Introduction to Generalized Parton Distributions

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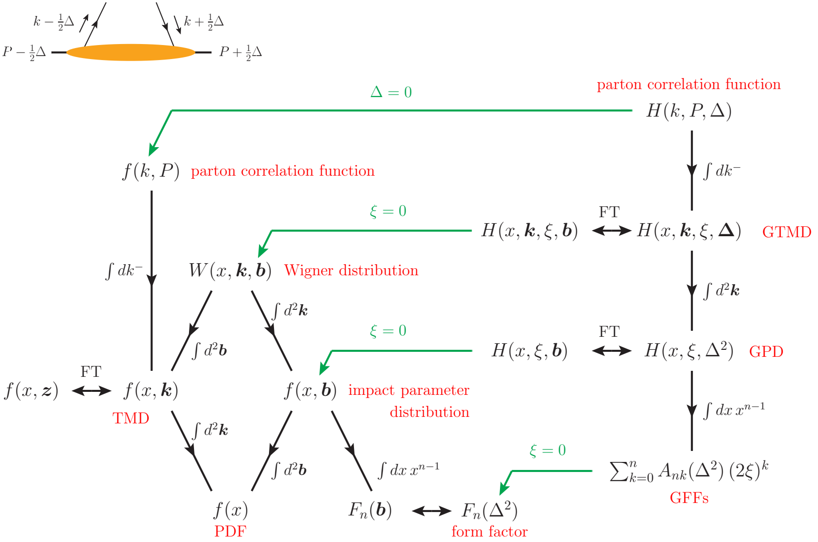
Family tree of hadron structure functions



[Fig. by Markus Diehl]

($\xi \to \eta$ from now on)

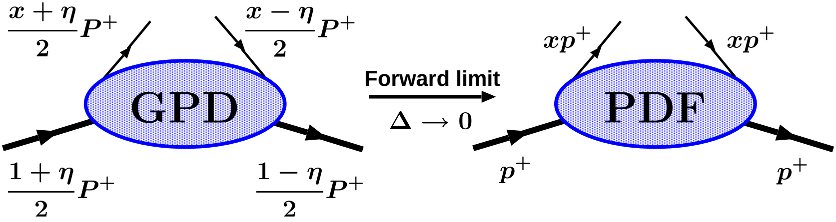
Definition of GPDs

- In QCD GPDs are defined as [Müller '92, et al. '94, Ji, Radyushkin '96]

$$F^q(x,\eta,t) = \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|\bar{q}(-z)\gamma^+ q(z)|P_1\rangle\Big|_{z^+=0,\,\mathbf{z}_\perp=\mathbf{0}}$$

$$\widetilde{F}^q(x,\eta,t) = \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|\bar{q}(-z)\gamma^+\gamma_5 q(z)|P_1\rangle\Big|_{z^+=0,\,\mathbf{z}_\perp=\mathbf{0}}$$

(and similarly for gluons F^g and $\widetilde{F}^g$).



$$P = P_1 + P_2 \; ; \qquad t = \Delta^2 = (P_2 - P_1)^2 \; ; \qquad \eta = -\frac{\Delta^+}{P^+} \;\; \text{(skewedness)}$$

Some properties of GPDs

- Decomposing into spin-non-flip and spin-flip part:

$$F^a = \frac{\bar{u}(P_2)\gamma^+ u(P_1)}{P^+} H^a + \frac{\bar{u}(P_2) i\sigma^{+\nu} u(P_1)\Delta_\nu}{2MP^+} E^a \qquad a = q, g$$

$$\tilde{F}^a = \frac{\bar{u}(P_2)\gamma^+ \gamma_5 u(P_1)}{P^+} \tilde{H}^a + \frac{\bar{u}(P_2)\gamma_5 u(P_1)\Delta^+}{2MP^+} \tilde{E}^a \qquad a = q, g$$

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- “Ji’s sum rule” (related to proton spin problem)

$$J^q = \frac{1}{2}\int_{-1}^{1} dx\, x \Big[H^q(x,\eta,t) + E^q(x,\eta,t) \Big]_{t\to 0} \qquad \text{[Ji '96]}$$

