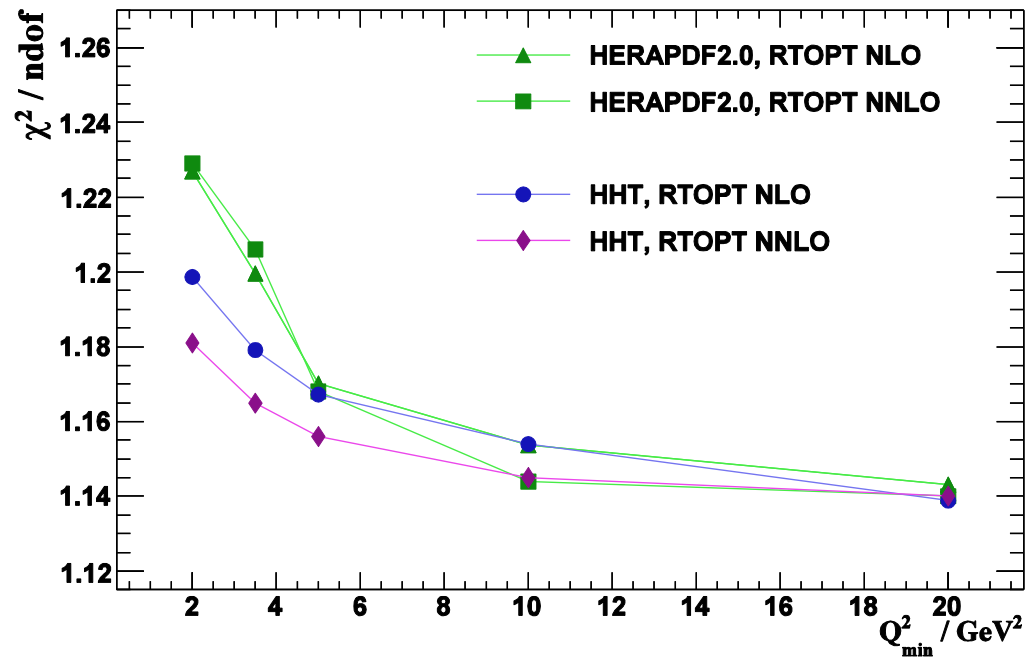
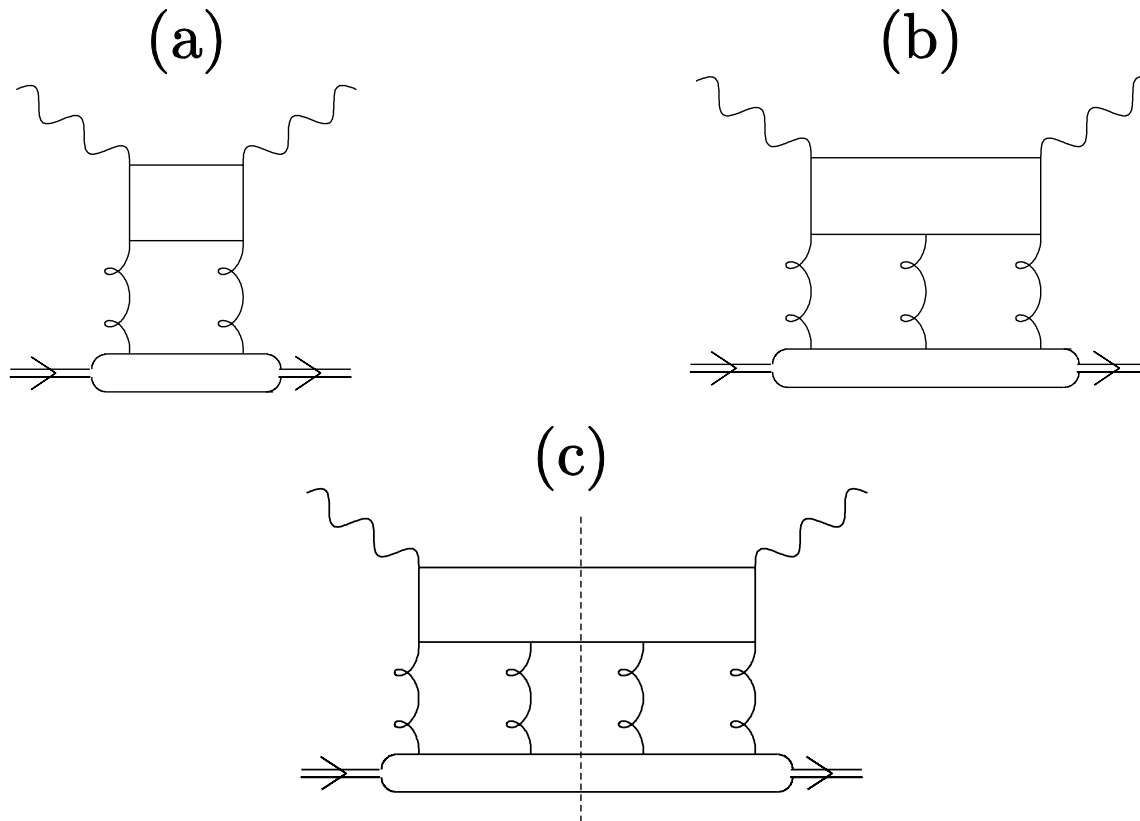


HERA data at low Q^2

We all remember that the χ^2/ndof is somewhat worse at low Q^2



One way to improve this is to add higher twist terms -HHT
BUT NOTE- these are not the high- x , low Q^2 terms we usually
associate with the terminology 'higher twist'



They are higher twist terms which act a low-x
 Their origin COULD be to do with the recombination of gluon ladders.
 Bartels, Golec-Biernat, Kowalski suggest that such higher twist terms would
 cancel between σ_L and σ_T in F2, but remain strong in FL

Try the simplest of possible modification to the structure functions F_2 and F_L

$$F_{2,L} = F_{2,L} (1 + A_{2,L}^{\text{HT}}/Q^2)$$

We find that such a modification of F_L is favoured, whereas for F_2 it is not.

