Four-jet production in single- and double-parton scattering within high-energy factorisation

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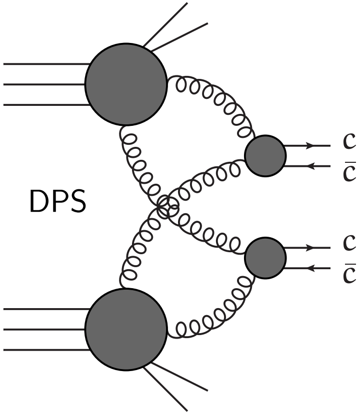
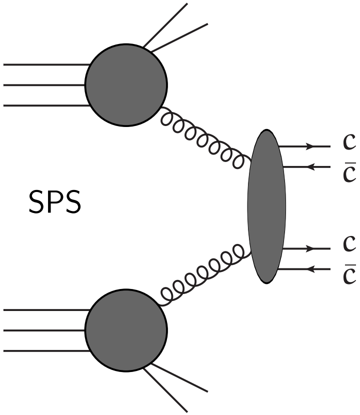
Outline

- DPS vs SPS for $\mathrm{pp} \to \mathrm{c\bar{c}\,c\bar{c}}$
- $\mathrm{k_T}$-factorization
- $\mathrm{pp} \to 4\mathrm{j}$
- Off-shell amplitudes
- BCFW recursion for amplitudes with off-shell partons
- Conclusions

DPS vs SPS for $pp \to c\bar{c}\,c\bar{c}$

- production of $c\bar{c}\,c\bar{c}$ is a good place to study DPS effects Łuszczak, Maciuła, Szczurek 2012
- DPS $c\bar{c}\,c\bar{c}$ cross section approaches $c\bar{c}$ cross section for large energies
- DPS $c\bar{c}\,c\bar{c}$ cross section is orders of magnitude larger than LO SPS $c\bar{c}\,c\bar{c}$ cross section Schäfer, Szczurek 2012, Maciuła, Szczurek, AvH 2014
- LHCb measured a surprisingly large cross section for the production of D-meson pairs JHEP 06 141 (2012)





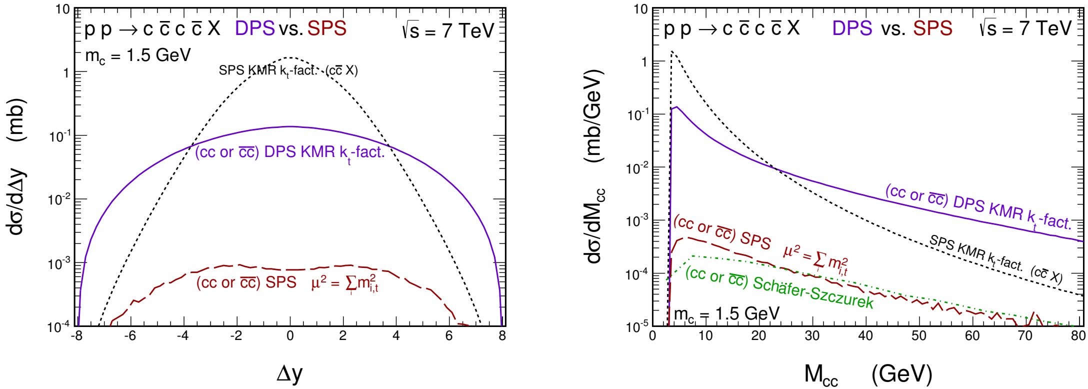
DPS vs SPS for $pp \to c\bar{c}\,c\bar{c}$

Maciuła, Szczurek, AvH 2014

Simple factorized model

$$d\sigma^{DPS}(pp \to c\bar{c}\,c\bar{c}X) = \frac{1}{2\sigma_{eff}}\, d\sigma^{SPS}(pp \to c\bar{c}X_1)\, d\sigma^{SPS}(pp \to c\bar{c}X_2)$$

with $\sigma_{eff} = 15\text{mb}$.



