Introduction to
TMD and Collinear Twist-3 Formalisms

(A. Metz, Temple University)

1. TMD approach
   - Motivation
   - Physics contained in TMDs
   - Phenomenology (flavor structure of Sivers and Collins functions)
   - Universality properties
   - Open issues and emerging fields

2. Collinear twist-3 approach
   - Double-spin asymmetry A_{LT} in $\vec{\ell}\, N^{\uparrow} \to \ell\, X$
   - Transverse single-spin asymmetry A_N in $p^{\uparrow}\, p \to h\, X$: data and flavor structure
   - Twist-3 formalism and sign-mismatch problem
   - Twist-3 fragmentation contribution to A_N in $p^{\uparrow}\, p \to h\, X$
   - Lorentz-invariance relations between twist-3 parton correlators
   - Transverse single-spin asymmetry A_N in $p^{\uparrow}\, p \to \gamma\, X$

3. Summary

Motivation 1: TMDs Appear Frequently

- Appear in QCD-description of many hard semi-inclusive reactions (→ many talks)

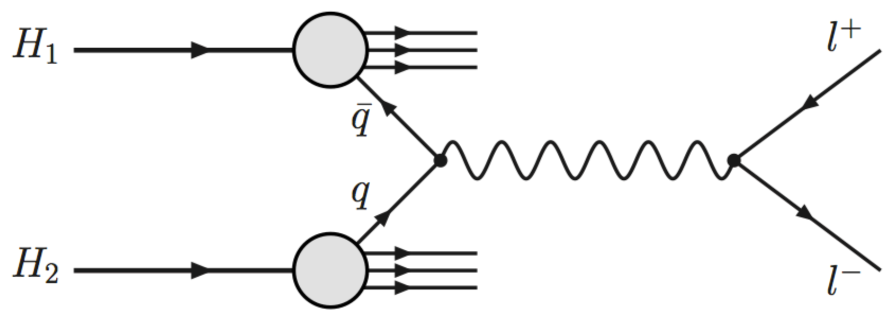
  $e^+ e^- \to h_1 h_2 X$, etc

  $\ell N \to \ell h X$, $\ell N \to \mathrm{jet\,jet\,X}$, etc

  $p\,p \to (\gamma^*, Z, W)$, $p\,p \to \gamma\,\gamma\,X$, $p\,p \to \mathrm{Higgs\,X}$, $p\,p \to (h\,\mathrm{jet})\,\mathrm{X}$, etc

  → rich phenomenology

- Example: TMDs in Drell-Yan process (two scales: q^2, q_T)



$$\frac{d\sigma_{\mathrm{DY}}}{dq_T} \sim \mathcal{H}_{\mathrm{DY}} \int d^2\vec{k}_{aT}\, d^2\vec{k}_{bT}\, \delta(\vec{q}_T - \vec{k}_{aT} - \vec{k}_{bT})\, f_1^q(x_a, \vec{k}_{aT}^{\,2})\, f_1^{\bar{q}}(x_b, \vec{k}_{bT}^{\,2}) + Y_{\mathrm{DY}}$$

Motivation 2: TMDs Provide 3-D Image

- Definition: unpolarized quarks in transversely polarized nucleon

$$\Phi^{[\gamma^+]q}(x,\vec{k}_T) = \frac{1}{2}\int \frac{d\xi^-}{2\pi}\frac{d^2\vec{\xi}_T}{(2\pi)^2}\, e^{ik\cdot\xi}\,\langle P,S|\,\bar{\psi}^q(0)\,\gamma^+\,\mathcal{W}_{TMD}\,\psi^q(\xi^-,\vec{\xi}_T)\,|P,S\rangle$$
$$= f_1^q(x,\vec{k}_T^{\,2}) - \frac{\vec{S}_T\cdot(\hat{P}\times\vec{k}_T)}{M}\, f_{1T}^{\perp q}(x,\vec{k}_T^{\,2})$$

- 3-D structure in $(x,\vec{k}_T)$-space

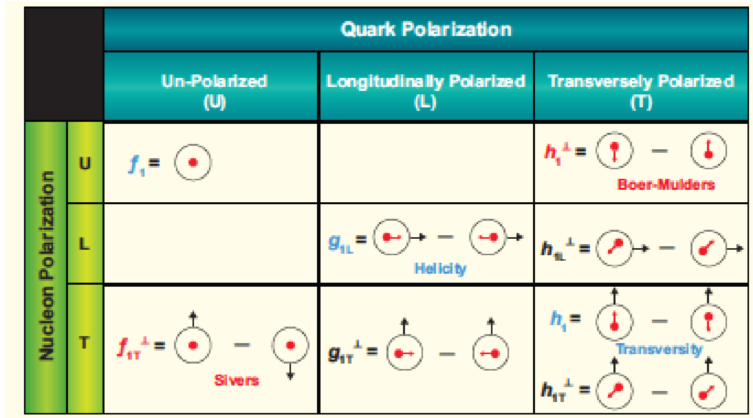
- Sivers function $f_{1T}^{\perp}$ describes strength of correlation $\vec{S}_T\cdot(\hat{P}\times\vec{k}_T)$ (Sivers, 1989)

- Also: TMD quark fragmentation functions (FFs) for $q(s_q,k)\to h(P_h)+X$
  Collins function $H_1^{\perp}$ describes strength of correlation $\vec{s}_{qT}\cdot(\hat{k}\times\vec{P}_{hT})$ (Collins, 1992)

- Sivers function and Collins function can give rise to SSAs in scattering processes

- In total: 8 leading-twist TMDs for both quarks and gluons (PDFs and FFs)

- Overview of leading-twist quark TMDs



(from arXiv:1212.1701)

- New physics aspects due to transverse momenta (confined motion)
  1. transverse momentum dependence of $f_1,\ g_1,\ h_1$
  2. new correlation between $\vec{S}_T,\ \vec{k}_T\ (f_{1T}^{\perp})$, and between $\vec{s}_T,\ \vec{k}_T\ (h_1^{\perp})$
  3. new correlation between $\vec{S}_T,\ \vec{s}_T,\ \vec{k}_T\ (h_{1T}^{\perp})$
  4. new correlation between $\vec{S}_T,\ \lambda,\ \vec{k}_T\ (g_{1T}^{\perp})$, and between $\Lambda,\ \vec{s}_T,\ \vec{k}_T\ (h_{1L}^{\perp})$
  5. connection to single-spin asymmetries and quark-gluon-quark correlations
  6. ideal playground for pQCD: factorization, universality, resummation
  7. allow one to directly study impact of local color gauge invariance of QCD
  8. etc

  $\rightarrow$ "new structures, new physics, new phenomena"

  (quote from X. Ji at 2014 JLab pre-town meeting)