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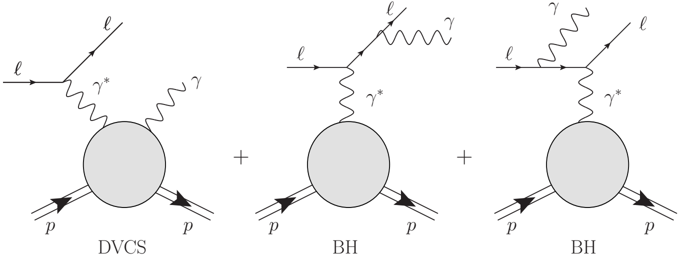
- Distribution of partons in transversal space

$$\rho(x, \vec{b}_\perp) = \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} H(x, 0, -\vec{\Delta}_\perp^2) \qquad \text{[Burkardt '00]}$$

Access to GPDs via DVCS

- Deeply virtual Compton scattering (DVCS) — “gold plated” process of exclusive physics
- DVCS is measured via leptoproduction of a photon



- Interference with Bethe-Heitler process gives unique access to both real and imaginary part of DVCS amplitude.

DVCS cross section

$$d\sigma \propto |\mathcal{T}|^2 = |\mathcal{T}_{\rm BH}|^2 + |\mathcal{T}_{\rm DVCS}|^2 + \mathcal{I} \,.$$

$$\mathcal{I} \quad \propto \quad \frac{-e_\ell}{\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)}\left\{ c_0^{\mathcal{I}} + \sum_{n=1}^{3}\left[c_n^{\mathcal{I}}\cos(n\phi) + s_n^{\mathcal{I}}\sin(n\phi)\right]\right\},$$

$$|\mathcal{T}_{\rm DVCS}|^2 \quad \propto \quad \left\{ c_0^{\rm DVCS} + \sum_{n=1}^{2}\left[c_n^{\rm DVCS}\cos(n\phi) + s_n^{\rm DVCS}\sin(n\phi)\right]\right\},$$

- Choosing polarizations (and charges) we focus on particular harmonics:

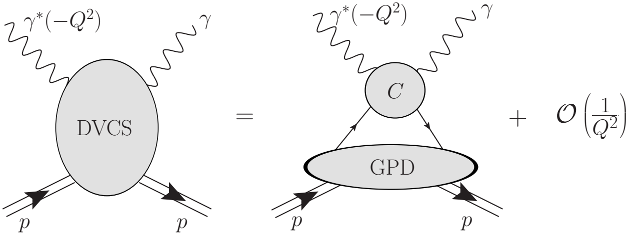
$$c^{\mathcal{I}}_{1,\rm unpol.} \propto \left[F_1\, \mathfrak{Re}\, \mathcal{H} - \frac{t}{4M_p^2} F_2\, \mathfrak{Re}\, \mathcal{E} + \frac{x_{\rm B}}{2-x_{\rm B}}(F_1+F_2)\, \mathfrak{Re}\, \widetilde{\mathcal{H}} \right]$$

[Belitsky, Müller et. al '01–'14]

- $\mathcal{H}(x_{\rm B}, t, \mathcal{Q}^2)$, . . . — four Compton form factors (CFFs)

Factorization of DVCS $\longrightarrow$ GPDs

- [Collins et al. '98]



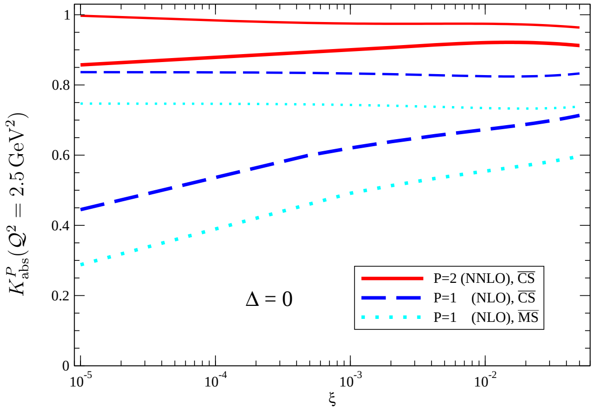
- Compton form factor is a convolution:

$$ {}^{a}\mathcal{H}(x_{\rm B}, t, \mathcal{Q}^2) = \int \mathrm{d}x \; C^a\Big(x, \frac{x_{\rm B}}{2-x_{\rm B}}, \frac{\mathcal{Q}^2}{\mathcal{Q}_0^2}\Big) \; H^a\Big(x, \frac{x_{\rm B}}{2-x_{\rm B}}, t, \mathcal{Q}_0^2\Big)_{a=q,G} $$

- $H^a(x, \eta, t, \mathcal{Q}_0^2)$ — Generalized parton distribution (GPD)

(N)NLO corrections

- [K.K., Müller and Passek-K. '07]

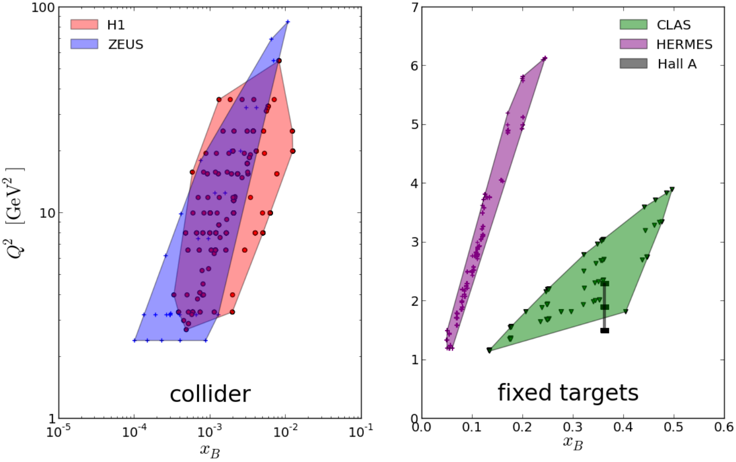


Thick lines:
"hard" gluon
$N_G = 0.4$
$\alpha_G(0){=}\alpha_\Sigma(0) + 0.05$

Thin lines:
"soft" gluon
$N_G = 0.3$
$\alpha_G(0){=}\alpha_\Sigma(0) - 0.02$

$$K^P_{\rm abs} \equiv \left| \frac{\mathcal{H}^{(P)}}{\mathcal{H}^{(P-1)}} \right|$$

Experimental coverage (1/2)



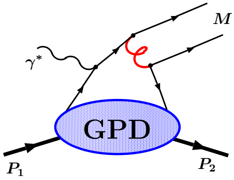
- Coming soon: COMPASS, JLab12, . . . EIC

Experimental coverage (2/2)