High Energy Factorization a.k.a. k_T-factorization

Catani, Ciafaloni, Hautmann 1991   Collins, Ellis 1991

$$\sigma_{h_1,h_2\to QQ} = \int d^2k_{1\perp}\,\frac{dx_1}{x_1}\,\mathcal{F}(x_1,k_{1\perp})\,d^2k_{2\perp}\,\frac{dx_2}{x_2}\,\mathcal{F}(x_2,k_{1\perp})\,\hat{\sigma}_{gg}\left(\frac{m^2}{x_1x_2s},\frac{k_{1\perp}}{m},\frac{k_{2\perp}}{m}\right)$$

- reduces to collinear factorization for $s \gg m^2 \gg k_\perp^2$, but holds al so for $s \gg m^2 \sim k_\perp^2$
- typically associated with small-x physics, forward physics, saturation, heavy-ions ...
- allows for higher-order kinematical effects at leading order

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- allows for higher-order kinematical effects at leading order
- $k_\perp$-dependent $\mathcal{F}$ may satisfy BFKL-eqn, CCFM-eqn, BK-eqn, KGBJS-eqn, ...
- in particular KMR-type unintegrated pdfs (Kimber, Martin, Ryskin 2000) contain essential hard scale dependence via Sudakov resummation

$$T_a(k^2,\mu^2) = \exp\left(-\int_{k^2}^{\mu^2}\frac{dp^2}{p^2}\frac{\alpha_S(p^2)}{2\pi}\sum_b\int_0^{k/(\mu+k)} dz\,P_{ba}(z)\right)$$

$$\mathcal{F}_a(x,k^2,\mu^2) = \partial_\lambda\left[T_a(\lambda,\mu^2)\,xg_a(x,\lambda)\right]_{\lambda=k^2}$$

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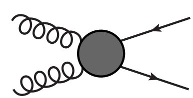
$$\mathcal{F}_a(x,k^2,\mu^2) = \partial_\lambda\big[T_a(\lambda,\mu^2)\,xg_a(x,\lambda)\big]_{\lambda=k^2}$$

- requires matrix elements with *off-shell* initial-state partons with $k_i^2 = k_{i\perp}^2 < 0$

$$k_1 = x_1p_1 + k_{1\perp}$$
$$k_2 = x_2p_2 + k_{2\perp}$$



DPS for four jets

$gg \to gggg$
$gg \to ggq\bar{q}$
$gg \to q\bar{q}q\bar{q}$
$gg \to q\bar{q}q'\bar{q}'$
$gq \to gggq$
$gq \to gqq\bar{q}$
$gq \to gqq'\bar{q}'$
$q\bar{q} \to gggg$
$q\bar{q} \to ggq\bar{q}$
$q\bar{q} \to ggq'\bar{q}'$
$q\bar{q} \to q\bar{q}q\bar{q}$
$q\bar{q} \to q\bar{q}q'\bar{q}'$
$q\bar{q} \to q'\bar{q}'q'\bar{q}'$
$qq \to ggqq$
$qq \to qqq\bar{q}$
$qq \to qqq'\bar{q}'$
$qq' \to ggqq'$
$qq' \to qq'q\bar{q}$

$gg \to gg$
$gg \to q\bar{q}$
$gq \to qg$
$q\bar{q} \to gg$
$q\bar{q} \to q\bar{q}$
$q\bar{q} \to q'\bar{q}'$
$qq \to qq$
$qq' \to qq'$

$$\sigma = \sum_{i,j;a,b;k,l;c,d} \frac{\mathcal{S}}{\sigma_{\text{eff}}}\, \sigma(i,j \to a,b)\, \sigma(k,l \to c,d)$$

$$\mathcal{S} = \begin{cases} 1/2 & \text{if} \quad i\,j = k\,l \quad \text{and} \quad a\,b = c\,d \\ 1 & \text{if} \quad i\,j \neq k\,l \quad \text{or} \quad a\,b \neq c\,d \end{cases}$$

