > 1$$

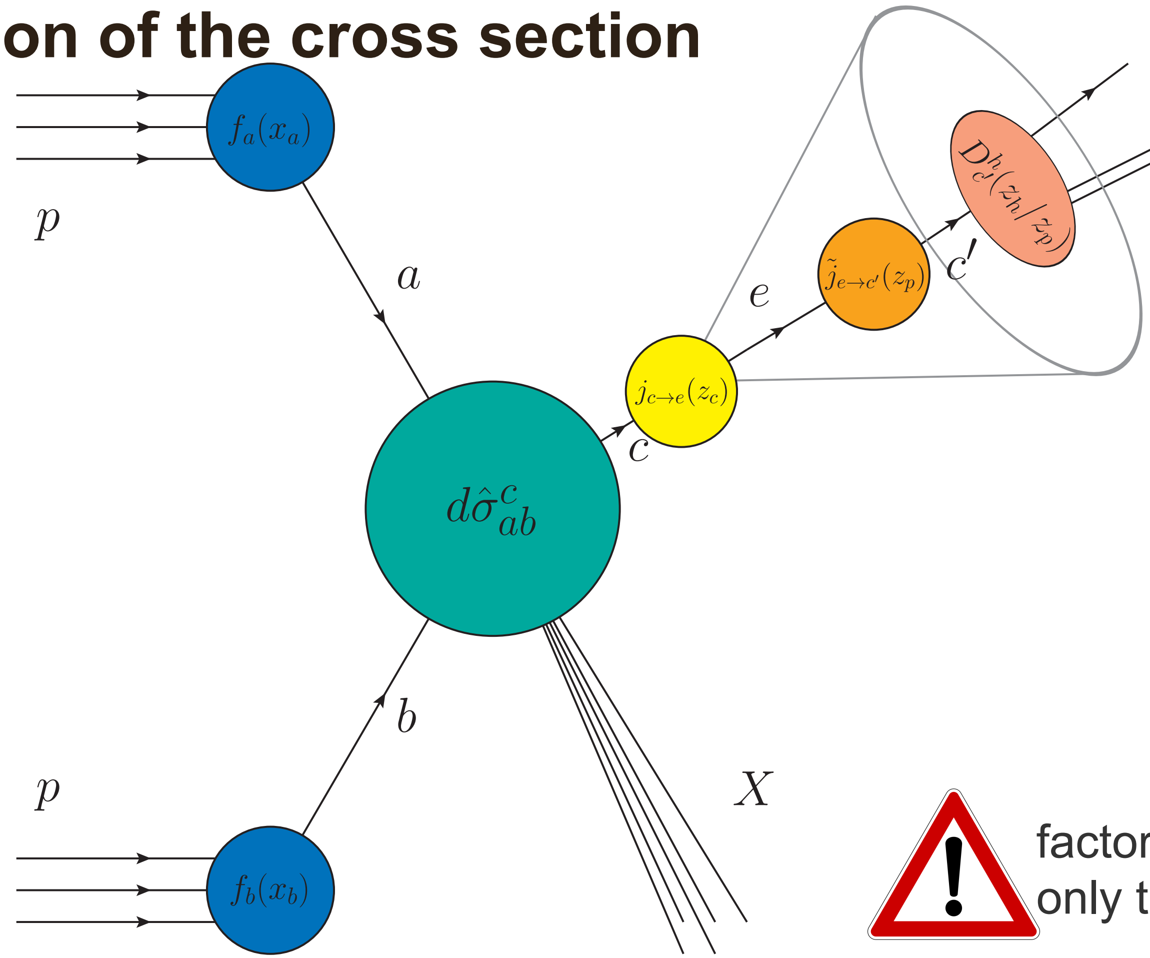
$J_{E_T}$

- interested in rather narrow jets

$$\mathcal{R} \equiv R, \frac{1}{\beta} \qquad \mathcal{R} \to 0$$

- allows analytical calculation of partonic cross sections
- cross section $d\hat{\sigma} \propto \mathcal{A} \log \mathcal{R} + \mathcal{B} + \mathcal{O}(\mathcal{R}^2)$
- good approximation up to $\mathcal{R} \approx 0.7$

# Calculation of the cross section




factorization “proved” only to NLO!

$$d\sigma \propto \left( \sum_{a,b,c} f_a \otimes f_b \otimes d\hat{\sigma}^c_{ab} \otimes \sum_e j_{c\to e} \right) \times \left( \sum_{c'} \tilde{j}_{e\to c'} \otimes D^h_{c'} \right)$$

# Calculation of the cross section

$$
\begin{aligned}
\frac{d\sigma^{H_1 H_2 \to (\mathrm{jet}\, h) X}}{dp_T^{\mathrm{jet}} d\eta^{\mathrm{jet}} dz_h} &= \frac{2 p_T^{\mathrm{jet}}}{S} \sum_{a,b,c} \int_{x_a^{\min}}^{1} \frac{dx_a}{x_a} f_a^{H_1}(x_a, \mu_F) \int_{x_b^{\min}}^{1} \frac{dx_b}{x_b} f_b^{H_2}(x_b, \mu_F) \\
&\times \int_{z_c^{\min}}^{1} \frac{dz_c}{z_c^2} \frac{d\hat{\sigma}_{ab}^{c}(\hat{s}, \hat{p}_T, \hat{\eta}, \mu_F, \mu'_F, \mu_R)}{v dv dw} \sum_e j_{c\to e}\left(z_c, \frac{\mathcal{R}\, p_T^{\mathrm{jet}}}{\mu'_F}, \mu_R\right) \\
&\times \sum_{c'} \int_{z_h}^{1} \frac{dz_p}{z_p} \tilde{j}_{e\to c'}\left(z_p, \frac{\mathcal{R}\, p_T^{\mathrm{jet}}}{\mu''_F}, \mu_R\right) D_{c'}^{h}\left(\frac{z_h}{z_p}, \mu''_F\right)
\end{aligned}
$$

# Calculation of the cross section

$$\frac{d\sigma^{H_1H_2\to(\text{jet}\,h)X}}{dp_T^{\text{jet}}d\eta^{\text{jet}}dz_h} = \frac{2p_T^{\text{jet}}}{S}\sum_{a,b,c}\int_{x_a^{\min}}^1 \frac{dx_a}{x_a} f_a^{H_1}(x_a,\mu_F)\int_{x_b^{\min}}^1 \frac{dx_b}{x_b} f_b^{H_2}(x_b,\mu_F)$$

$$\times \int_{z_c^{\min}}^1 \frac{dz_c}{z_c^2} \boxed{\frac{d\hat{\sigma}_{ab}^c(\hat{s},\hat{p}_T,\hat{\eta},\mu_F,\mu_F',\mu_R)}{vdvdw}} \sum_e j_{c\to e}\left(z_c, \frac{\mathcal{R}\,p_T^{\text{jet}}}{\mu_F'},\mu_R\right)$$

$$\times \sum_{c'}\int_{z_h}^1 \frac{dz_p}{z_p}\tilde{j}_{e\to c'}\left(z_p,\frac{\mathcal{R}\,p_T^{\text{jet}}}{\mu_F''},\mu_R\right) D_{c'}^h\left(\frac{z_h}{z_p},\mu_F''\right)$$

single-inclusive
parton cross section

[Jäger,Stratmann,Vogelsang; PRD67(2003)]
[Aversa,Chiappetta,Greco,Guillet; Nucl.Phys.B327(1989)]

# Calculation of the cross section

$$\frac{d\sigma^{H_1H_2\to(\text{jet}\,h)X}}{dp_T^{\text{jet}}\,d\eta^{\text{jet}}\,dz_h} = \frac{2p_T^{\text{jet}}}{S}\sum_{a,b,c}\int_{x_a^{\min}}^1 \frac{dx_a}{x_a} f_a^{H_1}(x_a,\mu_F)\int_{x_b^{\min}}^1 \frac{dx_b}{x_b} f_b^{H_2}(x_b,\mu_F)$$

$$\times \int_{z_c^{\min}}^1 \frac{dz_c}{z_c^2}\,\frac{d\hat{\sigma}_{ab}^c(\hat{s},\hat{p}_T,\hat{\eta},\mu_F,\mu_F',\mu_R)}{v\,dv\,dw}\,\boxed{\sum_e j_{c\to e}\left(z_c,\frac{\mathcal{R}\,p_T^{\text{jet}}}{\mu_F'},\mu_R\right)}$$

$$\times \sum_{c'}\int_{z_h}^1 \frac{dz_p}{z_p}\,\tilde{j}_{e\to c'}\left(z_p,\frac{\mathcal{R}\,p_T^{\text{jet}}}{\mu_F''},\mu_R\right) D_{c'}^h\left(\frac{z_h}{z_p},\mu_F''\right)$$

“jet-function”: formation of jet with $p_{\text{jet}} = z_c p_c$

# Calculation of the cross section

$$\frac{d\sigma^{H_1 H_2 \to (\text{jet}\, h) X}}{dp_T^{\text{jet}} d\eta^{\text{jet}} dz_h} = \frac{2 p_T^{\text{jet}}}{S} \sum_{a,b,c} \int_{x_a^{\min}}^{1} \frac{dx_a}{x_a} f_a^{H_1}(x_a, \mu_F) \int_{x_b^{\min}}^{1} \frac{dx_b}{x_b} f_b^{H_2}(x_b, \mu_F)$$

