QCD - N'16, July 16, 2016, Getxo, Spain

Evolution of GTMDs

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Institute for Theoretical Physics
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talk based on
M. G. Echevarria, A. Idilbi, K. Kanazawa, C. Lorcé, A. Metz, B. Pasquini, M. S.,
Phys. Lett. B 759 (2016), 336-341, [arXiv:1602.06953]

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Generalized Transverse Momentum Dependent Parton Distributions ?

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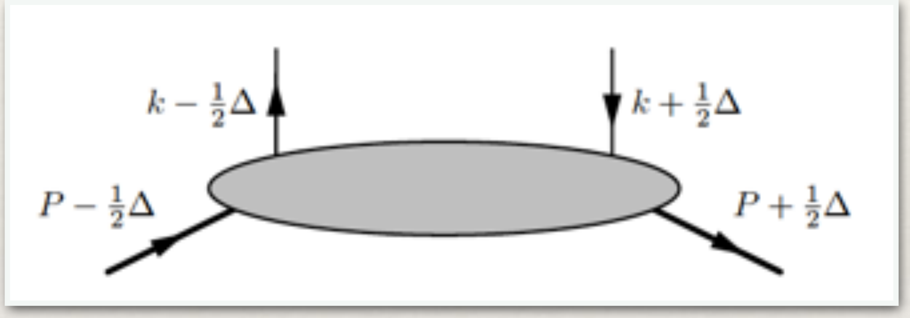
off-diagonal kinematics like GPDs



$$P = \tfrac{1}{2}(p + p') \qquad \Delta = p' - p$$

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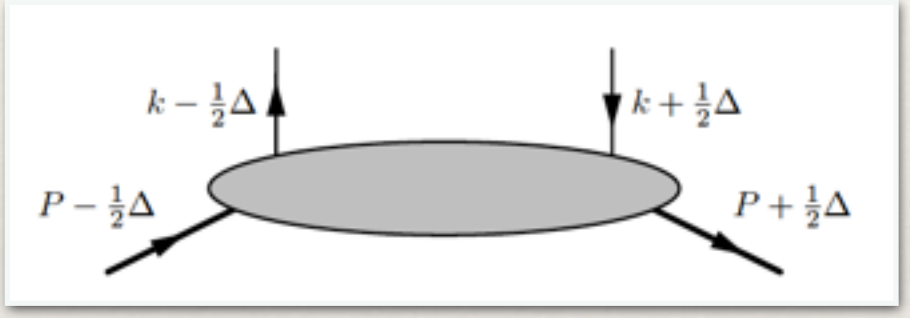

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Parametrization:

[Spin-0: Meißner, Metz, M.S., Goeke, JHEP 0808 (2008), 038 ; Spin-1/2: Meißner, Metz, M. S., JHEP 0908 (2009), 056; Gluons: Lorcé, Pasquini, JHEP 1309 (2013), 138]

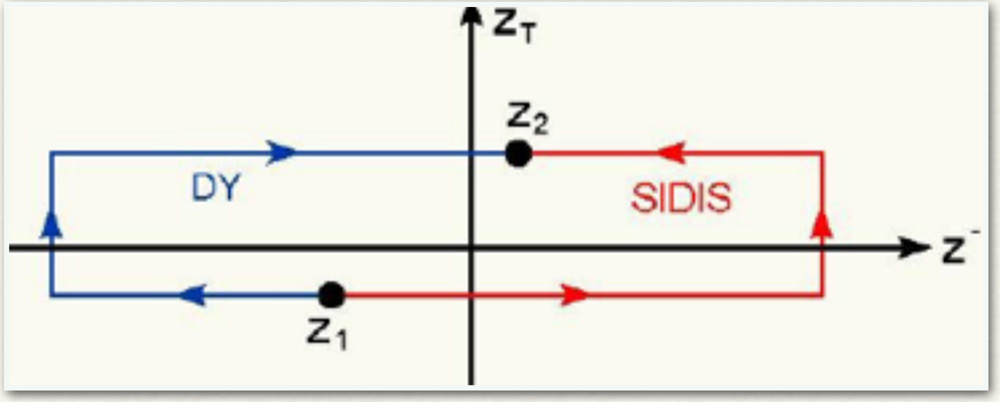
$$W^{[\gamma^+]} = \frac{1}{2M}\bar{u}(p') \left[F_{1,1} + \frac{k_T^\alpha}{P^+}\, i\sigma^{\alpha +}\, F_{1,2} + \frac{\Delta_T^\alpha}{P^+}\, i\sigma^{\alpha +}\, F_{1,3} + \frac{k_T^\alpha\, \Delta_T^\beta}{M^2}\, i\sigma^{\alpha\beta}\, F_{1,4} \right] u(p)$$

$$W^{[\gamma^+\gamma_5]} = \frac{1}{2M}\bar{u}(p') \left[-\frac{i\epsilon^{\alpha\beta} k_T^\alpha\, \Delta_T^\beta}{M^2} G_{1,1} + \frac{k_T^\alpha}{P^+}\, i\sigma^{\alpha +}\gamma_5\, G_{1,2} + \frac{\Delta_T^\alpha}{P^+}\, i\sigma^{\alpha +}\gamma_5\, G_{1,3} + i\sigma^{+-}\gamma_5\, G_{1,4} \right] u(p)$$

