

Phenomenology of transverse momentum distributions

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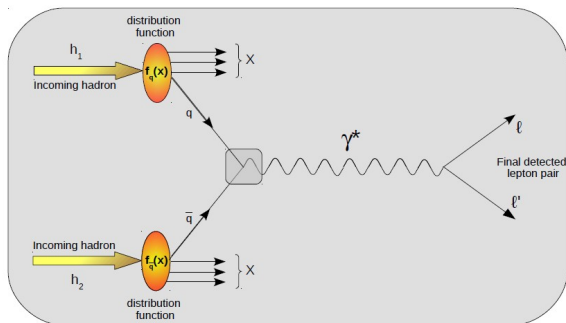
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***Where can we learn about
the 3D structure of matter ?***

Experimental data for TMD studies

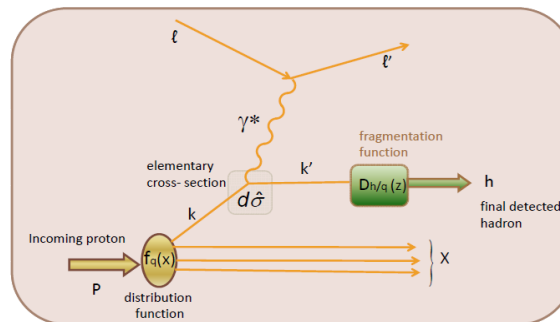
Unpolarized and Polarized Drell-Yan scattering



$$\sigma_{\text{Drell-Yan}} = f_q(x, k_{\perp}) \otimes f_{\bar{q}}(x, k_{\perp}) \otimes \hat{\sigma}^{q\bar{q} \rightarrow \ell\bar{\ell}}$$

Allows extraction of **distribution** functions

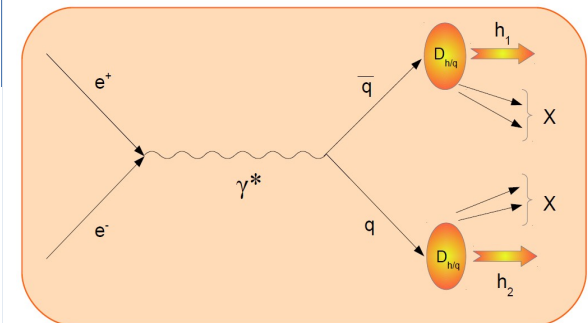
Unpolarized and Polarized SIDIS scattering



$$\sigma_{\text{SIDIS}} = f_q(x) \otimes \hat{\sigma} \otimes D_{h/q}(z)$$

Allows extraction of **distribution** and **fragmentation** functions

$e^+ e^- \rightarrow h_1 h_2 X$



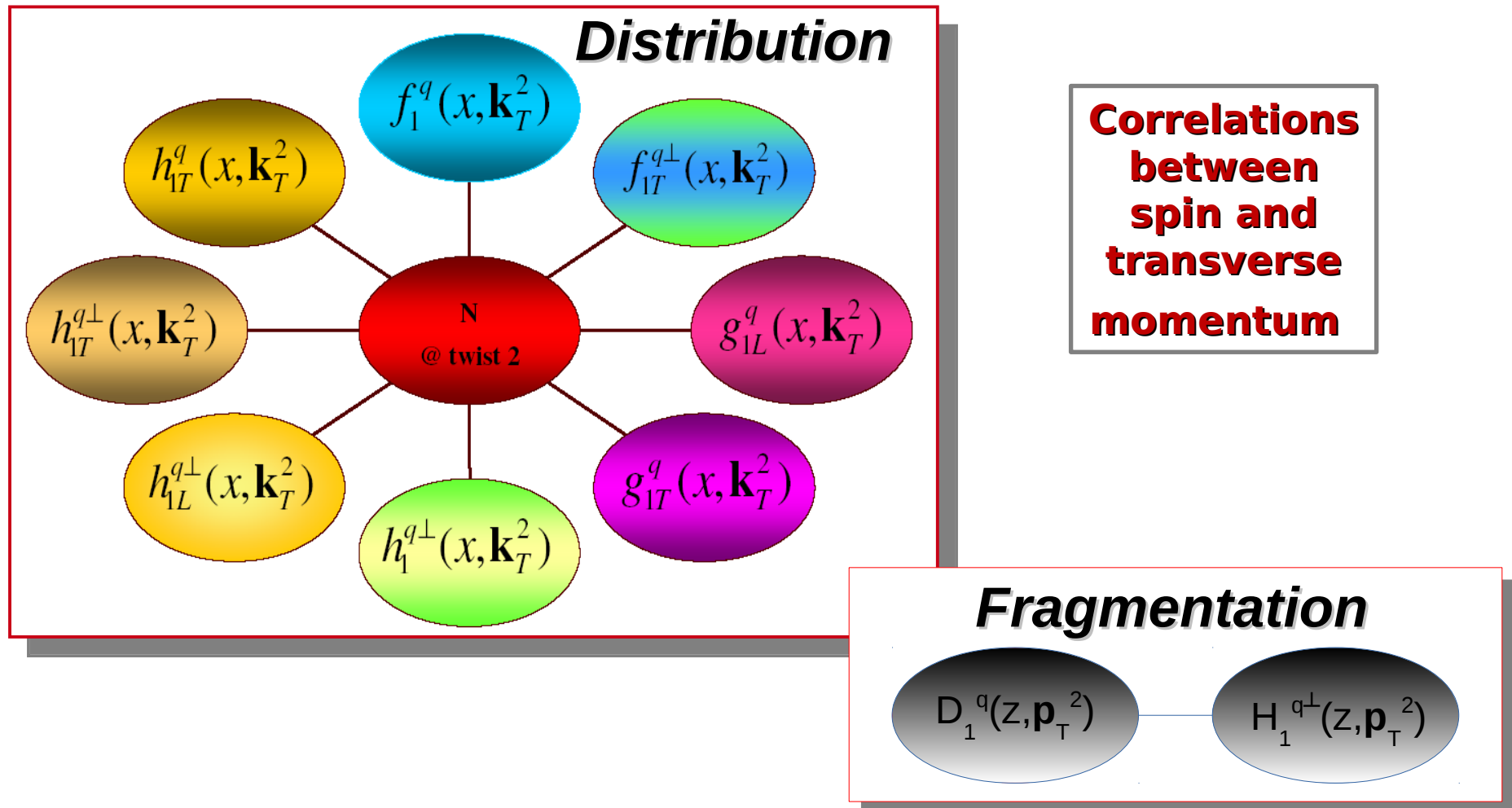
$$\sigma_{h_1 h_2} \propto D(z_1) \otimes D(z_2) \otimes \hat{\sigma}$$

Allows extraction of **fragmentation** functions

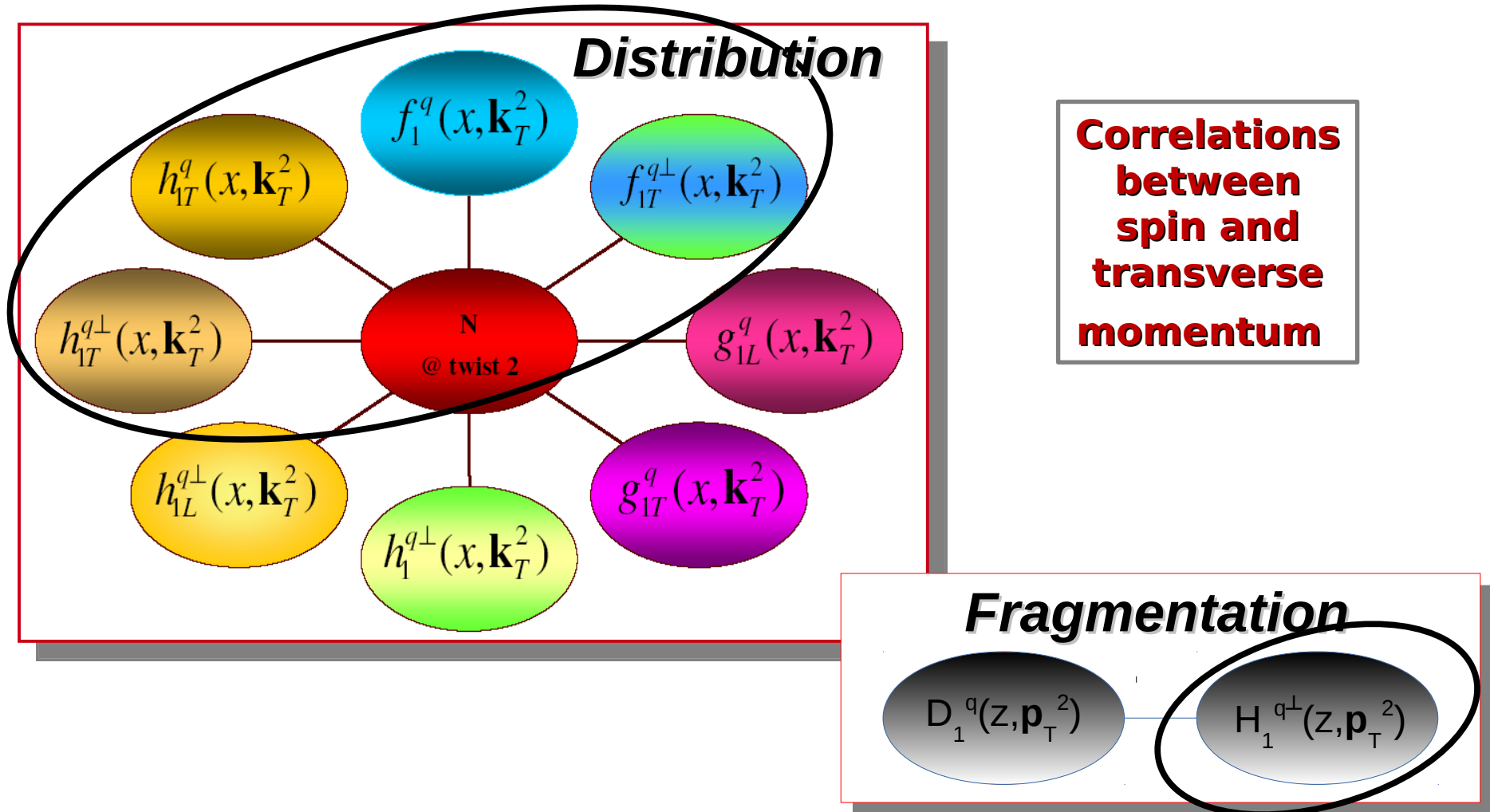


***Transverse momentum
dependent parton
distribution functions***

TMD distribution and fragmentation functions



TMD distribution and fragmentation functions



Extracting unpolarized TMDs from SIDIS data

Extracting the unpolarized TMD Gaussian widths from SIDIS multiplicities

M. Anselmino, M. Boglione, O. Gonzalez, S. Melis, A. Prokudin, JHEP 1404 (2014) 005, ArXiv:1312.6261

- Data: **Hermes** (p and d targets, π^+ , π^- , K^+ , K^- production)

2660 data points in (x , z , P_T , Q^2 bins)

A. Airapetian et al.,
Phys. Rev. D87
(2013) 074029

Compass (d target, h^+ , h^- production)

18627 data points in (x , z , P_T , Q^2 bins)

C. Adolph et al.,
Eur. Phys. J. C73,
2531 (2013)

- Parameterizations:

$$\hat{f}_{q/p}(x, k_{\perp}; Q) = f_{q/p}(x; Q) \frac{e^{-k_{\perp}^2 / \langle k_{\perp}^2 \rangle}}{\pi \langle k_{\perp}^2 \rangle}$$

CTEQ6L (DGLAP evolution)

1 free parameter
(no evolution)

$$D_{h/q}(z, p_{\perp}) = D_{h/q}(z) \frac{e^{-p_{\perp}^2 / \langle p_{\perp}^2 \rangle}}{\pi \langle p_{\perp}^2 \rangle},$$

DSS (DGLAP evolution)

1 free parameter
(no evolution)

Extracting the unpolarized TMD Gaussian widths from SIDIS multiplicities

In the simplest form of this model:

Flavor-independent average transverse momenta

No x-dependence

No z-dependence

Two parameters in total

Gaussian model:

$$\langle P_T^2 \rangle = \langle p_{\perp}^2 \rangle + z_h^2 \langle k_{\perp}^2 \rangle.$$