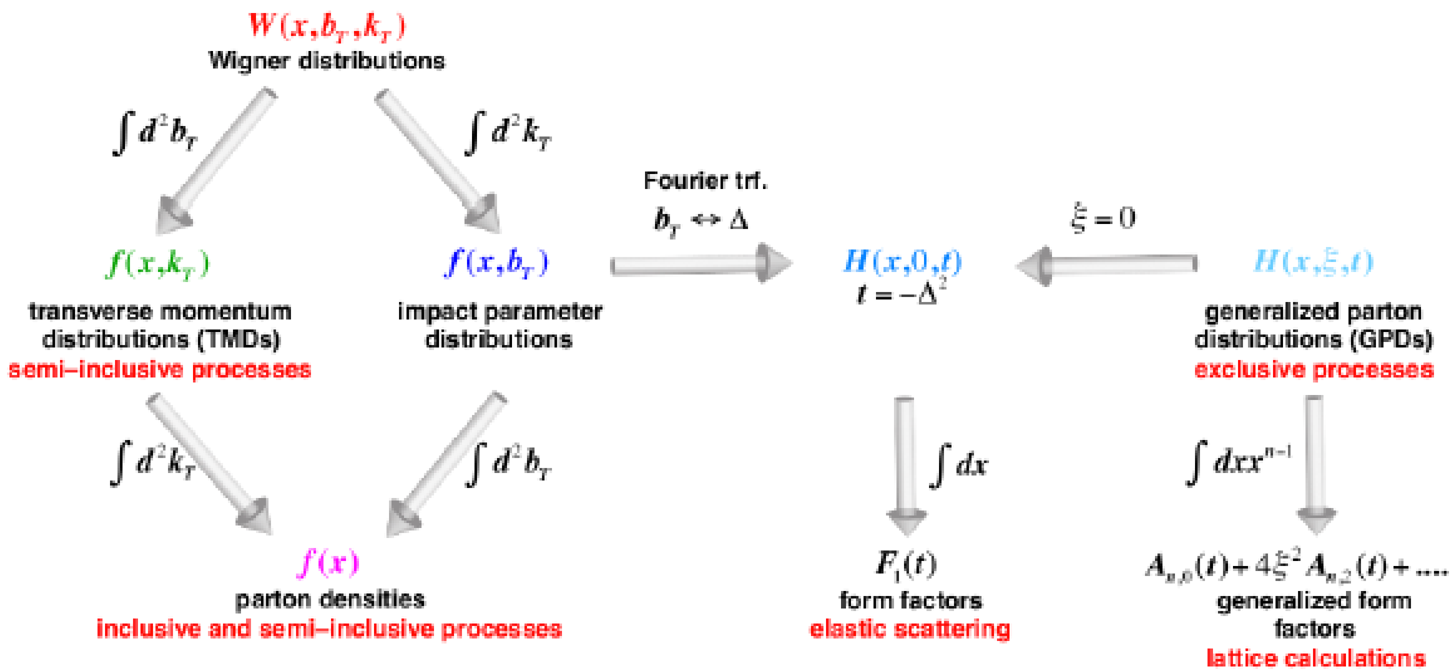
- “Stamp collection”? ... maybe yes ... but we are in good company
  - periodic table of elements

1 H																	2 He
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
55 Cs	56 Ba		72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra		104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Uut	114 Fl	115 Uup	116 Lv	117 Uus	118 Uuo
			57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu
			89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr

    don’t forget the isotopes ...

  - (supersymmetric) extensions of the Standard Model
  - materials science
  - etc.

3-D Imaging: Overview of Tools



(from arXiv:1212.1701)

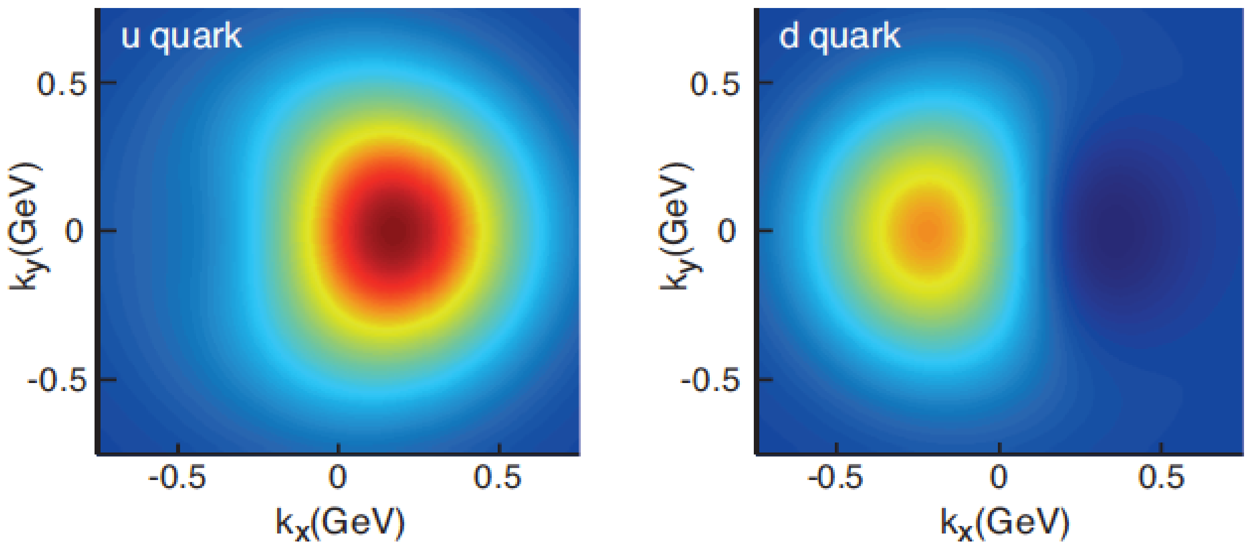
Objects of main interest for 3-D imaging

1. $f(x, \vec{k}_T)$ TMDs: in $(x, \vec{k}_T)$ space
2. $f(x, \vec{b}_T)$ GPDs: in $(x, \vec{b}_T)$ space
3. $W(x, \vec{b}_T, \vec{k}_T)$ Wigner distributions (5-D quasi-probability distribution)
   ($\rightarrow$ talks by Hatta, Schlegel)

Phenomenology: Sivers and Collins Functions

- Extraction of Sivers function

$$\Phi^{[\gamma^+]}(x, \vec{k}_T) = f_1^q(x, \vec{k}_T^2) - \frac{\vec{S}_T \cdot (\hat{P} \times \vec{k}_T)}{M} f_{1T}^{\perp q}(x, \vec{k}_T^2) \qquad (x = 0.1)$$



(from arXiv:1212.1701, based on Anselmino et al, 2011)

  - Sivers effect generates distorted distribution of unpolarized quarks
  - phenomenology agrees with large-N_c prediction $f_{1T}^{\perp u} = -f_{1T}^{\perp d}$ (Pobylitsa, 2003)

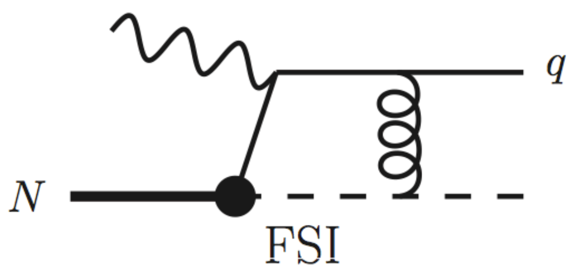
- Extraction of Collins function
  - phenomenology/theory provides/suggests for pion FFs: $H_1^{\perp,\mathrm{fav}} \sim -H_1^{\perp,\mathrm{dis}}$

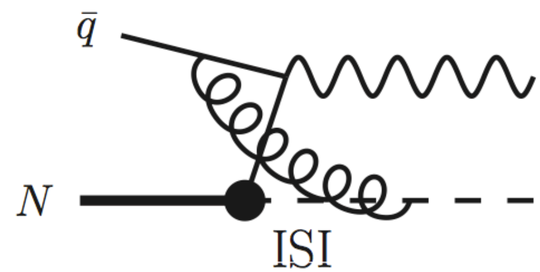
Universality Properties of TMDs

- Prediction based on operator definition in quantum field theory (Collins, 2002)

$$f_{1T}^{\perp}\big|_{\rm DY} = -\, f_{1T}^{\perp}\big|_{\rm SIDIS} \qquad\qquad h_1^{\perp}\big|_{\rm DY} = -\, h_1^{\perp}\big|_{\rm SIDIS}$$

- Underlying physics: re-scattering of active partons with hadron remnants:
  Final State Interaction in semi-inclusive DIS vs Initial State Interaction in Drell-Yan
  (Brodsky, Hwang, Schmidt, 2002)
  $\rightarrow$ change in the direction of $\mathcal{W}_{TMD}$