GTMDs functions of x, k_T, ξ, Δ_T

Wilson line

$$\mathcal{W}[a;b] = \mathcal{P}\mathrm{e}^{-ig\int_a^b ds\cdot A(s)}$$



GTMDs with staple-like Wilson line ⇒ complex functions

$$\Im[\mathrm{GTMD}]\Big|_{\mathrm{SIDIS}} = -\Im[\mathrm{GTMD}]\Big|_{DY}$$

Keep in mind: No hard process known (so far) where GTMDs are directly probed !

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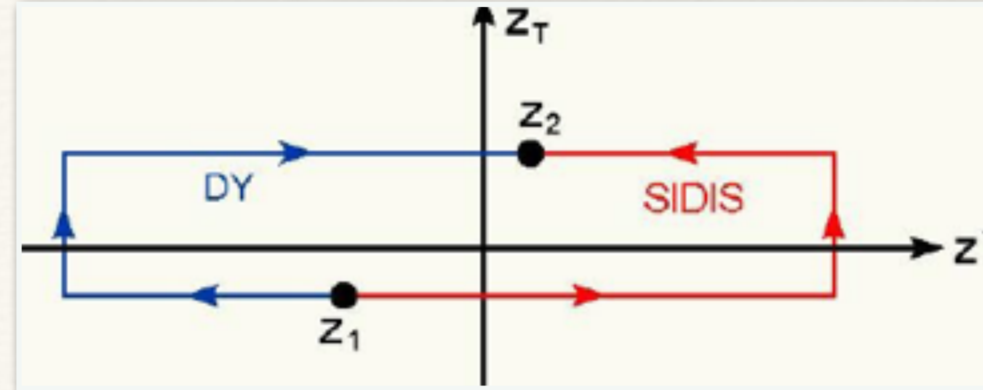

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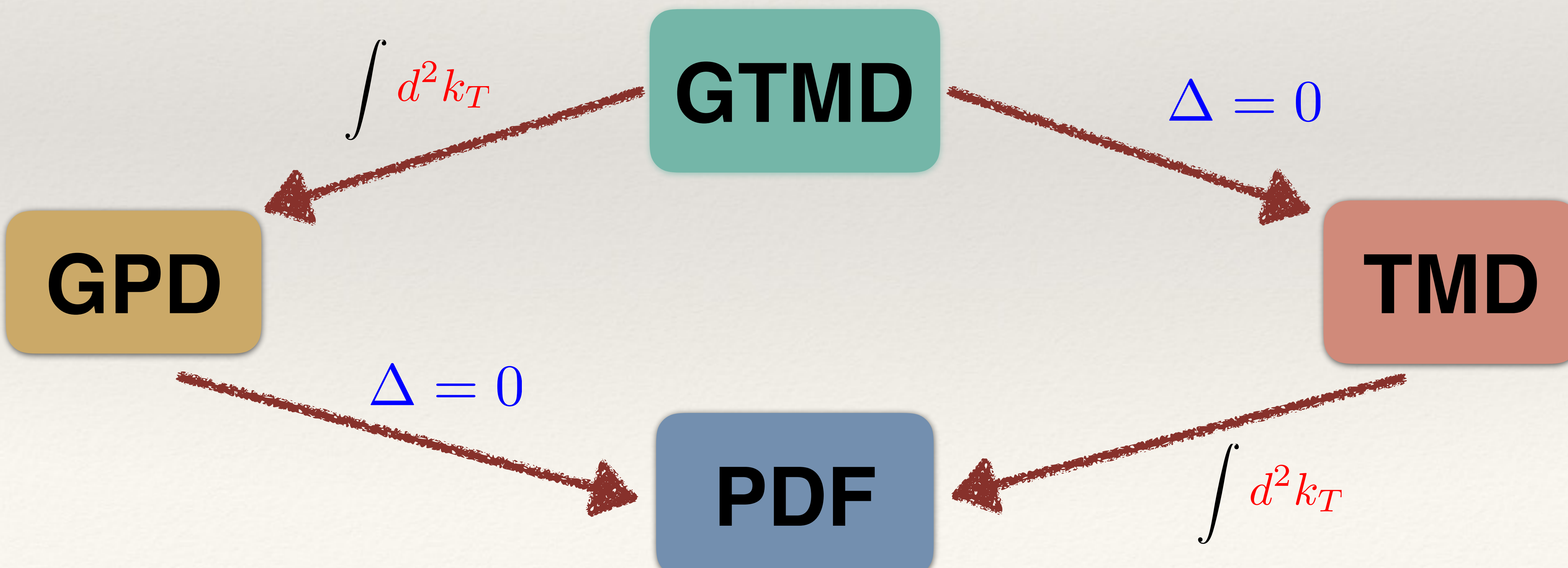
GTMDs with staple-like Wilson line: <u>'Unifying functions' of GPDs and TMDs</u>