Collab.	Year	Observables	Kinematics			No. of points	
			x_B	Q^2 [GeV^2]	$\|t\|$ [GeV^2]	total	indep.
HERMES	2001	$A_{\rm LU}^{\sin\phi}$	0.11	2.6	0.27	1	1
CLAS	2001	$A_{\rm LU}^{\sin\phi}$	0.19	1.25	0.19	1	1
CLAS	2006	$A_{\rm UL}^{\sin\phi}$	0.2–0.4	1.82	0.15–0.44	6	3
HERMES	2006	$A_{\rm C}^{\cos\phi}$	0.08–0.12	2.0–3.7	0.03–0.42	4	4
Hall A	2006	$\sigma(\phi)$, $\Delta\sigma(\phi)$	0.36	1.5–2.3	0.17–0.33	4×24+12×24	4×24+12×24
CLAS	2007	$A_{\rm LU}(\phi)$	0.11–0.58	1.0–4.8	0.09–1.8	62×12	62×12
HERMES	2008	$A_{\rm C}^{\cos(0,1)\phi}$, $A_{\rm UT,DVCS}^{\sin(\phi-\phi_S)}$, $A_{\rm UT,I}^{\sin(\phi-\phi_S)\cos(0,1)\phi}$, $A_{\rm UT,I}^{\cos(\phi-\phi_S)\sin\phi}$	0.03–0.35	1–10	<0.7	*12*+12+12 12+12 *12*	*4*+4+4 4+4 *4*
CLAS	2008	$A_{\rm LU}(\phi)$	0.12–0.48	1.0–2.8	0.1–0.8	66	33
HERMES	2009	$A_{\rm LU,I}^{\sin(1,2)\phi}$, $A_{\rm LU,DVCS}^{\sin\phi}$, $A_{\rm C}^{\cos(0,1,2,3)\phi}$	0.05–0.24	1.2–5.75	<0.7	18+18+18 18+18+*18*+*18*	6+6+6 6+6+*6*+*6*
HERMES	2010	$A_{\rm UL}^{\sin(1,2,3)\phi}$, $A_{\rm LL}^{\cos(0,1,2)\phi}$	0.03–0.35	1–10	<0.7	12+12+*12* 12+*12*+12	4+4+*4* 4+*4*+4
HERMES	2011	$A_{\rm LT,I}^{\cos(\phi-\phi_S)\cos(0,1,2)\phi}$, $A_{\rm LT,I}^{\sin(\phi-\phi_S)\sin(1,2)\phi}$, $A_{\rm LT,BH+DVCS}^{\cos(\phi-\phi_S)\cos(0,1)\phi}$, $A_{\rm LT,BH+DVCS}^{\sin(\phi-\phi_S)\sin\phi}$	0.03–0.35	1–10	<0.7	*12*+*12*+*12* *12*+*12* *12*+12 12	*4*+*4*+*4* *4*+*4* *4*+4 4
HERMES	2012	$A_{\rm LU,I}^{\sin(1,2)\phi}$, $A_{\rm LU,DVCS}^{\sin\phi}$, $A_{\rm C}^{\cos(0,1,2,3)\phi}$	0.03–0.35	1–10	<0.7	18+*18*+*18* 18+18+*18*+*18*	6+*6*+*6* 6+6+*6*+*6*
CLAS	2015	$A_{LU}(\phi)$, $A_{UL}(\phi)$, $A_{LL}(\phi)$	0.17–0.47	1.3–3.5	0.1–1.4	166+166+166	166+166+166
CLAS	2015	$\sigma(\phi)$, $\Delta\sigma(\phi)$	0.1–0.58	1–4.6	0.09–0.52	2640+2640	2640+2640
Hall A	2015	$\sigma(\phi)$, $\Delta\sigma(\phi)$	0.33–0.40	1.5–2.6	0.17–0.37	480+600	240+360

Alternative processes for GPD access

- Deeply virtual meson production (DVMP) $\gamma^* p \to Mp$.



  - Theory more "dirty" than for DVCS (second "soft" function appears: meson distribution amplitude)
  - Different mesons enable access to different flavours of GPDs [Goloskokov, Kroll]
- Wide-angle Compton scattering (WACS)
  - WACS: proton momentum transfer t is large (unlike DVCS, where photon virtuality is large: $Q^2 \gg t$!)
  - data reasonably described by GPD models [Diehl, Kroll, '13]
- double DVCS $\gamma^* p \to \gamma^* p$, timelike DVCS, . . . (see Trento workshop October 2016)

Extraction of GPDs/CFFs by fits to DVCS data

- In contrast to $PDFs(x)$, it is very difficult to perform truly model-independent extraction of $GPDs(x, \eta, t)$

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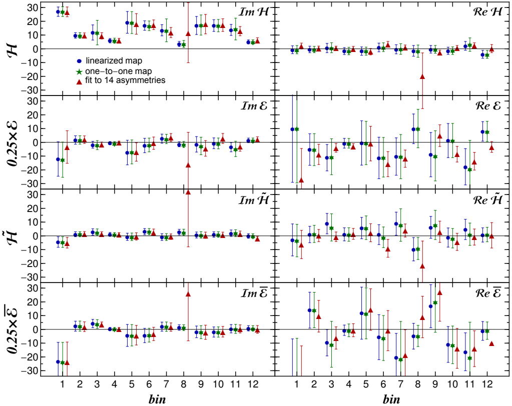
- (Dependence on additional variable, photon virtuality $\mathcal{Q}^2$, is in principle known — given by evolution equations.)