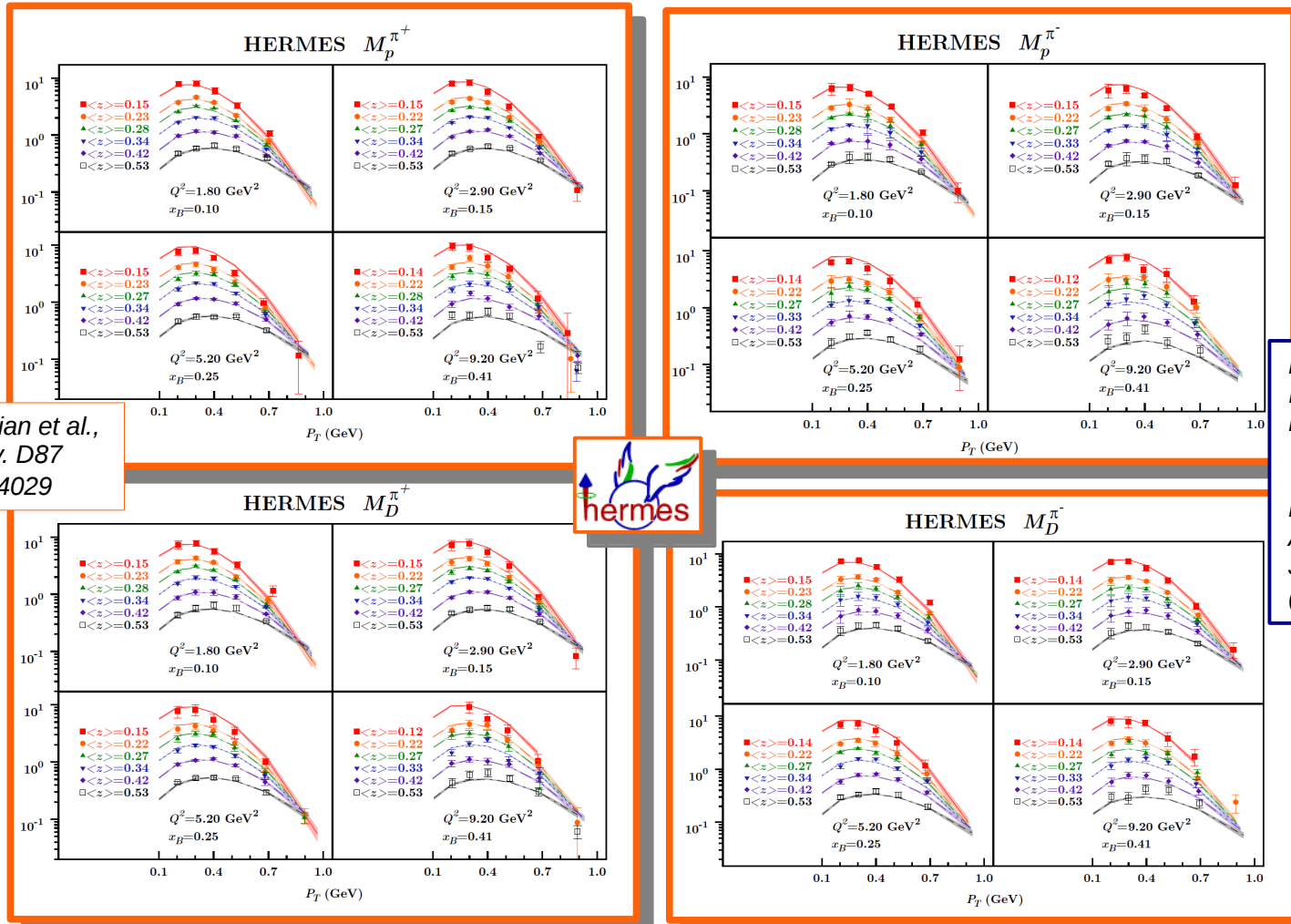
$$\sigma \propto \frac{1}{\pi \langle P_T^2 \rangle} e^{-P_T^2 / \langle P_T^2 \rangle}$$

Normalization

Gaussian width

Extracting the unpolarized TMD Gaussian widths from SIDIS multiplicities

M. Anselmino, M. Boglione, O. Gonzalez, S. Melis, A. Prokudin, JHEP 1404 (2014) 005, ArXiv:1312.6261



A. Airapetian et al.,
Phys. Rev. D87
(2013) 074029

NO flavour dep.
In distr. fns.,
MILD flavour dep.
in fragm. fns.,

Results agree with
A. Signori et al.,
JHEP 1311
(2013) 194

$$\langle k_{\perp}^2 \rangle = 0.57 \pm 0.08 \text{ GeV}^2$$

$$\langle p_{\perp}^2 \rangle = 0.12 \pm 0.01 \text{ GeV}^2$$

$$\chi_{\text{dof}}^2 = 1.69$$

Extracting the unpolarized TMD Gaussian widths from SIDIS multiplicities

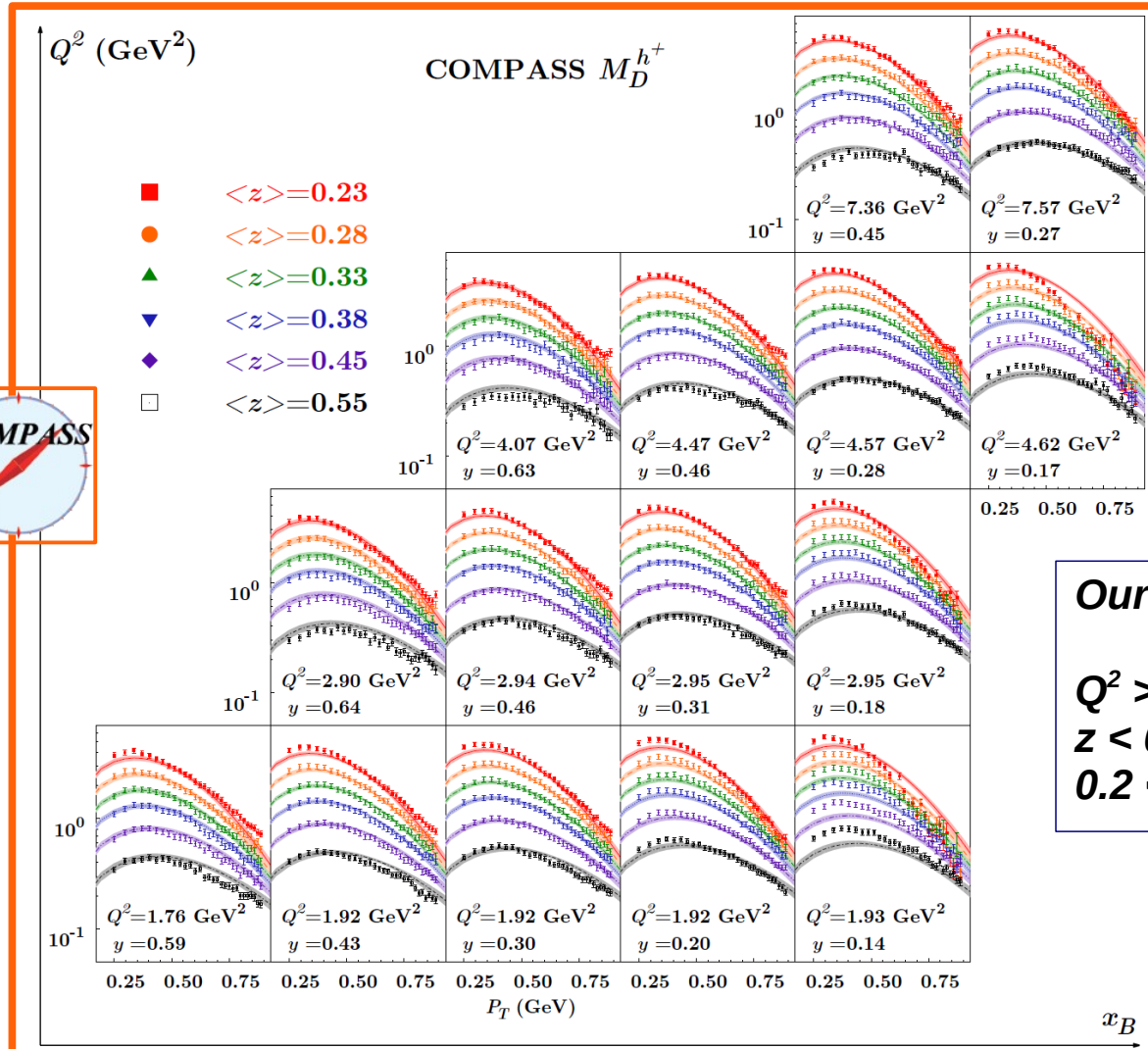
M. Anselmino, M. Boglione, O. Gonzalez, S. Melis, A. Prokudin, JHEP 1404 (2014) 005, ArXiv:1312.6261

$$\langle k_{\perp}^2 \rangle = 0.60 \pm 0.14 \text{ GeV}^2$$

$$\langle p_{\perp}^2 \rangle = 0.20 \pm 0.02 \text{ GeV}^2$$

$$\chi_{\text{dof}}^2 = 3.42$$

$$N_y = A + B y$$



NO flavour dep.
In distr. fns.,
NO flavour dep.
in fragm. fns.

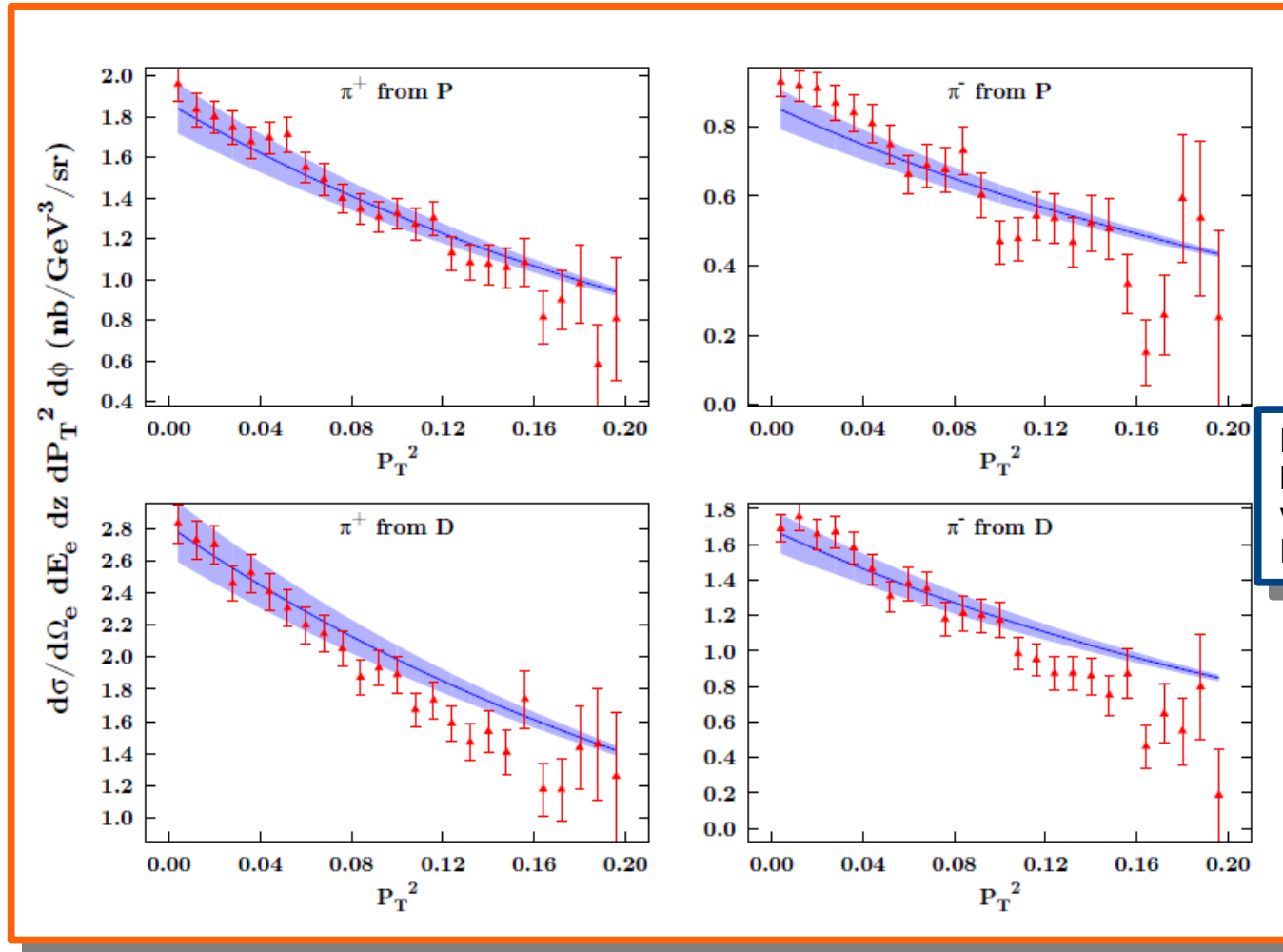
Our cuts:

$Q^2 > 1.6 \text{ GeV}^2$
 $z < 0.6$
 $0.2 < P_T < 0.9$

C. Adolph et al.,
Eur. Phys. J. C73, 2531
(2013)

Comparison with Jlab data HALL C

M. Anselmino, M. Boglione, O. Gonzalez, S. Melis, A. Prokudin, JHEP 1404 (2014) 005, ArXiv:1312.6261

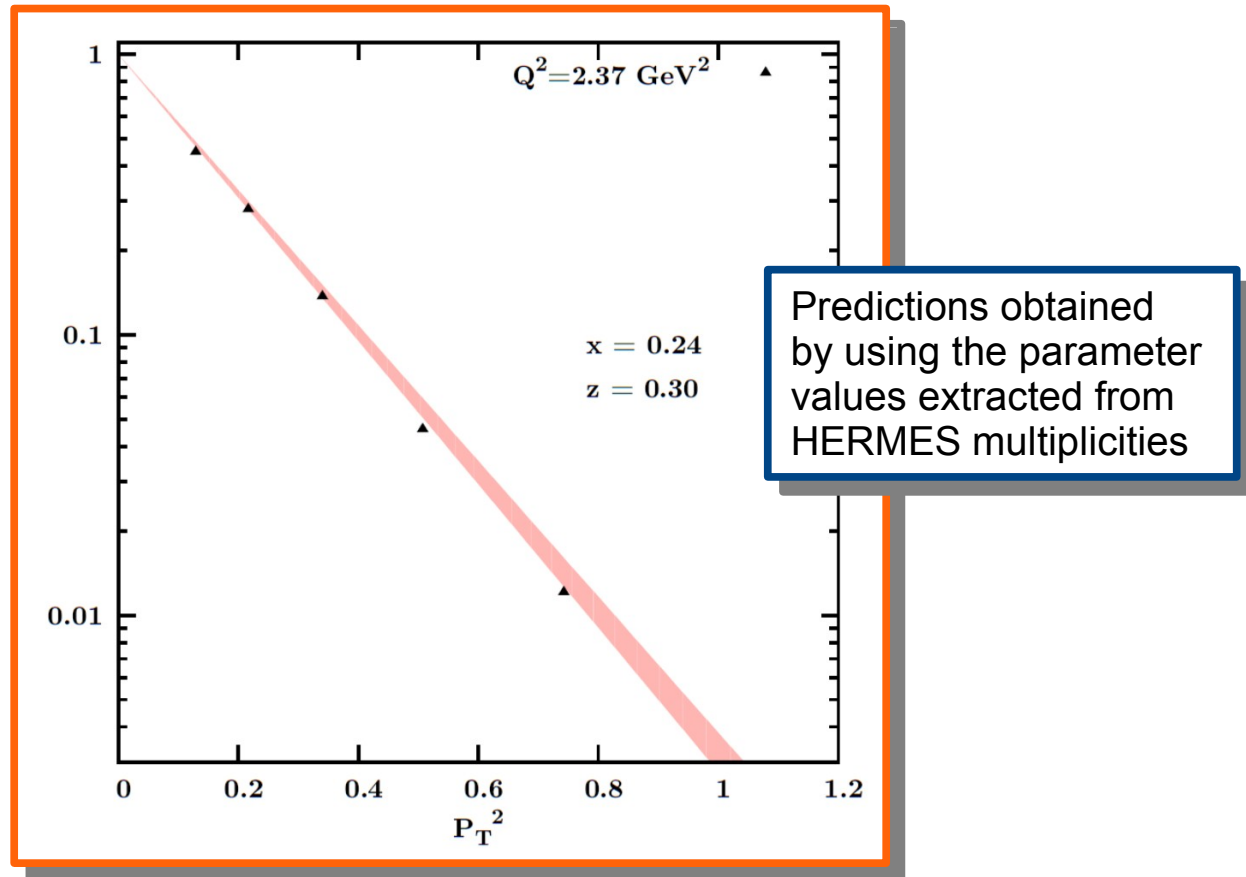


Predictions obtained by using the parameter values extracted from HERMES multiplicities