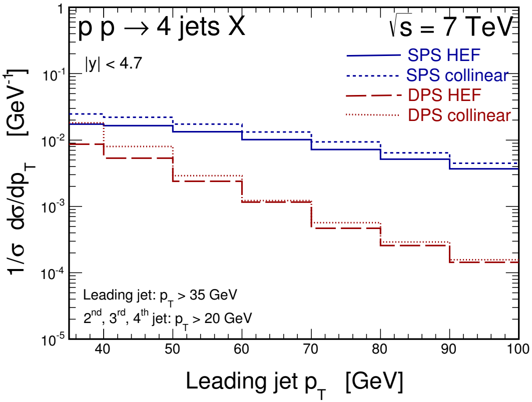
$$\sigma_{\text{eff}} = 15\text{mb}$$

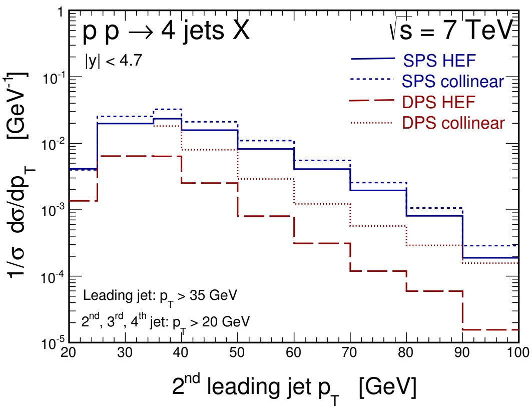
k_T-factorization

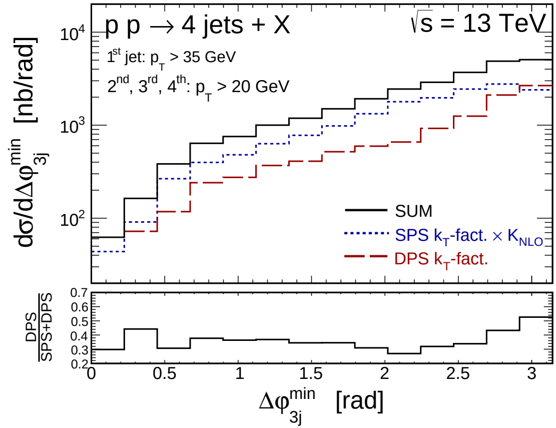
- PDFs and matrix elements are well-defined
- No rigorous factorization proof
- Reasonable description of data justifies the formula *a posteriori*

DPS for four jets

Maciuła, Szczurek, Kutak, Serino, AvH 2016





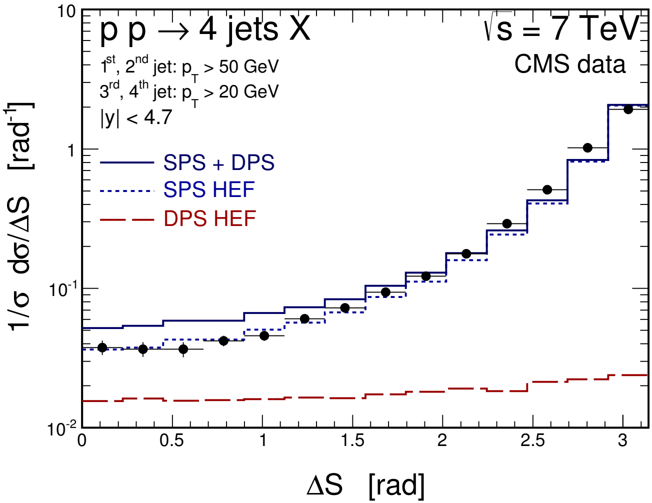


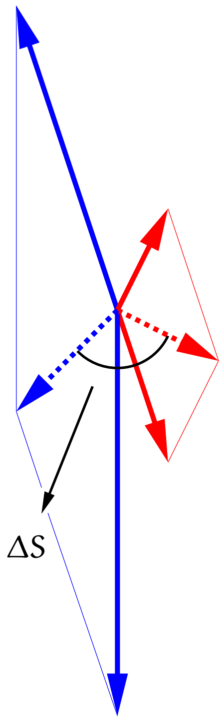
$$\Delta\varphi_{3j}^{\min} = \min_{i,j,k} \left(|\phi_i - \phi_j| + |\phi_j - \phi_k| \right)$$

Proposed by ATLAS in JHEP 12 105 (2015) for high-p_T analysis.