  - FSI and ISI provide imaginary part, but lead to opposite sign
  - check is crucial test of TMD factorization and collinear twist-3 factorization; mind matching of two approaches (Ji, Qiu, Vogelsang, Yuan, 2006)

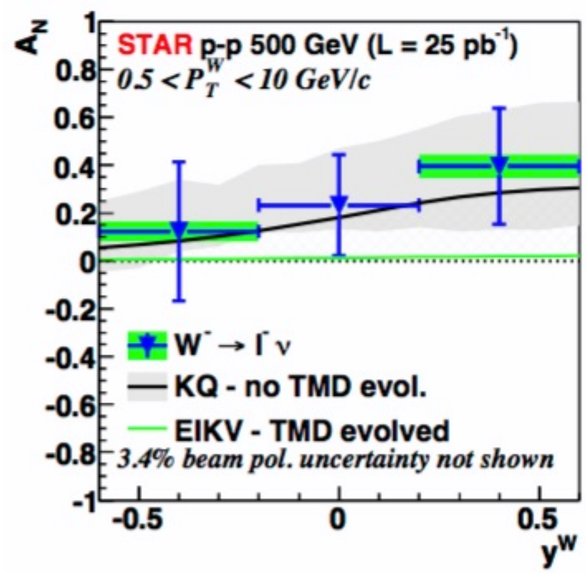
- Several labs worldwide aim at measurement of Sivers effect in Drell-Yan:
  BNL, CERN, FermiLab, GSI, IHEP, JINR, J-PARC

- Experimental verification of sign reversal is pending (DOE milestone HP13!)

- First indication on process dependence of $f_{1T}^{\perp}$ from analysis of A_N in $\ell\, N^{\uparrow} \to \ell\, X$
(A.M., Pitonyak, Schäfer, Schlegel, Vogelsang, Zhou, 2012)

- Process dependence of $f_{1T}^{\perp}$ compatible with AnDY data on A_N in $p^{\uparrow}p \to \text{jet}\, X$
(Gamberg, Kang, Prokudin, 2013)

- Measurement of A_N for $p^{\uparrow}p \to W^{\pm}X$ and $p^{\uparrow}p \to Z^0 X$ (STAR, 2015)



  - very interesting measurement
  - agrees with expected sign
  - however, theoretical prediction has large uncertainties (evolution, $f_{1T}^{\perp \bar{q}}$, ...)

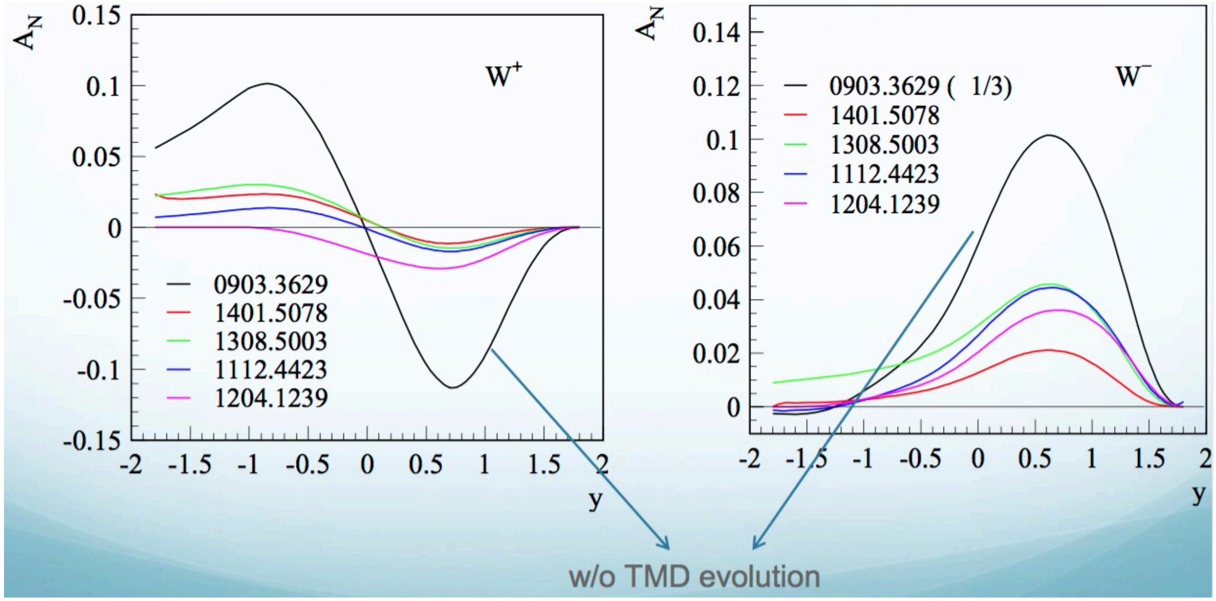
- Universality of TMD fragmentation functions (A.M., 2002 / Collins, A.M., 2004 / ...)

$$H_1^{\perp}\big|_{SIDIS} = H_1^{\perp}\big|_{e^+e^-}$$

  - nontrivial result
  - agrees with existing phenomenology

Open Issues and Emerging Fields (selection)

- TMD evolution (→ talks by Echevarria, Boglione, Signori, ...)
  - sensitivity to (still poorly constrained) non-perturbative physics
  - striking example: A_N for $p^{\uparrow} p \to W^{\pm} X$



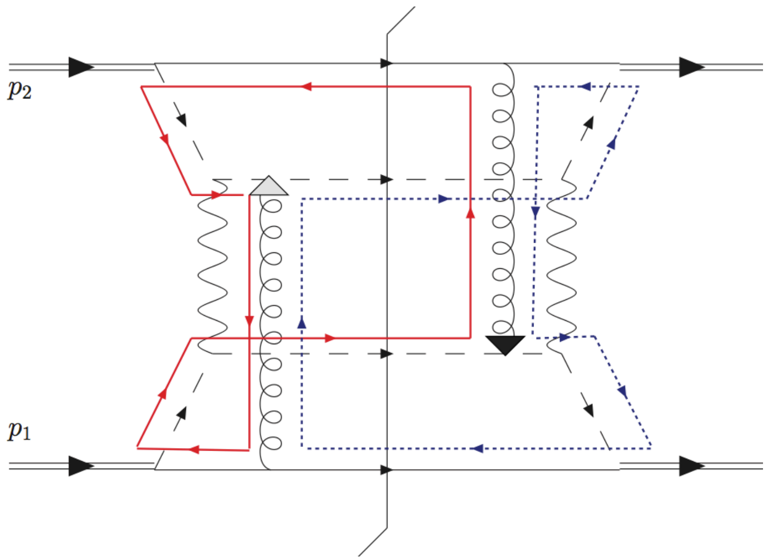
(compilation from Kang, 2015)

- Transverse momentum dependence of cross section for semi-inclusive processes

  (Boglione, Gonzales, Melis, Prokudin, 2014 / Collins, et al, 2016 / ...)