R. Asaturyan et al., Phys. Rev. C85, 015202 (2012)

Comparison with Jlab data CLAS 6

M. Anselmino, M. Boglione, O. Gonzalez, S. Melis, A. Prokudin, *JHEP* 1404 (2014) 005, ArXiv:1312.6261



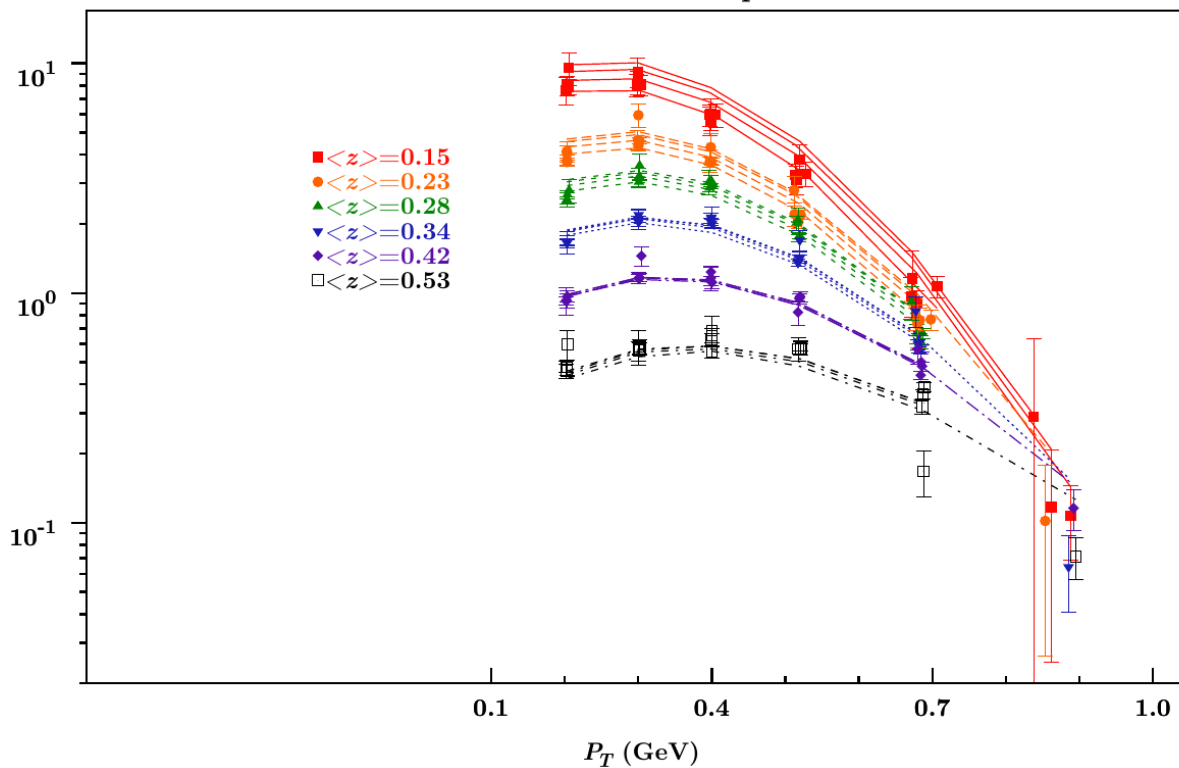
M. Osipenko et al., *Phys. Rev. D* 80, 032004 (2009)

Q^2 dependence of HERMES data...

$$F_{UU} = \sum_q e_q^2 f_{q/p}(x_B) D_{h/q}(z_h) \frac{e^{-P_T^2/\langle P_T^2 \rangle}}{\pi \langle P_T^2 \rangle}$$

$$\langle P_T^2 \rangle = \langle p_\perp^2 \rangle + z_h^2 \langle k_\perp^2 \rangle$$

HERMES $M_p^{\pi^+}$



Anselmino et al. JHEP 1404 (2014) 005

$$\langle k_\perp^2 \rangle = 0.57 \pm 0.08 \text{ GeV}^2$$

$$\langle p_\perp^2 \rangle = 0.12 \pm 0.01 \text{ GeV}^2$$

$$\chi_{\text{dof}}^2 = 1.69$$

All four bins have been overlapped in the same panel

Hard to decouple the Q^2 dependence from HERMES data alone

Scale Evolution of unpolarized multiplicities

HERMES and COMPASS multiplicities cover the same range in Q^2 ...

$$\langle k_{\perp}^2 \rangle = g_1 + g_2 \ln(Q^2 / Q_0^2) + g_3 \ln(10 e x)$$

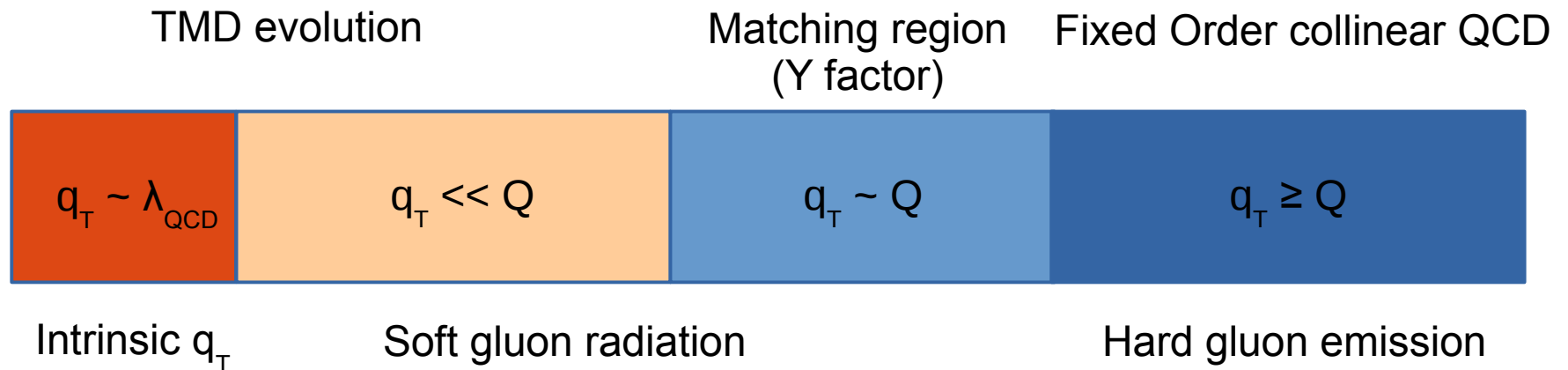
$$\langle p_{\perp}^2 \rangle = g'_1 + z^2 g'_2 \ln(Q^2 / Q_0^2)$$

$$\langle P_T^2 \rangle = g'_1 + z^2 [g_1 + g_2 \ln(Q^2 / Q_0^2) + g_3 \ln(10 e x)]$$

- HERMES multiplicities show no sensitivity to these parameters
- COMPASS fitting is much more involved.
After correcting for normalization,
we find that the total χ^2 decreases from 3.42 to 2.69.
- New COMPASS data on P_T dependent multiplicities will be of great help !

TMD regions

- For this scheme to work, 4 distinct kinematic regions have to be identified
- They should be large enough and well separated

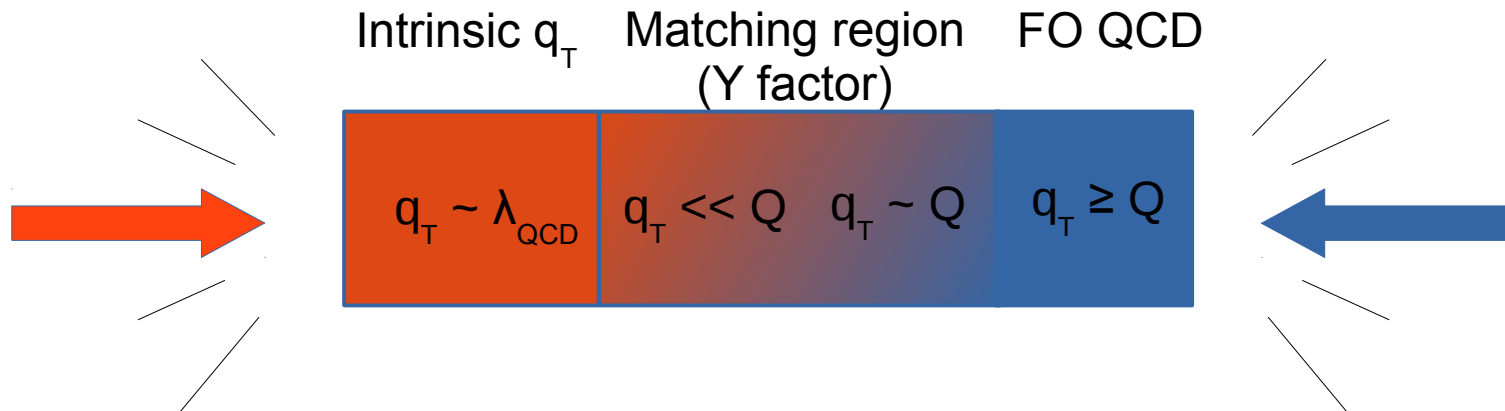


TMD regions

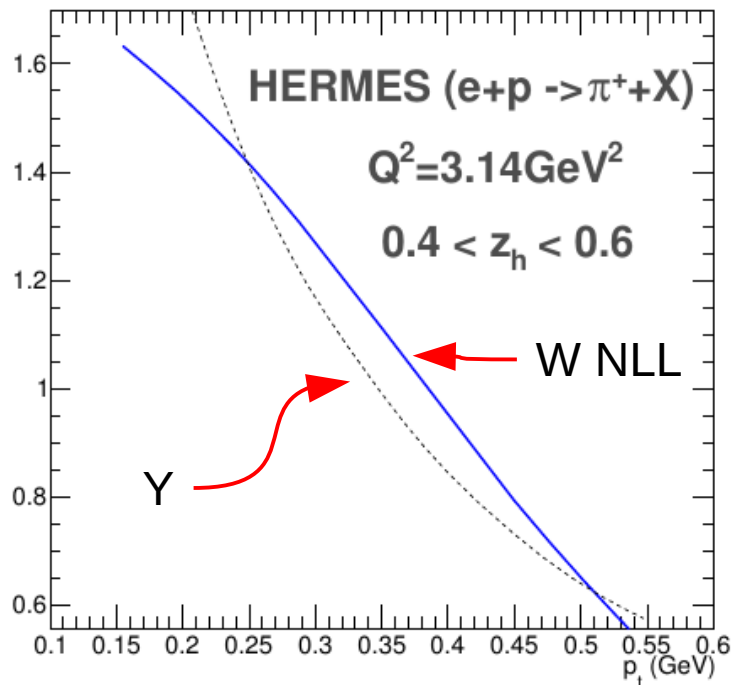
- For this scheme to work, 4 distinct kinematic regions have to be identified
- They should be large and well separated

Does not work in SIDIS !