At NNLO the $\chi^2/\text{ndof} = 1363/1131$ for HERAPDF2.0

If A_2^{HT} is added this becomes 1357/1130 and $A_2^{\text{HT}} = 0.12 \pm 0.07 \text{ GeV}^2$

If A_L^{HT} is added this becomes 1316/1130 and $A_L^{\text{HT}} = 5.5 \pm 0.6 \text{ GeV}^2$

If both A_L^{HT} and A_2^{HT} are added the result is consistent with just adding A_L^{HT}

So now concentrating on just F_L , we call these fits HHT

Fit at	with $Q_{\text{min}}^2 = 3.5 \text{ GeV}^2$	HERAPDF2.0	HHT	A_L^{HT}
NNLO	χ^2/ndof	1363/1145	1316/1145	5.5 ± 0.6
	Partial χ^2/ndof for NC e^+p : $Q^2 \geq Q_{\text{min}}^2$	451/377	422/377	
	χ^2/ndp for NC e^+p : $2.0 \leq Q^2 < Q_{\text{min}}^2$	41/25	32/25	
NLO	χ^2/ndof	1356/1145	1329/1145	4.2 ± 0.7
	Partial χ^2/ndof for NC e^+p : $Q^2 \geq Q_{\text{min}}^2$	447/377	431/377	
	χ^2/ndp for NC e^+p : $2.0 \leq Q^2 < Q_{\text{min}}^2$	46/25	46/25	

$$\Delta\chi^2 = -47$$

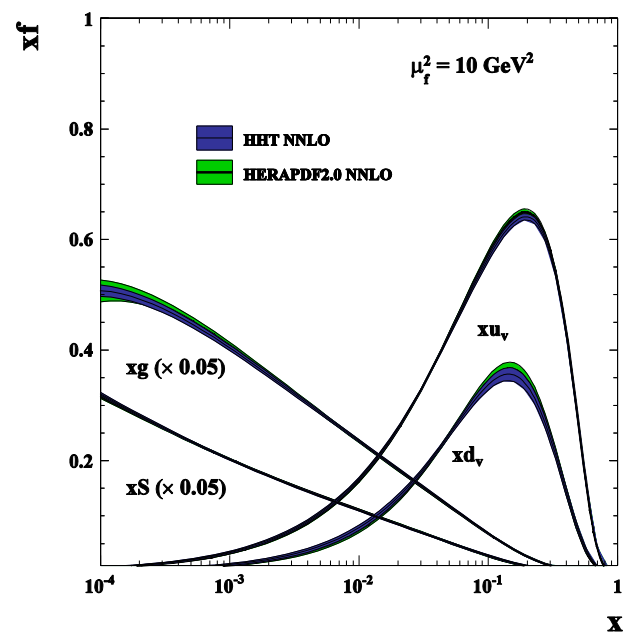
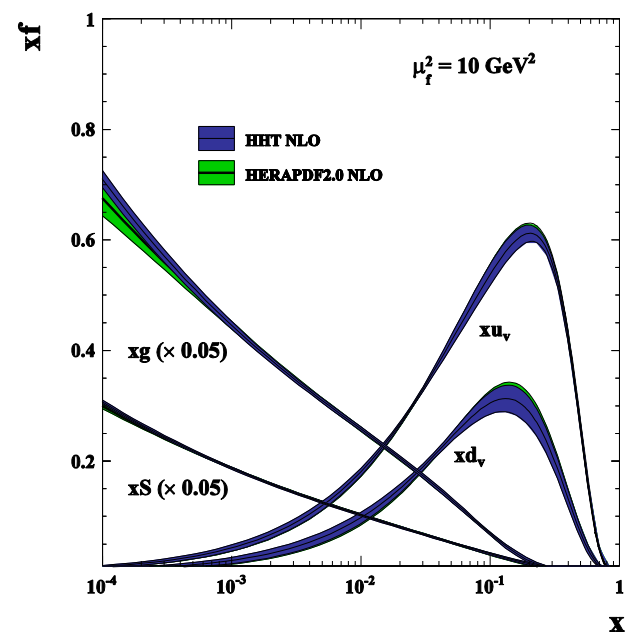
$$\Delta\chi^2 = -28$$

After HT is added the NNLO fit is better than the NLO fit

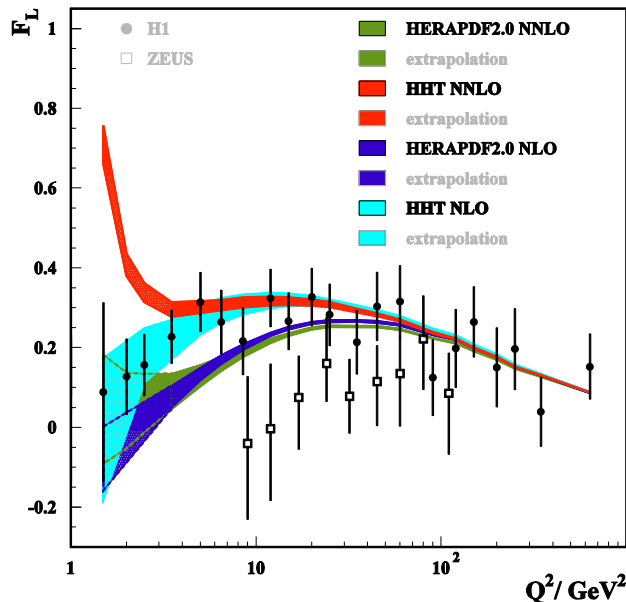
A substantial part of the improvement comes from the NCE⁺p 920 data

This persists even below Q_{min}^2

Note the HHT PDFs themselves barely change from HERAPDF2.0 – the higher twist modification does not affect high-scale LHC physics



The HHT fits tend to increase the value of F_L for both NLO and NNLO



Here's how F_L looks for both HERAPDF2.0 and our HHT analysis

You might think that -since F_L is related to the gluon - an easier way to obtain larger F_L would be to drop the negative term in the gluon PDF parametrisation.

So we did- we call this the AG parametrisation

This makes almost no difference for the NLO fits. However it is strongly disfavoured for the NNLO fits. At NNLO the fit wants a negative term in the gluon parametrization AND a higher twist term in F_L .