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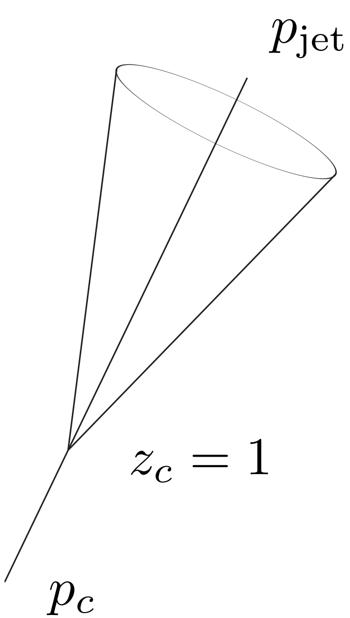


“jet-function”: formation of jet with $p_{\text{jet}} = z_c p_c$

# Calculation of the cross section

$$\frac{d\sigma^{H_1 H_2 \to (\text{jet}\, h) X}}{dp_T^{\text{jet}}\, d\eta^{\text{jet}}\, dz_h} = \frac{2p_T^{\text{jet}}}{S} \sum_{a,b,c} \int_{x_a^{\min}}^{1} \frac{dx_a}{x_a} f_a^{H_1}(x_a, \mu_F) \int_{x_b^{\min}}^{1} \frac{dx_b}{x_b} f_b^{H_2}(x_b, \mu_F)$$

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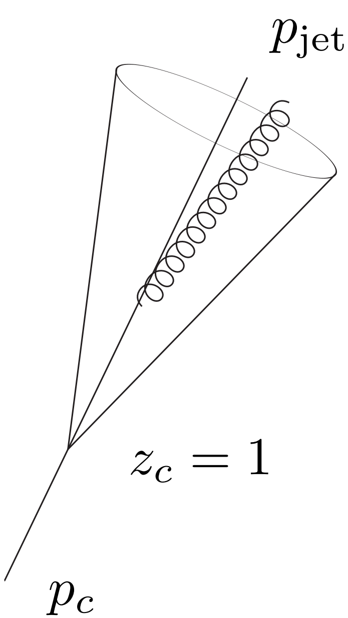


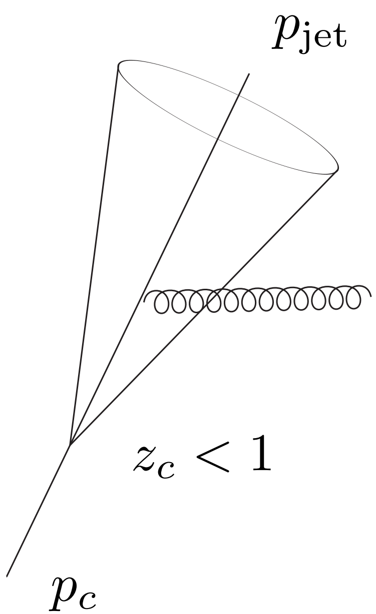

“jet-function”: formation of jet with $p_{\text{jet}} = z_c p_c$

# Calculation of the cross section

$$\frac{d\sigma^{H_1 H_2 \to (\text{jet}\, h) X}}{dp_T^{\text{jet}}\, d\eta^{\text{jet}}\, dz_h} = \frac{2 p_T^{\text{jet}}}{S} \sum_{a,b,c} \int_{x_a^{\min}}^{1} \frac{dx_a}{x_a} f_a^{H_1}(x_a, \mu_F) \int_{x_b^{\min}}^{1} \frac{dx_b}{x_b} f_b^{H_2}(x_b, \mu_F)$$

$$\times \int_{z_c^{\min}}^{1} \frac{dz_c}{z_c^2} \frac{d\hat{\sigma}_{ab}^{c}(\hat{s}, \hat{p}_T, \hat{\eta}, \mu_F, \mu_F', \mu_R)}{v dv dw} \boxed{\sum_e j_{c \to e}\left(z_c, \frac{\mathcal{R}\, p_T^{\text{jet}}}{\mu_F'}, \mu_R\right)}$$

$$\times \sum_{c'} \int_{z_h}^{1} \frac{dz_p}{z_p} \tilde{j}_{e \to c'}\left(z_p, \frac{\mathcal{R}\, p_T^{\text{jet}}}{\mu_F''}, \mu_R\right) D_{c'}^{h}\left(\frac{z_h}{z_p}, \mu_F''\right)$$

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“jet-function”: partonic fragmentation, $p_{c'} = z_p p_{\text{jet}}$

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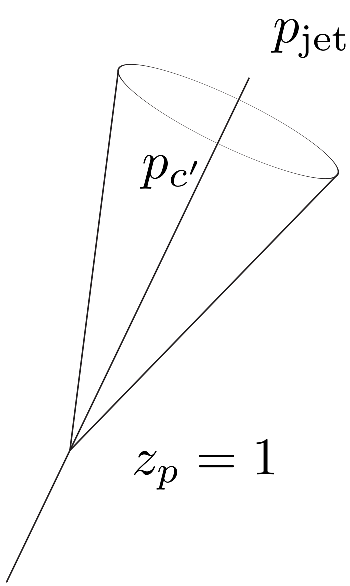


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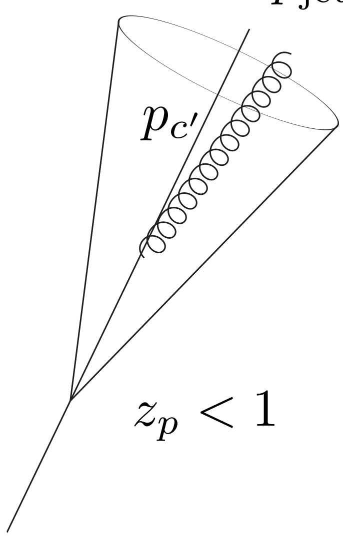


$$\times \int_{z_c^{\min}}^1 \frac{dz_c}{z_c^2}\,\frac{d\hat\sigma^c_{ab}(\hat s,\hat p_T,\hat\eta,\mu_F,\mu_F',\mu_R)}{v\,dv\,dw}\sum_e j_{c\to e}\left(z_c,\frac{\mathcal{R}\,p_T^{\text{jet}}}{\mu_F'},\mu_R\right)$$

$$\times \boxed{\sum_{c'}\int_{z_h}^1\frac{dz_p}{z_p}\,\tilde{j}_{e\to c'}\left(z_p,\frac{\mathcal{R}\,p_T^{\text{jet}}}{\mu_F''},\mu_R\right)}\, D_{c'}^h\left(\frac{z_h}{z_p},\mu_F''\right)$$

"jet-function": partonic fragmentation, $p_{c'} = z_p p_{\text{jet}}$

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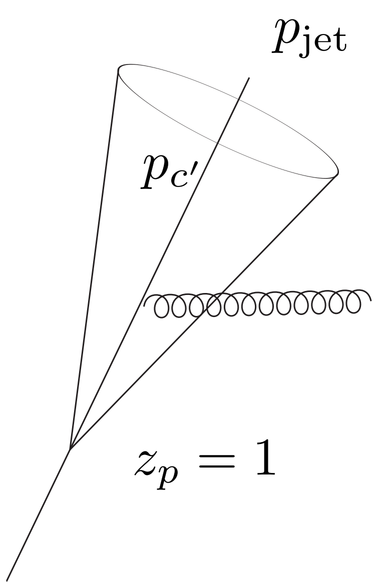


"jet-function": partonic fragmentation, $p_{c'} = z_p p_{\text{jet}}$

# Leading order

$$\frac{d\sigma^{H_1 H_2 \to (\text{jet}\, h) X}}{dp_T^{\text{jet}} d\eta^{\text{jet}} dz_h} = \frac{2 p_T^{\text{jet}}}{S} \sum_{a,b,c} \int_{x_a^{\min}}^{1} \frac{dx_a}{x_a} f_a^{H_1}(x_a, \mu_F) \int_{x_b^{\min}}^{1} \frac{dx_b}{x_b} f_b^{H_2}(x_b, \mu_F)$$

$$\times \int_{z_c^{\min}}^{1} \frac{dz_c}{z_c^2} \frac{d\hat{\sigma}_{ab}^{c}(\hat{s}, \hat{p}_T, \hat{\eta}, \mu_F, \mu_F', \mu_R)}{v dv dw} \sum_e j_{c \to e}\left(z_c, \frac{\mathcal{R}\, p_T^{\text{jet}}}{\mu_F'}, \mu_R\right)$$

$$\times \sum_{c'} \int_{z_h}^{1} \frac{dz_p}{z_p} \tilde{j}_{e \to c'}\left(z_p, \frac{\mathcal{R}\, p_T^{\text{jet}}}{\mu_F''}, \mu_R\right) D_{c'}^{h}\left(\frac{z_h}{z_p}, \mu_F''\right)$$

# Leading order

$$
\begin{aligned}
\frac{d\sigma^{H_1 H_2 \to (\text{jet}\, h) X}}{dp_T^{\text{jet}}\, d\eta^{\text{jet}}\, dz_h} = {} & \frac{2p_T^{\text{jet}}}{S} \sum_{a,b,c} \int_{x_a^{\min}}^{1} \frac{dx_a}{x_a} f_a^{H_1}(x_a, \mu_F) \int_{x_b^{\min}}^{1} \frac{dx_b}{x_b} f_b^{H_2}(x_b, \mu_F) \\
\times & \int_{z_c^{\min}}^{1} \frac{dz_c}{z_c^2} \frac{d\hat{\sigma}_{ab}^{c}(\hat{s}, \hat{p}_T, \hat{\eta}, \mu_F, \mu'_F, \mu_R)}{v\,dv\,dw} \sum_e \underbrace{\boxed{j_{c\to e}\left(z_c, \frac{\mathcal{R}\, p_T^{\text{jet}}}{\mu'_F}, \mu_R\right)}}_{\delta_{ce}\delta(1-z_c)} \\
\times & \sum_{c'} \int_{z_h}^{1} \frac{dz_p}{z_p} \underbrace{\boxed{\tilde{j}_{e\to c'}\left(z_p, \frac{\mathcal{R}\, p_T^{\text{jet}}}{\mu''_F}, \mu_R\right)}}_{\delta_{ec'}\delta(1-z_p)} D_{c'}^{h}\left(\frac{z_h}{z_p}, \mu''_F\right)
\end{aligned}
$$