  (→ talk by Wang)

- TMD factorization broken for processes like $p\,p \to \text{jet}\,\text{jet}\,X$ (Rogers, Mulders, 2010)

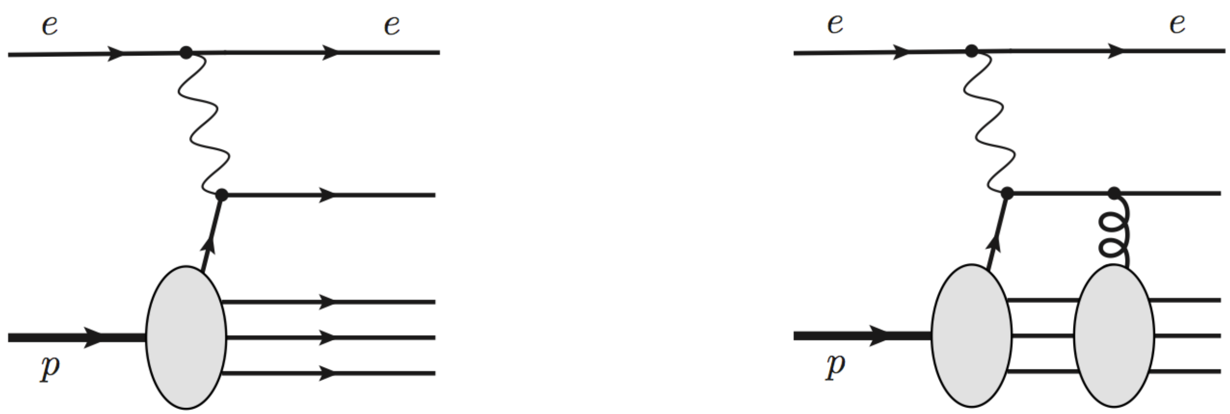


  - factorization breaking due to complicated color flow
  - numerical significance of factorization breaking ?

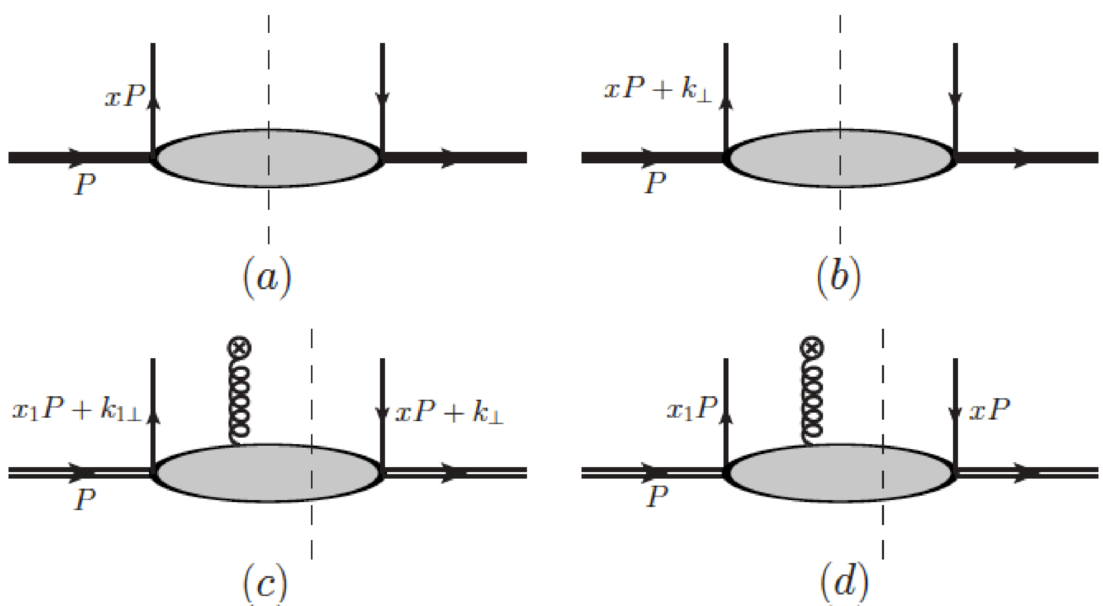
- Gluon TMDs at small x (regime of parton saturation) ($\to$ talk by Mulders)
  - relation between TMD factorization and Color Glass Condensate approach (Dominguez, Marquet, Xiao, Yuan, 2010, 2011 ...)
  - which of the gluon TMDs dominate at small x ? (AM, Zhou, 2011 / Domingez, Qiu, Xiao, Yuan, 2011 / Boer et al, 2015, 2016 / ...)
  - can (spin-dependent) TMDs be used to study parton saturation ?

Reminder: double-spin asymmetry A_{LT} for $\vec{\ell}\, N^{\uparrow} \to \ell\, X$

- Re-scattering of struck quark matters at twist-3 (gluon with physical polarization)



- Contributing correlators after factorization



- collinear quark-quark correlator at twist-3 $\quad \rightarrow \quad g_T(x)$
- k_T-dependent quark-quark correlator $\quad \rightarrow \quad \tilde{g}(x) = \int d^2\vec{k}_T \, \frac{\vec{k}_T^{\,2}}{2M^2} \, g_{1T}(x, \vec{k}_T^{\,2})$
- (collinear) quark-gluon-quark correlator $\quad \rightarrow \quad F_{FT}(x, x_1) \qquad G_{FT}(x, x_1)$

- Exploit relations between functions
  - relation due to QCD equation of motion

$$x \, g_T(x) = \int dx_1 \Big[G_{DT}(x, x_1) - F_{DT}(x, x_1) \Big]$$

- Final result

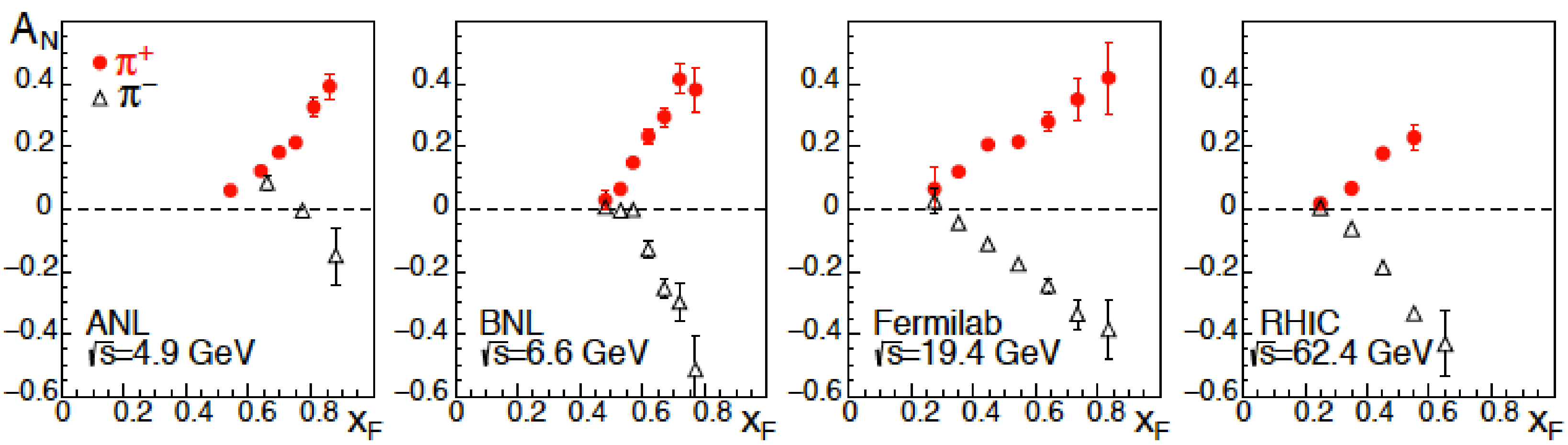
$$\frac{l'^0 d\sigma_{LT}}{d^3\vec{l}'} = -\frac{8 \, \alpha_{em}^2 \, x_B^2 \, \sqrt{1-y} \, M}{Q^5} \, \lambda_\ell \, |\vec{S}_\perp| \cos\phi_S \, \sum_q e_q^2 \, g_T^q(x_B)$$

  - twist-3 effect
  - final result looks rather simple
  - comparable twist-3 observables may have more complicated structure