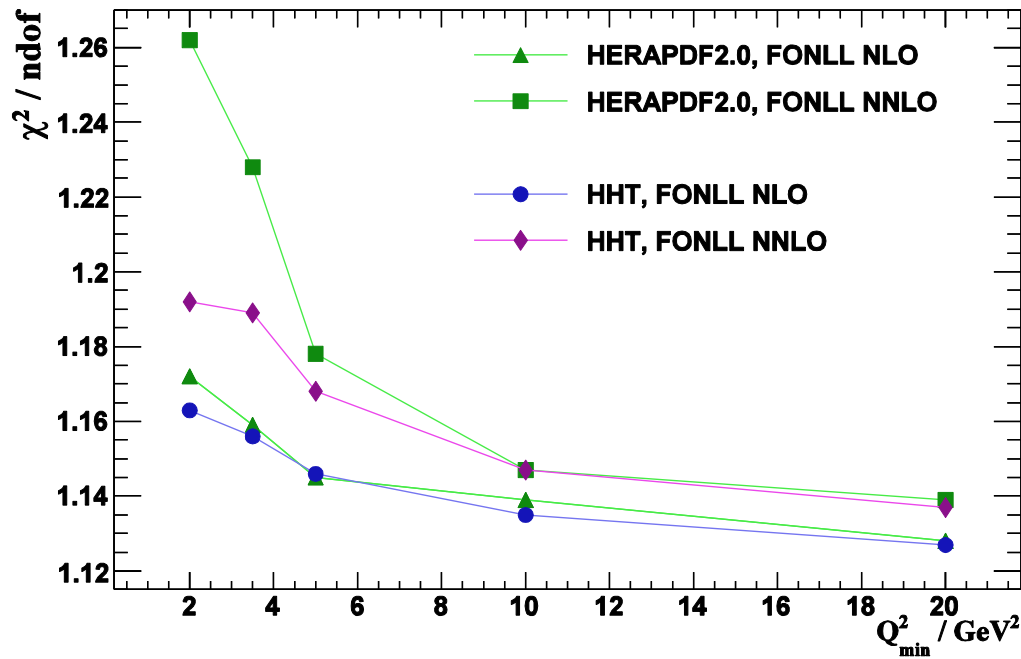
For HERAPDF2.0 AG the χ^2/ndof = 1389/1131 cf	1363/1130 for the standard fit
For HHT AG the χ^2/ndof = 1350/1130 cf	1316/1130 for the standard fit

These two contributions clearly affect the fit in different ways

Another consideration is that we know that the rate of decrease χ^2/ndof with increasing Q_{\min}^2 differs with the heavy flavour scheme used AND with the order in α_s to which F_L is evaluated

So let's take a look at FONLL

For FONLL at NNLO a higher twist term in FL brings a substantial decrease in the χ^2/ndof with a similar value of $A_L^{\text{HT}} = 6.0 \pm 0.7 \text{ GeV}^2$ to that for the RTOPT scheme. For FONLL at NLO a higher twist term in FL brings almost no decrease in χ^2/ndof . This is probably related to the order in α_s to which F_L is evaluated



For FONLL/RTOPT at NNLO, F_L is evaluated to $O(\alpha_s^2)/O(\alpha_s^3)$

For FONLL/RTOPT at NLO, F_L is evaluated to $O(\alpha_s)/O(\alpha_s^2)$

The value of F_L at $O(\alpha_s)$ is relatively large in any scheme and thus there is little need for higher twist.

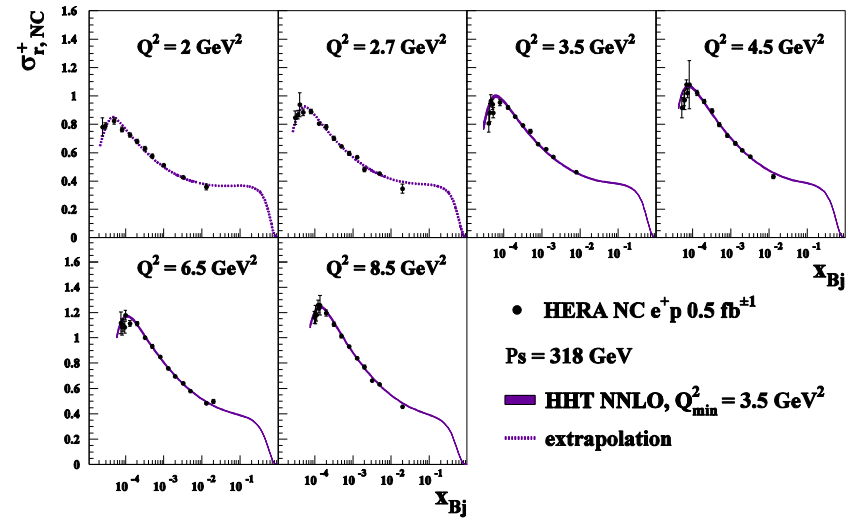
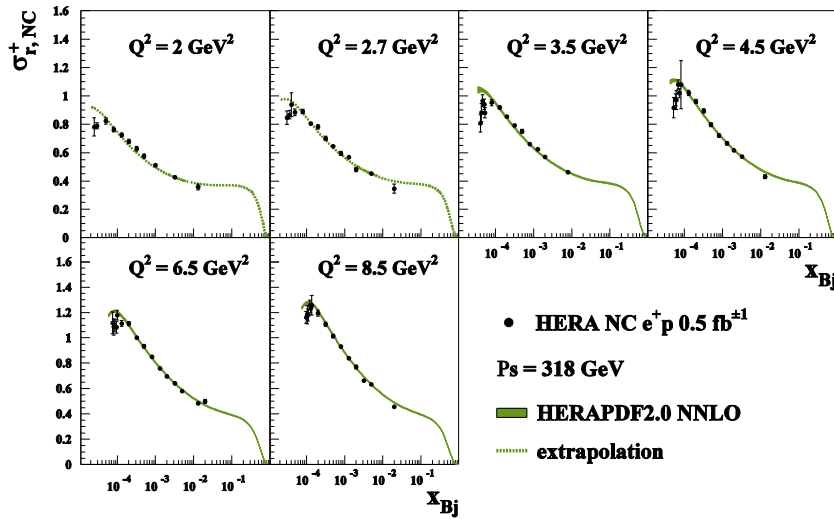
However as soon as F_L is evaluated to $O(\alpha_s^2)$ or higher the need for higher twist appears