# Leading order

$$\left.\frac{d\sigma^{H_1 H_2 \to (\text{jet}\, h) X}}{dp_T^{\text{jet}}\, d\eta^{\text{jet}}\, dz_h}\right|_{\text{LO}} = \frac{2p_T^{\text{jet}}}{S} \sum_{a,b,c} \int_{x_a^{\min}}^{1} \frac{dx_a}{x_a} f_a^{H_1}(x_a, \mu_F) \int_{x_b^{\min}}^{1} \frac{dx_b}{x_b} f_b^{H_2}(x_b, \mu_F)$$

$$\times \frac{d\hat{\sigma}_{ab}^{c,\text{Born}}(\hat{s}, \hat{p}_T, \hat{\eta}, \mu_F, \mu_F', \mu_R)}{v dv dw} \times D_c^h(z_h, \mu_F'')$$

$$\propto D_c^h(z_h, \mu_F'') \,.$$

# pQCD vs SCET

- Consistent at NLO
- Instead of the factorized structure with $j_{c\to e}(z_c)$ and $\tilde{j}_{e\to c'}(z_p)$ consider the combinations

$$\mathcal{K}_{c\to c'}(z, z_p; \lambda, \kappa) \equiv \sum_e j_{c\to e}(z, \lambda)\, \tilde{j}_{e\to c'}(z_p, \kappa)$$

[Dai,Kim,Leibovich; arXiv:1606.07411]
[Kang,Ringer,Vitev; arXiv:1606.06732]

- Same functions found in SCET $\mathcal{J}_{cc'} = \mathcal{K}_{c\to c'}$
- semi-inclusive fragmenting jet functions (FJFs)

$$\mathcal{G}_q^h(z, z_h, \omega_J, \mu_\mathcal{G}) = \int_{z_h}^1 \frac{dz'_h}{z'_h} \left[\mathcal{J}_{qq}(z, z'_h, \omega_J, \mu_\mathcal{G}) D_q^h\left(\frac{z_h}{z'_h}, \mu_\mathcal{G}\right) + \mathcal{J}_{qg}(z, z'_h, \omega_J, \mu_\mathcal{G}) D_g^h\left(\frac{z_h}{z'_h}, \mu_\mathcal{G}\right)\right]$$

$$\mathcal{G}_g^h(z, z_h, \omega_J, \mu_\mathcal{G}) = \int_{z_h}^1 \frac{dz'_h}{z'_h} \left[\mathcal{J}_{gg}(z, z'_h, \omega_J, \mu_\mathcal{G}) D_g^h\left(\frac{z_h}{z'_h}, \mu_\mathcal{G}\right) + \sum_{i=q,\bar{q}} \mathcal{J}_{gi}(z, z'_h, \omega_J, \mu_\mathcal{G}) D_i^h\left(\frac{z_h}{z'_h}, \mu_\mathcal{G}\right)\right]$$

# pQCD vs SCET

[Kang,Ringer,Vitev; arXiv:1606.06732]

- DGLAP like evolution equations in $(z,\mu)$-space. $z_h$ is unaffected

$$\frac{d}{d\log\mu^2}\begin{pmatrix}\mathcal{G}_S^h(z,z_h,\omega_J,\mu)\\ \mathcal{G}_g^h(z,z_h,\omega_J,\mu)\end{pmatrix} = \frac{\alpha_s(\mu)}{2\pi}\begin{pmatrix}P_{qq}(z) & 2N_f P_{gq}(z)\\ P_{qg}(z) & P_{gg}(z)\end{pmatrix}\otimes\begin{pmatrix}\mathcal{G}_S^h(z,z_h,\omega_J,\mu)\\ \mathcal{G}_g^h(z,z_h,\omega_J,\mu)\end{pmatrix}$$

with the singlet given by

$$\mathcal{G}_S^h(z,z_h,\omega_J,\mu) = \sum_{i=q,\bar{q}}\mathcal{G}_i^h(z,z_h,\omega_J,\mu) = 2N_f\mathcal{G}_q^h(z,z_h,\omega_J,\mu)$$

- $\Rightarrow$ $\log\mathcal{R}$ resummation
- Beyond LL: techniques analogue to evolution of FFs with NLO kernels, i.e. expansion around LO solution (PEGASUS)

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- $\Rightarrow$ $\log\mathcal{R}$ resummation
- Beyond LL: tec… …ue to evolution of FFs with NLO kernels, i.e. expansion ar… …(PEGASUS)

Details in talk by Daniele Anderle

# Outline

- Overview
- Calculation of the cross section
- **Numerical Results**
- Photon-in-jet
- Summary & Outlook

# Numerical Results

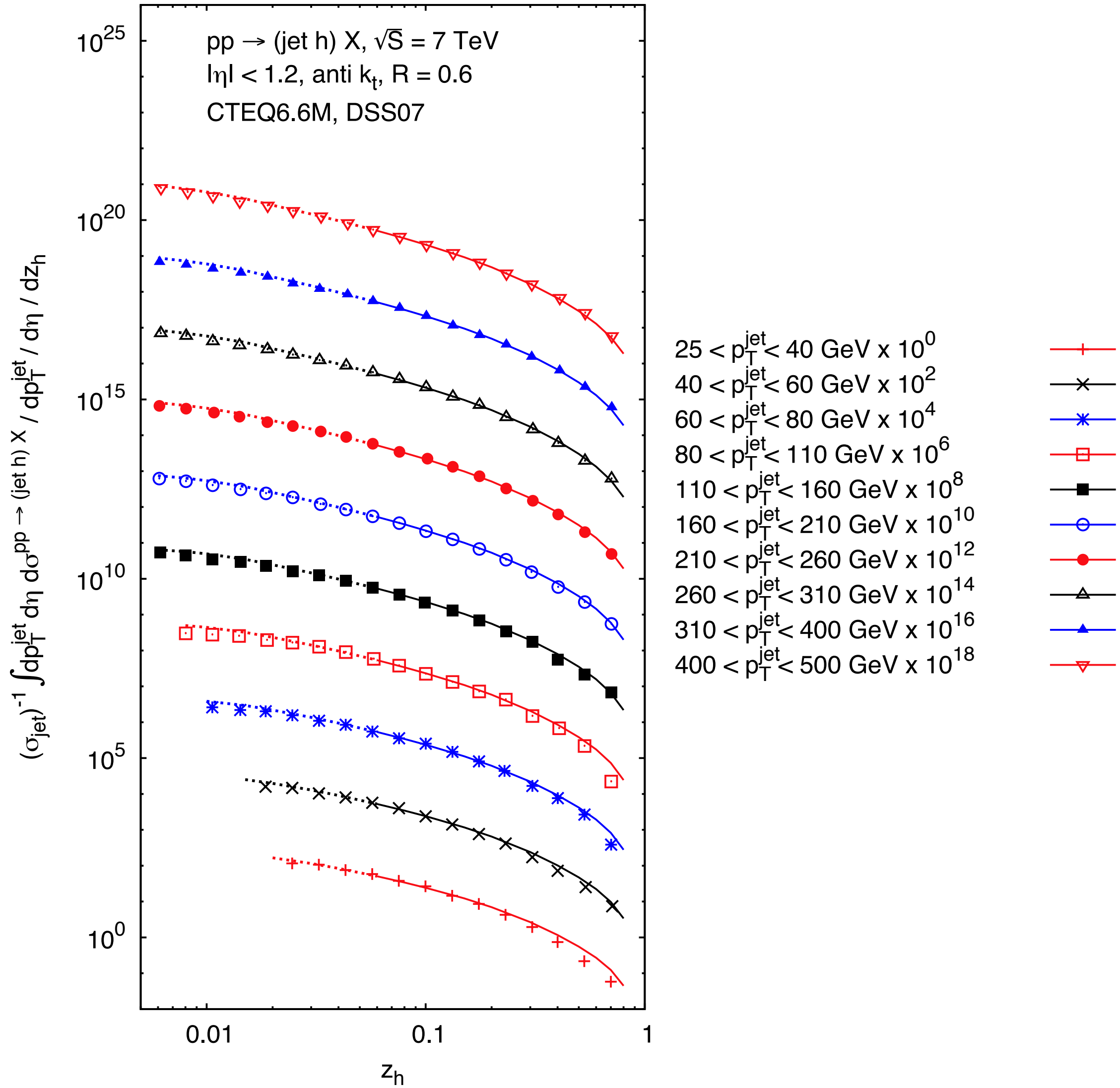
- Usually, cross section differential in $z_h$ and normalized to the single-inclusive jet spectrum, i.e.

$$F(z_h) \equiv \left( \int dp_T^{\rm jet} d\eta^{\rm jet} \frac{d\sigma^{{\rm jet}\, X}}{dp_T^{\rm jet} d\eta^{\rm jet}} \right)^{-1} \times \int dp_T^{\rm jet} d\eta^{\rm jet} \frac{d\sigma^{({\rm jet}\, h) X}}{dp_T^{\rm jet} d\eta^{\rm jet} dz_h}$$