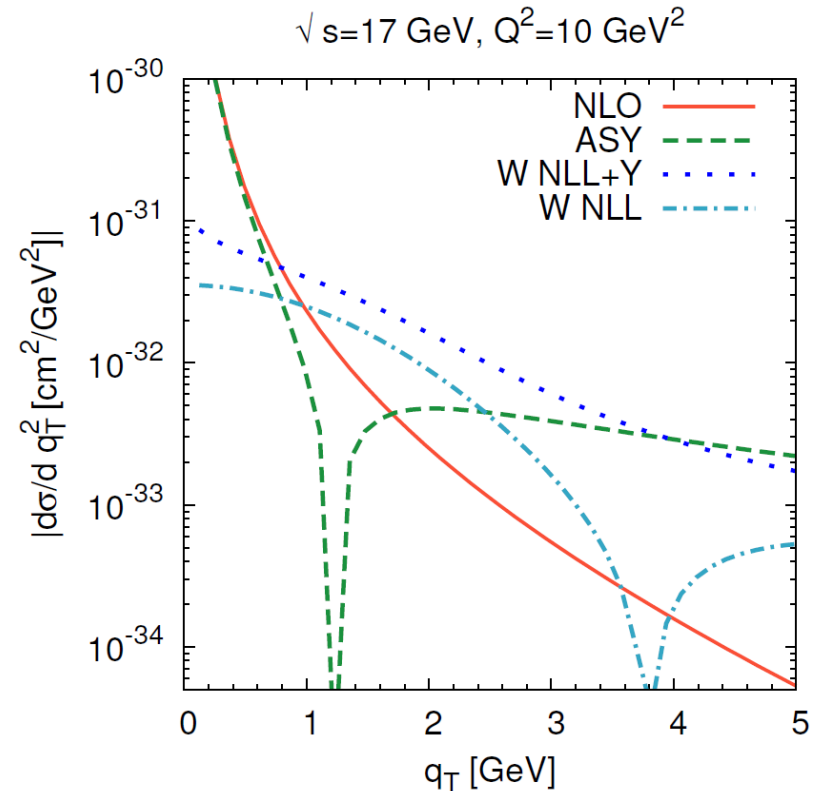
TMD evolution



SIDIS - Y factor



Sun et al arXiv:1406.3073

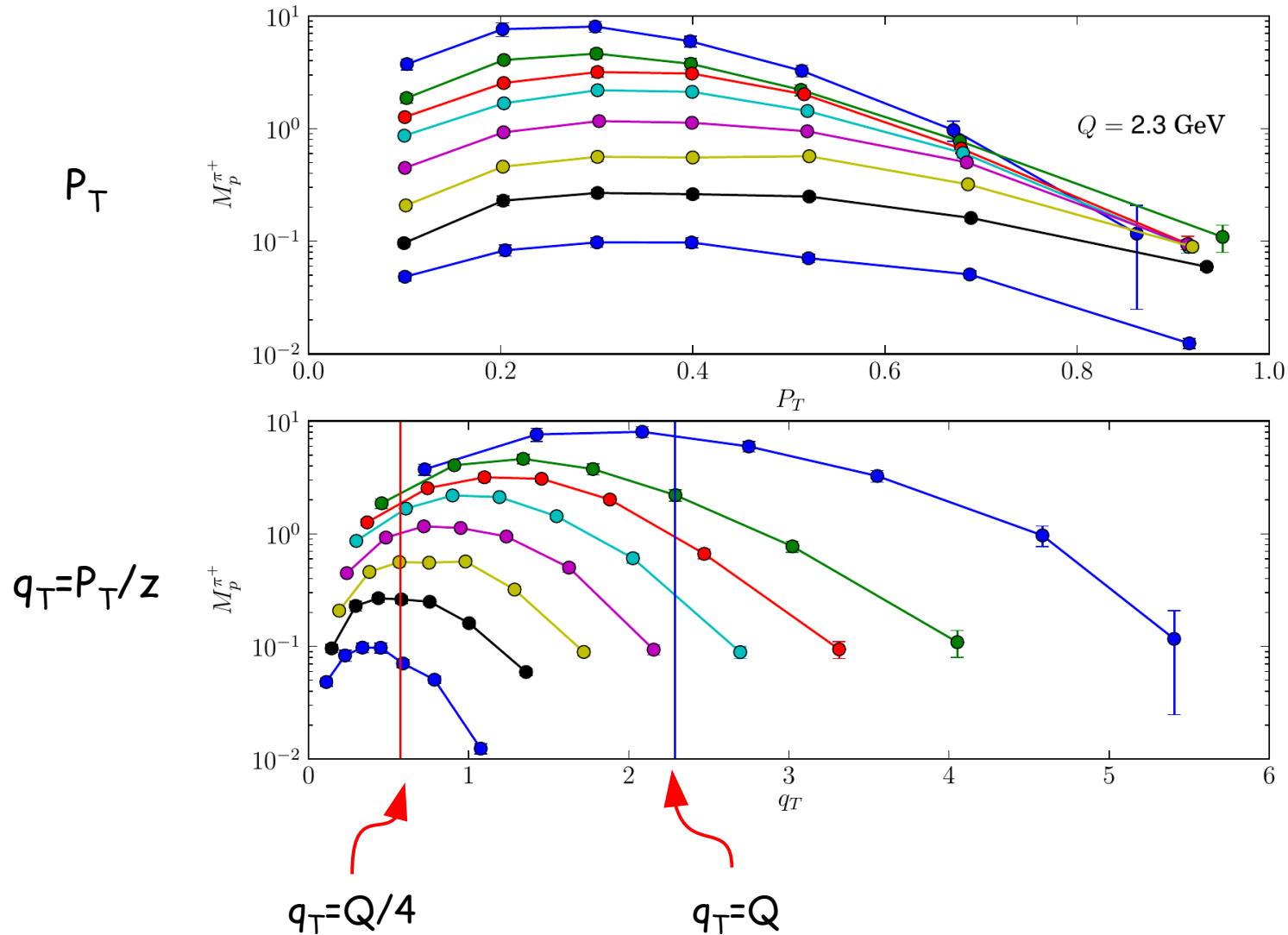


Boglione et al, JHEP 02 (2015) 095

$$Y = \frac{d\sigma^{\text{NLO}}}{dx dy dz dq_T^2} - \frac{d\sigma^{\text{ASY}}}{dx dy dz dq_T^2}$$

- Y factor is very large, larger or as large as the resummed cross section

TMD regions



Summary

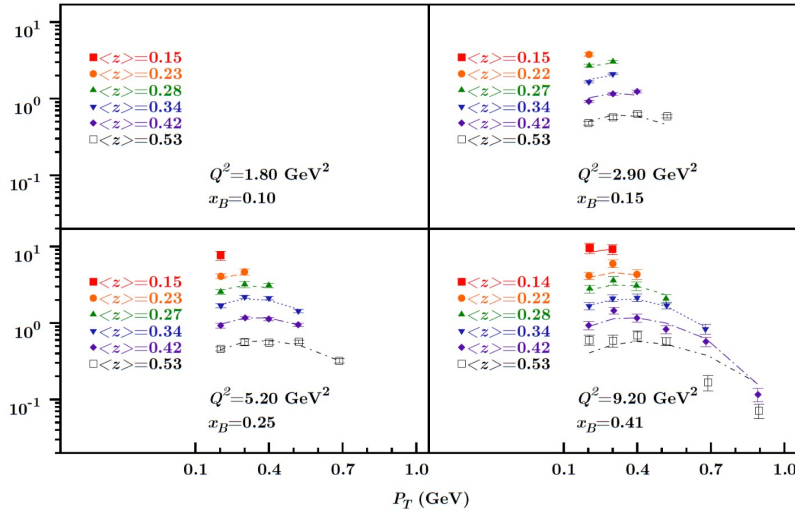
- Naive TMD Models can describe HERMES data
- Similarly to DY the Q^2 dependence is not clearly visible in the shape of the spectrum
- TMD resummation is difficult
 - ✗ no information on TMD fragmentations
 - ✗ issues with normalization
 - ✗ in SIDIS most of the data is at intermediate q_T , however both q_T and Q are small compared to DY processes
 - ✗ the non-perturbative behavior is dominant

Fit of HERMES and COMPASS data Attempting "Resummation" in SIDIS ...

J. Osvaldo Gonzalez Hernandez, work in progress

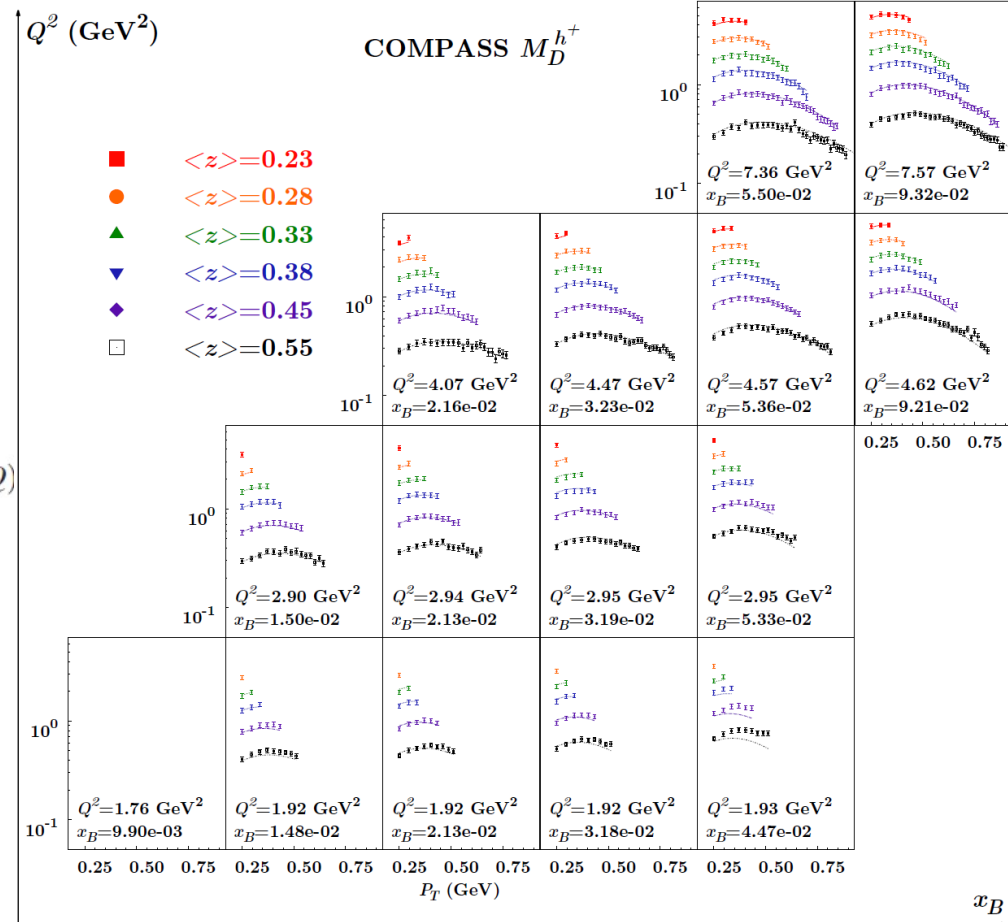
$$\chi^2_{\text{HERMES}} = 1.32$$

HERMES $M_p^{\pi^+}$



$$\chi^2_{\text{tot}} = 1.17$$

$$\chi^2_{\text{COMPASS}} = 1.12$$



$$\frac{d\sigma}{dx dy dz dq_T^2} = \pi \sigma_0^{DIS} \left\{ \int \frac{d^2 \mathbf{b}_T e^{i\mathbf{q}_T \cdot \mathbf{b}_T}}{(2\pi)^2} W^{SIDIS}(x, z, b_*, Q, C_1, C_2, C_3) F_{NP}^{SIDIS}(x, z, b_T, Q) + Y^{SIDIS}(x, z, q_T, Q, C_4) \right\}$$

$$F_{NP}^{SIDIS}(x, z, Q) = \exp \left\{ \left[-\frac{g_1 + g_{1f}/z^2}{2} - g_2 \ln(Q/(2Q_0)) - g_1 g_3 \ln(10x) \right] b_T^2 \right\}$$

- N ~ 2 (One overall normalization parameter is required)
- g1 ~ 0.5 (too large compared to the value extracted from DY data)
- g2 ~ 0.5
- g3 ~ - 0.03

Possible issues ...

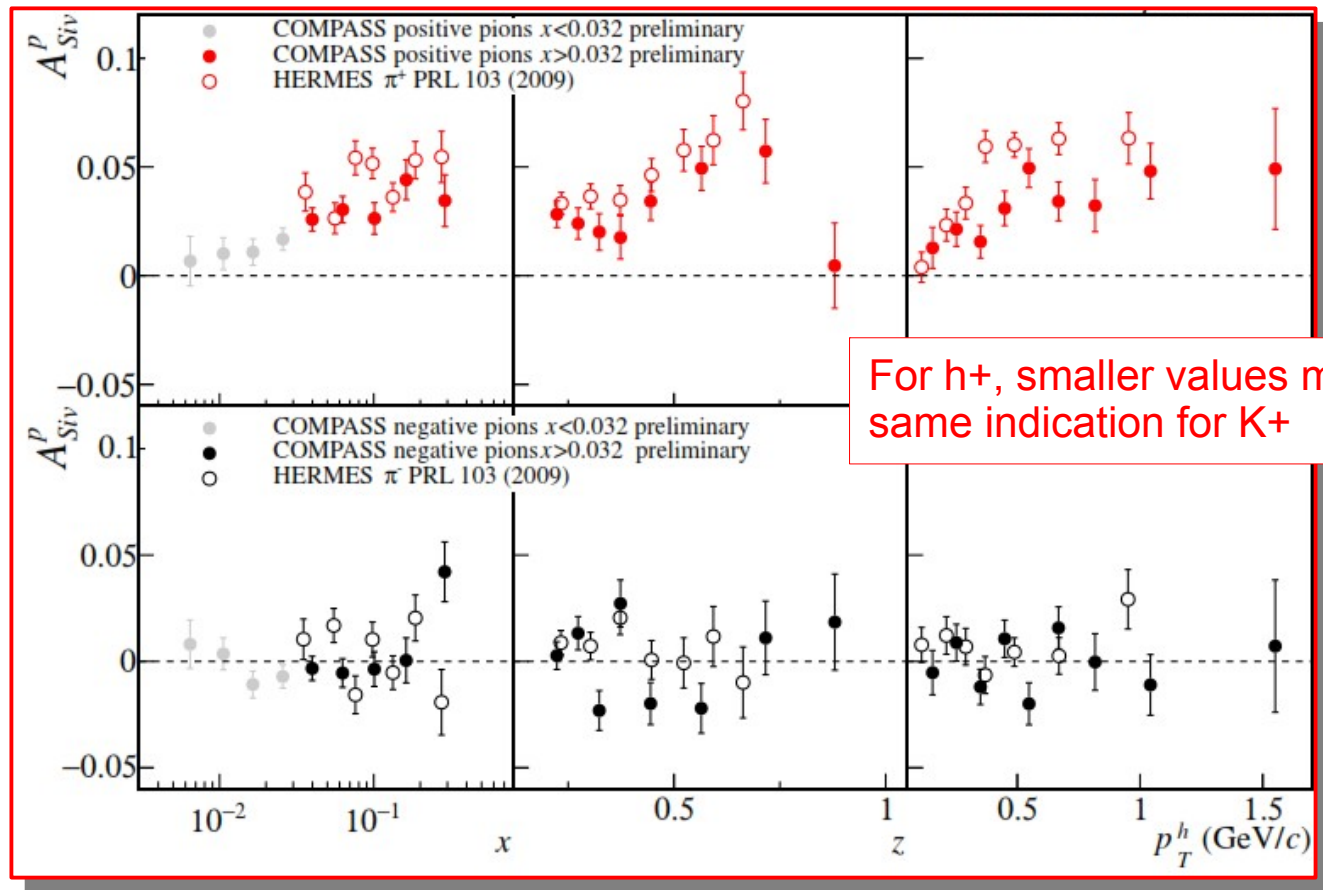
- This fit gives a very high quality description of a wide amount of data points
- However, there are a few issues that are worth mentioning:
 - ★ The NLL SIDIS cross section is not correctly normalized $\rightarrow N \sim 2$
 - ★ The Y factor has been neglected
 - ★ More work required to include Drell-Yan data into a global fit

TMD evolution phenomenology


Does most recent SIDIS data suggest TMD evolution ?