So now let's look at why the HHT fits do so well

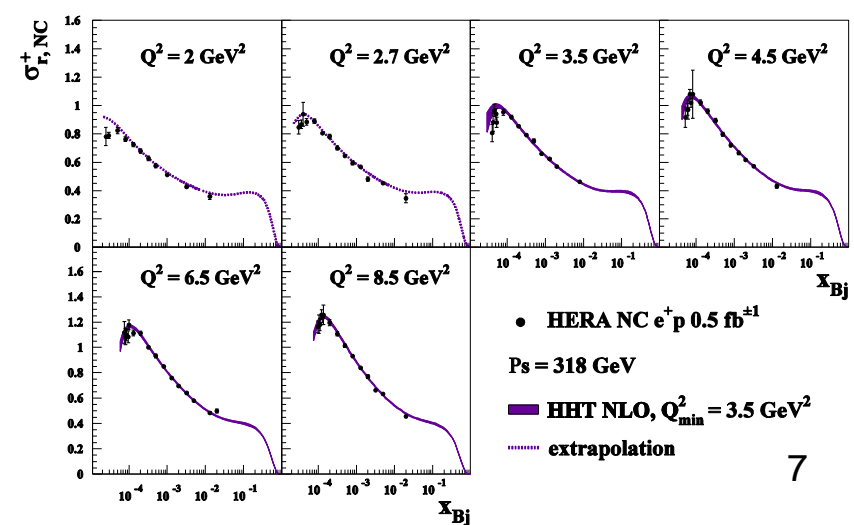
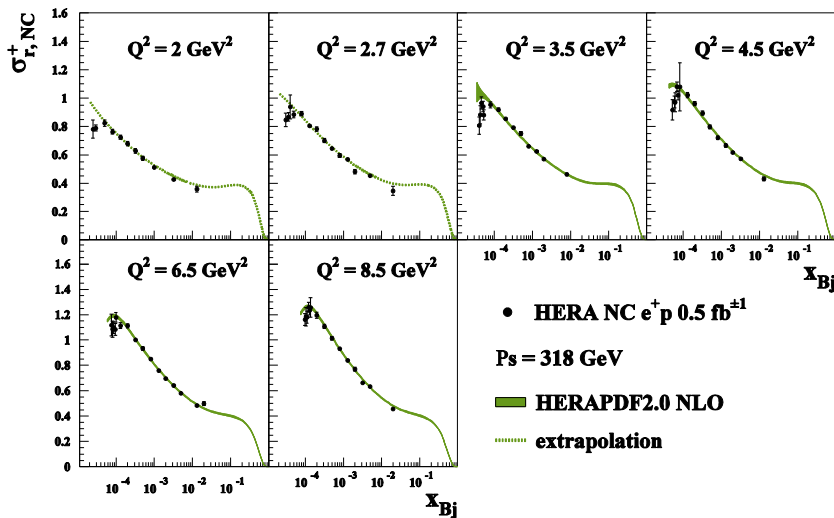
It is because they describe the turn over at low x , Q^2 much better

$$\sigma_{\text{red}} = F_2 - y^2/Y_+ F_L$$

The data clearly wants a larger F_L and this is what the higher twist term provides



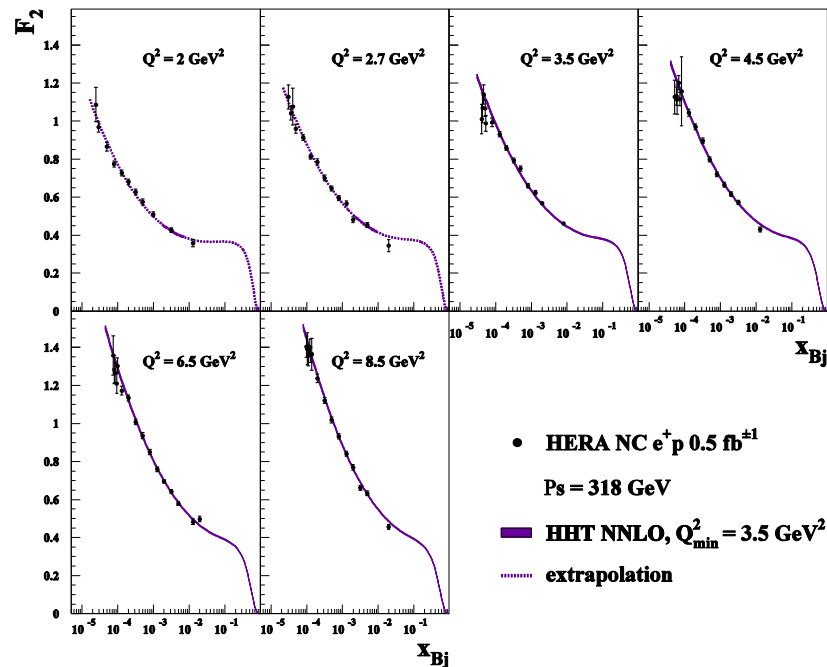
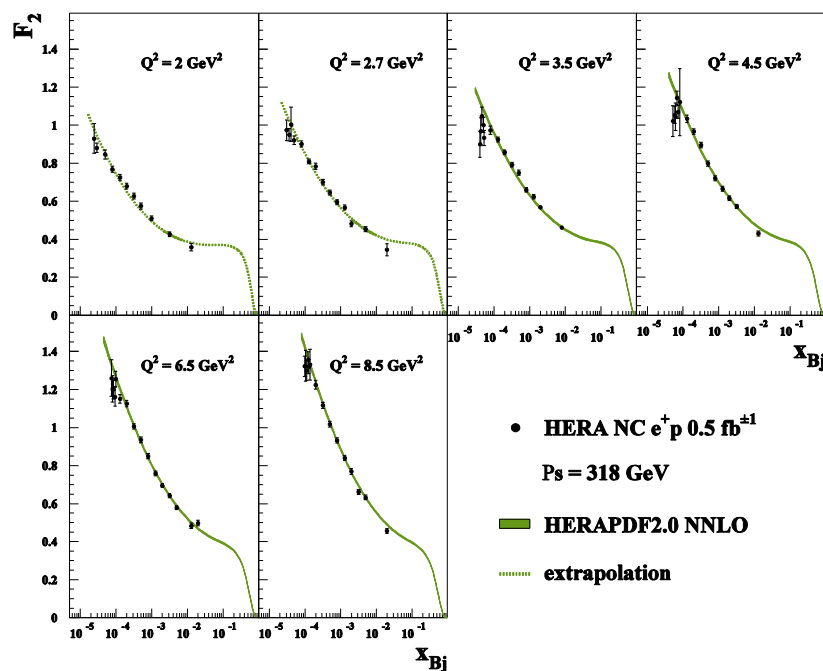
You can also see that NNLO does better than NLO