Relation to Wigner distributions

[Ji, PRL 91, 062001 (2003); Belitsky, Ji, Yuan, PRD 69, 074014 (2004); Lorcé, Pasquini, PRD 84, 014015 (2011)]

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Wigner function in Quantum Mechanics:

$$P(x,p) = \frac{1}{2\pi\hbar}\int dy\,\psi^*(x-\tfrac{y}{2})\,\psi(x+\tfrac{y}{2})\,\mathrm{e}^{-ipy/\hbar}$$

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<u>Wigner Distributions in QCD:</u>

$$\rho^{[\Gamma]}(x_T, p_T, x, S) = \int \frac{d^2\Delta_T}{(2\pi)^2}\, \langle p', S| \left(\int \frac{dy^-\, d^2y_T}{2(2\pi)^3}\, \bar{q}(x_T - \tfrac{y}{2})\, \Gamma\, \mathcal{W}\, q(x_T + \tfrac{y}{2})\, \mathrm{e}^{ip\cdot y} \right)_{y^+=0} |p, S\rangle \Big|_{\xi=0}$$

⇒ Fourier transform of GTMDs

$$\rho^{[\Gamma]}(B_T, k_T, x, S) = \int \frac{d^2\Delta_T}{(2\pi)^2}\, \mathrm{e}^{-i\Delta_T\cdot B_T}\, W^{[\Gamma]}(x, \xi=0, k_T, \Delta_T)$$

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Formulation of Quark OAM and Spin-Orbit Correlations:

$$\langle \hat{L}_z^q \rangle = \int dx\, d^2k_T\, d^2B_T\, (\vec{B_T} \times \vec{k_T})_z\, \rho^{[\gamma^+]}(B_T, k_T, x, S) = -\int dx\, d^2k_T\, \frac{k_T^2}{M^2}\, F_{1,4}^q(x, \xi=0, k_T, \Delta_T=0)$$

$$C_z^q = \int dx\, d^2k_T\, d^2B_T\, (\vec{B_T} \times \vec{k_T})_z\, \rho^{[\gamma^+\gamma_5]}(B_T, k_T, x, S) = \int dx\, d^2k_T\, \frac{k_T^2}{M^2}\, G_{1,1}^q(x, \xi=0, k_T, \Delta_T=0)$$

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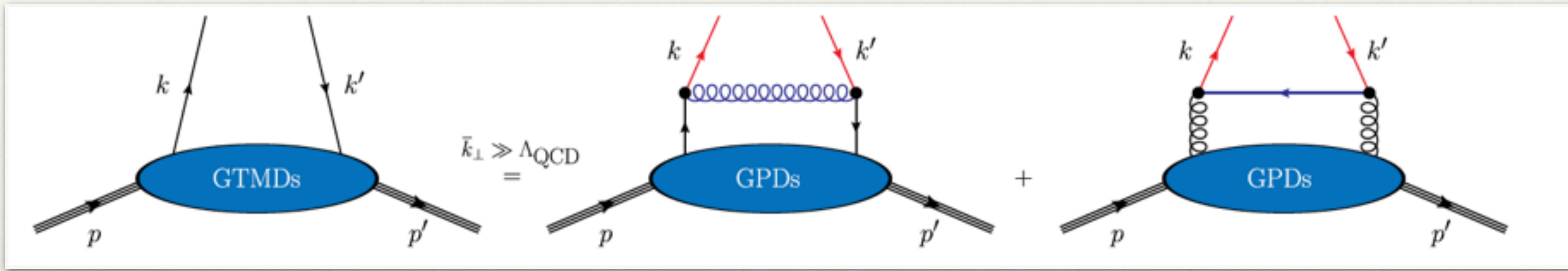
$\Longrightarrow$ behaviour of GTMDs are large transverse momenta $k_T \gg \Lambda_{QCD}$



naive GTMD definition in gauge $v \cdot A(x) = 0$

[cf. TMDs: Bacchetta, Boer, Diehl, Mulders, JHEP 0808, 023 (2008)]

$$n^\mu = [1^+, 0^-, 0_T] \to v^\mu = [1, -\zeta/(2(P^+)^2)]$$

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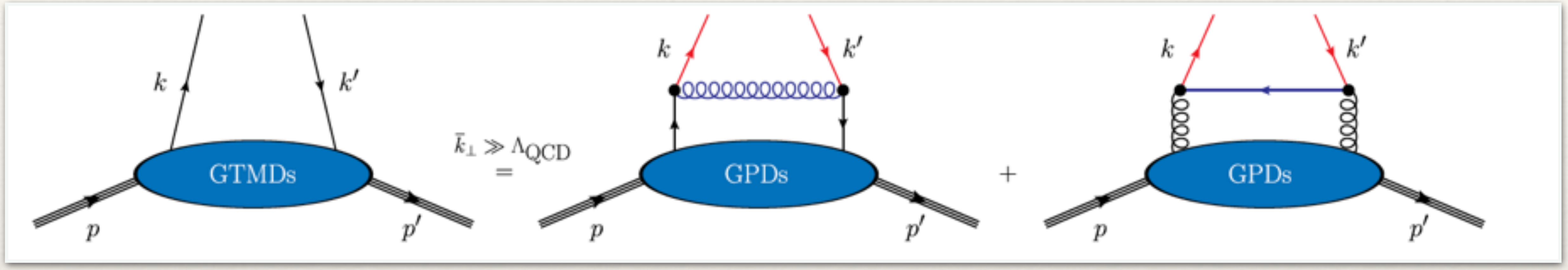

