Effective Field Theory

- arxiv:1006.2142 (W. Skiba)
- hep-th/0701053 (C.P. Burgess)

- Intro
- Tree-level matching
- Loop-level matching
- RGE's

$$\int \mathcal{D}\varphi_H \, e^{i\int \mathcal{L}(\varphi_L, \varphi_H)} = e^{i\int \mathcal{L}_{eff}(\varphi_L)}$$

$$\mathcal{L}_{eff}(\varphi_L) = \mathcal{L}_{d\leq 4} + \sum_i \frac{\mathcal{O}_i}{\Lambda^{\dim(\mathcal{O}_i)-4}}$$

weakly coupled:

$$\dim(\mathcal{O}_i) = \sum \dim(\text{fields}) + \text{derivatives}.$$

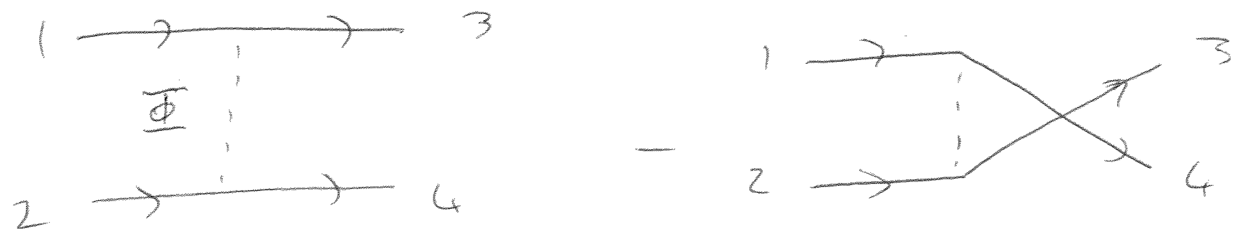
- Theory has a breakdown scale
- Heavy fields can also contribute to $\mathcal{L}_{d\leq 4}$ (e.g. sym. breaking effects).

Tree-level matching

Consider

$$\mathcal{L}_F = i\bar{\psi}\not{\partial}\psi + \frac{1}{2}(\partial_\mu \Phi)^2 - \frac{M^2}{2}\Phi^2 + \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{m^2}{2}\varphi^2 - \lambda\bar{\psi}\psi\Phi - \eta\bar{\psi}\psi\varphi$$

Assume $M \gg m$. Examine $\psi\psi \to \psi\psi$



$$\mathcal{M}_{UV} = \bar{u}_3 u_1 \bar{u}_4 u_2 \times (-i\lambda)^2 \frac{i}{(p_3-p_1)^2 - M^2} - \{3 \leftrightarrow 4\}$$

Now

$$(-i\lambda)^2 \frac{i}{(p_3-p_1)^2 - M^2} = \frac{i\lambda^2}{M^2} \frac{1}{1 - \frac{(p_3-p_1)^2}{M^2}}$$

$$\simeq \frac{i\lambda^2}{M^2}\left(1 + \frac{(p_3-p_1)^2}{M^2} + O\left(\frac{p^4}{M^4}\right)\right) \quad (*)$$

$$\mathcal{L}_E = i\bar{\psi}\partial\!\!\!/\psi + \frac{c}{2}\bar{\psi}\psi\bar{\psi}\psi + d\,\partial_\mu\bar{\psi}\partial_\mu\psi\bar{\psi}\psi$$

$$\mathcal{M}_{IR,c} = \bar{u}_3 u_1 \bar{u}_4 u_2 (ic) - \{3 \leftrightarrow 4\}$$

Cf. $(*)$ $\qquad c = \dfrac{\lambda^2}{M^2}$

$$\mathcal{M}_{IR,d} = id(p_1 \cdot p_3 + p_2 \cdot p_4)\bar{u}_3 u_1 \bar{u}_4 u_2 - \{3 \leftrightarrow 4\}$$

Cf. $\dfrac{i\lambda^2}{M^4}(p_3-p_1)^2 - \{3 \leftrightarrow 4\}$

Can take on-shell momenta (for any small external mom. IR theory must match UV).