It is also interesting to look at F_2 , where the data points are extracted as

$$F_2^{\text{extracted}} = F_2^{\text{predicted}} \frac{\sigma_F^{\text{measured}}}{\sigma_F^{\text{predicted}}}$$

Since F_2 is the dominant part of the reduced cross section this is a reasonable procedure



This essentially means that we get F_2 by correcting σ_{red} with our predicted F_L

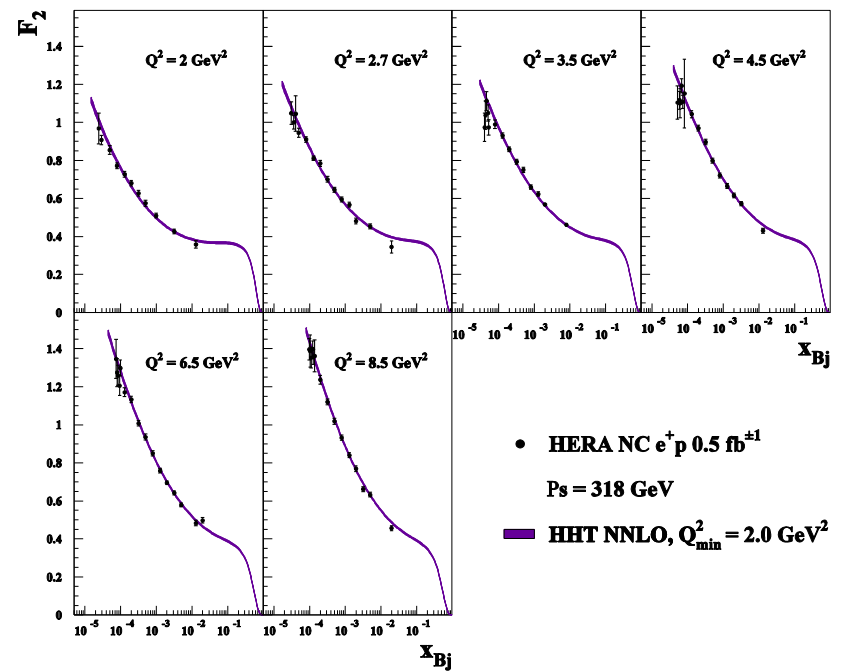
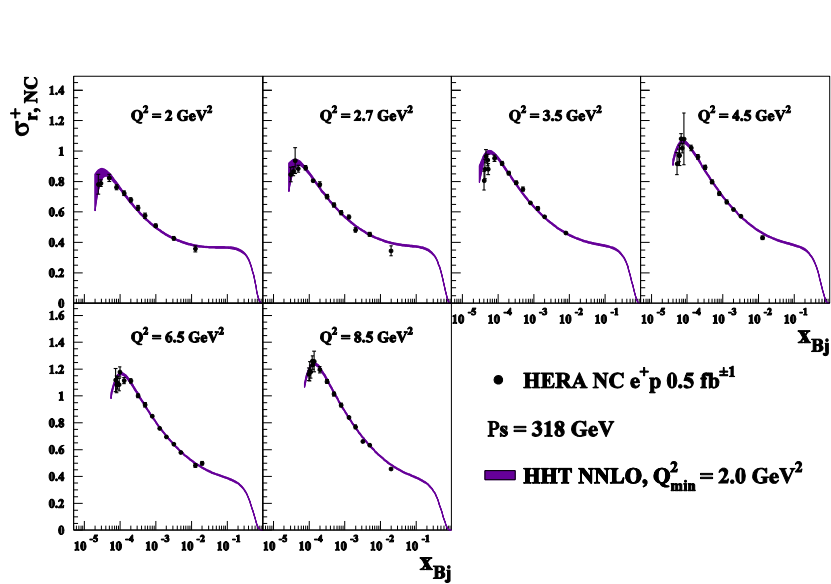
$$F_2 = \sigma_{\text{red}} + y^2/Y_+ F_L$$

If our predicted F_L is too small the F_2 will also be too small and this is what we see in HERAPDF2.0 F_2 at low x, Q^2 . The extracted F_2 takes a turn over!

This is not the pQCD F_2 predictions say.

If we use the HHT predictions for F_L then the F_2 extracted is much closer to the F_2 predictions— and these F_2 predictions are very similar for HERAPDF2.0 and HHT because they depend ONLY on the very similar PDFs.

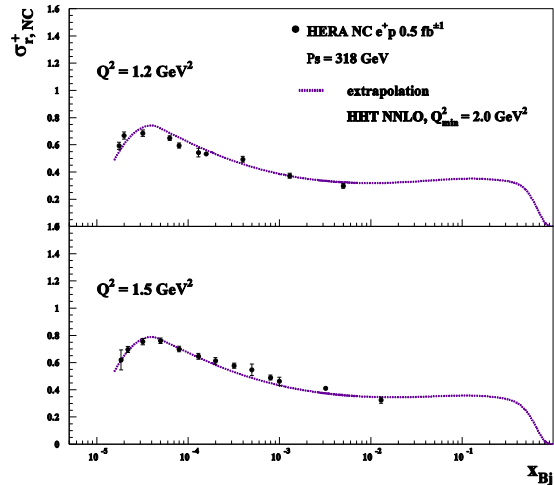
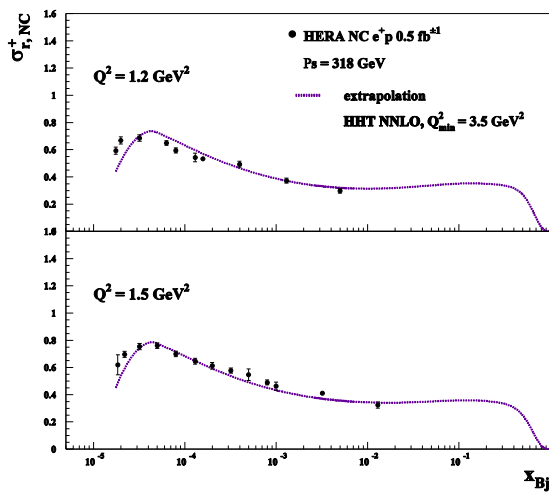
(The picture is similar but not quite so good for NLO- see back-up)