Four jets with k_T-factorization

Maciuła, Szczurek, Kutak, Serino, AvH 2016





- ΔS is the azimutal angle between the sum of the two hardest jets and the sum of the two softest jets.
- This variable has no distribution at LO in collinear factorization: pairs would have to be back-to-back.
- Our (KMR-type) updfs DLC2016v2 describe data remarkably well.

Amplitudes with off-shell gluons

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n-parton amplitude is a function of n momenta $k_1, k_2, \ldots, k_n$
and n *directions* $p_1, p_2, \ldots, p_n$

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$$\begin{aligned} k_1^\mu + k_2^\mu + \cdots + k_n^\mu &= 0 && \text{momentum conservation} \\ p_1^2 = p_2^2 = \cdots = p_n^2 &= 0 && \text{light-likeness} \\ p_1\cdot k_1 = p_2\cdot k_2 = \cdots = p_n\cdot k_n &= 0 && \text{eikonal condition} \end{aligned}$$

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With the help of an auxiliary four-vector q^μ with $q^2 = 0$, we define

$$k_T^\mu(q) = k^\mu - x(q) p^\mu \quad \text{with} \quad x(q) \equiv \frac{q \cdot k}{q \cdot p}$$

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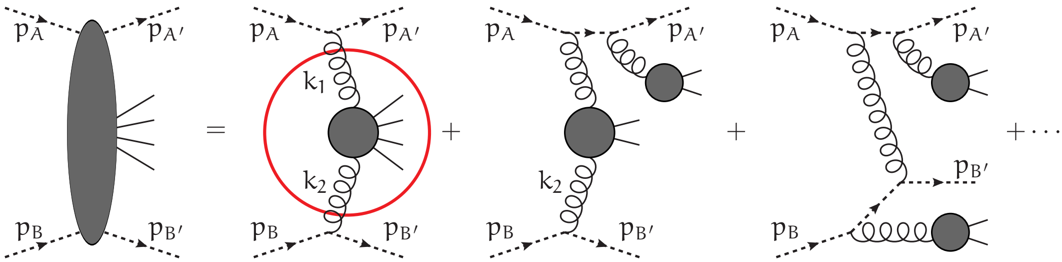
Construct k_T^μ explicitly in terms of p^μ and q^μ:

$$k_T^\mu(q) = -\frac{\kappa}{2}\,\varepsilon^\mu - \frac{\kappa^*}{2}\,\varepsilon^{*\mu} \quad \text{with} \quad \begin{cases} \varepsilon^\mu = \dfrac{\langle p|\gamma^\mu|q]}{[pq]} \quad , \quad \kappa = \dfrac{\langle q|\not{k}|p]}{\langle qp\rangle} \\ \varepsilon^{*\mu} = \dfrac{\langle q|\gamma^\mu|p]}{\langle qp\rangle} \quad , \quad \kappa^* = \dfrac{\langle p|\not{k}|q]}{[pq]} \end{cases}$$

$k^2 = -\kappa\kappa^*$ is independent of q^μ, but also individually κ and κ^* are independent of q^μ.