Transverse SSA in $p^{\uparrow} p \to \pi X$: Data

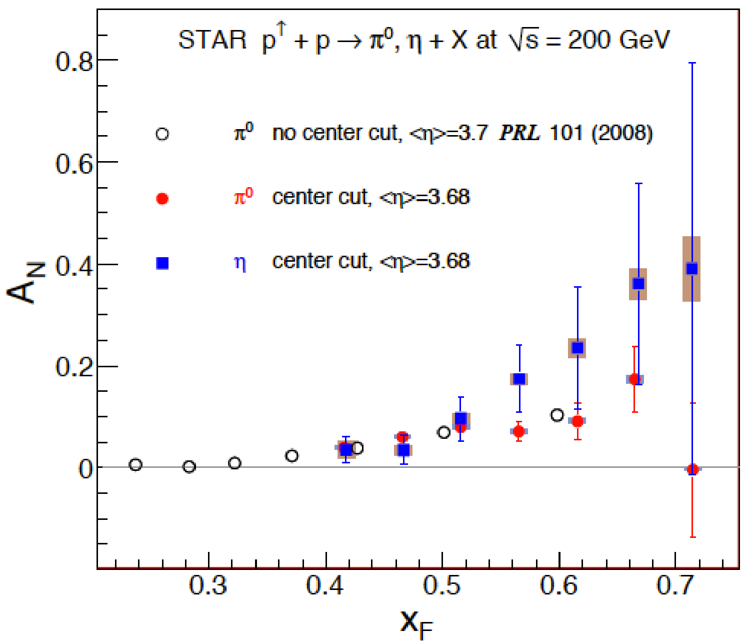
$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} \sim \frac{d\sigma_L - d\sigma_R}{d\sigma_L + d\sigma_R}$$

- Charged pions: sample data

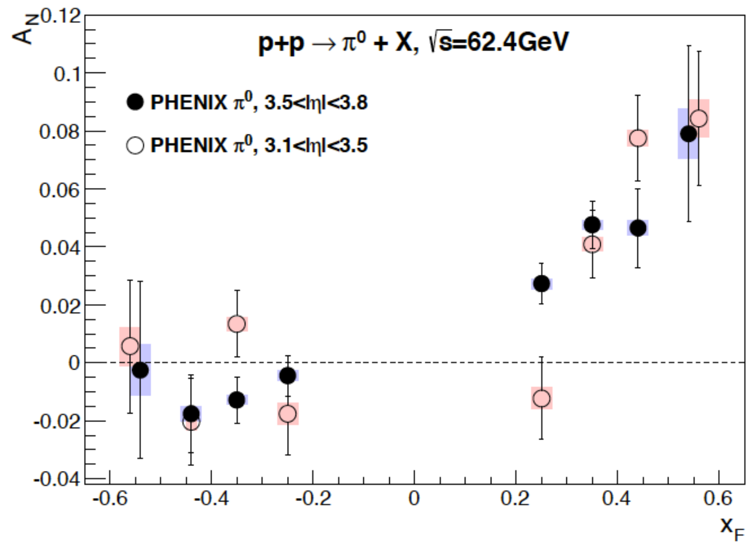


(from Aidala, Bass, Hasch, Mallot, 2012)

- Neutral pions: sample data



STAR, 2012 $\sqrt{s} = 200\,\mathrm{GeV}$



PHENIX, 2013 $\sqrt{s} = 62.4\,\mathrm{GeV}$

- General features
  - very striking effects at large x_F
  - A_N survives at large $\sqrt{s}$
  - $A_N^{\pi^+}$ and $A_N^{\pi^-}$ have roughly same magnitude but opposite sign
  - $A_N^{\pi^0}$ systematically smaller than $A_N^{\pi^\pm}$
  - A_N is twist-3 observable and cannot be explained in collinear parton model
  - data on transverse SSAs represent 40-year old puzzle

Generalized Parton Model and Flavor Structure of A_N

(Torino-Cagliari group, 1994 ... / $\rightarrow$ talk by Murgia)

- Assumes TMD factorization for unpolarized and polarized cross section in $p\,p \rightarrow h\,X$

$$d\sigma = H \otimes \Phi(x_a, \vec{k}_{Ta}) \otimes \Phi(x_b, \vec{k}_{Tb}) \otimes \Delta(z, \vec{k}_{Tc})$$

- Main advantages
  - decent description of twist-2 unpolarized cross section at LO
  - can mimic effects of higher-order corrections of collinear treatment
  - contains certain kinematical higher-twist effects that may be important
  - provides simple intuitive picture of A_N (through Sivers and Collins mechanisms)

- Main drawbacks
  - no derivation of TMD factorization
  - (arbitrary) infrared cutoff for k_T integrations needed
  - physics of ISI/FSI for Sivers effect not included ($\rightarrow$ different source? $\rightarrow$ possibly)
  - analytical results in GPM and collinear twist-3 approach differ

Example: $\sigma_{LT,DIS}^{\text{twist}-3} \sim g_T \qquad \sigma_{LT,DIS}^{\text{GPM}} \sim g_{1T}$

- Flavor structure of A_N (use: no antiquarks, dominance of $qg \to qg$ channel)
  - Sivers contribution

$$d\sigma^{\uparrow}_{\rm Siv}(\pi^+) \;\sim\; f_{1T}^{\perp u} \otimes f_1^g \otimes D_1^{\rm fav} + f_{1T}^{\perp d} \otimes f_1^g \otimes D_1^{\rm dis}$$

$$d\sigma^{\uparrow}_{\rm Siv}(\pi^-) \;\sim\; f_{1T}^{\perp d} \otimes f_1^g \otimes D_1^{\rm fav} + f_{1T}^{\perp u} \otimes f_1^g \otimes D_1^{\rm dis}$$

    - can explain reversed sign for $A_N^{\pi^+}$ and $A_N^{\pi^-}$
    - partial cancellation btw. contributions from favored and disfavored fragmentation

  - Collins contribution

$$d\sigma^{\uparrow}_{\rm Col}(\pi^+) \;\sim\; h_1^u \otimes f_1^g \otimes H_1^{\perp,\rm fav} + h_1^d \otimes f_1^g \otimes H_1^{\perp,\rm dis}$$

$$d\sigma^{\uparrow}_{\rm Col}(\pi^-) \;\sim\; h_1^d \otimes f_1^g \otimes H_1^{\perp,\rm fav} + h_1^u \otimes f_1^g \otimes H_1^{\perp,\rm dis}$$

    - h_1^u and h_1^d have opposite signs
    - can explain reversed sign for $A_N^{\pi^+}$ and $A_N^{\pi^-}$, and nonzero $A_N^{\pi^0}$ as $|h_1^u| > |h_1^d|$
    - no cancellation btw. contributions from favored and disfavored fragmentation
    - Collins contribution can be larger than Sivers contribution

Transverse SSA in $p^\uparrow p \to h\,X$ in Twist-3 Factorization

- Estimate in naïve (twist-2) parton model (Kane, Pumplin, Repko, 1978)

$$A_N \sim \alpha_s \frac{m_q}{P_{h\perp}} \qquad \text{Note: } A_N \not\sim \alpha_s \frac{m_q}{\sqrt{s}}$$

  - α_s due to NLO graphs needed for imaginary part
  - transverse spin effects proportional to mass of polarized particle
  - calculation clearly reveals twist-3 nature of A_N

- Collinear twist-3 factorization in full glory ($P_{h\perp}$ is the only large scale)
  (Ellis, Furmanski, Petronzio, 1983 / Efremov, Teryaev, 1983, 1984 /
  Qiu, Sterman, 1991, 1998 / Koike et al, 2000, ... / etc.)
  - Generic structure of cross section

$$\begin{aligned} d\sigma^\uparrow &= H \otimes f_{a/A(3)} \otimes f_{b/B(2)} \otimes D_{C/c(2)} \quad \to \quad \text{Sivers-type} \\ &+ H' \otimes f_{a/A(2)} \otimes f_{b/B(3)} \otimes D_{C/c(2)} \quad \to \quad \text{Boer-Mulders-type} \\ &+ H'' \otimes f_{a/A(2)} \otimes f_{b/B(2)} \otimes D_{C/c(3)} \quad \to \quad \text{“Collins-type”} \end{aligned}$$