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$$F^q_{1,4}(x, \xi = 0, k_T \gg \Lambda_{QCD}, \Delta_T) = \frac{\alpha_s}{2\pi^2} \int_x^1 \frac{y}{y} \frac{M^2 \left(C_F\, \tilde{H}^q(x/y, 0, \Delta_T) - T_R(1-y)^2\, \tilde{H}^g(x/y, 0, \Delta_T)\right)}{\prod_\pm \left[(k_T \pm (1-y)\Delta_T/2)^2 + y(1-y)\Delta_T^2/4\right]}$$

seems to work for $F_{1,4}$ (at this order α_s), but…

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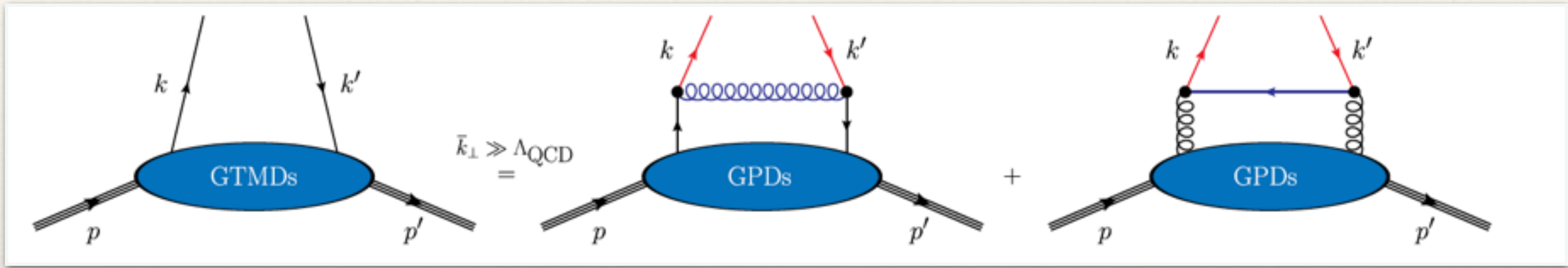

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$$F^q_{1,1} = \frac{\alpha_s}{2\pi^2}\left[C_F \left(\log\left|\frac{x^2\zeta}{k_T^2}\right| - \frac{3}{2}\right)\frac{1}{k_T^2} \, H^q(x, 0, \Delta) \; + \int_x^1 \frac{dy}{y} \, C(y, k_T, \Delta_T) \, \mathrm{GPDs}(x/y, 0, \Delta_T)\right]$$

Logarithmic divergence for Wilson lines along the light-cone!

Alternative Definition of GTMDs

[Echevarria, Idilbi, Kanazawa, Lorcé, Metz, Pasquini, M.S., Phys. Lett. B 759 (2016), 336-341]

Modify definition of GTMDs in the same way as TMDs (same operator!)

1) Renormalizable Matrix Element
2) Wilson Coefficients without divergences

Inclusion of Soft Function $\Longrightarrow$

$$S(z_T) = \frac{\mathrm{Tr}_c}{N_c} \langle 0 | \mathcal{W}_n^\dagger(-z_T/2)\, \mathcal{W}_{\bar{n}}(-z_T/2)\, \mathcal{W}_{\bar{n}}^\dagger(z_T/2)\, \mathcal{W}_n(z_T/2)\,] 0 \rangle$$

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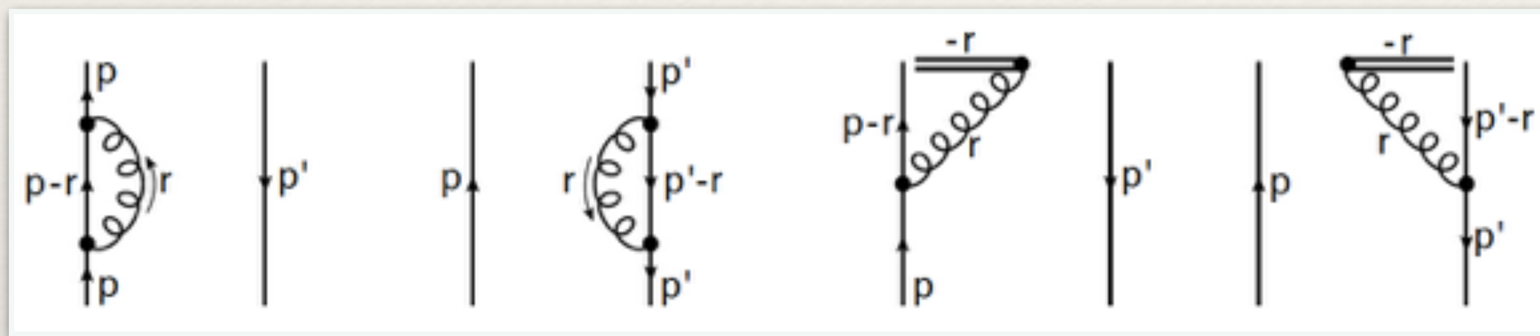
Renormalization of GTMDs with massless quark targets:



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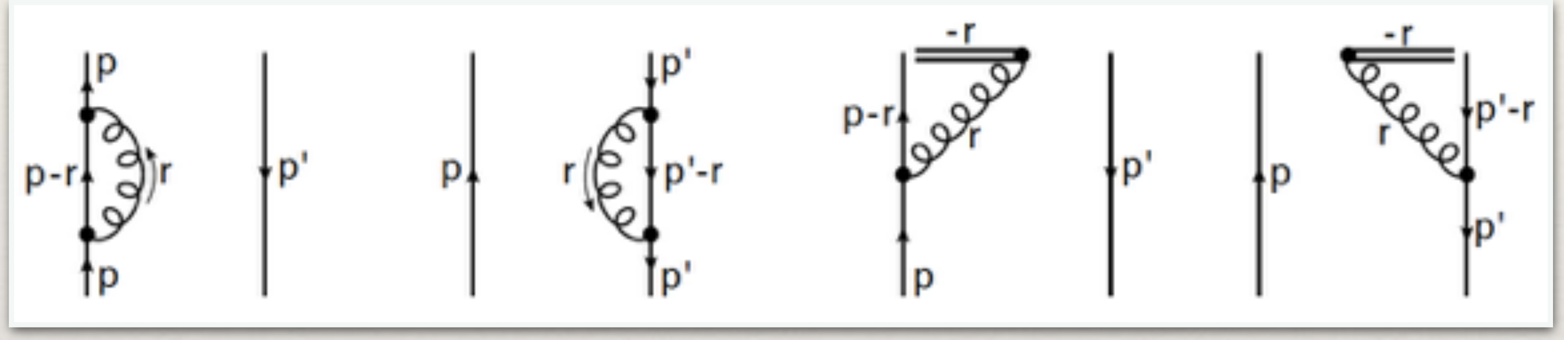

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naive GTMDs without soft function

$$W^{[\Gamma],\mathrm{virt}} = \frac{\bar{u}(p')\,\Gamma\, u(p)}{2P^+}\, \delta(1-x)\, \delta^{(d-2)}(k_T) \left(1 + \frac{C_F\,\alpha_s}{2\pi} S_\varepsilon \left[\frac{3}{2\varepsilon} + \frac{2}{\varepsilon} \ln\left(\frac{\delta}{\Lambda^2}\right) + \mathcal{O}(\varepsilon^0) \right] + \mathcal{O}(\alpha_s^2) \right)$$

overlapping UV and soft divergence

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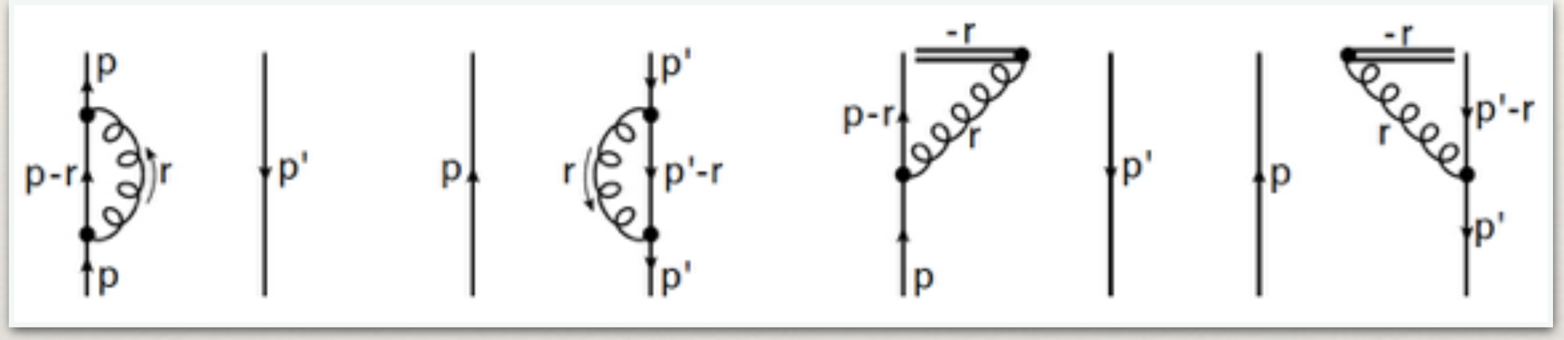