Amplitudes with off-shell gluons

AvH, Kutak, Kotko 2013:
Embed the process in an on-shell process with auxiliary partons



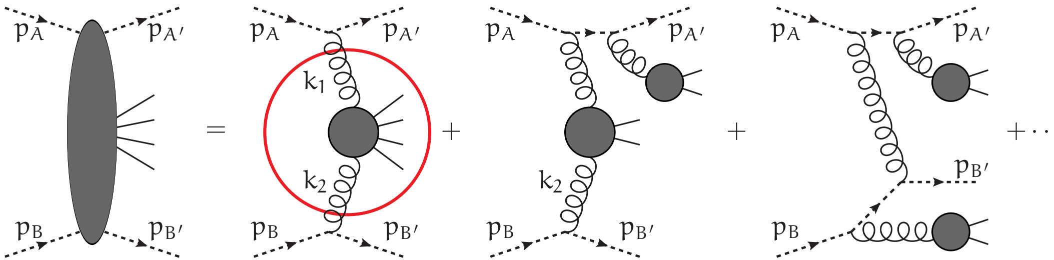
$$p_A^\mu = \Lambda p_1^\mu - \frac{\kappa_1^*}{2}\,\varepsilon_1^{*\mu}$$

$$p_{A'}^\mu = -(\Lambda - x_1)p_1^\mu - \frac{\kappa_1}{2}\,\varepsilon_1^\mu$$

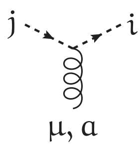
Amplitudes with off-shell gluons

AvH, Kutak, Kotko 2013:

Embed the process in an on-shell process with auxiliary partons and eikonal Feynman rules.



$$\left.\begin{aligned} p_A^\mu &= \Lambda p_1^\mu - \frac{\kappa_1^*}{2}\,\varepsilon_1^{*\mu} \\ p_{A'}^\mu &= -(\Lambda - x_1)p_1^\mu - \frac{\kappa_1}{2}\,\varepsilon_1^\mu \\ \Lambda &\to \infty \end{aligned}\right\} \Rightarrow$$

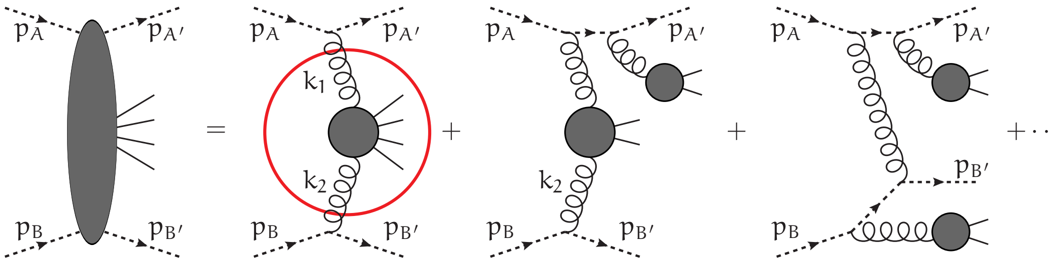


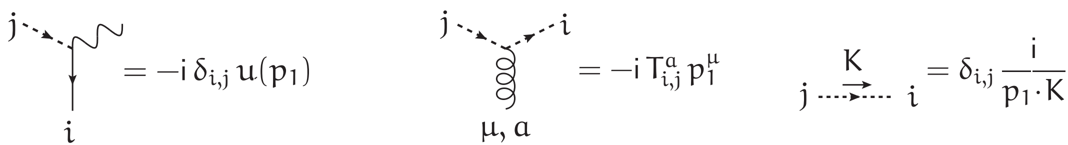
$= -i\,T^a_{i,j}\,p_1^\mu$

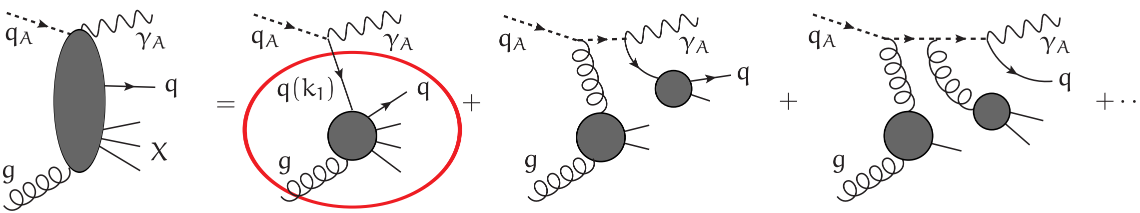
$$j \xrightarrow{K} i = \delta_{i,j}\,\frac{i}{p_1\cdot K}$$

Amplitudes with off-shell partons

AvH, Kutak, Kotko 2013, AvH, Kutak, Salwa 2013:
Embed the process in an on-shell process with auxiliary partons and eikonal Feynman rules.