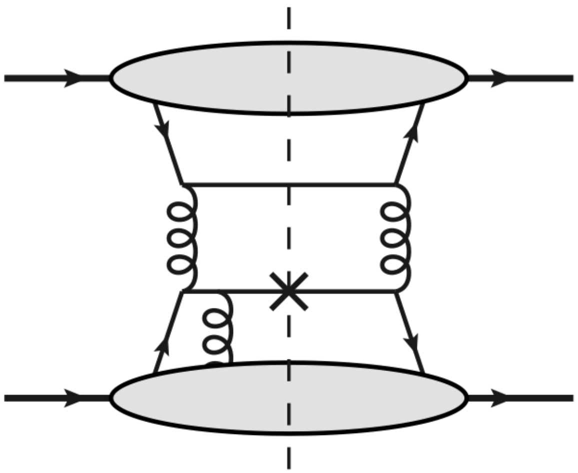
– Sivers-type contribution

∗ contribution from QS function T_F (Qiu, Sterman,1991)

$$\int \frac{d\xi^- d\zeta^-}{4\pi} e^{ixP^+\xi^-} \langle P,S|\bar{\psi}^q(0)\,\gamma^+\,F_{QCD}^{+i}(\zeta^-)\,\psi^q(\xi^-)|P,S\rangle = -\varepsilon_T^{ij} S_T^j\, T_F^q(x,x)$$

vanishing gluon momentum → soft gluon pole matrix element

∗ sample diagram for $qq \to qq$ channel



→ quark propagator goes on-shell for vanishing gluon momentum

→ provides required imaginary part

→ attach extra gluon in all possible ways and consider all graphs and channels

→ contributions from both ISI and FSI

* generic structure of $d\sigma^{\uparrow}_{\rm Siv}$

$$d\sigma^{\uparrow}_{\rm Siv} \sim \sum_i \sum_{a,b,c} H^i \otimes T_F^a(x_a, x_a) \otimes f_1^b \otimes D_1^c \qquad \rightarrow \quad \text{SGPs}$$

$$+ \sum_i \sum_{a,b,c} \tilde{H}^i \otimes \big(T_F^a(0, x_a) + \tilde{T}_F^a(0, x_a)\big) \otimes f_1^b \otimes D_1^c \quad \rightarrow \quad \text{SFPs}$$

→ soft gluon pole (SGP) contribution has relation to TMD approach

→ soft fermion pole (SFP) contribution has no relation to TMD approach

→ SFP matrix elements may be small (Kang et al, 2010 / Braun et al, 2011)

→ H^i and $\tilde{H}^i$ contain physics of ISI/FSI

* relation between QS function and Sivers function (Boer, Mulders, Pijlman, 2003)

$$g\, T_F(x, x) = -\int d^2\vec{k}_T\, \frac{\vec{k}_T^{\,2}}{M}\, f_{1T}^{\perp}(x, \vec{k}_T^{\,2})\Big|_{SIDIS} \sim \langle k_T(x)\rangle$$

→ provides very intuitive interpretation of T_F

→ relation between $A^{\rm Siv}_{\rm SIDIS}$ in SIDIS and A_N in $p^{\uparrow}p \rightarrow h\, X$ possible

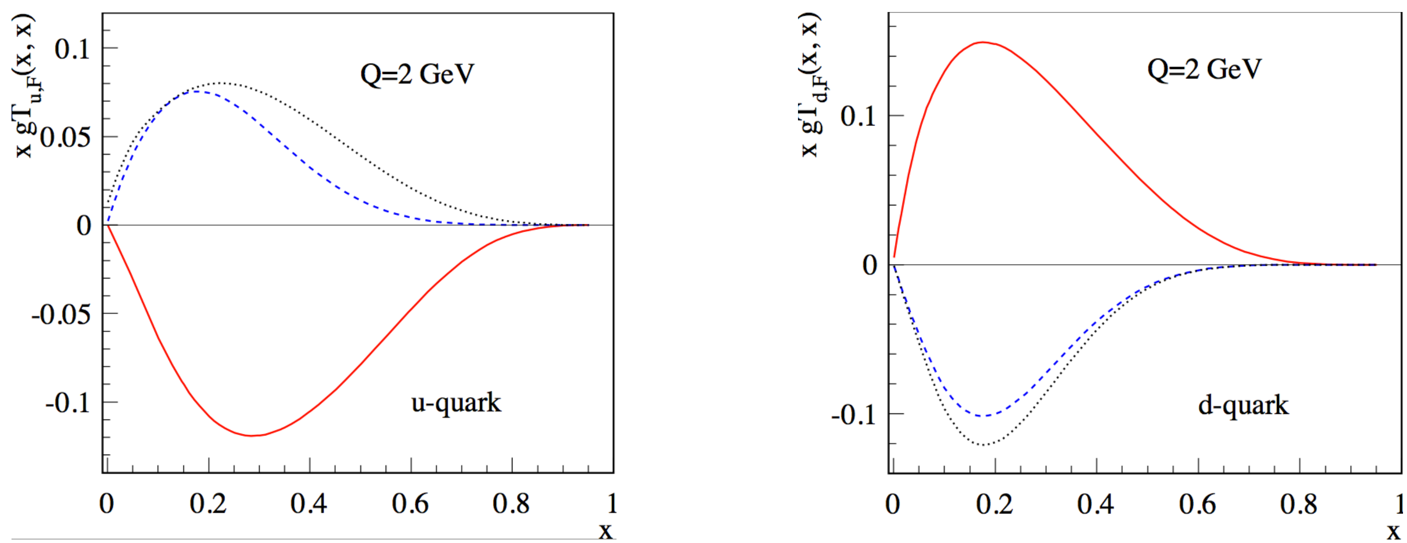
→ flavor structure of A_N like in TMD approach

→ magnitude and sign of A_N may differ from TMD approach due to ISI/FSI

* early successful phenomenology (Kouvaris, et al, 2006 / Kanazawa, Koike, 2010, 2011)

– Sivers-type contribution and sign-mismatch problem (Kang, Qiu, Vogelsang, Yuan, 2011)

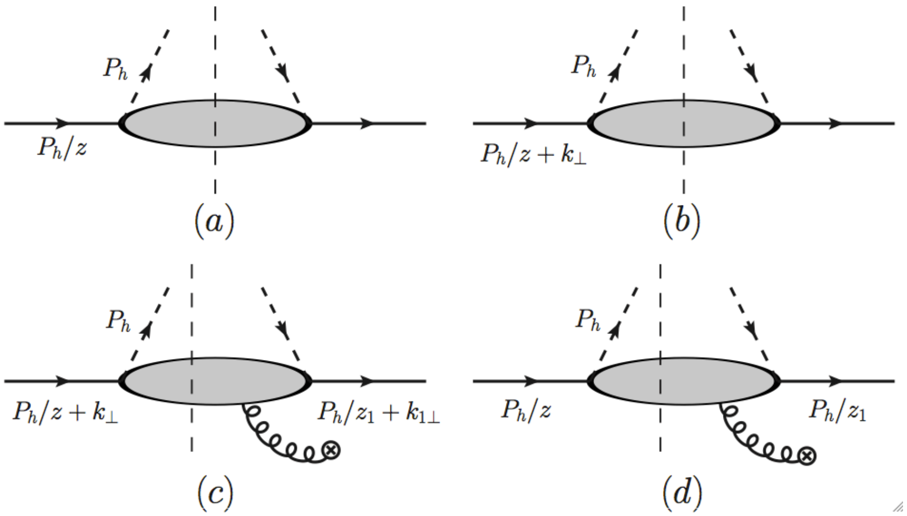
- ∗ assume SSA in $p^{\uparrow}p \to h\,X$ is dominated by Sivers-type contribution
- ∗ T_F can be extracted from different sources (direct extraction vs Sivers input)



- ∗ striking sign-mismatch !
- ∗ model calculation favors sign coming from Sivers input (Braun et al, 2011)
- ∗ one may doubt the dominance of the Sivers-type contribution in A_N
- ∗ doubts supported by analysis of A_N in $\ell\,N^{\uparrow} \to \ell\,X$
  (A.M., Pitonyak, Schäfer, Schlegel, Vogelsang, Zhou, 2012)
- ∗ Boer-Mulders type contribution small (Koike, Kanazawa, 2000)
- ∗ can the large A_N in $p^{\uparrow}p \to H\,X$ be caused by the "Collins-type" contribution ?