- Many theoretical uncertainties cancel out, e.g. choice of PDFs or initial state factorization scale

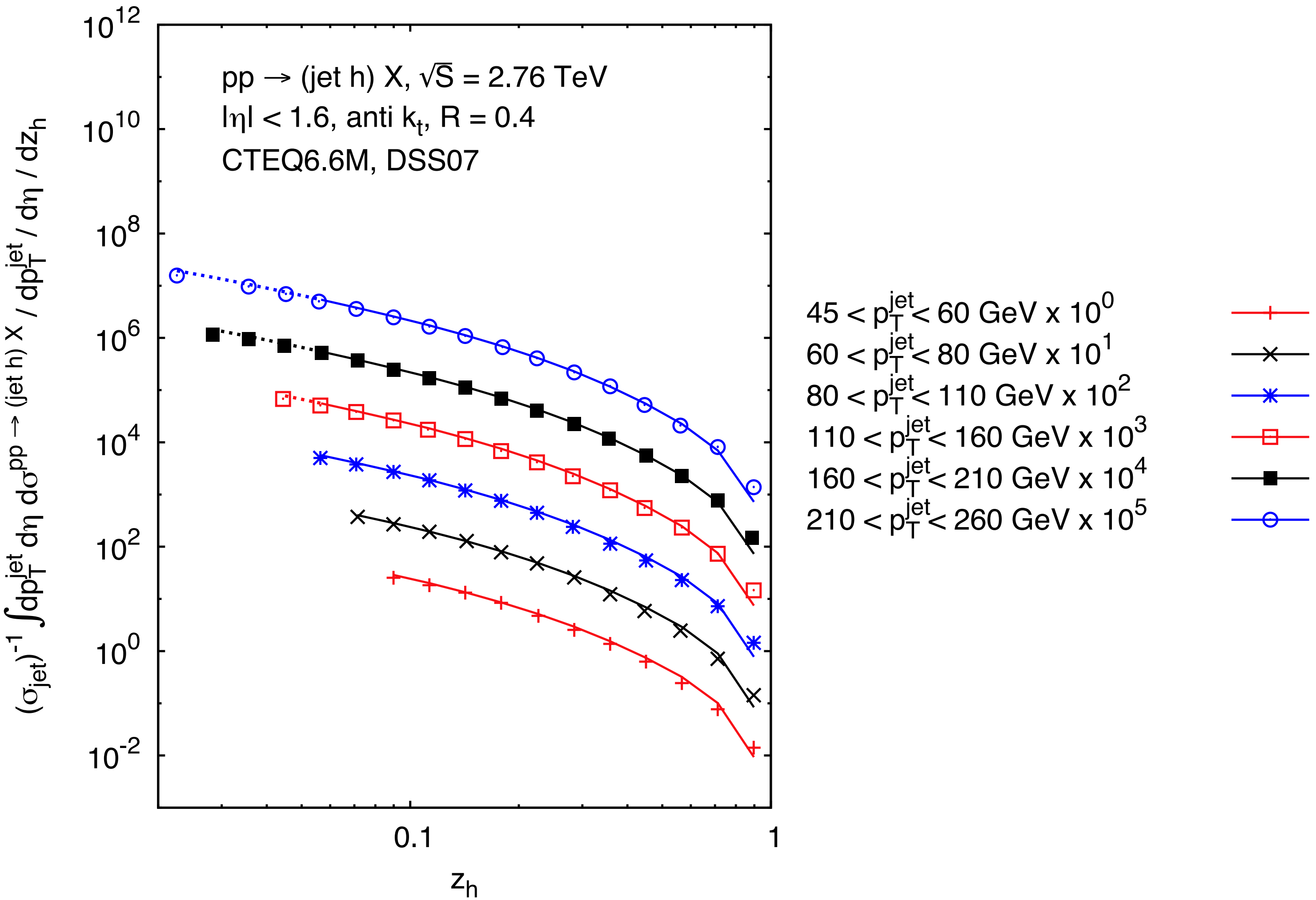
# Numerical Results

[Aad et al: Eur.Phys.J. C71,1795 (2011)]



# Numerical Results

[ATLAS-CONF-2015-022]



# Outline

- Overview
- Calculation of the cross section
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- **Photon-in-jet**
- Summary & Outlook

# Photon-in-Jet

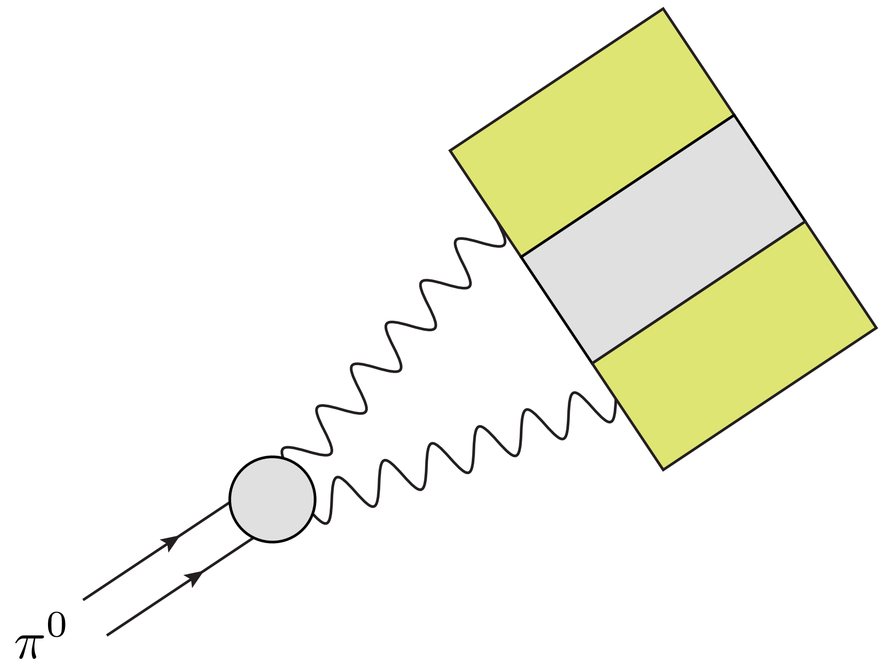
[Kaufmann, Mukherjee, Vogelsang: PRD93:114021]

- Poorly known photon fragmentation functions $D_c^\gamma(z,\mu_F)$
- process $pp \to (\gamma\ \mathrm{jet})X$
- "democratic jet algorithm": photon is part of the jet!
- additional direct part, however not @LO for $z_\gamma < 1$

$$d\sigma = \sum_{a,b} f_a \otimes f_b \otimes \Big[ \underbrace{d\hat{\sigma}^\gamma_{ab}}_{\text{direct}} + \sum_c \underbrace{d\hat{\sigma}^c_{ab} \otimes D^\gamma_c}_{\text{fragmentation}} \Big]$$

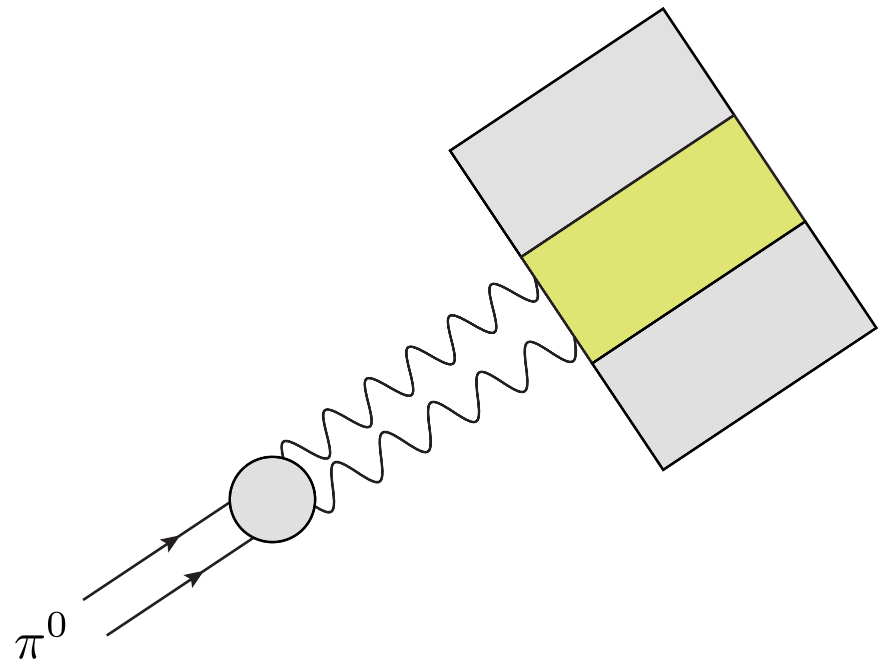
# Background $\pi^0 \to \gamma\gamma \quad (\approx 98.8\%)$

- As long as both photons are detected, pion may be reconstructed.