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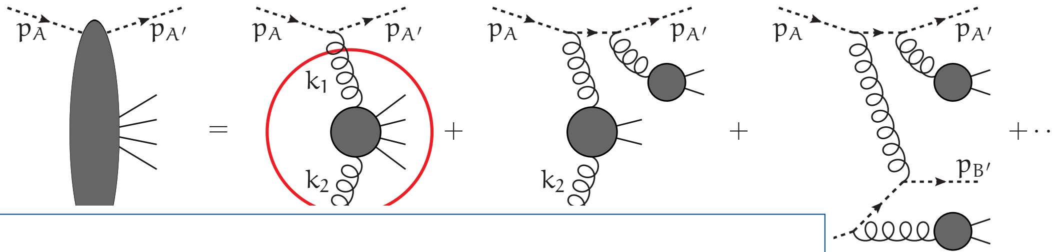
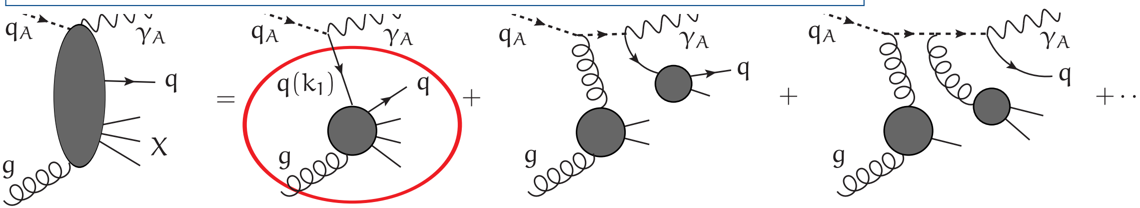


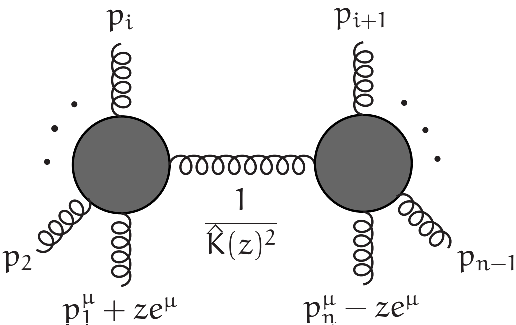
In agreement with the effective action approach of
Lipatov 1995, Antonov, Lipatov, Kuraev, Cherednikov 2005
Lipatov, Vyazovsky 2000, Nefedov, Saleev, Shipilova 2013
and the Wilson-line approach of
Kotko 2014

$$\xrightarrow{K}\ {}_i = \delta_{i,j}\,\frac{i}{p_1\cdot K}$$



BCFW recursion for on-shell amplitudes

Gives compact expression through recursion of *on-shell* amplitudes.



$$\hat{K}^\mu(z) = p_1^\mu + \cdots + p_i^\mu + z e^\mu$$
$$= -p_{i+1}^\mu - \cdots - p_n^\mu + z e^\mu$$

$$e^\mu = \tfrac{1}{2}\langle p_1|\gamma^\mu|p_n]$$

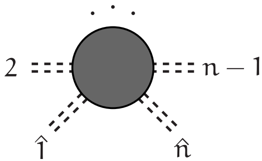
$$\hat{K}(z)^2 = 0 \quad \Leftrightarrow \quad z = -\frac{(p_1 + \cdots + p_i)^2}{2(p_2 + \cdots + p_i)\cdot e}$$

$$\mathcal{A}(1^+, 2, \ldots, n-1, n^-) = \sum_{i=2}^{n-1} \sum_{h=+,-} \mathcal{A}(\hat{1}^+, 2, \ldots, i, -\hat{K}_{1,i}^h)\, \frac{1}{K_{1,i}^2}\, \mathcal{A}(\hat{K}_{1,i}^{-h}, i+1, \ldots, n-1, \hat{n}^-)$$