Fragmentation Contribution to Transverse SSA in $p^{\uparrow} p \to h\, X$

1. Contributing effects (compare σ_{LT} in inclusive DIS)



- Collinear twist-3 quark-quark correlator: $H(z)$
- Transverse momentum effect from quark-quark correlator: $\hat{H}(z)$

$\to$ has relation with Collins function: $\hat{H}(z) = z^2 \int d^2\vec{k}_\perp \, \frac{\vec{k}_\perp^{\,2}}{2M_h^2} \, H_1^\perp(z, z^2\vec{k}_\perp^{\,2})$

- Collinear twist-3 quark-gluon-quark correlator: $\hat{H}_{FU}^{\Im}(z, z_1)$

2. Analytical results (A.M., Pitonyak, 2012)

$$\frac{P_h^0 d\sigma(\vec{S}_\perp)}{d^3\vec{P}_h} = -\frac{2\alpha_s^2 M_h}{S}\epsilon_{\perp,\alpha\beta}\, S_\perp^\alpha P_{h\perp}^\beta$$

$$\times \sum_i \sum_{a,b,c} \int_{z_{min}}^1 \frac{dz}{z^3} \int_{x'_{min}}^1 \frac{dx'}{x'} \frac{1}{x} \frac{1}{x'S + T/z} \frac{1}{-x'\hat{t} - x\hat{u}} h_1^a(x)\, f_1^b(x')$$

$$\times \left\{ \left[\hat{H}^c(z) - z\frac{d\hat{H}^c(z)}{dz} \right] S_{\hat{H}}^i + \frac{1}{z} H^c(z)\, S_H^i \right.$$

$$\left. + 2z^2 \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{c,\Im}(z, z_1)\, \frac{1}{\xi}\, S_{\hat{H}_{FU}}^i \right\}$$

- $\hat{H},\ H,\ \hat{H}_{FU}^{\Im}$ related

- Derivative term for $\hat{H}$ computed previously (Kang, Yuan, Zhou, 2010)
  $\rightarrow$ does not necessarily dominate

- $S_H^i \sim 1/\hat{t}^3$ and $S_{\hat{H}_{FU}}^i \sim 1/\hat{t}^3$ suggest that contributions from H and $\hat{H}_{FU}^{\Im}$ might dominate in the forward region (large positive x_F); color suppression for $S_{\hat{H}_{FU}}^i$

- Imaginary part provided by (non-perturbative) fragmentation

3. Numerical results (Kanazawa, Koike, A.M., Pitonyak, 2014)

- Relation between fragmentation functions due to QCD equation of motion

$$\hat{H}^{h/q}(z) = -\frac{1}{2z} H^{h/q}(z) + z^2 \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{h/q,\Im}(z, z_1)$$

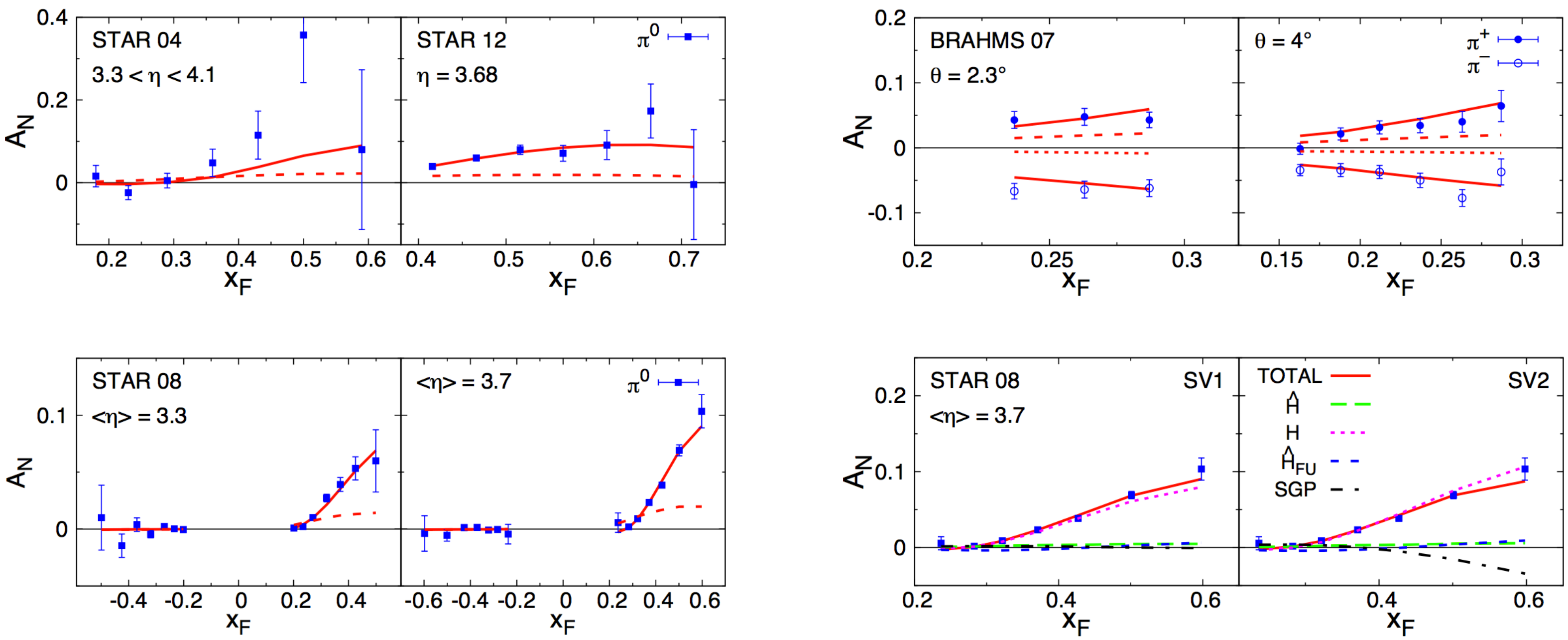
- Ansatz for 3-parton fragmentation function

$$\frac{\hat{H}_{FU}^{\pi^+/(u,\bar{d}),\Im}(z, z_1)}{D_1^{\pi^+/(u,\bar{d})}(z)\, D_1^{\pi^+/(u,\bar{d})}(z/z_1)} \sim N_{\rm fav}\, z^{\alpha_{\rm fav}} (z/z_1)^{\alpha'_{\rm fav}} (1-z)^{\beta_{\rm fav}}\, (1-z/z_1)^{\beta'_{\rm fav}}$$

  - likewise for disfavored fragmentation
  - 8-parameter fit to data for A_N from RHIC

- Input for transversity h_1, Collins function $H_1^{\perp}$ $(\hat{H})$, and Sivers function $f_{1T}^{\perp}$ from $A_{\rm SIDIS}^{\rm Siv}$, $A_{\rm SIDIS}^{\rm Col}$, $A_{e^+e^-}^{\cos(2\phi)}$ (Anselmino et al, 2008, 2013)