Looking at the extrapolations of our fits below $Q_{\min}^2 = 3.5$ GeV 2 made us bold enough to extend the fit down to $Q_{\min}^2 = 2.0$ GeV 2

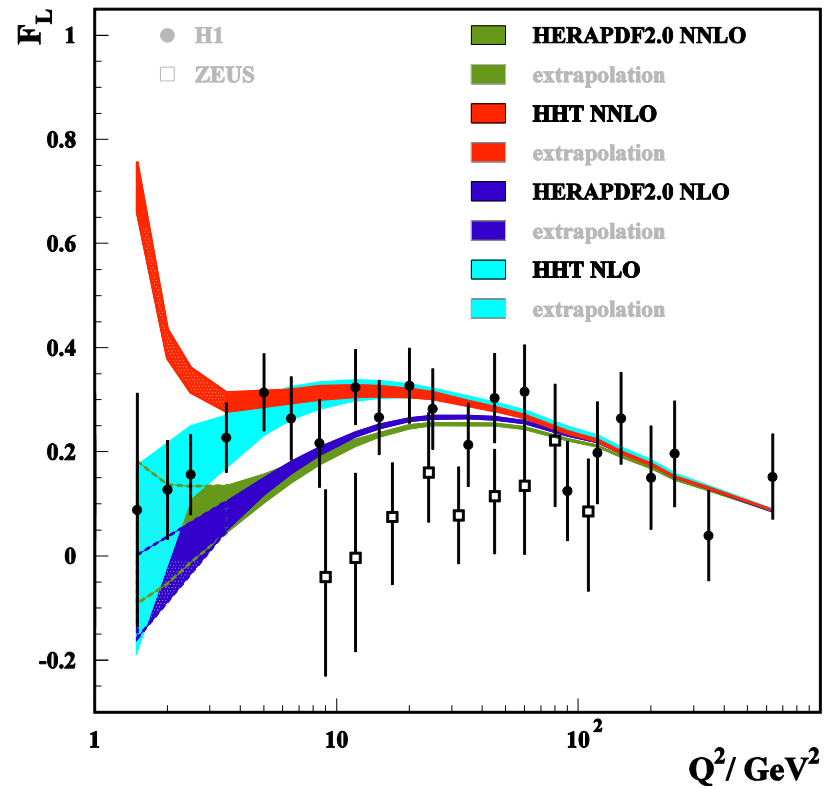
Fit at	with $Q_{\min}^2 = 2.0$ GeV 2	HERAPDF2.0	HHT	$A_{\text{fit}}^{\text{fit}}$
NNLO	χ^2/ndof	1437/1185	1381/1188	5.2 ± 0.7
	Partial χ^2/ndof for NC e^+p : $Q^2 \geq Q_{\min}^2$	486/402	457/402	
	Partial χ^2/ndof NC e^+p : $Q_{\min}^2 \leq Q^2 < 3.5$ GeV 2	31/25	26/25	
NLO	χ^2/ndof	1433/1185	1398/1188	4.0 ± 0.6
	Partial χ^2/ndof for NC e^+p : $Q^2 \geq Q_{\min}^2$	487/402	466/402	
	Partial χ^2/ndof NC e^+p : $Q_{\min}^2 \leq Q^2 < 3.5$ GeV 2	40/25	31/25	

Where not much changes for the NNLO fit, and the NLO fit improves a little
 See back-up



So we got even bolder and looked at lower Q^2 - by backward evolution

But beware...is this actually reasonable?
 What does FL itself look like?



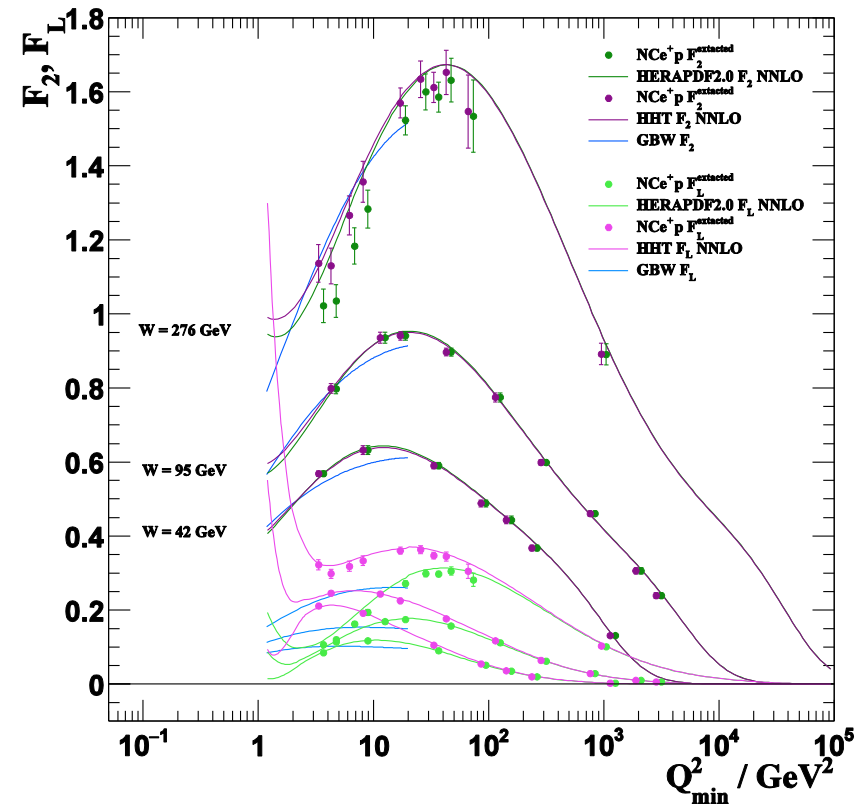
NNLO HHT F_L prediction is becoming untamed at low Q^2 — this approach cannot be pushed too far.
 This comes from NNLO coefficient functions and the $1/Q^2$ term just makes it worse

Another interesting way to look at this is by looking at plots of F_2 and F_L at fixed W as a function of Q^2 (This is the Golec-Biernat Wusthoff dipole model way of looking at it)

First look at the top three curves for F_2

Compare the HHT F_2 extracted points to the F_2 predictions – the description is good. Then compare the HERAPDF2.0 F_2 extracted points to the F_2 predictions the description is not so good.