Local fits — some results

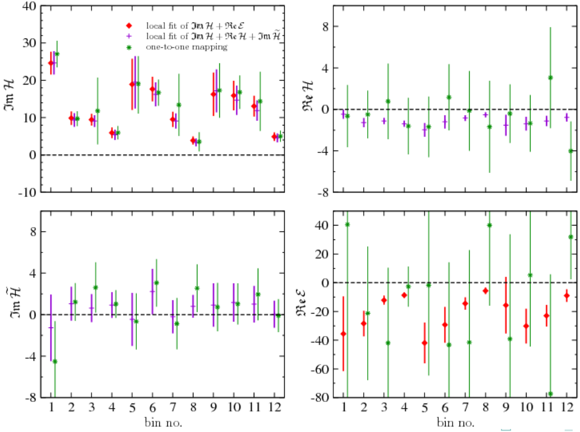
- [K.K., Müller, Murray '13]: using HERMES set of 10+ observables measured at same 12 kinematical points and extracting 8 leading "CFFs"



- Most CFFs are not reliably constrained.

Reducing the number of CFFs

- Scenario 1: Fit of $\mathfrak{Im}\,\mathcal{H}$, $\mathfrak{Re}\,\mathcal{H}$ and $\mathfrak{Im}\,\widetilde{\mathcal{H}}$. $\chi^2/n_{\rm d.o.f.} = 148.8/144$. (In good agreement with local fit of [Guidal '10])
- Scenario 2: Fit of $\mathfrak{Im}\,\mathcal{H}$ and $\mathfrak{Re}\,\mathcal{E}$. $\chi^2/n_{\rm d.o.f.} = 134.2/144$.



Hybrid GPD models for global fits

- Sea quarks and gluons modelled using SO(3) partial wave expansion in conformal GPD moment space + $\mathcal{Q}^2$ evolution.
- Valence quarks — model CFFs directly (ignoring $\mathcal{Q}^2$ evolution):

$$\mathfrak{Im}\,\mathcal{H}(\xi,t) = \pi\left[\frac{4}{9}H^{u_{\rm val}}(\xi,\xi,t) + \frac{1}{9}H^{d_{\rm val}}(\xi,\xi,t) + \frac{2}{9}H^{\rm sea}(\xi,\xi,t)\right]$$

$$H(x,x,t) = n\,r\,2^{\alpha}\left(\frac{2x}{1+x}\right)^{-\alpha(t)}\left(\frac{1-x}{1+x}\right)^{b}\frac{1}{\left(1-\frac{1-x}{1+x}\frac{t}{M^2}\right)^{p}}\,.$$

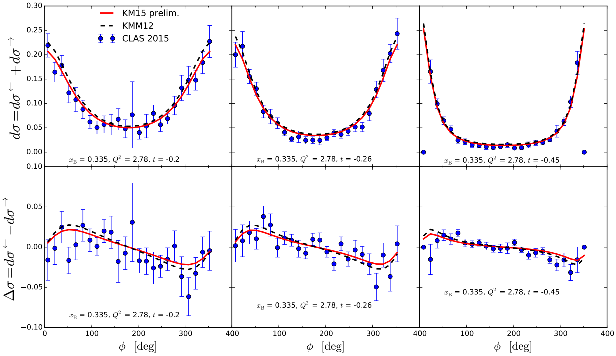
- $\mathfrak{Re}\,\mathcal{H}$ determined by dispersion relations
- 15 free parameters in total for H, $\tilde{H}$, E, $\tilde{E}$.