- Comparison with data



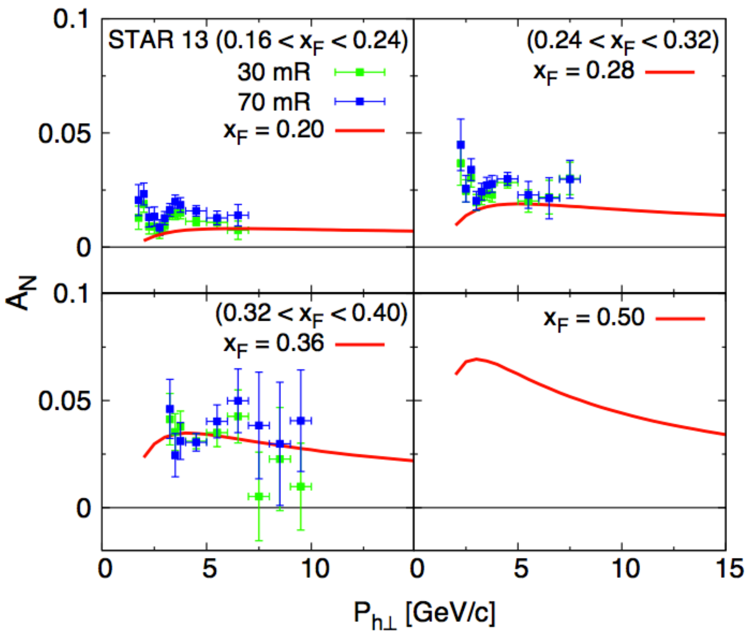
– good fit can be obtained $(\chi^2/\mathrm{d.o.f.} = 1.03)$

– data cannot be described without 3-parton fragmentation function $\hat{H}_{FU}^{\Im}$

– numerics dominated by contribution from H (fixed by $\hat{H}$ and $\hat{H}_{FU}^{\Im}$)

– fit is rather flexible $(\chi^2/\mathrm{d.o.f.} = 1.10$ for SV2 input)

- Transverse momentum dependence of A_N



  - preliminary STAR data show rather flat $P_{h\perp}$ dependence of A_N
  - collinear twist-3 calculation can describe this trend
  - note: data not included in fit, only statistical errors shown

- Overall outcome
  - simultaneous description of A_N, and $A^{\rm Siv}_{\rm SIDIS}$, $A^{\rm Col}_{\rm SIDIS}$, $A^{\cos(2\phi)}_{e^+e^-}$ possible
  - breakthrough in understanding A_N (?)
  - information on $\hat{H}^{\Im}_{FU}$ from other sources required
  - some support from model calculation (Lu, Schmidt, 2015)

4. Lorentz-invariance relations (Kanazawa, Koike, A.M., Pitonyak, Schlegel, 2015)

- Additional constraint, beyond QCD equation of motion

- Both $\hat{H}$ and H can be expressed through $\hat{H}_{FU}^{\Im}$

- Example

$$\hat{H}^{h/q}(z) = -\frac{2}{z}\int_z^1 dz_1 \int_{z_1}^{\infty} \frac{dz_2}{z_2^2} \frac{\frac{2}{z_1} - \frac{1}{z_2}}{\left(\frac{1}{z_1} - \frac{1}{z_2}\right)^2} \hat{H}_{FU}^{h/q,\Im}(z_1, z_2) \sim \langle P_{hT}(z) \rangle$$

  - fragmentation contribution to A_N given by 3-parton correlator $\hat{H}_{FU}^{h/q,\Im}(z_1, z_2)$
  - intuitive interpretation for twist-3 fragmentation contribution
  - Schäfer-Teryaev sum rule suggests flavor structure of A_N

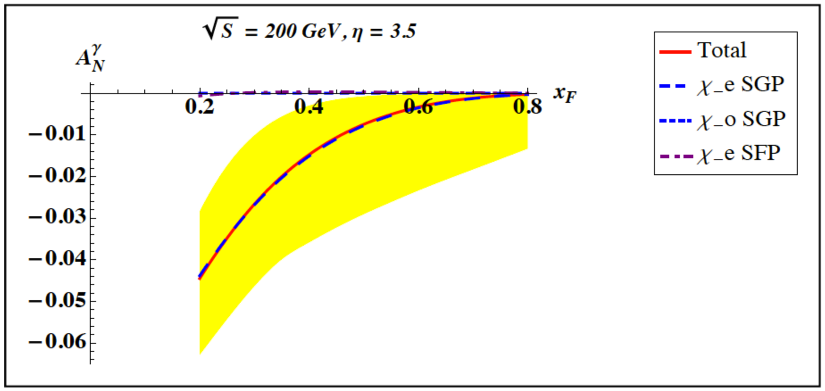
- Updated phenomenology needed for
  - A_N in $p^{\uparrow} p \to h X$ (Kanazawa, Koike, A.M., Pitonyak, 2014)
  - A_N in $\ell N^{\uparrow} \to h X$ (Gamberg, Kang, A.M., Pitonyak, Prokudin, 2014)

Transverse SSA in $p^\uparrow p \to \gamma\, X$ in Twist-3 Factorization

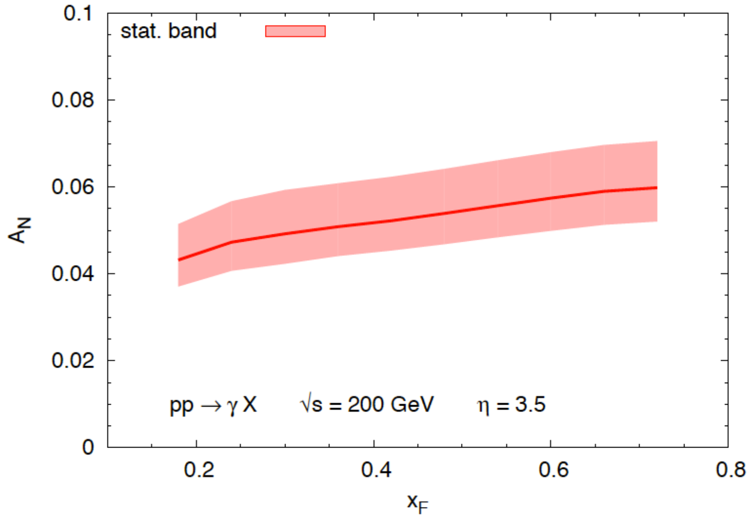
(Kanazawa, Koike, A.M., Pitonyak, 2014)

- Will be measured at RHIC
- Numerical results

Collinear twist-3 (Kanazawa et al, 2014)



GPM (Anselmino et al, 2013)



- dominated by SGP contribution related to polarized proton $\longrightarrow$ clean access to T_F
- physics of ISI/FSI enters $\longrightarrow$ process-dependence of Sivers function can be checked
- seems ideal for discriminating between collinear twist-3 approach and GPM (different signs)

Summary

- TMD approach
  - TMDs appear in many processes and have rich phenomenology
  - tremendous progress with regard to concepts and phenomenology
  - is intuitive
  - can be used for processes like SIDIS and Drell-Yan
  - indications about process-dependence of Sivers function
  - has conceptual problems for twist-3 observables like A_N in $p^\uparrow p \to h\, X$ (this is not a statement about phenomenology)

- Collinear twist-3 approach
  - is also intuitive (to some extent)
  - takes into account physics of ISI/FSI for twist-3 observables
  - fragmentation contribution may play crucial role for A_N in $p^\uparrow p \to h\, X$
    $\to$ can also solve sign-mismatch problem
  - simultaneous description of various SSAs possible
  - updated phenomenology for twist-3 fragmentation effects needed
  - A_N for $p^\uparrow p \to \gamma\, X$ may provide critical new insights