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overlapping UV and soft divergence

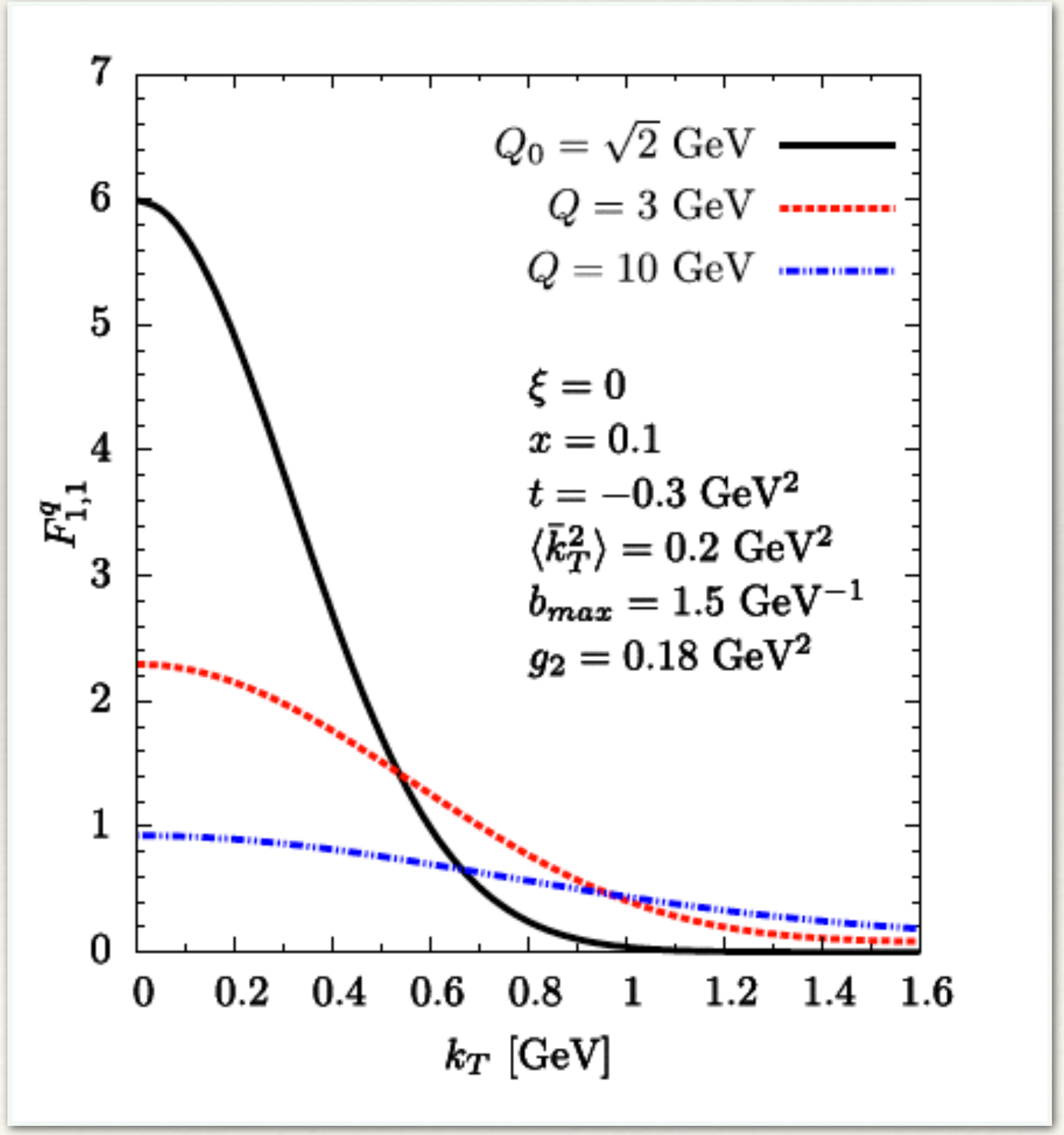
Soft function: $$W^{[\Gamma],\mathrm{SF}} = \frac{\bar{u}(p')\,\Gamma\, u(p)}{2P^+}\, \delta(1-x)\, \delta^{(d-2)}(k_T) \left(1 + \frac{C_F\, \alpha_s}{2\pi} S_\varepsilon \left[\frac{1}{\varepsilon^2} - \frac{2}{\varepsilon} \ln\left(\frac{\delta}{\Lambda\, \mu}\right) + \mathcal{O}(\varepsilon^0)\right] + \mathcal{O}(\alpha_s^2)\right)$$

Renormalization Constant: $$Z^{\overline{\mathrm{MS}}} = 1 - \frac{C_F\, \alpha_s(\mu)}{2\pi} \left(\tfrac{1}{\varepsilon^2} + \tfrac{1}{\varepsilon}\left(\tfrac{3}{2} - 2\ln(\Lambda/\mu)\right) + \mathcal{O}(\alpha_s^2)\right)$$