Model	KM09a	KM09b	KM10	KM10a	KM10b	KMS11	KMM12	KM15
free params.	{3}+(3)+5	{3}+(3)+6	{3}+15	{3}+10	{3}+15	NNet	{3}+15	{3}+15
χ^2/d.o.f.	32.0/31	33.4/34	135.7/160	129.2/149	115.5/126	13.8/36	123.5/80	240./275
F_2	{85}	{85}	{85}	{85}	{85}		{85}	{85}
$\sigma_{\rm DVCS}$	(45)	(45)	51	51	45		11	11
$d\sigma_{\rm DVCS}/dt$	(56)	(56)	56	56	56		24	24
$A_{LU}^{\sin\phi}$	12+12	12+12	12	16	12+12		4	13
$A_{LU,I}^{\sin\phi}$			18	18		18	6	6
$A_C^{\cos 0\phi}$							6	6
$A_C^{\cos\phi}$	12	12	18	18	12	18	6	6
$\Delta\sigma^{\sin\phi,w}$			12				12	63
$\sigma^{\cos 0\phi,w}$			4				4	58
$\sigma^{\cos\phi,w}$			4				4	58
$\sigma^{\cos\phi,w}/\sigma^{\cos 0\phi,w}$		4			4			
$A_{UL}^{\sin\phi}$							10	17
$A_{LL}^{\cos 0\phi}$							4	14
$A_{LL}^{\cos\phi}$								10
$A_{UT,I}^{\sin(\phi-\phi_S)\cos\phi}$							4	4

- [K.K., Müller, et al. '09–'15]
- These models are publicly available (google for "gpd page")
- Other approach: recursive fit [Goldstein, Gonzalez, Liuti '11] focuses on fixed-target data

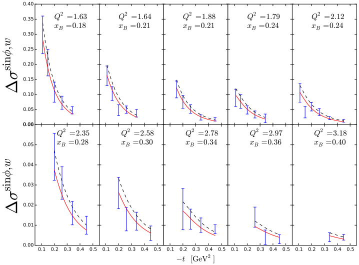
2015 CLAS cross-sections (1/2)

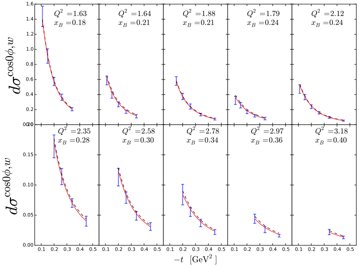
- Restriction to kinematics where leading-order framework should be valid: $-t/Q^2 < 0.25$ with $Q^2 > 1.5\,\mathrm{GeV}^2$, means using 48 out of measured 110 x_{B}–Q^2–t bins.

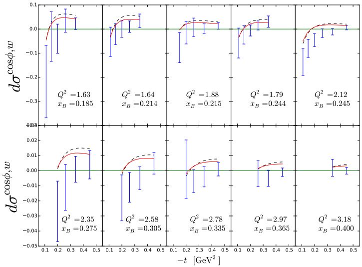


- $\chi^2/\mathrm{npts} = 1032.0/1014$ for $d\sigma$ and $936.1/1012$ for $\Delta\sigma$