This is essentially what we saw in the F_2 curves on slide 8 but it emphasizes that the discrepancy comes at low x . Only the top curve $W=276\text{GeV}$ involves data at really low x
 $x = Q^2/(W^2+Q^2)$



Now look at the bottom three curves for F_L . The predictions for HHT go crazy at very low Q^2 .

In fact this upturn happens in HERAPDF as well- and it is starting to happen in F_2 . It is a feature of the low- x coefficient functions

Here the extracted F_L points are got from

$$F_L^{\text{extracted}} = \frac{F_L^{\text{predicted}} \sigma_{\text{measured}}}{\sigma_{\text{predicted}}}$$

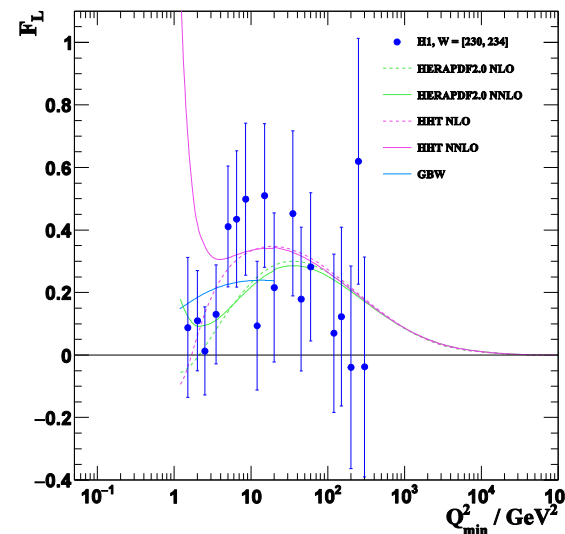
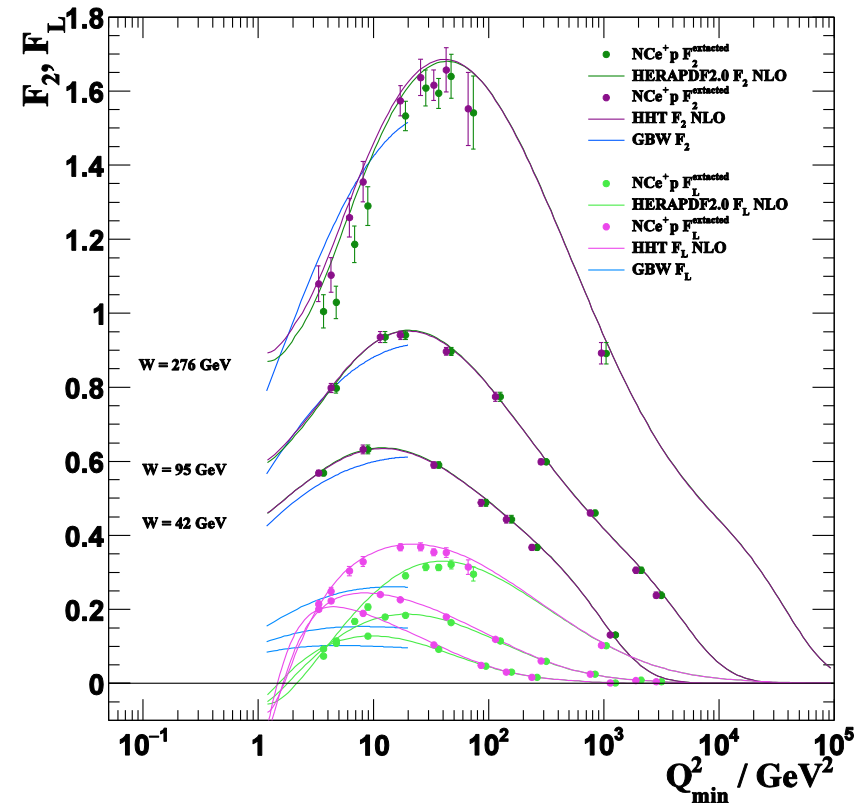
Since F_L is not the dominant part of the reduced cross section these cannot be considered as measurements and they simply follow the predictions

It is not just the NNLO F_L which is becoming unacceptable at low Q^2 , the NLO predictions also have problems. They are becoming negative. This is not allowed for a structure function (as opposed to a PDF)