# Background $\pi^0 \to \gamma\gamma \qquad (\approx 98.8\%)$

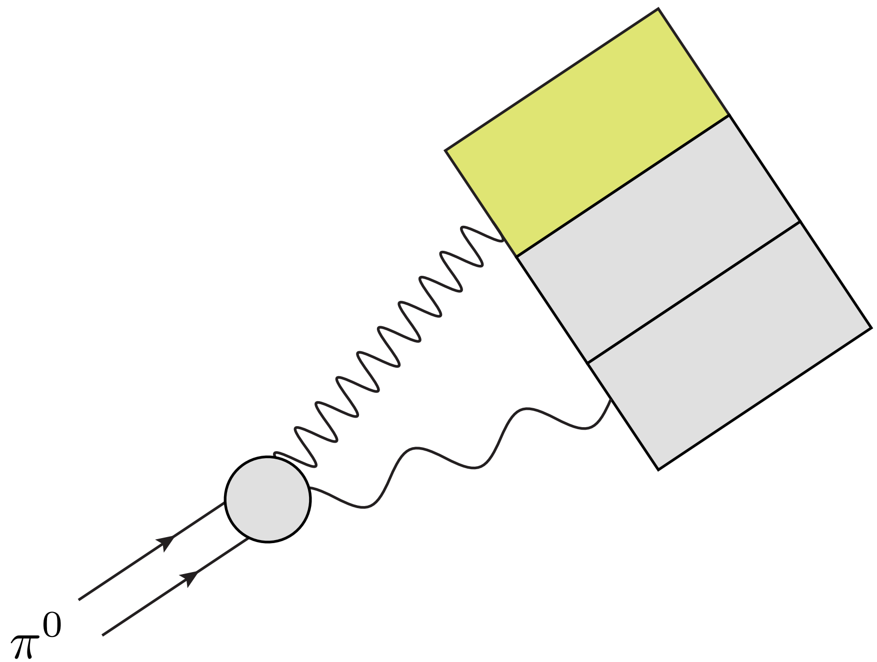
- 2 collinear photons mimic 1 e.m. signal




- with $\delta\eta = \delta\phi \approx 0.01$ relevant for $p_T^\pi > 20\,\mathrm{GeV}$

# Background $\pi^0 \to \gamma\gamma \quad (\approx 98.8\%)$

- Decay asymmetric in energy, one photon below energy threshold



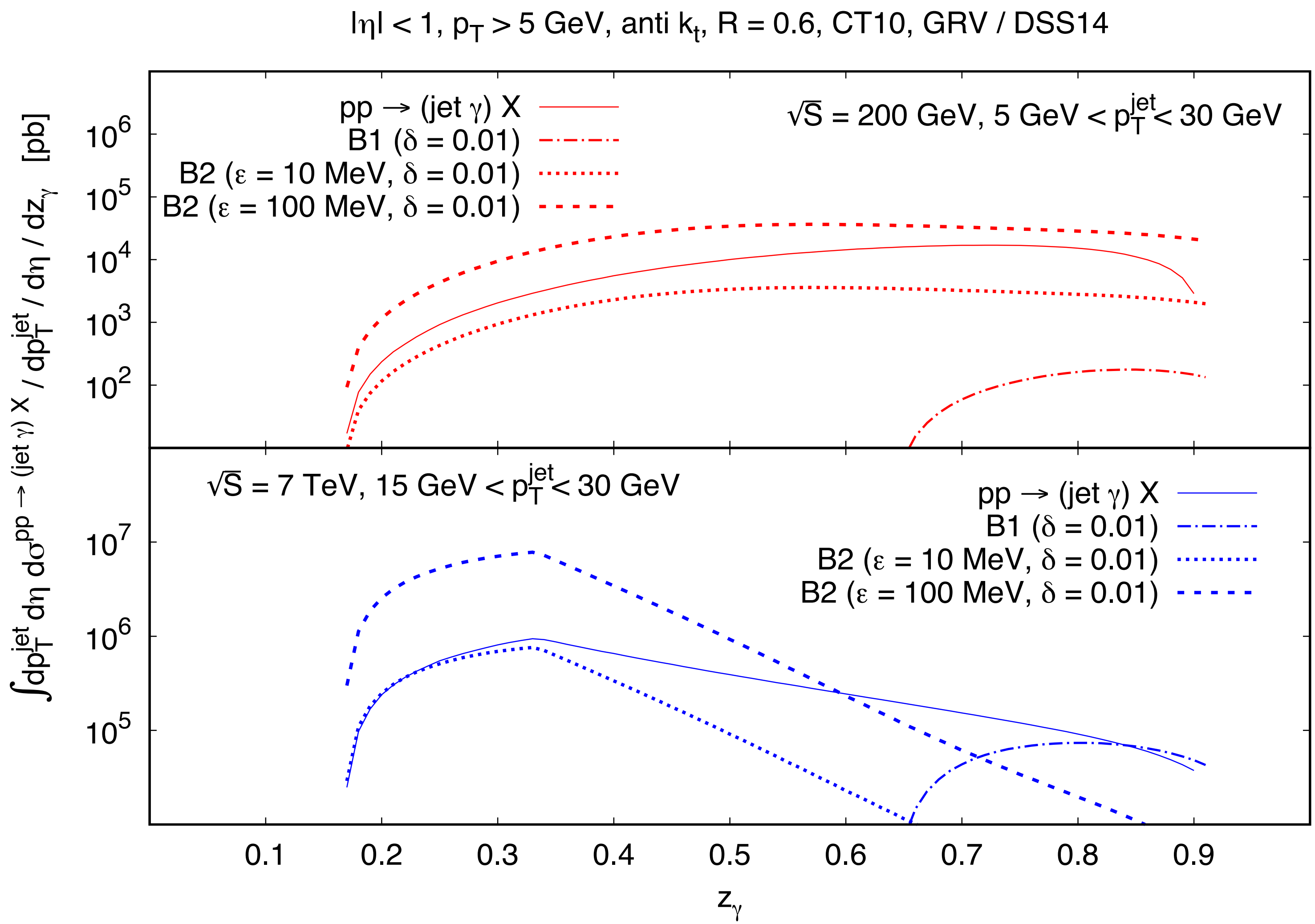

- Energy threshold $\varepsilon < 10\,\mathrm{MeV}$ required $\rightsquigarrow$ main experimental challenge

# Background - Note

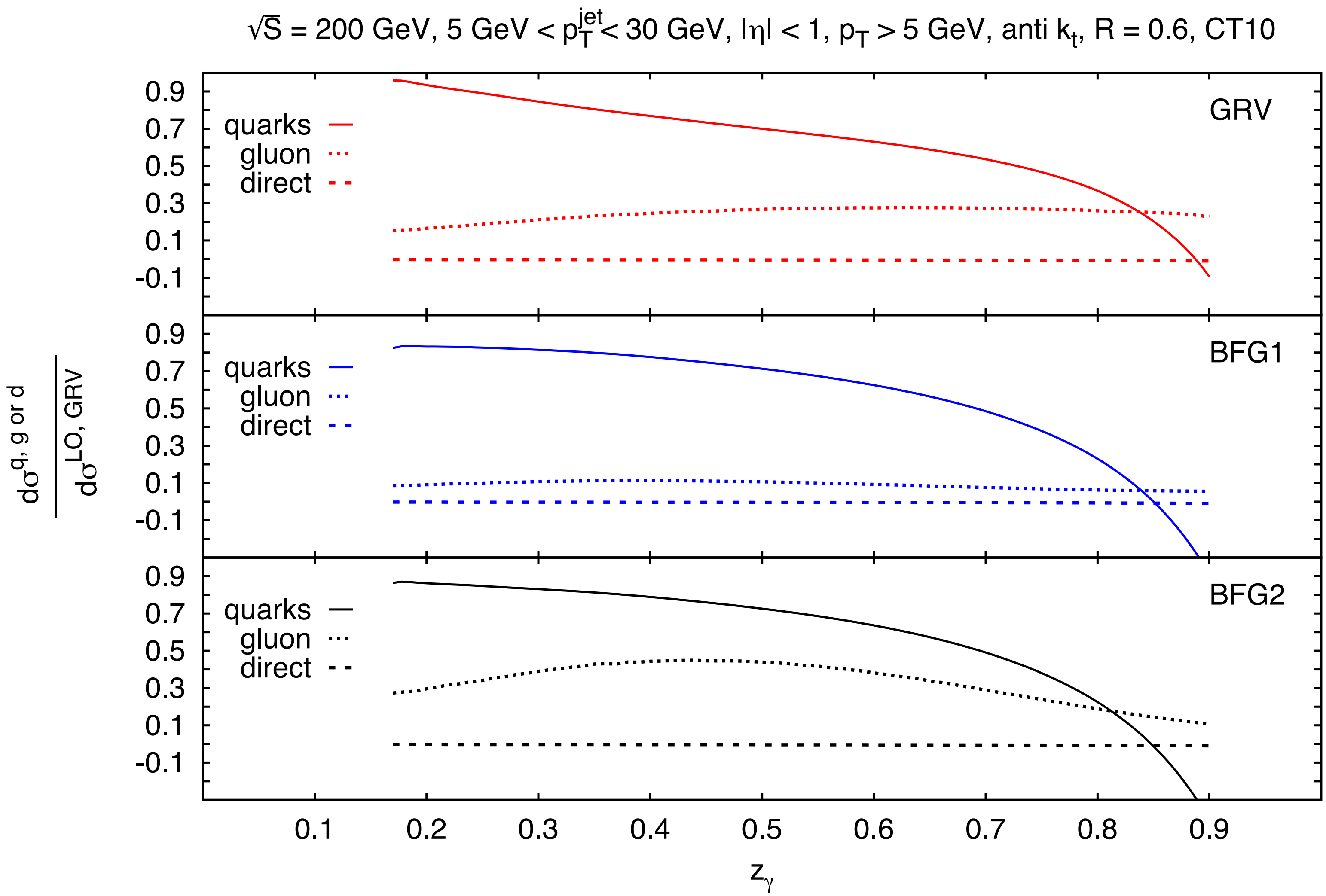
[Kaufmann, Mukherjee, Vogelsang: PRD93:114021]

- Most photon observables suffer from $\pi^0$ background
- Background calculation not limited to photon-in-jet
- Corresponding pion cross section is needed, e.g. background for single-inclusive photon productions requires single-inclusive pion cross section, etc.