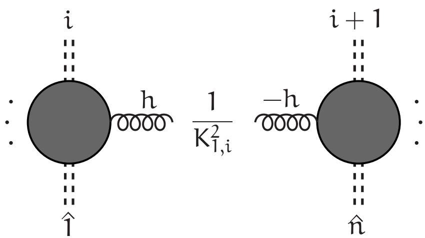
$$\mathcal{A}(1^+, 2^-, 3^-) = \frac{\langle 23\rangle^3}{\langle 31\rangle\langle 12\rangle} \quad , \quad \mathcal{A}(1^-, 2^+, 3^+) = \frac{[32]^3}{[21][13]}$$

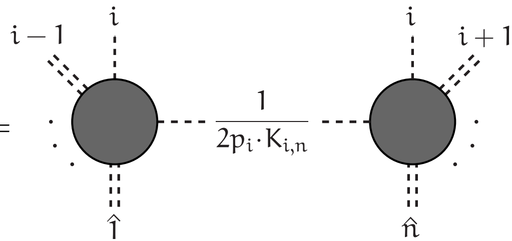
BCFW recursion for off-shell amplitudes

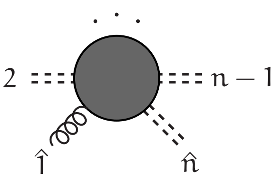
The BCFW recursion formula becomes

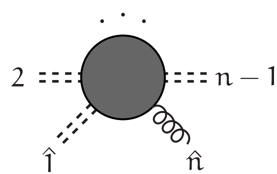


$$= \sum_{i=2}^{n-2} \sum_{h=+,-} A_{i,h} + \sum_{i=2}^{n-1} B_i + C + D\,,$$

$A_{i,h} =$ 

$B_i =$ 

$C = \frac{1}{\kappa_1}$ 

$D = \frac{1}{\kappa_1^*}$ 

The hatted numbers label the shifted external gluons.

Example of a 4-gluon amplitude

$$\mathcal{A}(1^*,2^-,3^*,4^+) = \frac{\langle 13\rangle^3[13]^3}{\langle 34\rangle\langle 41\rangle\langle 1|\not{k}_3+\not{p}_4|3]\langle 3|\not{k}_1+\not{p}_4|1][32][21]}$$
$$+\frac{1}{\kappa_1^*\kappa_3}\frac{\langle 12\rangle^3[43]^3}{\langle 2|\not{k}_3|4]\langle 1|\not{k}_3+\not{p}_4|3](k_3+p_4)^2}+\frac{1}{\kappa_1\kappa_3^*}\frac{\langle 23\rangle^3[14]^3}{\langle 2|\not{k}_1|4]\langle 3|\not{k}_1+\not{p}_4|1](k_1+p_4)^2}$$

- Eventual matrix element needs factor $k_1^2k_3^2 = |\kappa_1|^2|\kappa_3|^2$.
  This *must not* be included at the amplitude level not to spoil analytic structure.