Sivers asymmetry on **proton** ($x > 0.032$)

Charged pions (and kaons), 2010 data
Comparison with HERMES results




π^+

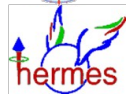
 $\langle Q^2 \rangle = 3.2 \text{ GeV}^2$

 $\langle Q^2 \rangle = 2.4 \text{ GeV}^2$

For h^+ , smaller values measured by COMPASS;
same indication for K^+

π^-

 $\langle Q^2 \rangle = 3.2 \text{ GeV}^2$

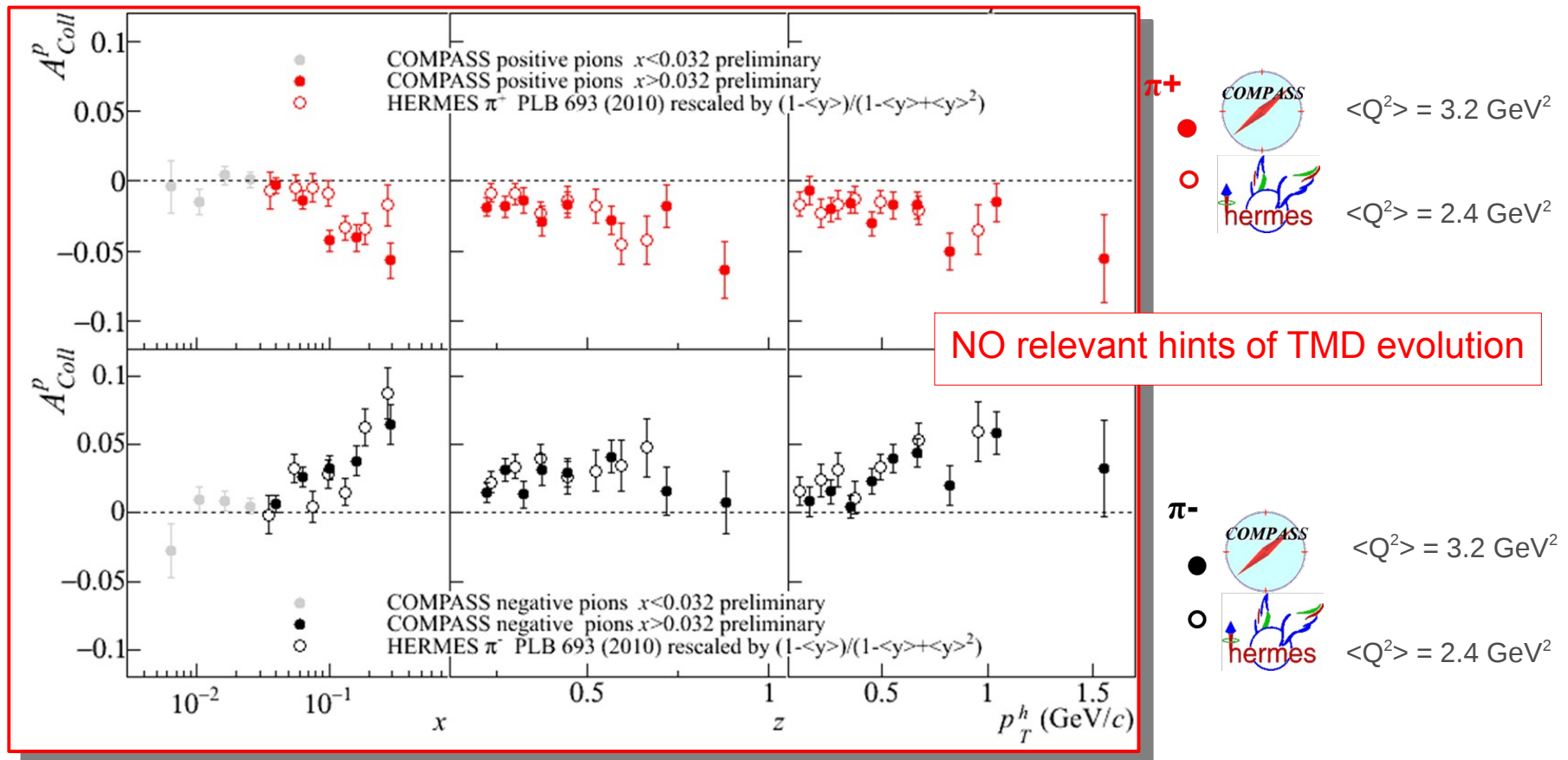
 $\langle Q^2 \rangle = 2.4 \text{ GeV}^2$

Does most recent SIDIS data suggest TMD evolution ?

Collins asymmetry on **proton** ($x > 0.032$)

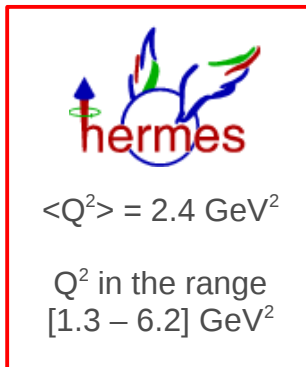
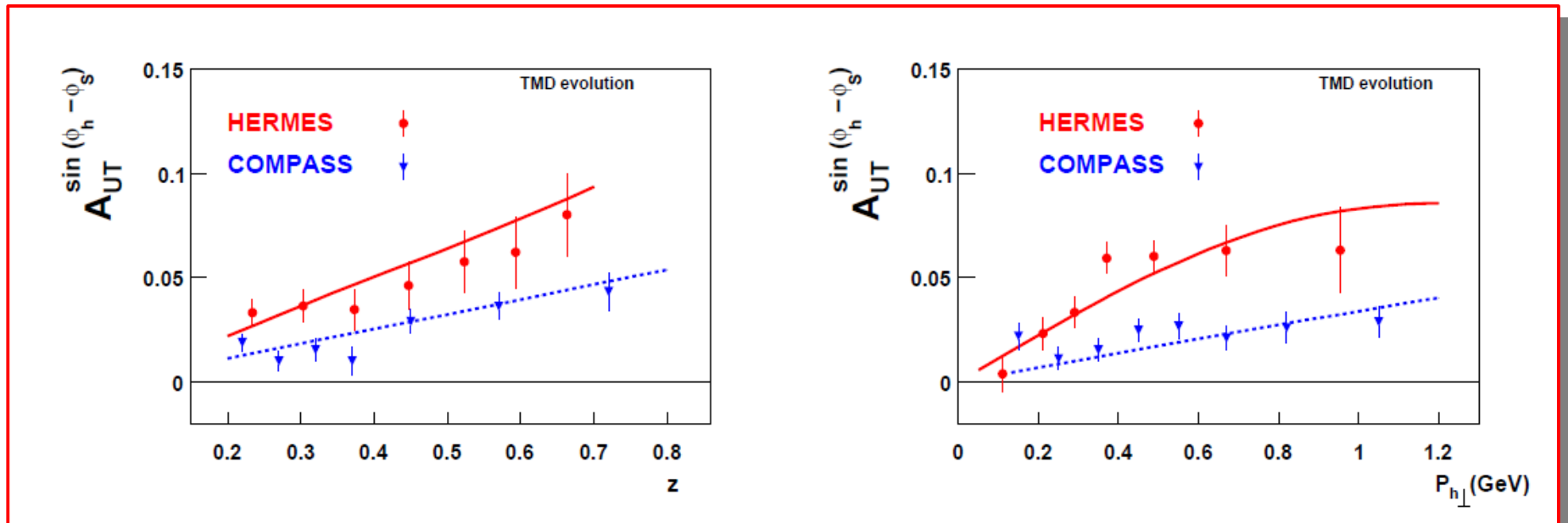
Charged pions (and kaons), 2010 data

Comparison with HERMES results

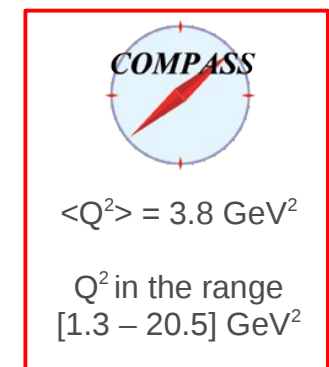


Sivers TMD evolution: phenomenological results

Aybat, Prokudin, Rogers, *Phys. Rev. Lett.* 108, (2011) 242003



- No x dependence taken into account
- Sivers A_{UT} calculated at two fixed different values of Q^2 : 2.4 and 3.8 GeV^2
- Evolution effects are then compared.



Sivers function from HERMES and COMPASS SIDIS data

Anselmino, Boglione, Melis, *Phys. Rev. D*86 (2012) 014028

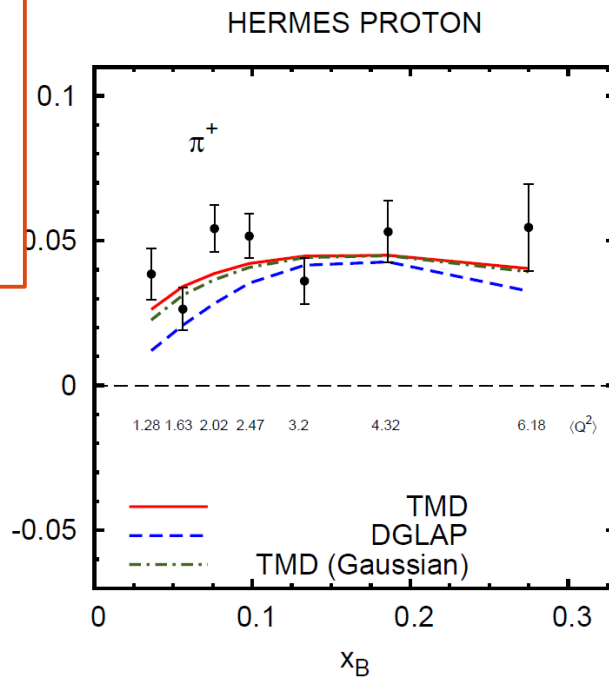
- Q^2 and x dependence rigorously taken into account
- **2 different fits:**
- TMD-fit (computing TMD evolution equations numerically)
- DGLAP evolution equation for the collinear part of the TMD)



$\langle Q^2 \rangle = 2.4 \text{ GeV}^2$

Q^2 in the range
[1.3 – 6.2] GeV^2

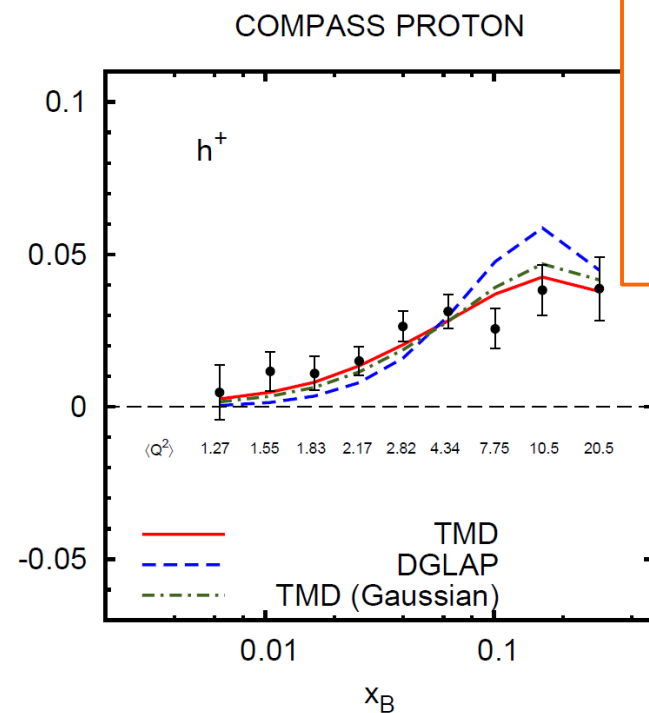
$A_{UT}^{\sin(\phi_h - \phi_s)}$



$\langle Q^2 \rangle = 3.8 \text{ GeV}^2$

Q^2 in the range
[1.3 – 20.5] GeV^2

$A_{UT}^{\sin(\phi_h - \phi_s)}$



A. Airapetian et al., *Phys. Rev. Lett.* 103, (2009) 152002

C. Adolph et al., *Phys. Lett. B*717 (2012) 383

g_2 alert !