Evolution of GTMDs

Operator of GTMDs = Operator of TMDs ⇒ identical evolution

$$F^q_{1,1}(\ldots;\mu,\Lambda) = e^{-S_{\rm pert}}\, e^{-S_{\rm non-pert}\ln(\Lambda/\Lambda_0)}\, F^q_{1,1}(\ldots;\mu_0,\Lambda_0)$$



Input function:

Gaussian model at low Q_0

$$F^q_{1,1}(x,\xi=0,k_T,\Delta_T;\mu=Q_0,\Lambda=Q_0) = H^q(x,\xi=0,t=-\Delta_T^2;\mu=Q_0)\,\frac{e^{-k_T^2/\langle k_T^2\rangle}}{\pi\langle k_T^2\rangle}$$

GPD model:

$$H^q(x,\xi=0,t;\mu=Q_0) = f_1^q(x;Q_0)\,e^{\lambda t}$$

Evolution:

k_T - distribution flattens out at larger Q just as TMDs do…

Wilson Coefficients

Relation to collinear GPD: Operator Product Expansion in coordinate space $z_T = b_T$

$$F_{1,1}^{q,\mathrm{ren}}(x, \xi = 0, b_T, \Delta_T) = \int_x^1 dy\, C^{q/q}(y, b_T, \Delta_T, \mu, \Lambda)\, H^{q,\mathrm{ren}}(x/y, 0, \Delta_T)$$

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Calculate Wilson Coefficient for quark target:

$$C^{q/q} = \delta(1-y) + \frac{C_F\,\alpha_s}{2\pi} C^{[1]}(y) + \ldots \implies C^{[1]} = F_{1,1}^{[1]} - H^{[1]}$$

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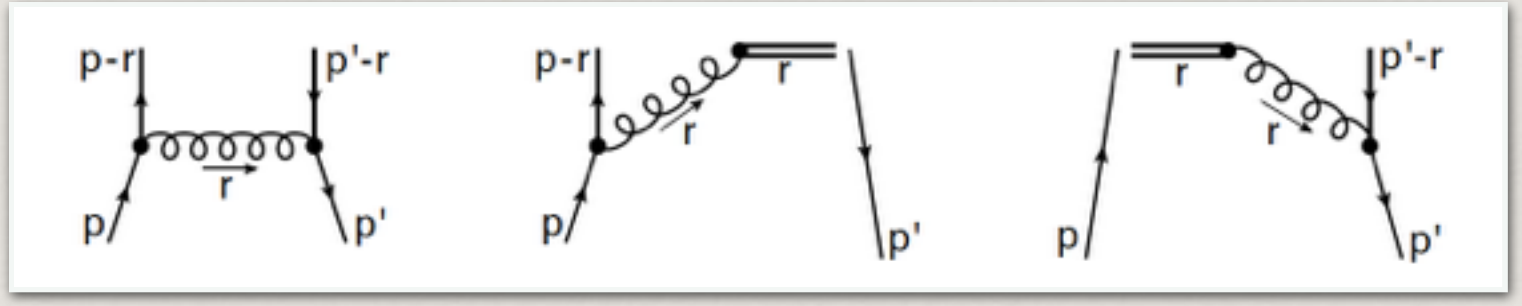
Need GTMD $F_{1,1}$ in coordinate space and GPD H $\implies$ real diagrams



$$F_{1,1}^{q,\mathrm{real}}(x,\xi=0,k_T,\Delta_T) = \frac{C_F\,\alpha_s}{2\pi^2}\left[\frac{\frac{1-x}{(1-x)^2+\delta/\Lambda^2}\left(k_T^2(1+x^2)+2x(\delta/\Lambda^2)(k_T\cdot\Delta_T)-(1-x)^2\Delta_T^2/2\right)}{[(k_T-(1-x)\Delta_T/2)^2-(1-x)i\delta]\,[(k_T+(1-x)\Delta_T/2)^2+(1-x)i\delta]} \quad -\delta(1-x)\frac{2\ln(k_T\,\Lambda/\delta)}{k_T^2-\delta^2/\Lambda^2}\right]$$

Transverse Momentum Transfer $\Delta_T \implies$ complicates Fourier - Transform

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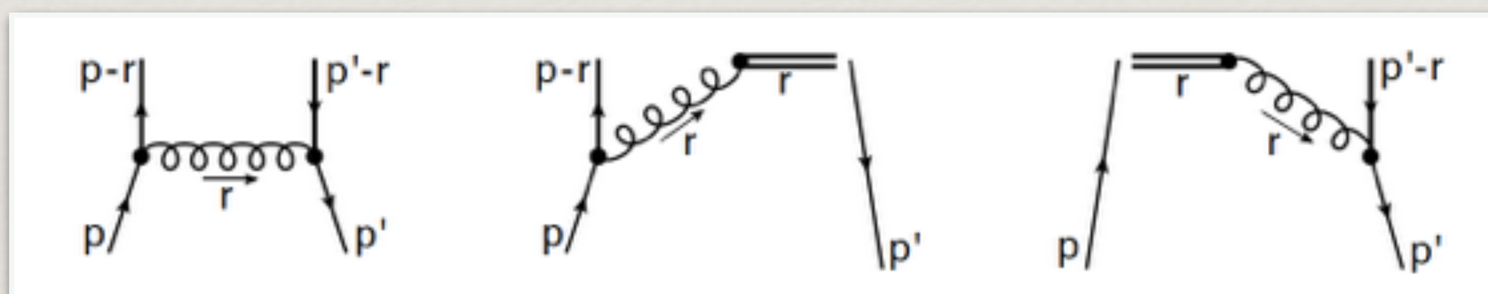