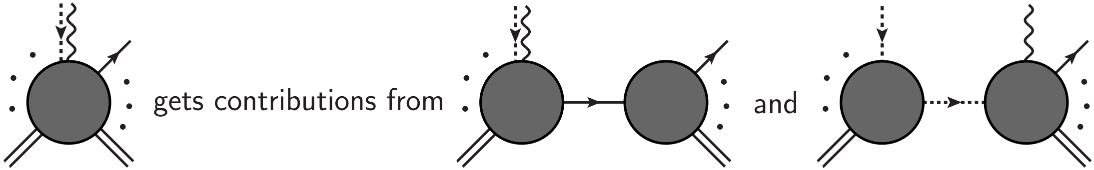
- Last two terms dominate for $|k_1| \to 0$ and $|k_3| \to 0$, and give the on-shell helicity amplitudes in that limit.

$$\mathcal{A}(1^*,2^-,3^*,4^+) \xrightarrow{|k_1|,|k_3|\to 0} \frac{1}{\kappa_1^*\kappa_3}\mathcal{A}(1^-,2^-,3^+,4^+)+\frac{1}{\kappa_1\kappa_3^*}\mathcal{A}(1^+,2^-,3^-,4^+)$$

- Coherent sum of amplitudes becomes incoherent sum of squared amplitudes via angular integrations for $\vec{k}_{1T}$ and $\vec{k}_{3T}$.

BCFW recursion with (off-shell) quarks

- on-shell case treated in Luo, Wen 2005
- any off-shell parton can be shifted: propagators of "external" off-shell partons give the correct power of z in order to vanish at infinity
- different kinds of contributions in the recursion



- many of the MHV amplitudes come out as expected
- some more-than-MHV amplitudes do not vanish, but are sub-leading in k_T

$$\mathcal{A}(1^+, 2^+, \ldots, n^+, \bar{q}^*, q^-) = \frac{-\langle \bar{q}q\rangle^3}{\langle 12\rangle\langle 23\rangle \cdots \langle n\bar{q}\rangle\langle \bar{q}q\rangle\langle q1\rangle}$$

- off-shell quarks have helicity

$$\mathcal{A}(1, 2, \ldots, n, \bar{q}^{*(+)}, q^{*(-)}) \neq \mathcal{A}(1, 2, \ldots, n, \bar{q}^{*(-)}, q^{*(+)})$$

Conclusions

- Double-parton scattering gives an important contribution to the cross section for the process $\mathrm{pp} \to \mathrm{c}\bar{\mathrm{c}}\,\mathrm{c}\bar{\mathrm{c}}$.
- This is confirmed by comparing with single-parton scattering at tree-level both in collinear factorization and $\mathrm{k_T}$-factorization.
- $\mathrm{k_T}$-factorization allows for the description of kinematical situations inaccessible with LO collinear factorization with parton shower, eg. ΔS for four jets.
- Factorization prescriptions with explicit $\mathrm{k_T}$ dependence in the pdfs ask for hard matrix elements with off-shell initial-state partons.
- The necessary amplitudes can be defined in a manifestly gauge invariang manner that allows for Dyson-Schwinger recursion and BCFW recursion, both for off-shell gluons and off-shell quarks.
- Progress towards NLO.

Public programs http://bitbucket.org/hameren/

AVHLIB (A Very Handy LIBrary)

- complete Monte Carlo program for tree-level calculations
- any process within the Standard Model
- any initial-state partons on-shell or off-shell
- employs numerical Dyson-Schwinger recursion to calculate helicity amplitudes
- automatic phase space optimization
- flexibility at the cost of user-friendliness

AMP4HEF (AvH, M.Bury, K.Bilko, H.Milczarek, M.Serino)

- only provides tree-level matrix elements (or color-ordered helicity amplitudes)
- available processes (plus those with fewer on-shell gluons and fewer off-shell partons):

$$
\begin{array}{lll}
\emptyset \to g^* \, g^* + 5g & \emptyset \to \bar{q} \, q^* + 3g & \emptyset \to \bar{q}^* \, q^* + 2g \\
 & \emptyset \to \bar{q}^* \, q + 3g & \emptyset \to g^* \, \bar{q}^* + q \, g \\
 & \emptyset \to g^* + \bar{q} \, q + 2g & \emptyset \to q^* \, g^* + g \, \bar{q}
\end{array}
$$

- employs BCFW recursion to calculate color-ordered helicity amplitudes
- easy to use, both in Fortran and C++

Backup