$$(p_3 - p_1)^2 = -2\, p_3 \cdot p_1$$

Cons. of mom. $\quad p_1 + p_2 = p_3 + p_4$

$$\Rightarrow p_2 \cdot p_4 = p_1 \cdot p_3$$

$$\Rightarrow d = -\frac{\lambda^2}{M^4}$$

Could have also included

$(\partial^2 \bar\psi)\psi\bar\psi\psi + \text{h.c.} \rightarrow$ vanishes on-shell

$\partial_\mu \bar\psi \psi \bar\psi \partial^\mu \psi \rightarrow$ not needed at tree-level.

One-loop matching

$$\mathcal{L} = i\bar\psi \not\partial \psi - \sigma \bar\psi\psi + \frac{1}{2}(\partial_\mu \Phi)^2 - \frac{M^2}{2}\Phi^2$$

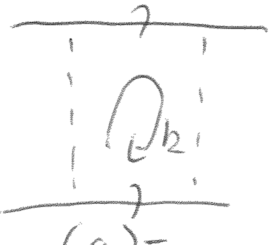
$$- \lambda \bar\psi \psi \Phi + \phi \text{ terms.}$$

Add σ to (i) avoid IR-divergences.

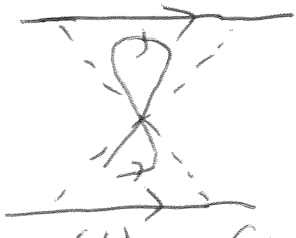
(ii) find non-zero terms prop. to $\frac{1}{M^4}$.

As an example find λ^4 terms.

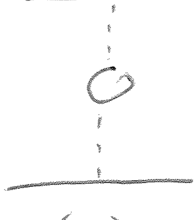
$\psi\psi \rightarrow \psi\psi$



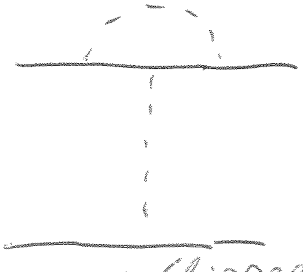
$(a)_F$



$(b)_F$ ③



$(c)_F$



+ flipped

$(d)_F$

Suppress $\{3 \leftrightarrow 4\}$ terms.

$(a)_F$ and $(b)_F$ converge

$(c)_F$ and $(d)_F$ diverge

$$U_S \equiv \bar{u}_3 u_1 \bar{u}_4 u_2$$

$$\frac{1}{\bar{\varepsilon}} = \frac{1}{\varepsilon} - \gamma + \log(4\pi)$$

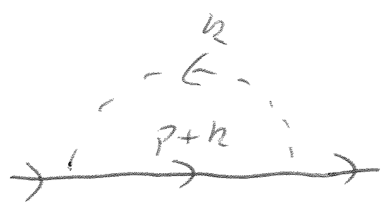
μ is the ren. scale.

$\lambda \to \mu^{\varepsilon}\lambda$ in dim. reg.

Sum is

$$(a + \dots + d)_F = \frac{2i\lambda^4 U_S}{(4\pi)^2 M^2}\Big\{ -\frac{1}{\bar{\varepsilon}} - 1 - \log\left[\frac{\mu^2}{M^2}\right] + \frac{\sigma^2}{M^2}\left(-\frac{6}{\bar{\varepsilon}} - 6\log\left[\frac{\mu^2}{\sigma^2}\right] - 6 + 4\log\left[\frac{M^2}{\sigma^2}\right]\right)\Big\}$$