- g_2 controls the b_T gaussian width and its spreading as b_T varies.

$$g_K(b_T) = \frac{1}{2} g_2 b_T^2 \quad \text{with} \quad g_2 = 0.68$$

$$b_{\max} = 0.5 \text{ GeV}^{-1}$$

- We do not extract the value of g_2 from our fit
- We use a fixed value, previously determined in a fit of D-Y data. Landry, Brock, Nadolsky, Yuan, Phys. Rev. D67(2003) 073016
We could have extracted it, and probably got a smaller value, but it is important to remember that **SIDIS data are very little sensitive to the precise value of g_2** .
- D-Y data, instead, are extremely sensitive to it: this requires a new, careful, global analysis on all SIDIS and D-Y, re-starting from unpolarized cross sections.

Exploring the proton Siverts sea

Anselmino, Boglione, D'Alesio, Murgia, Prokudin, in preparation ...

- A new Siverts fit has recently been performed, motivated by the necessity to explore the sea contributions in a more detailed way.
- In this fit the Siverts function depends on Q through its collinear part, which evolves according to DGLAP equations. No TMD evolution is considered.
- This fit is based on a different parametrization of the sea Siverts functions, in which we assume them to be directly proportional to their unpolarized counterparts:

$$\Delta^N f_{q/p\uparrow}(x, Q_0) = 2 \mathcal{N}_q(x) f_{q/p}(x, Q_0)$$

Previous fits

$$\mathcal{N}_q(x) = N_q x^{\alpha_q} (1-x)^{\beta_q} \frac{(\alpha_q + \beta_q)^{(\alpha_q + \beta_q)}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}}$$

Same for valence and sea contributions

New fit

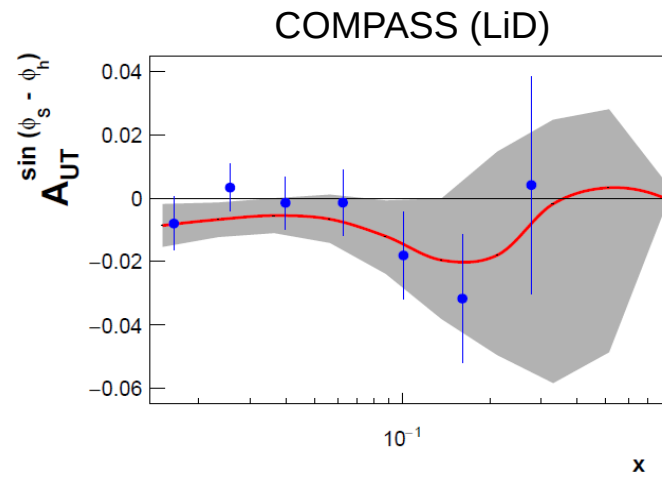
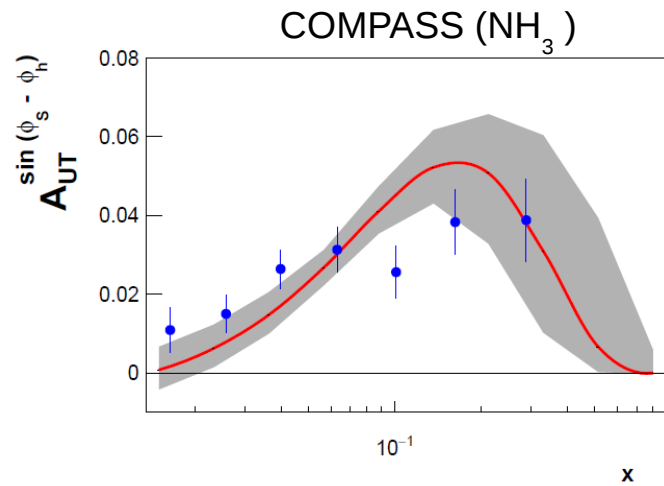
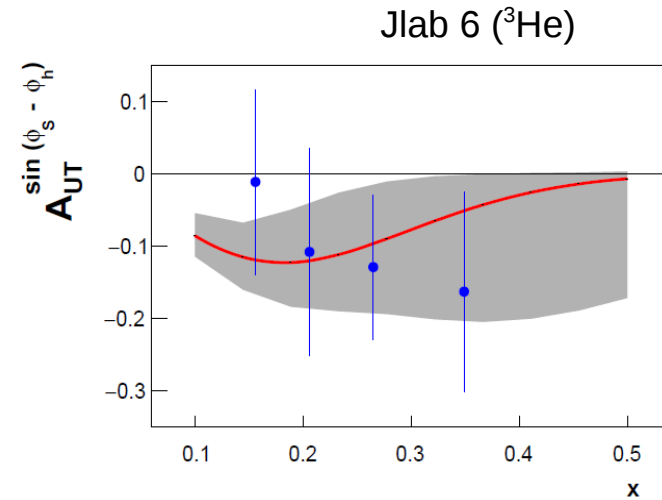
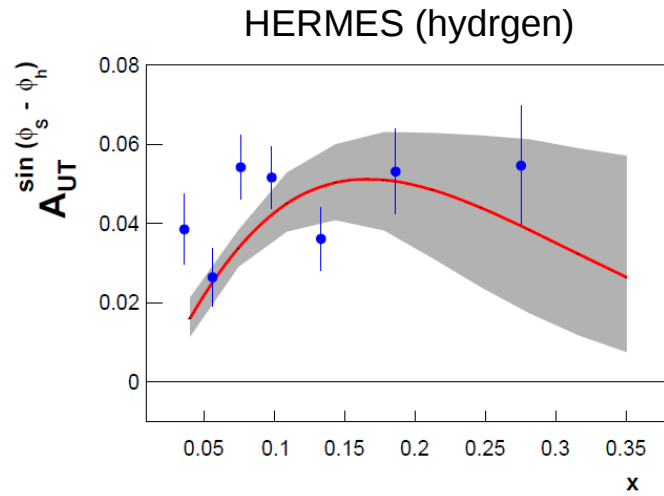
$$\mathcal{N}_q(x) = N_q x^{\alpha_q} (1-x)^{\beta_q} \frac{(\alpha_q + \beta_q)^{(\alpha_q + \beta_q)}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}}$$

$$\mathcal{N}_{\bar{q}}(x) = N_{\bar{q}}$$

- HERMES (hydrogen target), COMPASS (NH₃ and LiD targets) and JLAB (³He target) SIDIS data are fitted

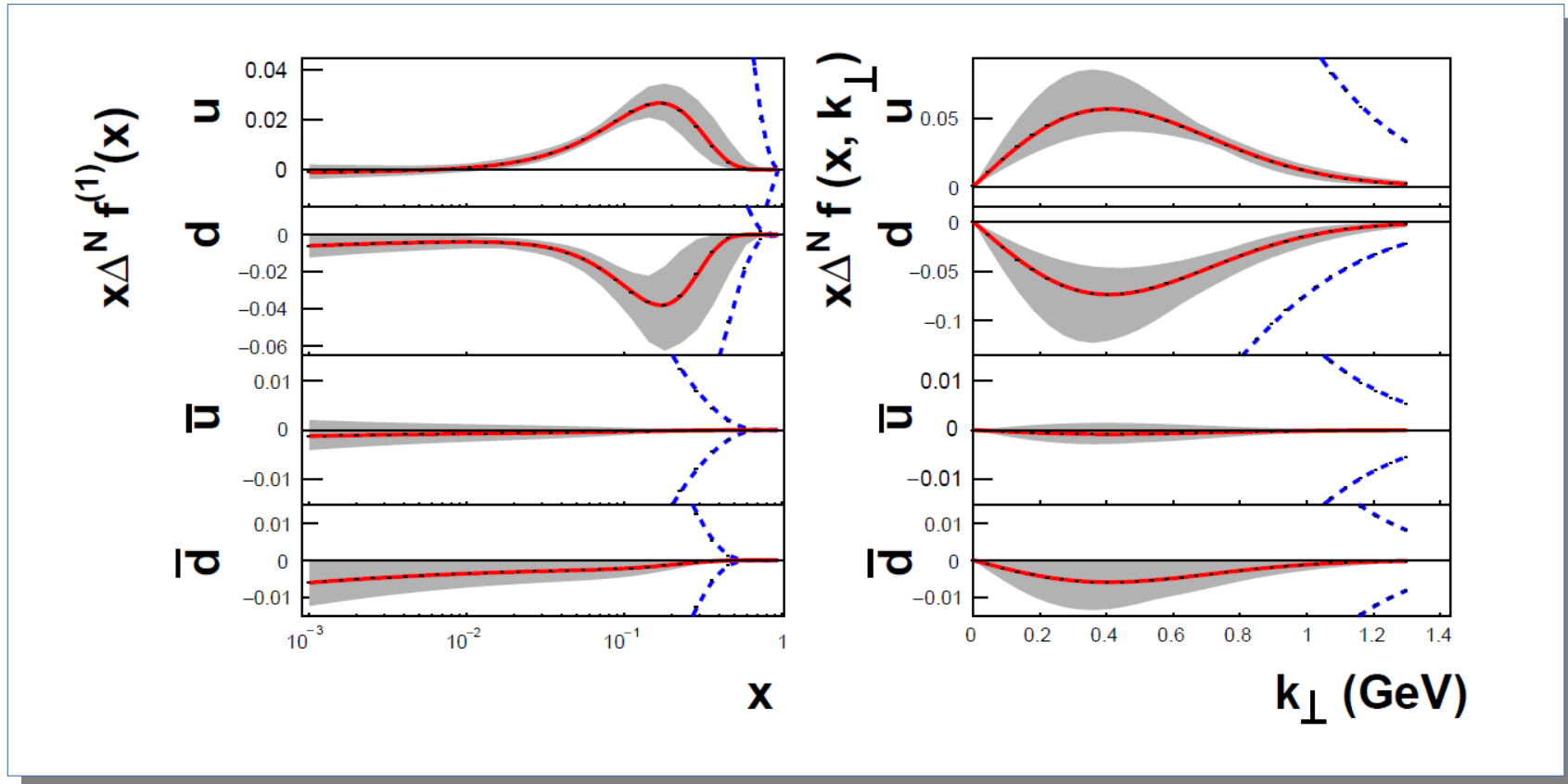
New Sivers fit

Anselmino, Boglione, D'Alesio, Murgia, Prokudin, in preparation ...



New Sivers fit

Anselmino, Boglione, D'Alesio, Murgia, Prokudin, in preparation ...



- The \bar{d} contribution to the Sivers function is negative and has definite sign, while the \bar{u} is small and has no definite sign.

Testing the Sivers sign change: a crucial test of factorization

Sivers function from Drell Yan data on the J/ψ peak at COMPASS

Anselmino, Barone, Boglione, arXiv:1607.00275

$$A_N^{J/\Psi}(\pi^-; x_1, x_2, q_T) \simeq \frac{\int d^2\mathbf{k}_{\perp 1} d^2\mathbf{k}_{\perp 2} \delta^2(\mathbf{k}_{\perp 1} + \mathbf{k}_{\perp 2} - \mathbf{q}_T) \mathbf{S} \cdot (\hat{\mathbf{p}}_2 \times \hat{\mathbf{k}}_{\perp 2}) f_{\bar{u}/\pi^-}(x_1, k_{\perp 1}) \Delta^N f_{u/p^\uparrow}(x_2, k_{\perp 2})}{2 \int d^2\mathbf{k}_{\perp 1} d^2\mathbf{k}_{\perp 2} \delta^2(\mathbf{k}_{\perp 1} + \mathbf{k}_{\perp 2} - \mathbf{q}_T) f_{\bar{u}/\pi^-}(x_1, k_{\perp 1}) f_{u/p}(x_2, k_{\perp 2})}$$

$$A_N^{J/\Psi}(\pi^+; x_1, x_2, q_T) \simeq \frac{\int d^2\mathbf{k}_{\perp 1} d^2\mathbf{k}_{\perp 2} \delta^2(\mathbf{k}_{\perp 1} + \mathbf{k}_{\perp 2} - \mathbf{q}_T) \mathbf{S} \cdot (\hat{\mathbf{p}}_2 \times \hat{\mathbf{k}}_{\perp 2}) f_{\bar{d}/\pi^+}(x_1, k_{\perp 1}) \Delta^N f_{d/p^\uparrow}(x_2, k_{\perp 2})}{2 \int d^2\mathbf{k}_{\perp 1} d^2\mathbf{k}_{\perp 2} \delta^2(\mathbf{k}_{\perp 1} + \mathbf{k}_{\perp 2} - \mathbf{q}_T) f_{\bar{d}/\pi^+}(x_1, k_{\perp 1}) f_{d/p}(x_2, k_{\perp 2})}$$

$\pi^- p^\uparrow \rightarrow \ell^+ \ell^- X$

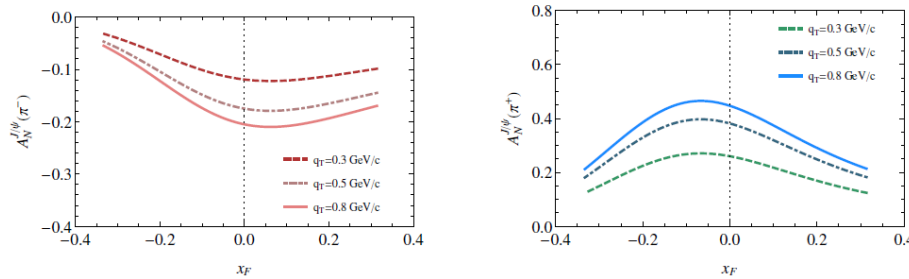


FIG. 1: Plots of $A_N^{J/\Psi}(\pi^-; x_2, q_T)$ (left) and $A_N^{J/\Psi}(\pi^+; x_2, q_T)$ (right) versus x_F , for three different values of q_T . These estimates are obtained according to Eqs. (17)–(20) of the text, using the parameters of Ref. [11], with a sign change for the Sivers functions.