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Transverse Momentum Transfer $\Delta_T \Longrightarrow$ complicates Fourier - Transform

1) Perform k_T - integration $\Longrightarrow$ GPD H $\Longrightarrow$ analytic expression 😀

2) Perform Fourier - Transform $\Longrightarrow$ GTMD $F_{1,1} \Longrightarrow$ numerics 🙄

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$$+ \int_0^1 d\alpha \, \frac{\mathrm{e}^{i(1-2\alpha)(1-x) b_T \cdot \Delta_T/2}}{S(\alpha, x, \Delta_T^2, \delta)} \left(a + b \frac{1}{b_T} \frac{\partial}{\partial b_T} + c \frac{\partial^2}{\partial b_T^2} \right) \left(b_T \, K_1(b_T \, S(\alpha, x, \Delta_T, \delta)) \right) \Big]$$

$$S(\alpha, x, \Delta_T, \delta) = \sqrt{\alpha(1-\alpha)(1-x)^2 \Delta_T^2 - (1-2\alpha)(1-x) i\delta}$$

modified Bessel function

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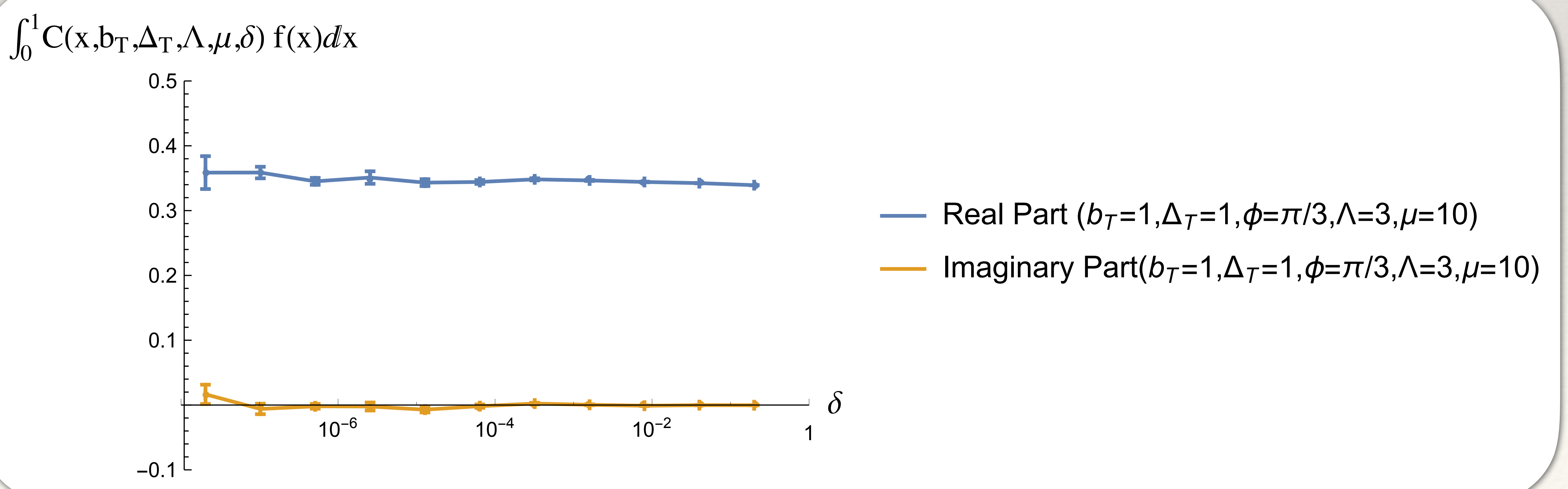
What can be shown analytically:

1) Wilson Coefficients are real (ξ=0)
2) Δ_T = 0: Recover the TMD result

What can be shown numerically:

1) Limit $\delta \longrightarrow 0$ seems to exist
2) no Δ_T - dependence (work in progress)



Summary

- GTMDs with staple-like Wilson line:
  Unifying functions of GPDs and TMDs
- GTMDs allow for a quantitative formulation of OAM and Spin - Orbit Correlations in the nucleon
- perturbative QCD: renormalization & evolution identical to TMDs, Soft Function crucial
- Wilson Coefficients for small b_T OPE (= large k_T): more complicated due to spurious Δ_T (& ξ) - dependence