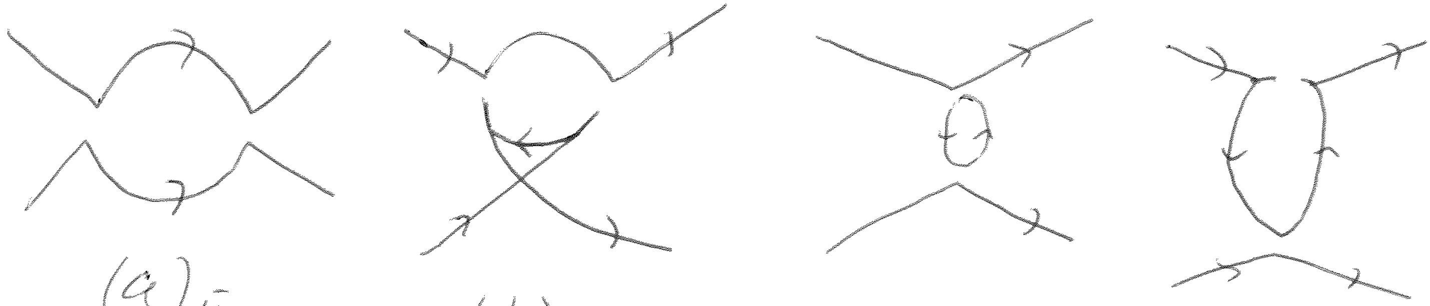
Two-point function



$$= \frac{i\not{p}\,\lambda^2}{2(4\pi)^2}\left(\frac{1}{\bar{\varepsilon}} + \log\left(\frac{\mu^2}{M^2}\right) + \frac{1}{2} + \dots\right)$$

<u>Effective theory</u>

$$\mathcal{L} = iZ\bar{\psi}\not{\partial}\psi - \sigma\bar{\psi}\psi + \frac{C}{2}\bar{\psi}\psi\bar{\psi}\psi$$



$(a)_E$ $(b)_E$ $(c)_E$ + flipped $(d)_E$

(Two point loop amplitude vanishes in the EFT.)

Four-point amplitude → all diagrams diverge.

$$(a+\dots+d)_E = -\frac{2ic^2\sigma^2}{(4\pi)^2}\,u_s\left[\frac{2}{\bar{\varepsilon}}+2\log\left(\frac{\mu^2}{\sigma^2}\right)+1\right]$$

- IR and UV theories have different counterterms to cancel divergences.
- We use $\overline{MS}$ and cancel the $\frac{1}{\bar{\varepsilon}}$ poles.

~~Also~~ • Now set $\mu=M$ and use $c=\frac{\lambda^2}{M^2}$

$$(a+\dots+d)_F-(a+\dots+d)_E$$
$$=\frac{2i\lambda^4}{(4\pi)^2M^2}\,u_s\left\{-1+\frac{\sigma^2}{M^2}\left(-6\log\left[\frac{M^2}{\sigma^2}\right]-6+4\log\left[\frac{M^2}{\sigma^2}\right]\right)+\frac{\sigma^2}{M^2}\left(1+2\log\left[\frac{M^2}{\sigma^2}\right]\right)\right\}$$

$$=i u_s\left\{-\frac{2\lambda^4}{(4\pi)^2M^2}-\frac{10\lambda^4\sigma^2}{(4\pi)^2M^4}\right\}$$

$$\Rightarrow c(\mu=M)=\frac{\lambda^2}{M^2}-\frac{2\lambda^4}{(4\pi)^2M^2}-\frac{10\lambda^4\sigma^2}{(4\pi)^2M^4}$$

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$Z=1+\frac{\lambda^2}{4(4\pi)^2}$, $\sqrt{Z}\,4\to 4$. Also have to check "d"!