# Numerical Results



# Numerical Results



# Summary

- $pp \to (\text{jet}\, h/\gamma)\, X$ are promising for $D_c^{h/\gamma}(z)$
- large amount of data
- various theoretical calculations in different frameworks
- Different definitions for $z_h$ , charged jets, exclusive/inclusive,…

# Outlook

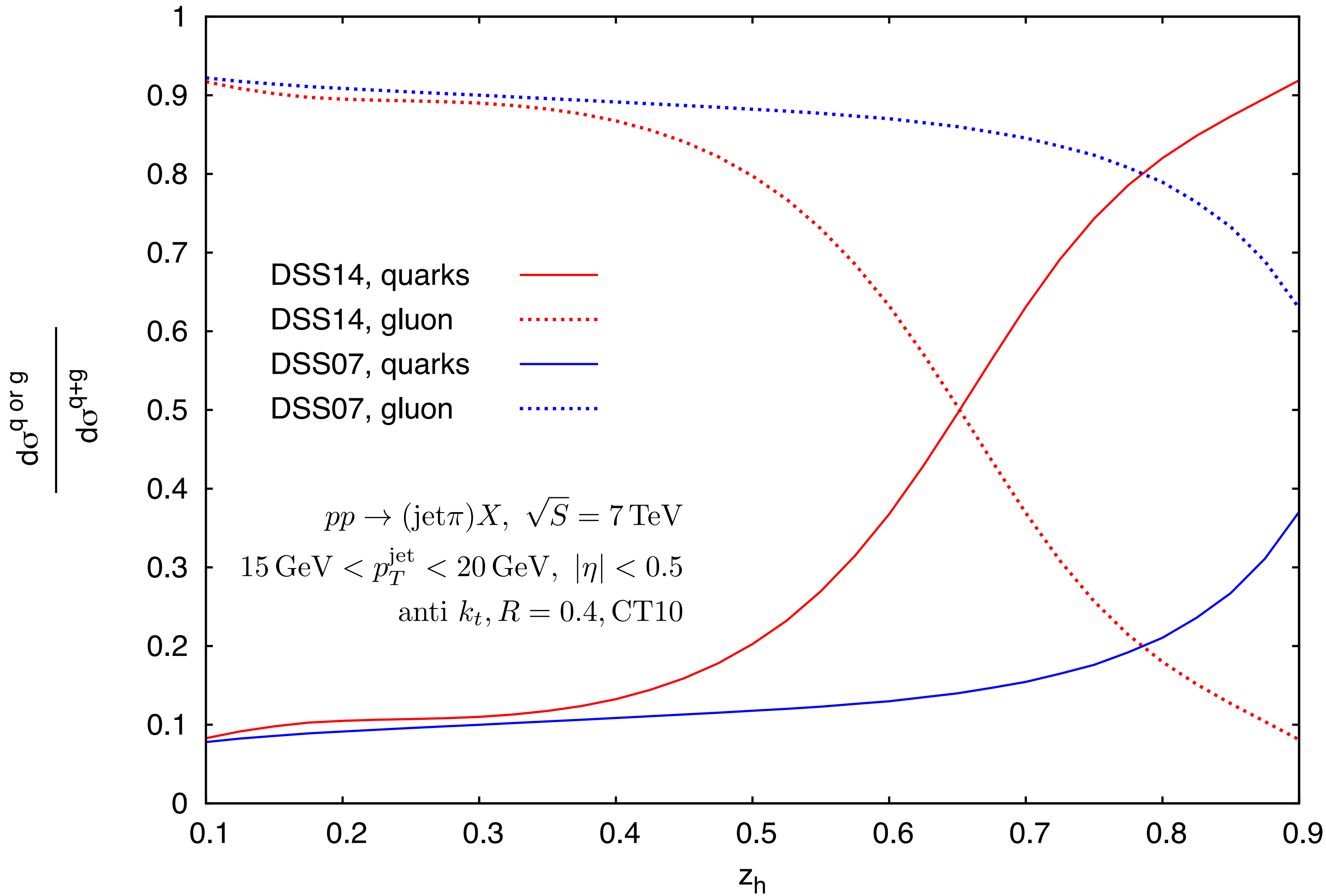
- Detailed comparison Experiment vs. Theory — Need for higher precision?
- new (global) FF fits including the jet+hadron data
- Jet+Photon? An experimental challenge…

EBERHARD KARLS UNIVERSITÄT TÜBINGEN

# Thank you.

# Backup Slides

# Numerical Results



# Calculation of the cross section

$$
\frac{d\sigma^{H_1 H_2 \to (\text{jet}\, h) X}}{dp_T^{\text{jet}} d\eta^{\text{jet}} dz_h} = \frac{2p_T^{\text{jet}}}{S} \sum_{a,b,c} \int_{x_a^{\min}}^{1} \frac{dx_a}{x_a} f_a^{H_1}(x_a, \mu_F) \int_{x_b^{\min}}^{1} \frac{dx_b}{x_b} f_b^{H_2}(x_b, \mu_F)
$$

$$
\times \int_{z_c^{\min}}^{1} \frac{dz_c}{z_c^2} \frac{d\hat{\sigma}_{ab}^{c}(\hat{s}, \hat{p}_T, \hat{\eta}, \mu_F, \mu_F', \mu_R)}{v dv dw} \sum_e j_{c\to e}\left(z_c, \frac{\mathcal{R}\, p_T^{\text{jet}}}{\mu_F'}, \mu_R\right)
$$

$$
\times \sum_{c'} \int_{z_h}^{1} \frac{dz_p}{z_p} \tilde{j}_{e\to c'}\left(z_p, \frac{\mathcal{R}\, p_T^{\text{jet}}}{\mu_F''}, \mu_R\right) D_{c'}^{h}\left(\frac{z_h}{z_p}, \mu_F''\right)
$$

# Calculation of the cross section

$$
\begin{aligned}
\frac{d\sigma^{H_1 H_2 \to (\text{jet}\, h) X}}{dp_T^{\text{jet}} d\eta^{\text{jet}} dz_h} = \; & \frac{2p_T^{\text{jet}}}{S} \sum_{a,b,c} \int_{x_a^{\min}}^{1} \frac{dx_a}{x_a} f_a^{H_1}(x_a, \mu_F) \int_{x_b^{\min}}^{1} \frac{dx_b}{x_b} f_b^{H_2}(x_b, \mu_F) \\
& \times \boxed{\int_{z_c^{\min}}^{1} \frac{dz_c}{z_c^2} \frac{d\hat{\sigma}_{ab}^{c}(\hat{s}, \hat{p}_T, \hat{\eta}, \mu_F, \mu_F', \mu_R)}{v dv dw} \sum_e j_{c\to e}\left(z_c, \frac{\mathcal{R}\, p_T^{\text{jet}}}{\mu_F'}, \mu_R\right)} \\
& \times \boxed{\sum_{c'} \int_{z_h}^{1} \frac{dz_p}{z_p} \tilde{j}_{e\to c'}\left(z_p, \frac{\mathcal{R}\, p_T^{\text{jet}}}{\mu_F''}, \mu_R\right)} D_{c'}^{h}\left(\frac{z_h}{z_p}, \mu_F''\right)
\end{aligned}
$$

analytical

# Scales and Cancellations

$$\frac{d\sigma^{H_1 H_2 \to (\text{jet}\,h)X}}{dp_T^{\text{jet}}\,d\eta^{\text{jet}}\,dz_h} = \frac{2p_T^{\text{jet}}}{S}\sum_{a,b,c}\int_{x_a^{\min}}^1 \frac{dx_a}{x_a} f_a^{H_1}(x_a,\mu_F)\int_{x_b^{\min}}^1 \frac{dx_b}{x_b} f_b^{H_2}(x_b,\mu_F)$$

$$\times \int_{z_c^{\min}}^1 \frac{dz_c}{z_c^2}\,\frac{d\hat{\sigma}^c_{ab}(\hat{s},\hat{p}_T,\hat{\eta},\mu_F,\mu_F',\mu_R)}{v\,dv\,dw}\sum_e j_{c\to e}\left(z_c,\frac{\mathcal{R}\,p_T^{\text{jet}}}{\mu_F'},\mu_R\right)$$

$$\times \sum_{c'}\int_{z_h}^1 \frac{dz_p}{z_p}\,\tilde{j}_{e\to c'}\left(z_p,\frac{\mathcal{R}\,p_T^{\text{jet}}}{\mu_F''},\mu_R\right) D_{c'}^h\left(\frac{z_h}{z_p},\mu_F''\right)$$

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$$

$$
\times \int_{z_c^{\min}}^{1} \frac{dz_c}{z_c^2} \frac{d\hat{\sigma}_{ab}^{c}(\hat{s}, \hat{p}_T, \hat{\eta}, \mu_F, \cancel{\mu'_F}, \mu_R)}{v dv dw} \sum_e j_{c \to e}\left(z_c, \frac{\mathcal{R}\, p_T^{\text{jet}}}{\cancel{\mu'_F}}, \mu_R\right)
$$

$$
\times \sum_{c'} \int_{z_h}^{1} \frac{dz_p}{z_p} \tilde{j}_{e \to c'}\left(z_p, \frac{\mathcal{R}\, p_T^{\text{jet}}}{\mu''_F}, \mu_R\right) D_{c'}^{h}\left(\frac{z_h}{z_p}, \mu''_F\right)
$$