2015 CLAS cross-sections (2/2)



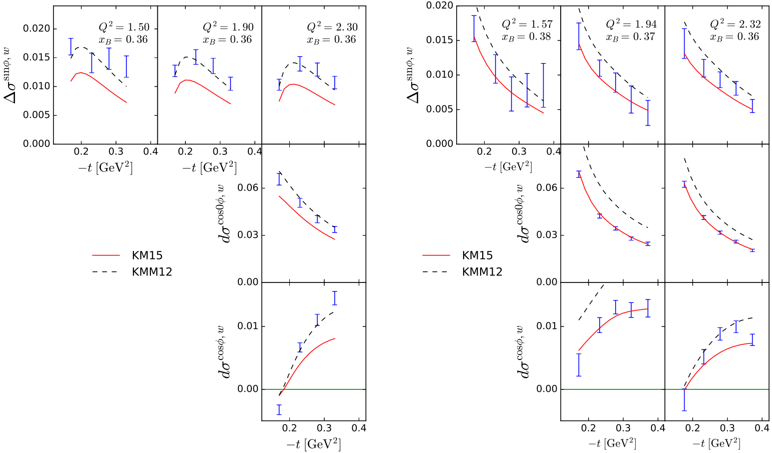




- $\chi^2/\mathrm{npts} = 62.2/48$ for $d\sigma^{\cos\phi,w}$

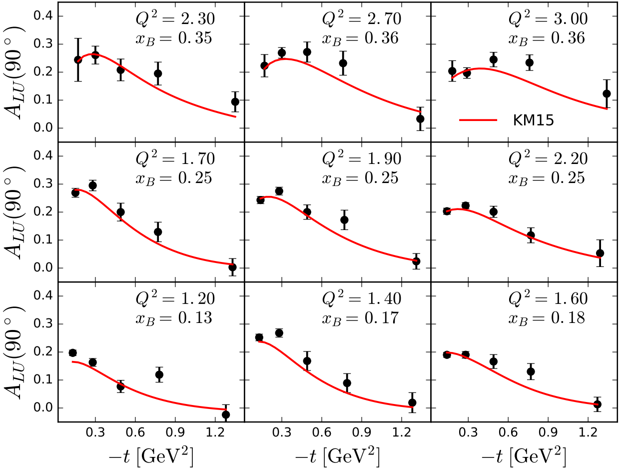
(O.K. but not so perfect as in ϕ-space)

2006 vs 2015 Hall A cross-sections



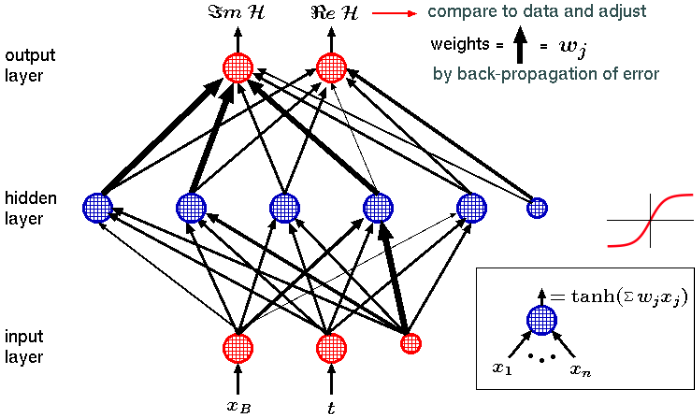
- Improvement of global χ^2/d.o.f. 123.5/80 → 240./275

2007 CLAS beam spin asymmetry



- Only data with $|t| \leq 0.3\,\mathrm{GeV}^2$ used for fits.

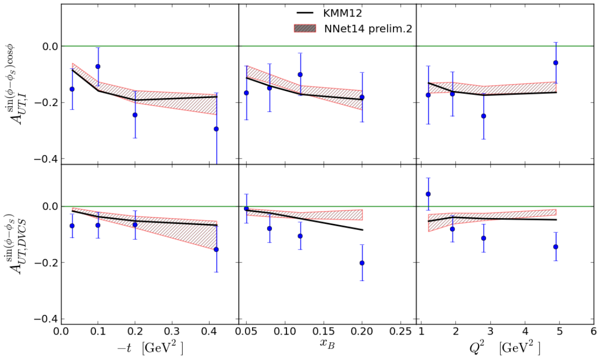
Neural networks



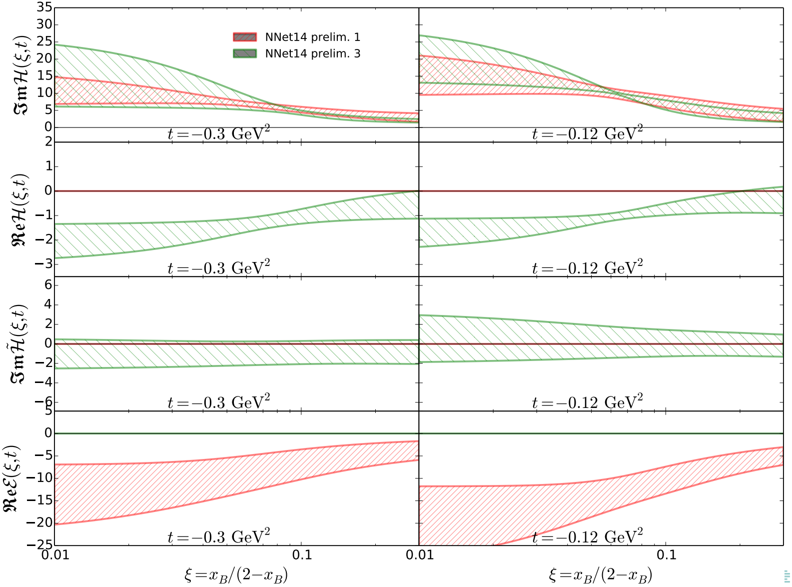
- Essentially a least-squares fit of a complicated many-parameter function. $f(x) = \tanh(\sum w_i \tanh(\sum w_j \cdots))$
  $\Rightarrow$ no theory bias