RG Running

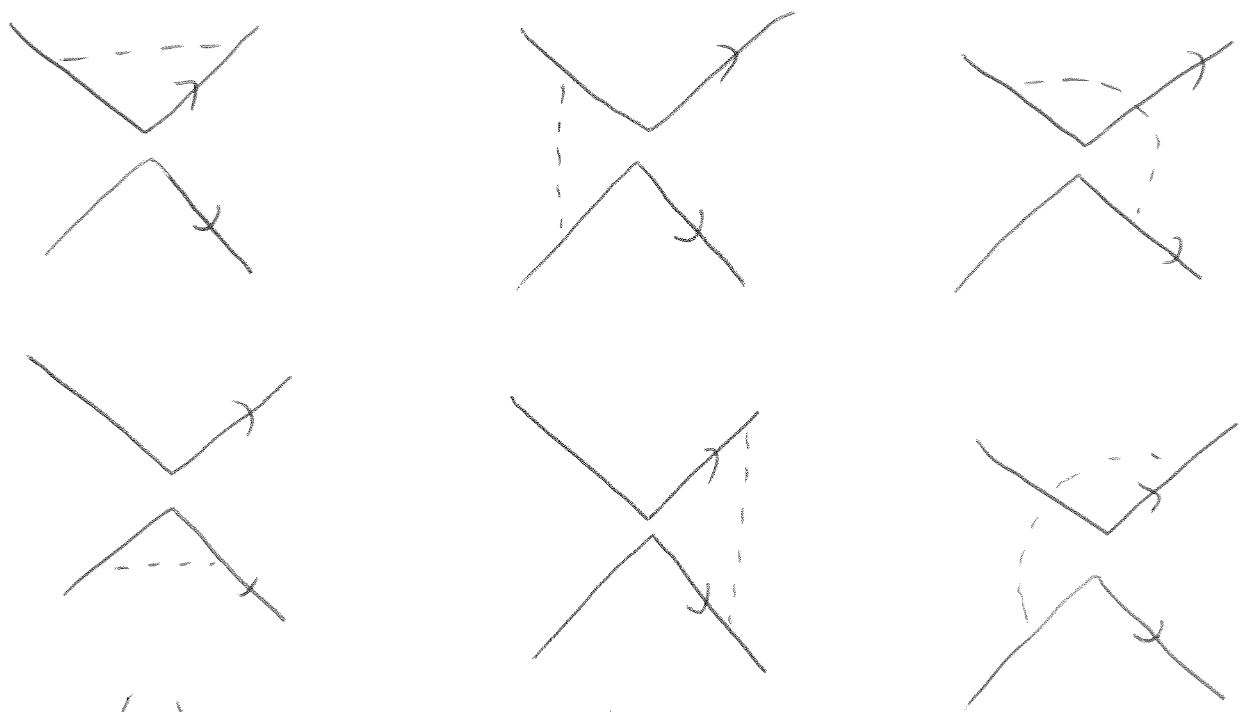
$$\mathcal{L} = i\bar\psi \partial\!\!\!/ \psi + \frac{c}{2}\bar\psi\psi\bar\psi\psi + d\,\partial_\mu\bar\psi\,\partial^\mu\psi\,\bar\psi\psi$$
$$+ \frac{1}{2}(\partial_\mu\phi)^2 - \frac{m^2}{2}\phi^2 - \eta\bar\psi\psi\phi$$

- To compute $\psi\psi \to \psi\psi$ amplitude to lowest order in p and $c^2\eta^2$ in all couplings.
- Neglect d term in $\mathcal{L}$.
- Use dim-reg with $d = \cancel{4} - 2\varepsilon$

2-point:

$$= \frac{i\eta^2 p\!\!\!/}{2(4\pi)^2}\frac{1}{\varepsilon} + \text{finite.}$$

4-point:



(a) (b) (d)

$$(a) = \left(\frac{-2icy^2}{(4\pi)^2} \frac{1}{\varepsilon} + \text{finite} \right) \bar{u}_3 u_1 \bar{u}_4 u_2$$

$$(b) = \frac{icy^2}{2(4\pi)^2} \frac{1}{\varepsilon} \bar{u}_3 \gamma^\mu u_1 \bar{u}_4 \gamma_\mu u_2 + \text{finite}$$

$$(c) = -(b)$$

$$\psi_0 = \sqrt{Z_\psi}\, \psi \qquad c_0 = c \mu^{2\varepsilon} Z_c$$

$$\mathcal{L} = i \bar{\psi}_0 \not{\partial} \psi_0 + \frac{c_0}{2} \bar{\psi}_0 \psi_0 \bar{\psi}_0 \psi_0 = i Z_\psi \bar{\psi} \not{\partial} \psi + \frac{c}{2} Z_c Z_\psi^2 \mu^{2\varepsilon} \bar{\psi}\psi\bar{\psi}\psi$$

$$= i \bar{\psi} \not{\partial} \psi + \mu^{2\varepsilon} \frac{c}{2} \bar{\psi}\psi\bar{\psi}\psi$$

$$+ i (Z_\psi - 1) \bar{\psi} \not{\partial} \psi + \mu^{2\varepsilon} \frac{c}{2} (Z_c Z_\psi^2 - 1) \bar{\psi}\psi\bar{\psi}\psi$$