# Scales and Cancellations

- $\mu'_F$ is an artifact of the organization
- $d\hat{\sigma}^c_{ab}(\mu'_F)$ due to collinear factorization
- cancellation with $j(\mu'_F)$ because we require a jet in the final state
- powerful check, analytical AND numerical!
- $\mu''_F$ because of final state hadron
- needed to subtract collinear singularities when two partons jointly form the jet together
- $\log \mu''_F$ are standard scale logs that compensate evolution of FFs

# Jet Functions

$$j_{a\to b}(z,\lambda) \equiv \delta_{ab}\delta(1-z) - \frac{\alpha_s}{2\pi}\left[\left\{P_{ba}(z)\log(\lambda^2(1-z)^2)\right\}_+ - P_{ba}^{(\epsilon)}(z) + \delta_{ab}\delta(1-z)I_a^{\text{algo}}\right]$$

- splitting functions for in $D = 4 - 2\epsilon$ dimensions:

$$\tilde{P}_{ij}(z) = P_{ij}(z) + \epsilon P_{ij}^{(\epsilon)}$$

- $\{\dots\}_+$ regularizes in the diagonal case, e.g. for $P_{qq}$

$$2C_F(1+z)^2\left(\frac{\log(1-z)}{1-z}\right)_+ + P_{qq}(z)\log\lambda^2$$

- algorithm dependent quantities $I_j^{\text{algo}}$ (pure numbers)
- $\lambda = \dfrac{\mathcal{R}p_T^{\text{jet}}}{\mu'_F}$

# Jet Functions

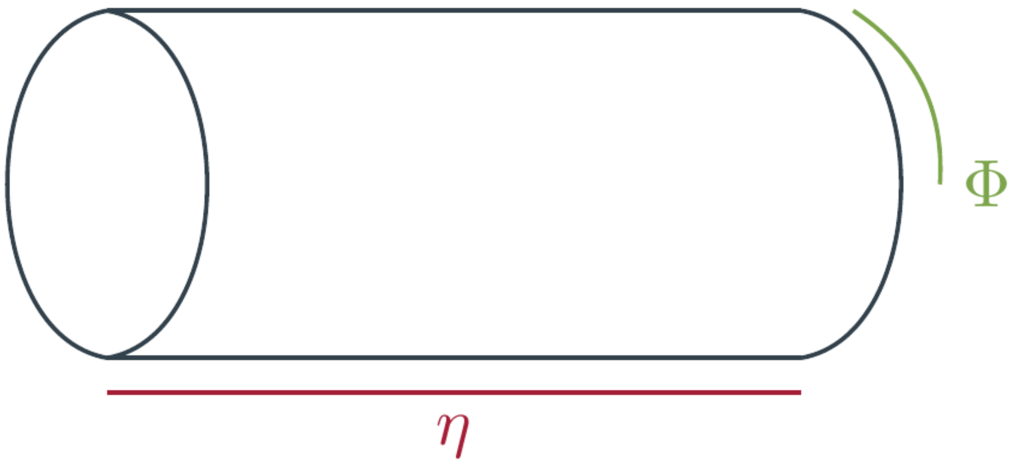
$$\tilde{j}_{a\to b}(z_p,\kappa) \equiv \delta_{ab}\delta(1-z_p) + \frac{\alpha_s}{2\pi}\left[\left\{P_{ba}(z_p)\log(\kappa^2(1-z_p)^2)\right\}_+ - P_{ba}^{(\epsilon)}(z_p) + \delta_{ab}\delta(1-z_p)I_a^{\text{algo}} + \mathcal{I}_{ba}^{\text{algo}}(z_p)\right]$$

- algorithm dependent functions

$$\mathcal{I}_{c'c}^{\text{algo}}(z) = \begin{cases} 2P_{c'c}(z)\log\left(\frac{z}{1-z}\right)\Theta(1/2-z) & \text{cone algorithm}\,, \\ 2P_{c'c}(z)\log z & (\text{anti}-)k_t\ \text{algorithm}\,, \\ P_{c'c}(z)\left[\log(z)+\log\left(\frac{z}{1-z}\right)\Theta(1/2-z)\right] & J_{E_T}\ \text{algorithm}\,. \end{cases}$$

- $\kappa = \dfrac{\mathcal{R}p_T^{\text{jet}}}{\mu_F''}$

# Rapidity and Azimuth

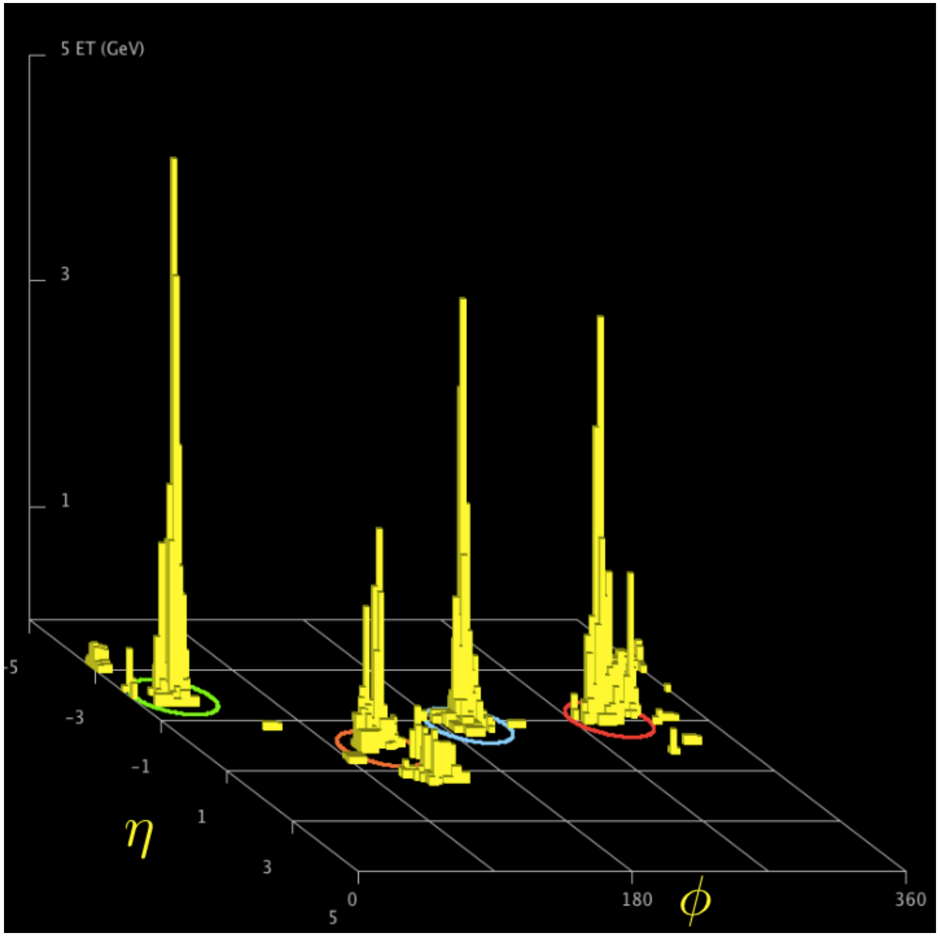



$$\eta = -\log\tan\frac{\theta}{2}\,, \qquad [0,\pi] \to \mathbb{R}$$

# Jet Algorithms

- no uniquely defined objects
- definition needed
- fulfill experimental and theoretical needs

- must be valid for all “levels”
- some examples:
    - cone
    - (anti-) kT
    - $J_{E_T}$
    - …




[Soper, CTEQ Summerschool 2013]

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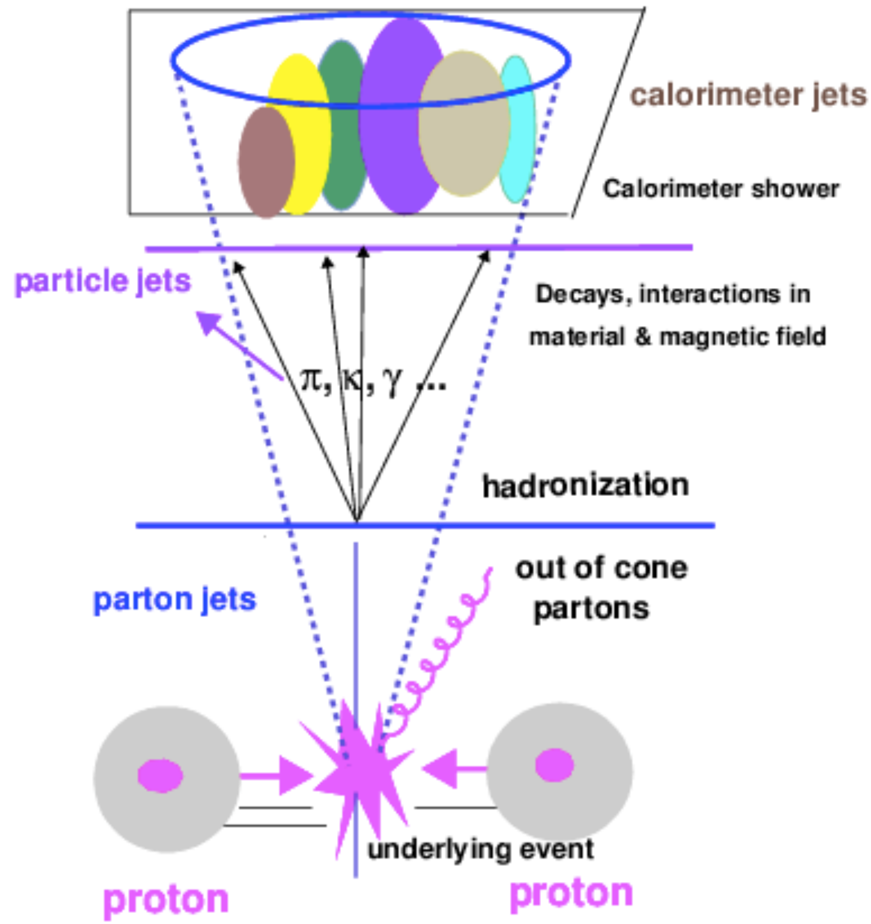