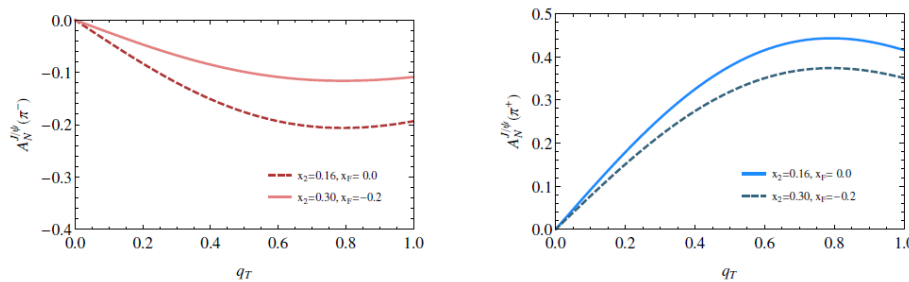


FIG. 2: Plots of $A_N^{J/\Psi}(\pi^-; x_2, q_T)$ (left) and $A_N^{J/\Psi}(\pi^+; x_2, q_T)$ (right) versus q_T , for two different values of x_F . These estimates are obtained according to Eqs. (17)–(20) of the text, using the parameters of Ref. [11], with a sign change for the Sivers functions.

$$f_{q/p}(x, k_\perp) = f_q(x) \frac{1}{\pi \langle k_\perp^2 \rangle} e^{-k_\perp^2 / \langle k_\perp^2 \rangle}$$

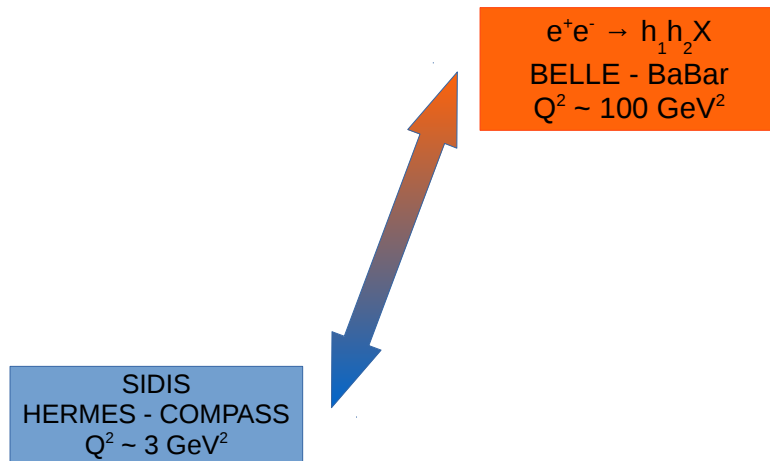
$$\Delta^N f_{q/p^\uparrow}(x, k_\perp) = 2 \mathcal{N}_q(x) h(k_\perp) f_{q/p}(x, k_\perp)$$

$$h(k_\perp) = \sqrt{2} e \frac{k_\perp}{M_1} e^{-k_\perp^2 / M_1^2},$$

The Sivers asymmetry is large has a definite sign, even when uncertainty bands are considered.

***Simultaneous extraction
of transversity
and the Collins function***

What about Q^2 evolution ?



Simultaneous fits of SIDIS and $e^+e^- \rightarrow h_1 h_2 X$
Involve data sets at very different Q^2 scales

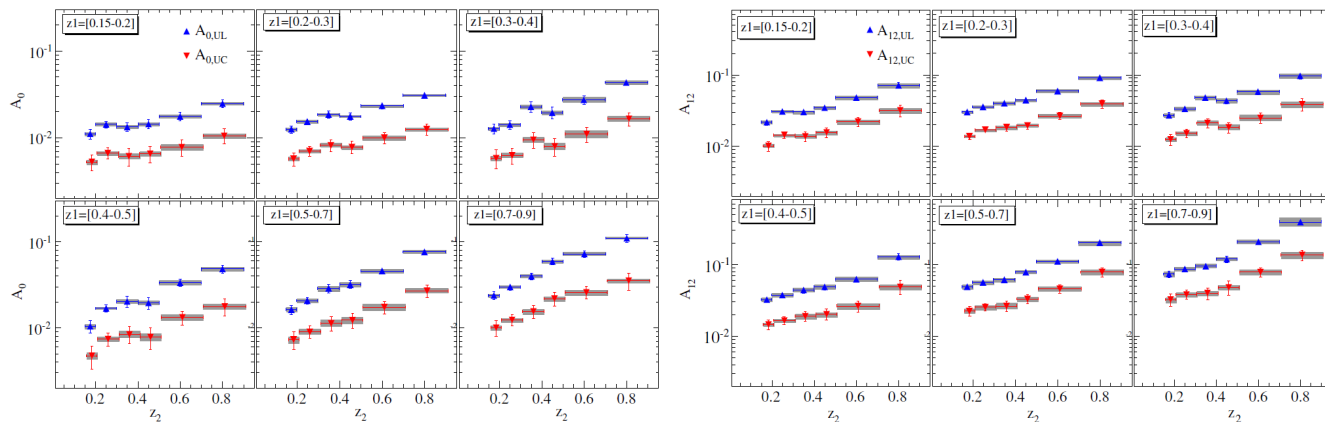
In our computation the Collins TMD function evolves according to DGLAP evolution equations, through its $D_{h/q}(z, p_t, Q^2)$ component

- Could TMD evolution be an issue ?
- Could TMD evolution affect our results ?

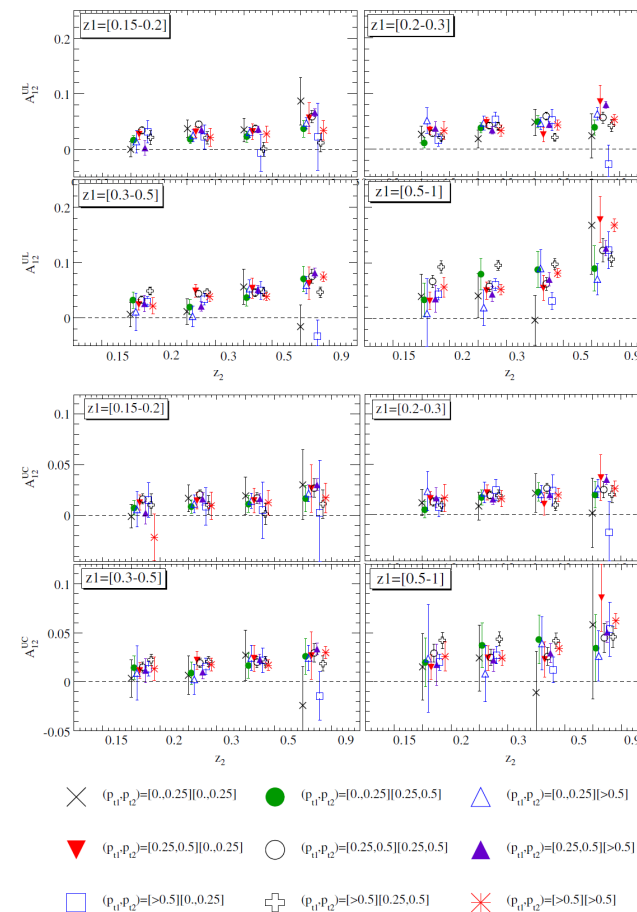


New BaBar data

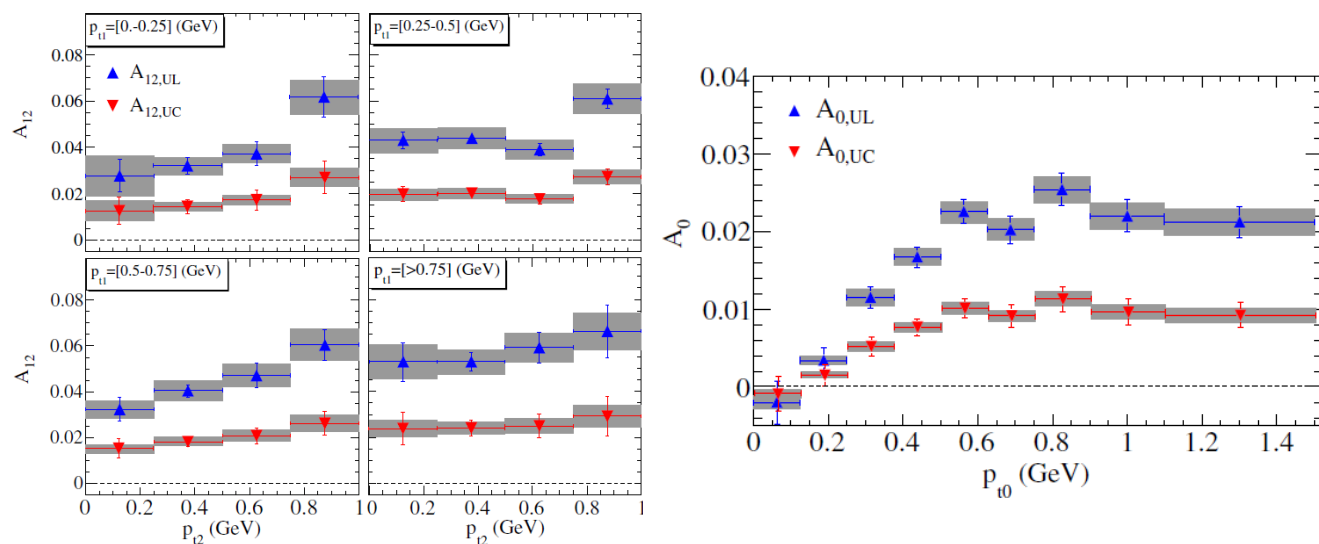
BaBar measurements of A_0 and A_{12} as a function of z_1 and z_2



BaBar multidimensional data on A_{12} in bins of $(z_1, z_2, p_{t1}, p_{t2})$

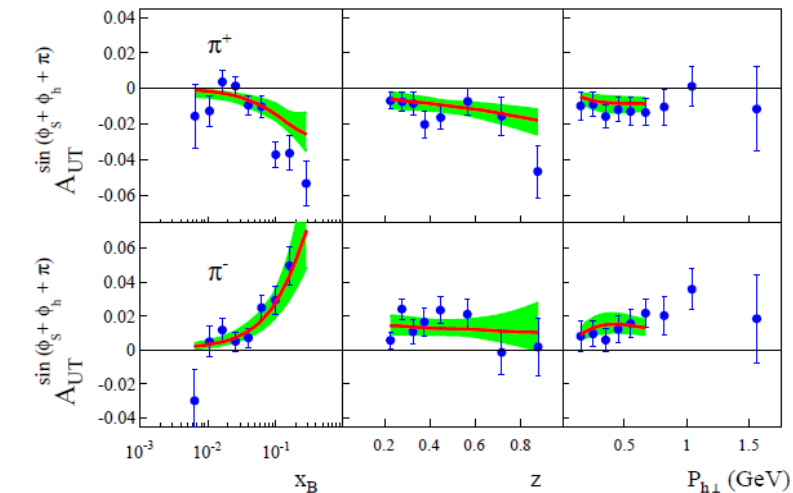
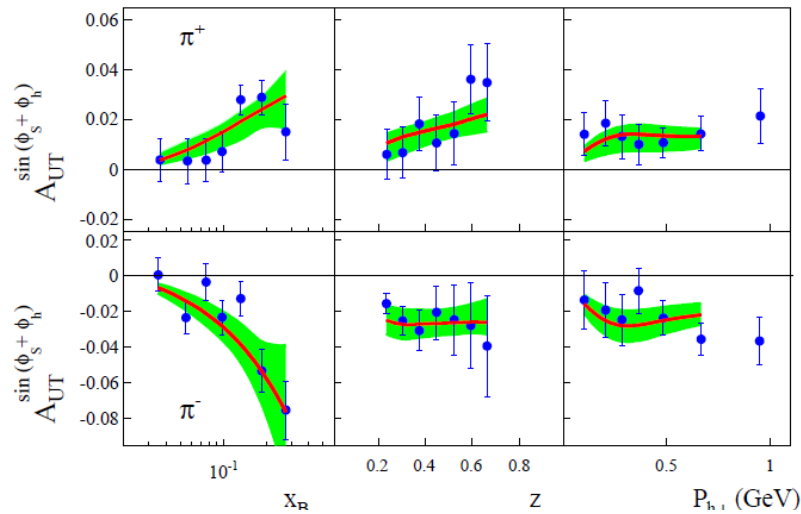