Counterterms (MS)

$$Z_\psi - 1 = \frac{-y^2}{2(4\pi)^2} \frac{1}{\varepsilon} \qquad c(Z_c Z_\psi^2 - 1) = \frac{2cy^2}{(4\pi)^2} \frac{1}{\varepsilon}$$

$$\Rightarrow \quad Z_c = 1 + \frac{3y^2}{(4\pi)^2} \frac{1}{\varepsilon} + O(y^4)$$

To compute RGE's use

$$0 = \mu \frac{d}{d\mu} c_0 = \mu \frac{d}{d\mu} (c \mu^{2\varepsilon} Z_c)$$

$$= \mu^{2\varepsilon} Z_c \left(\mu \frac{d}{d\mu} c \right) + 2\varepsilon c \mu^{2\varepsilon} Z_c + c \mu^{2\varepsilon} \mu \frac{d}{d\mu} Z_c$$

After some work end up with

$$\beta_c = \mu \frac{d}{d\mu} c = \frac{6 \eta^2}{(4\pi)^2} c$$

$$\Rightarrow c(m) = c(M) \left[1 - \frac{6\eta^2}{(4\pi)^2} \log\left(\frac{M}{m}\right) \right]$$

$$= \underbrace{\frac{\lambda^2}{M^2}}_{\text{tree-level matching.}} \left[1 - \frac{6\eta^2}{(4\pi)^2} \log\left(\frac{M}{m}\right) \right]$$

<u>Operator Mixing</u>

Consider if we had a heavy vector instead.

$$\mathcal{L} = i \bar{\psi} \not{\partial} \psi + \frac{c_V}{2} \bar{\psi} \gamma^\mu \psi \bar{\psi} \gamma_\mu \psi$$
$$+ \frac{1}{2} (\partial_\mu \phi)^2 - \frac{m^2}{2} \phi^2 - \eta \bar{\psi} \psi \phi$$

Four-fermion vertex now contains γ^μ term in ϕ-exchange diagram

(a) gives div. cont. to $\bar{\psi} \gamma^\mu \psi \bar{\psi} \gamma_\mu \psi$

(b)+(c) give " " " $\bar{\psi} \sigma^{\mu\nu} \psi \bar{\psi} \sigma_{\mu\nu} \psi$

where $\sigma^{\mu\nu} = i [\gamma^\mu, \gamma^\nu]$

Have to include a $\frac{c_T}{2} \bar{\psi} \sigma^{\mu\nu} \psi \bar{\psi} \sigma_{\mu\nu} \psi$ term in $\mathcal{L}$.

$c_T(\mu = M) = 0 \qquad c_V(\mu = M) \neq 0.$

Coefficients of the counterterms

$$C_V (Z_V Z_\psi^2 - 1) = \frac{\eta^2}{(4\pi)^2} (-C_V + 6 C_T) \frac{1}{\varepsilon}$$

$$C_T (Z_T Z_\psi^2 - 1) = \frac{\eta^2}{(4\pi)^2} C_V \frac{1}{\varepsilon}$$

$$\beta_{C_V} = 12 C_T \frac{\eta^2}{(4\pi)^2}$$

$$\beta_{C_T} = 2 (C_T + C_V) \frac{\eta^2}{(4\pi)^2}$$

$$\mu \frac{d}{d\mu} \begin{pmatrix} C_V \\ C_T \end{pmatrix} = \frac{2\eta^2}{(4\pi)^2} \begin{pmatrix} 0 & 6 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} C_V \\ C_T \end{pmatrix}$$

Eigenvectors satisfy uncoupled RGE's.

$\rightarrow$ correspond to lin. comb.'s of the operators that do not mix under 1-loop renormalization.