[CMS-PAS-QCD-08-005]

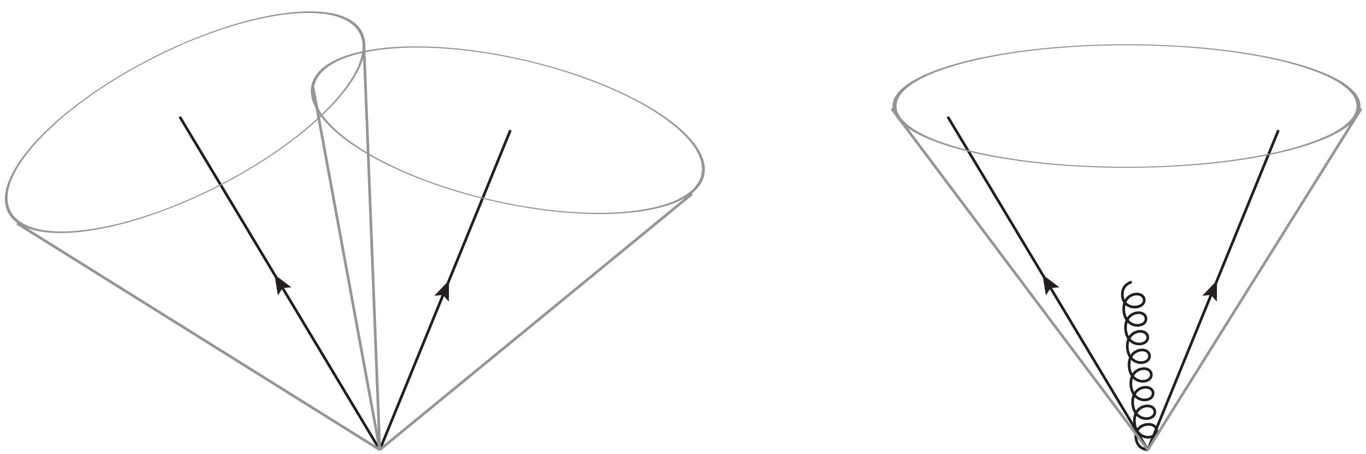
# Cone algorithms

- all particles inside a geometrical cone

$$i \in \text{Jet} \quad \Leftrightarrow \quad (\phi_J - \phi_i)^2 + (\eta_J - \eta_i)^2 \leq R^2$$

Problem

- iterative search (seeds) for “stable” cones $\Rightarrow$ not IRC safe



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Problem

- iterative search (seeds) for “stable” cones $\Rightarrow$ not IRC safe
- improvements (midpoint,…) only shift to higher orders
- IR safe: exact seedless algorithms, solve mathematical problem
- naive implementation: unusable $\mathcal{O}(N2^N)$
- improved implementation: SISCone $\mathcal{O}(N^2 \log N)$ [Salam, Soyez: JHEP0705:086]

# Cone algorithms

"size"

- all particles inside a geometrical cone

$$i \in \text{Jet} \quad \Leftrightarrow \quad (\phi_J - \phi_i)^2 + (\eta_J - \eta_i)^2 \leq R^2$$

Problem

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# kT-Algorithms

- distance between two objects

$$d_{ij} \equiv \min\left((k_T^i)^{2p}, (k_T^j)^{2p}\right) \frac{(\phi_i - \phi_j)^2 + (\eta_i - \eta_j)^2}{R^2}$$

- distance between object and beam

$$d_{iB} \equiv (k_T^i)^{2p}$$

- parameter *p* specifies algorithm:
  - *p* = 1 the kT algorithm [Ellis, Soper: PRD.48.3160],[Catani, Dokshitzer, Seymour, Webber: NuclPhys.B406]
  - *p* = 0 Cambridge-Aachen [Dokshitzer, Leder, Moretti, Webber: JHEP9708:001]
  - *p* = -1 anti-kT [Cacciari, Salam, Soyez: JHEP0804:063]

# kT-Algorithms

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  - *p* = -1 anti-kT [Cacciari, Salam, Soyez: JHEP0804:063]

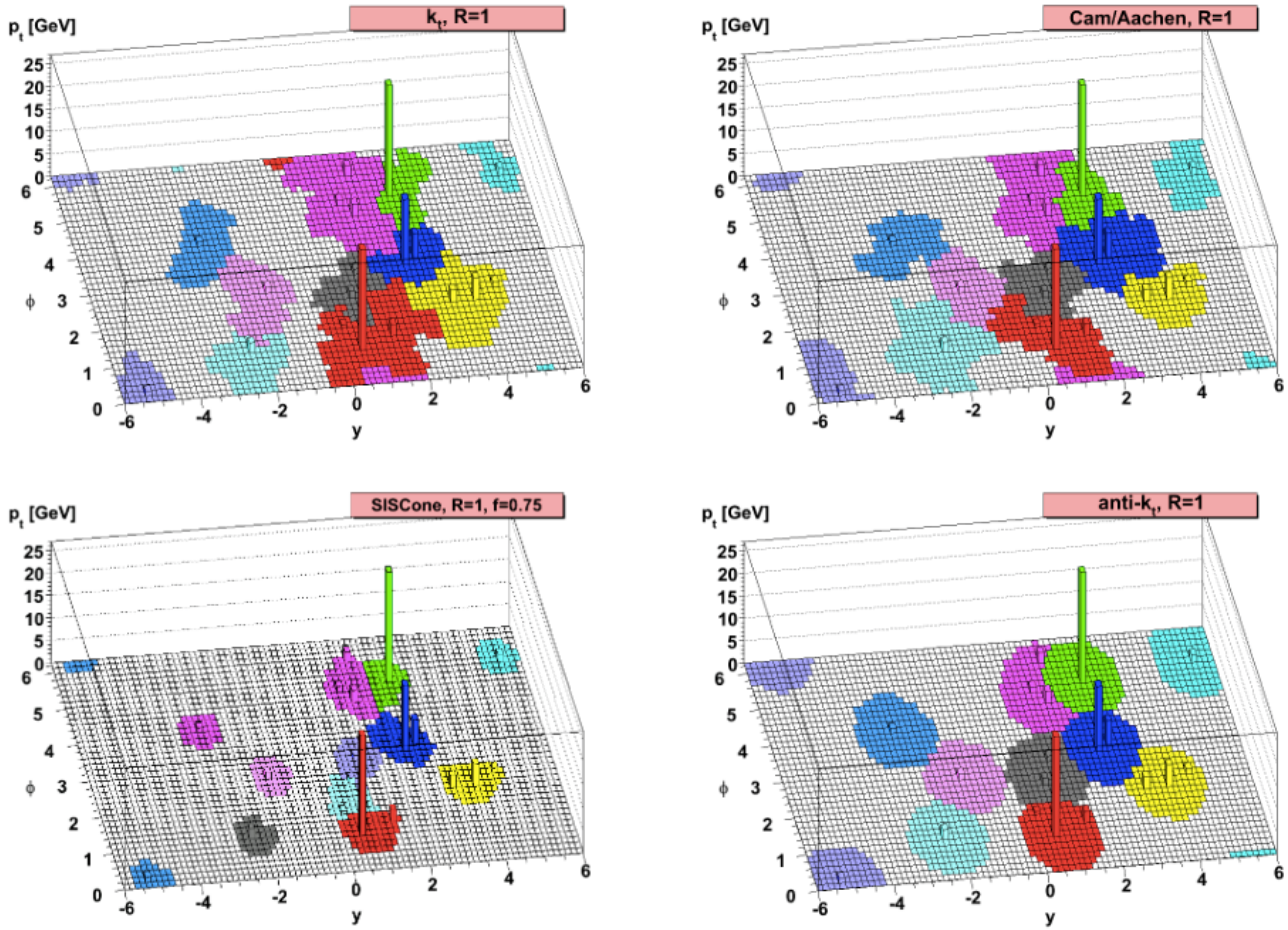
# kT-Algorithms

Iterative procedure

- initially: list of all particles
- while list not empty
  - for all objects in list: compute distances
  - if minimum distance is beam: object is a jet, remove from list
  - else, merge the two objects

# kT- vs cone algorithms

[Cacciari, Salam, Soyez: JHEP0804:063]



# Maximized Jet Function

[Georgi: arXiv:1408.1161; Kaufmann, Mukherjee, Vogelsang: PRD91:034001]

- jet as a sum of particles, increases jet energy
- not gain too much mass = be collimated

$$m^2 \equiv p_{\text{jet}}^2 = 2p_j \cdot p_k = 2E_j E_k (1 - \cos\theta_{jk})$$

- define jet function

$$J_\beta(p) \equiv E - \beta \frac{m^2}{E} \,, \quad \beta > 1$$

- increases with increasing energy
- decreases with increasing mass squared over energy
- highest J jet maximizes the jet function
- iterative procedure

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“inverse size”

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