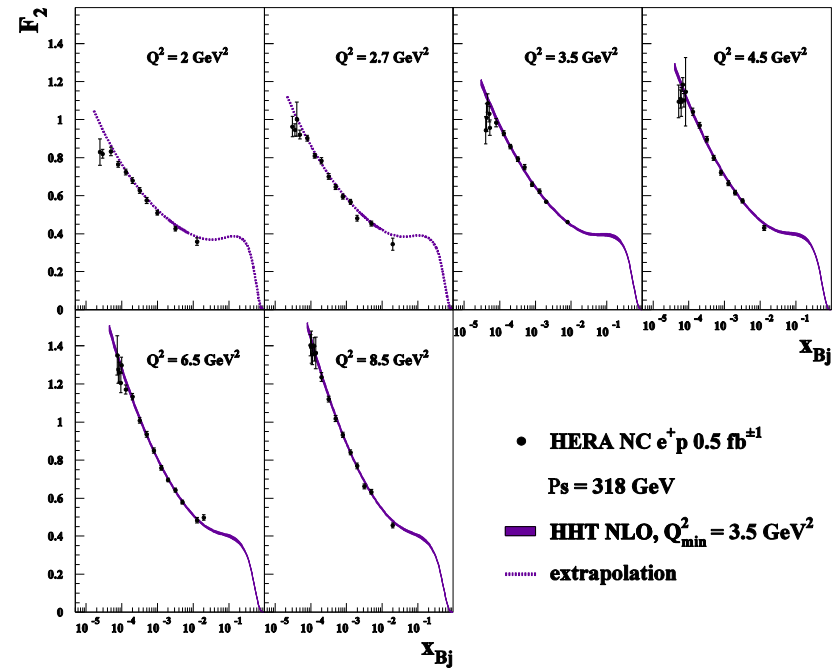
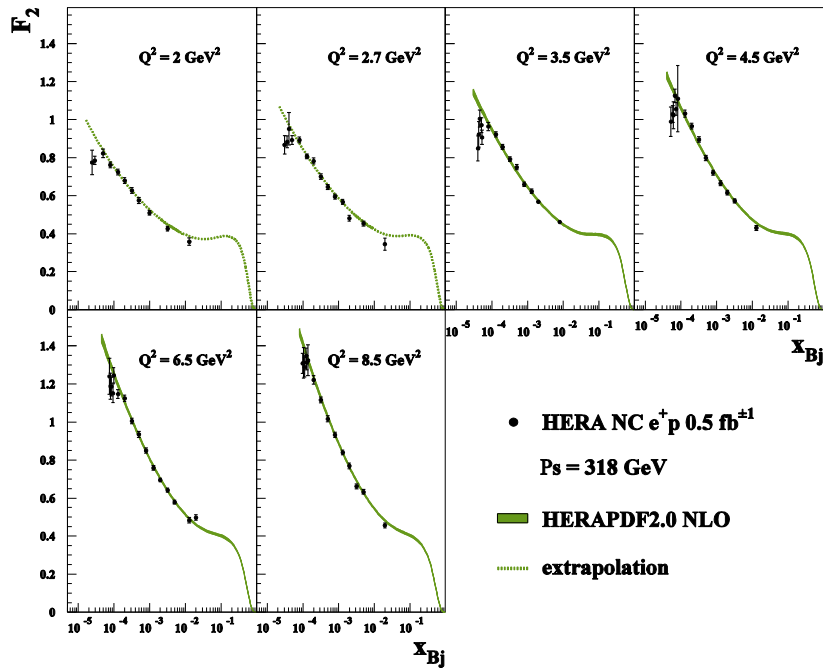
The GBW predictions at both NNLO and NLO are also compared to the extracted data points in these figures. They are broadly compatible with the HHT predictions for F_2 for $Q^2 < 10 \text{ GeV}^2$

Finally we look at the F_L predictions for HERAPDF2.0 and HHT at NNLO as compared to the H1 direct measurements at $W = 232 \text{ GeV}$.

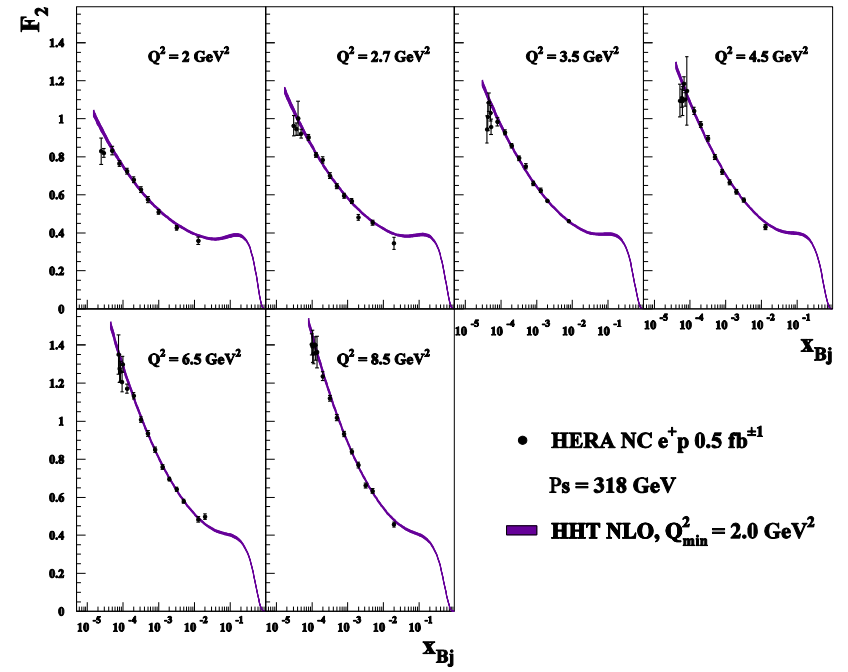
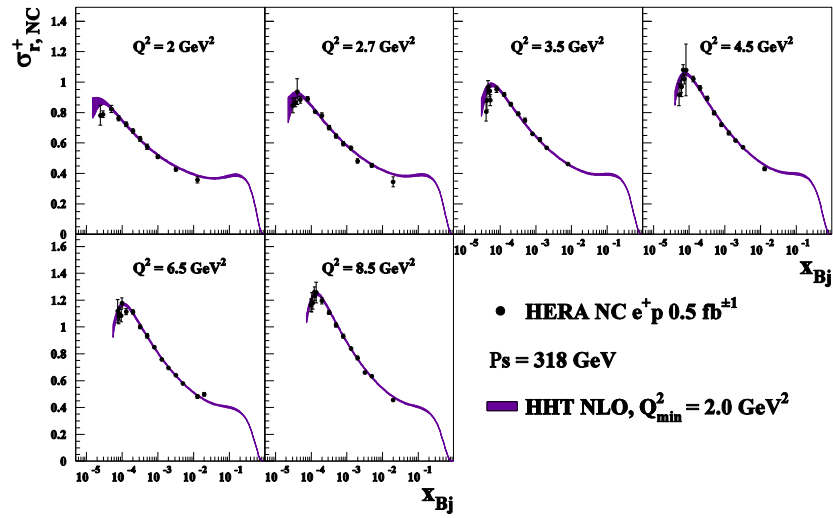
The data are able to exclude the extreme behaviour of the HHT prediction for $Q^2 < 3.5 \text{ GeV}^2$



Back-up



And at NLO –the F_2 down to
 $Q_{\min}^2 = 3.5$



And at NLO down to $Q_{\min}^2 = 2.0$