BaBar measurements of A_0 and A_{12} as a function of p_{t0}, p_{t1} and p_{t2}



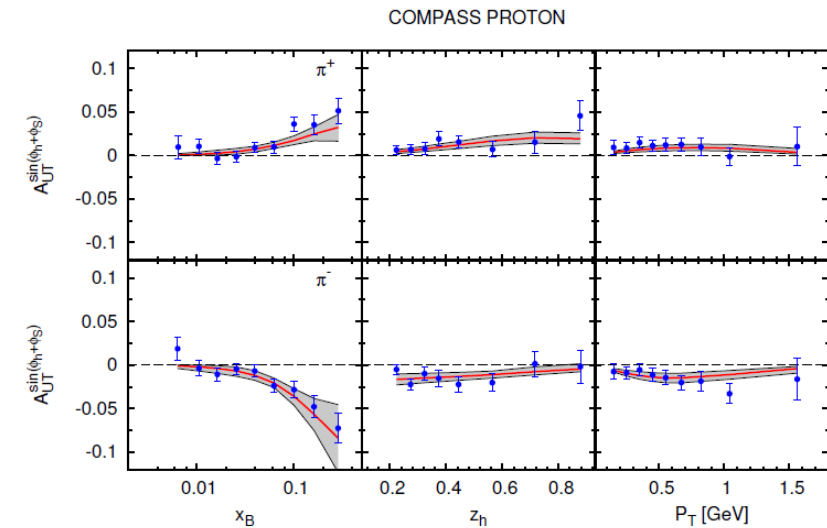
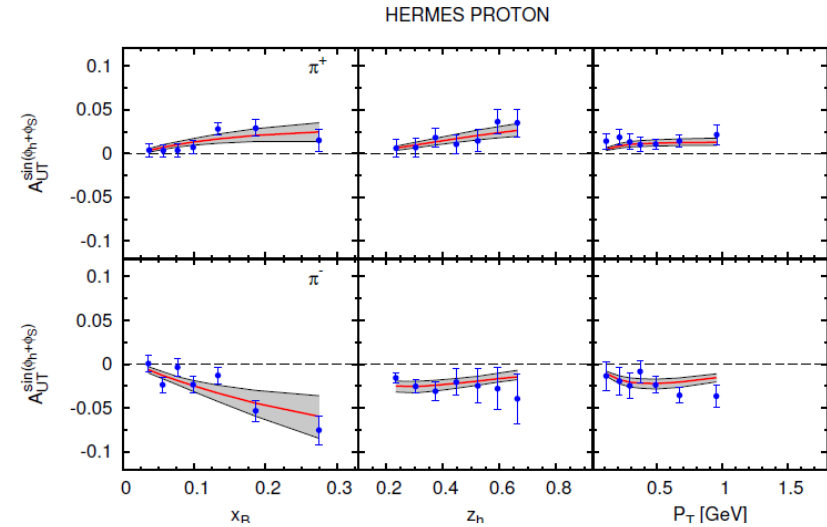
CSS/TMD evolution and Collins/Transversity

➤ TMD evolution



Kang et al: *Phys. Rev. D*93 (2016) 014009 (a)

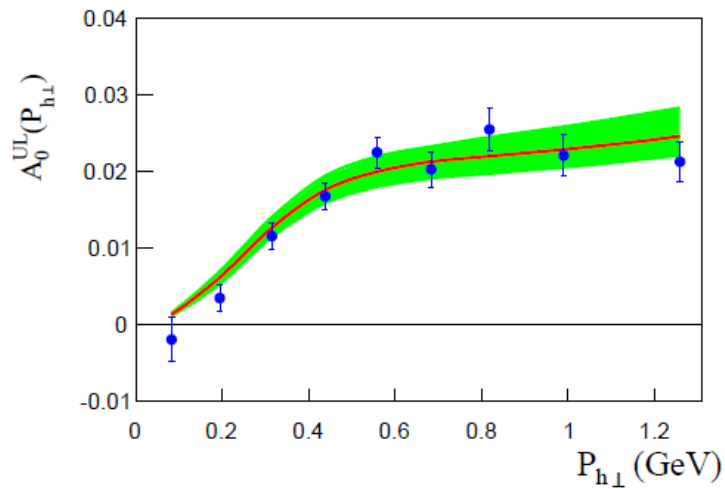
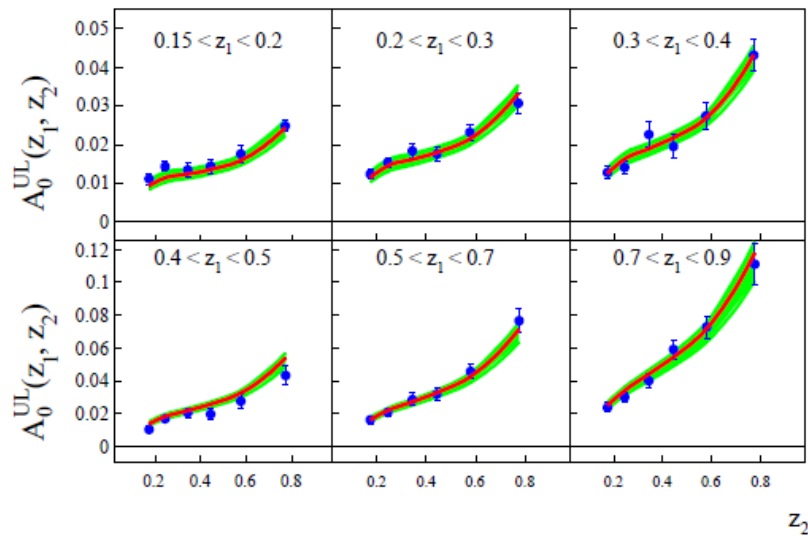
➤ Naive TMD



Anselmino et al., *Phys. Rev. D*92 (2015), 114023

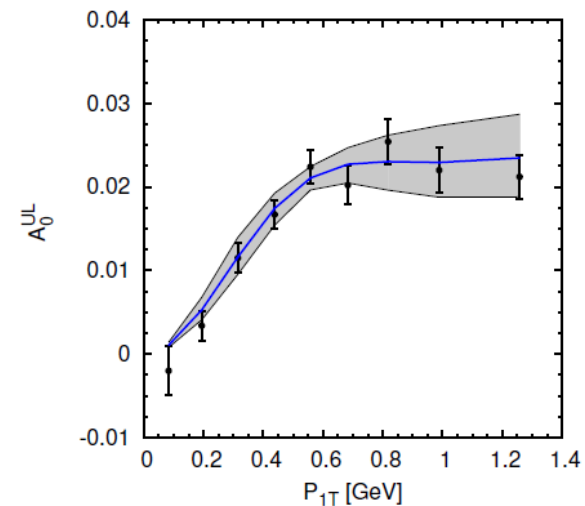
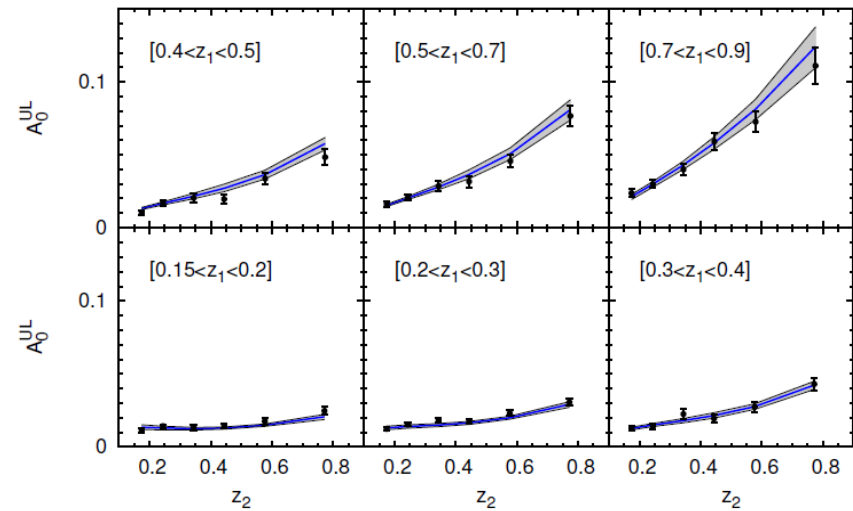
CSS/TMD evolution and Collins/Transversity

➤ TMD evolution



Kang et al: *Phys. Rev. D*93 (2016) 014009

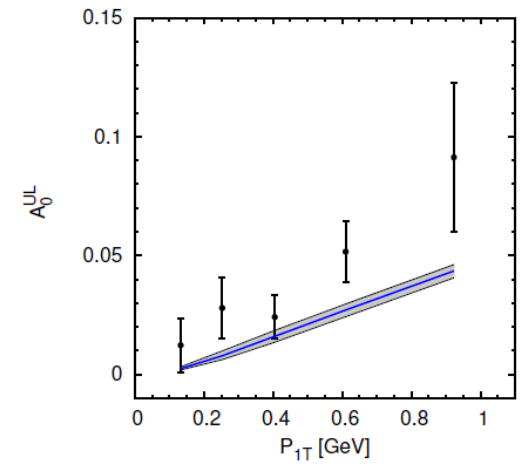
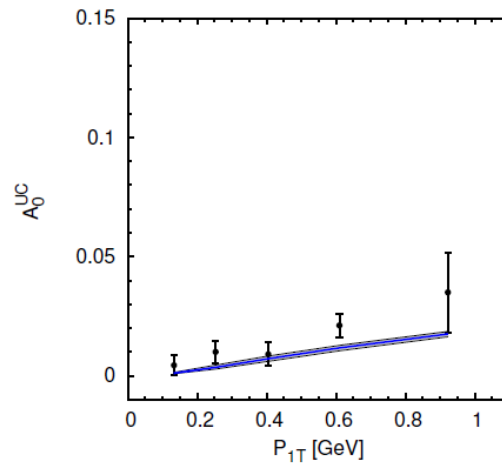
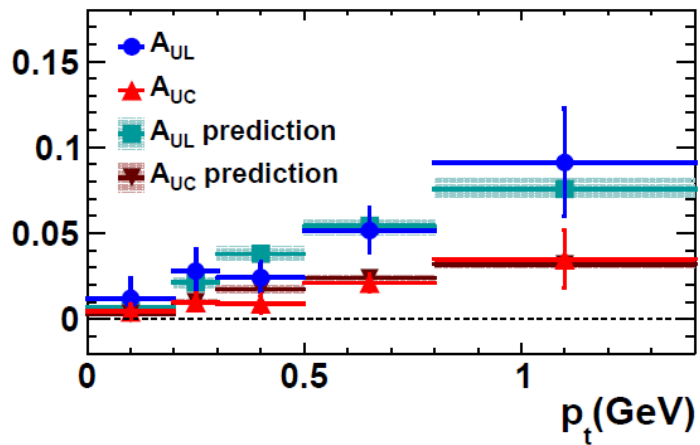
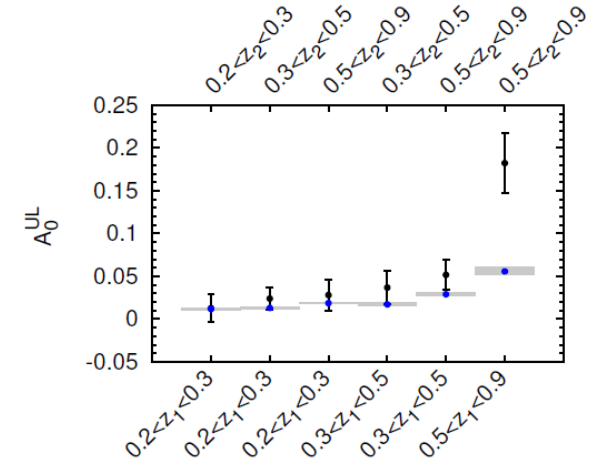
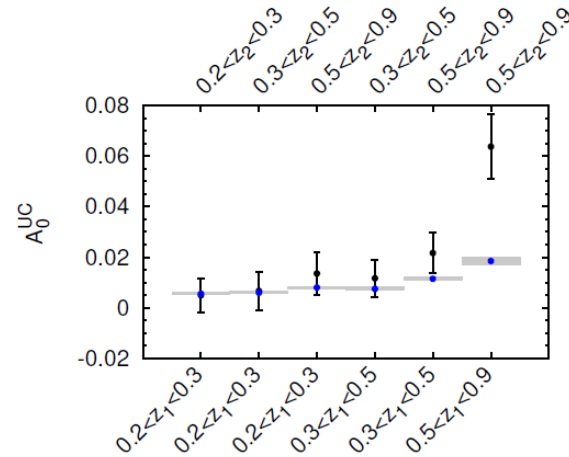
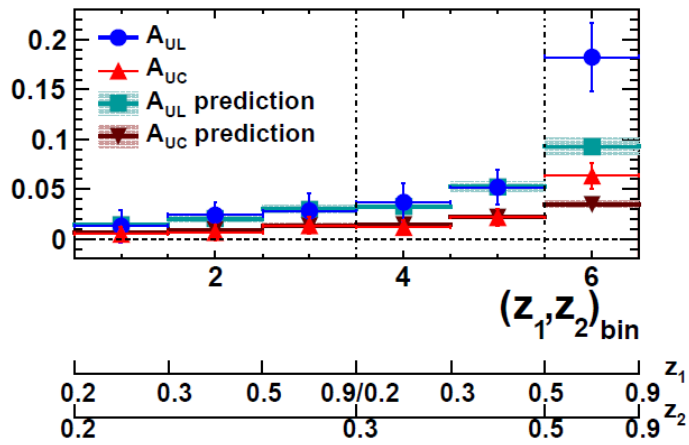
➤ Naive TMD



Anselmino et al., *Phys. Rev. D*92 (2015), 114023

➤ TMD evolution effects at BESIII ?

➤ Naive TMD



BESIII, Ablikim et al., PRL116 (2016) 042001

Kang et al: Phys. Rev. D93 (2016) 014009

BESIII, Ablikim et al., PRL116 (2016) 042001

Anselmino et al., Phys. Rev. D92 (2015), 114023

Summary

- Naive TMD models and TMD evolution give similar results at different Q .
- Asymmetries do not seem to be strongly sensitive to TMD evolution effects (due to cancellations in ratios ?)
- Open points:
 - ✗ We do not have unpolarized data for e^+e^- processes...
 - ✗ Full evolution of Twist 3 function far from trivial...
 - ✗ BESIII is a little bit extreme in the kinematics...

Outlook and Conclusions

- We are now ready to enter a new phase of high precision studies of TMDs
- Need cross section or multiplicity data to extract unpolarized PDF and FF TMDs
- Drell Yan studies show that the cross section can be well reproduced over a very wide (full) q_T range
- SIDIS studies are presently being performed including TMD evolution and resummation – **issues with matching and Y factor**
- Global analysis of SIDIS and e^+e^- high statistics and high precision new data sets delivers very satisfactory results, although they seem to be little sensitive to TMD evolution effects.
- Global analyses of SIDIS and Drell Yan data are presently under investigation – **issues ...**