Preliminary neural net HERMES fit

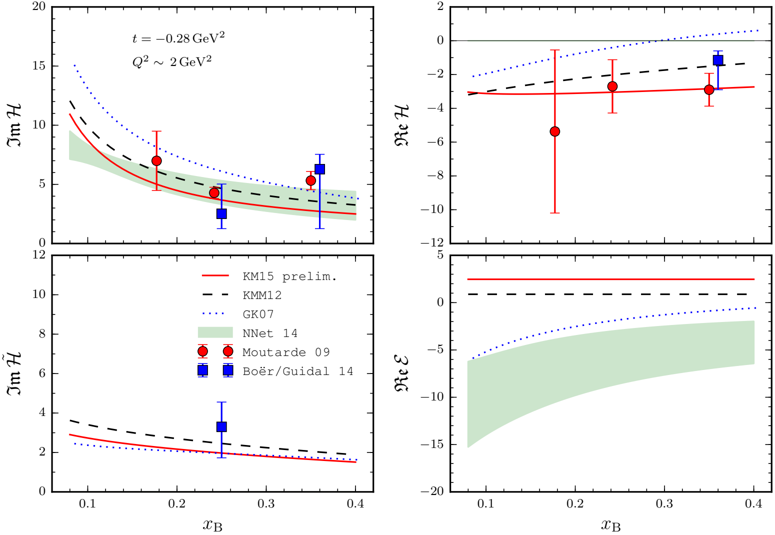
- Fit to all HERMES DVCS data with two types of neural nets
  - (x_B, t) – (7 neurons) – $(\mathfrak{Im}\,\mathcal{H},\ \mathfrak{Re}\,\mathcal{H},\ \mathfrak{Im}\,\tilde{\mathcal{H}})$: $\chi^2/n_{\rm pts} = 135.4/144$
  - (x_B, t) – (7 neurons) – $(\mathfrak{Im}\,\mathcal{H},\ \mathfrak{Re}\,\mathcal{E})$: $\chi^2/n_{\rm pts} = 120.2/144$



Neural net HERMES fit — CFFs

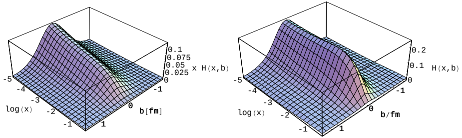


CFFs from various fits

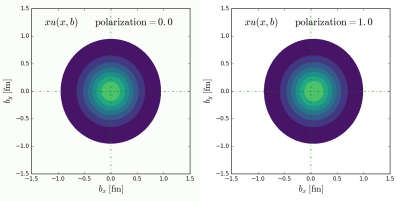


Tomography

- Quark and gluon sea 2D distributions $H(x, \vec{b}_\perp)$ ([KM] model)



- Sivers effect for valence quarks ([GK] model)



- See also [Dupré, Guidal, Vanderhaeghen '16]
- Tomography is still very much model-dependent; e.g. some extrapolation from $H(x, x, t)$ to $H(x, 0, t)$ is needed.

Summary

- Global fits of all proton DVCS data using flexible hybrid models are in healthy shape
- Data clearly restrict $H(x, x, t)$, and to some extent $\tilde{H}$, but any information about E is very model-dependent
- New 2015 JLab data relieve some old tensions
- Neural networks are very promising method for GPD/CFF extraction

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The End