Constructing Gravity Theories from Gauge Theories

Henrik Johansson

Uppsala U. & Nordita

Sept 29, 2016

DESY Theory Workshop:
Rethinking Quantum
Field Theory

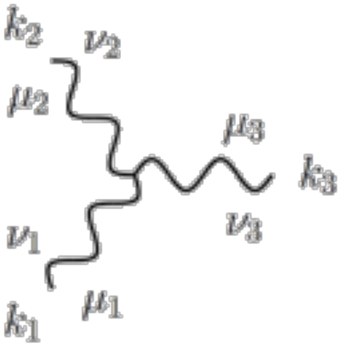
Based on work with
Marco Chiodaroli, Murat Gunaydin, Radu Roiban
[1408.0764, 1511.01740, 1512.09130]
and Josh Nohle [1610.xxxx]

Textbook perturbative gravity:

$$\mathcal{L} = \frac{2}{\kappa^2}\sqrt{g}R, \qquad g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

$$= \frac{1}{2}\left[\eta_{\mu_1\nu_1}\eta_{\mu_2\nu_2} + \eta_{\mu_1\nu_2}\eta_{\nu_1\mu_2} - \frac{2}{D-2}\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}\right]\frac{i}{p^2 + i\epsilon}$$

de Donder gauge

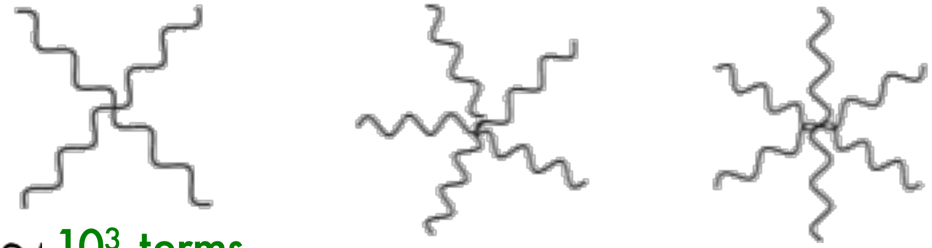


$$\begin{aligned}
= \mathrm{sym}[&-\frac{1}{2}P_3(k_1\cdot k_2\eta_{\mu_1\nu_1}\eta_{\mu_2\nu_2}\eta_{\mu_3\nu_3}) - \frac{1}{2}P_6(k_{1\mu_1}k_{1\nu_2}\eta_{\mu_1\nu_1}\eta_{\mu_3\nu_3}) + \frac{1}{2}P_3(k_1\cdot k_2\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}\eta_{\mu_3\nu_3})\\
&+P_6(k_1\cdot k_2\eta_{\mu_1\nu_1}\eta_{\mu_2\mu_3}\eta_{\nu_2\nu_3}) + 2P_3(k_{1\mu_2}k_{1\nu_3}\eta_{\mu_1\nu_1}\eta_{\nu_2\mu_3}) - P_3(k_{1\nu_2}k_{2\mu_1}\eta_{\nu_1\mu_1}\eta_{\mu_3\nu_3})\\
&+P_3(k_{1\mu_3}k_{2\nu_3}\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}) + P_6(k_{1\mu_3}k_{1\nu_3}\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}) + 2P_6(k_{1\mu_2}k_{2\nu_3}\eta_{\nu_2\mu_1}\eta_{\nu_1\mu_3})\\
&+2P_3(k_{1\mu_2}k_{2\mu_1}\eta_{\nu_2\mu_3}\eta_{\nu_3\nu_1}) - 2P_3(k_1\cdot k_2\eta_{\nu_1\mu_2}\eta_{\nu_2\mu_3}\eta_{\nu_3\mu_1})]
\end{aligned}$$

After symmetrization

$\sim$ 100 terms !

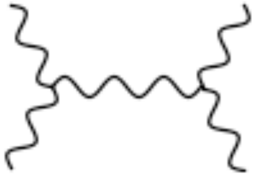
higher order vertices...



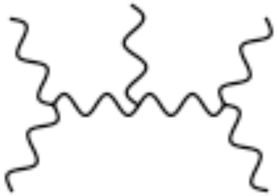
$\sim 10^3$ terms

· · ·

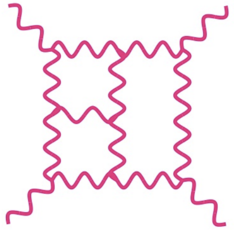
complicated diagrams:



$\sim 10^4$ terms



$\sim 10^7$ terms



$\sim 10^{21}$ terms

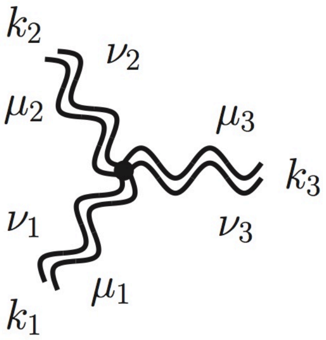
On-shell simplifications

Graviton plane wave:

$$\varepsilon^{\mu}(p)\varepsilon^{\nu}(p)\, e^{ip\cdot x}$$

Yang-Mills polarization

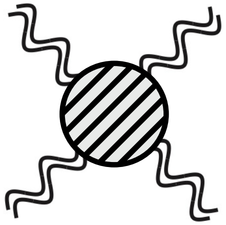
On-shell 3-graviton vertex:



$$= \quad i\kappa\Big(\eta_{\mu_1\mu_2}(k_1-k_2)_{\mu_3}+\text{cyclic}\Big)\Big(\eta_{\nu_1\nu_2}(k_1-k_2)_{\nu_3}+\text{cyclic}\Big)$$

Yang-Mills vertex

Gravity scattering amplitude:



Yang-Mills amplitude

$$M^{\text{GR}}_{\text{tree}}(1,2,3,4)=\frac{st}{u}A^{\text{YM}}_{\text{tree}}(1,2,3,4)\otimes A^{\text{YM}}_{\text{tree}}(1,2,3,4)$$

Gravity processes = “squares” of gauge theory ones

$$\text{gravity}=(\text{gauge th})\otimes(\text{gauge th})$$

Generic gravities are double copies

Amplitudes in familiar theories are secretly related, for example:

- $(\mathcal{N}=4\ \mathrm{SYM})\otimes(\mathcal{N}=4\ \mathrm{SYM})=(\mathcal{N}=8\ \mathrm{SUGRA})$
- $(\mathcal{N}=4\ \mathrm{SYM})\otimes(\mathrm{pure\ YM})=(\mathcal{N}=4\ \mathrm{SUGRA})$
- $(\mathrm{pure\ YM})\otimes(\mathrm{pure\ YM})=\mathrm{GR}+\phi+B^{\mu\nu}$ (dilaton-axion)
- $\mathrm{QCD}\otimes\mathrm{QCD}=\mathrm{GR}+\mathrm{matter}$ (Maxwell-Einstein)
- $(\mathrm{YM})\otimes(\mathrm{YM}+\phi^3)=\mathrm{GR}+\mathrm{YM}$ (Yang-Mills-Einstein)

and many more...

→ $(\mathrm{gauge\ sym})\otimes(\mathrm{gauge\ sym})=\mathrm{diffeo\ sym}$

Kawai-Lewellen-Tye Relations ('86)

spin-2: $|2\rangle = |1\rangle \otimes |1\rangle$ "1+1=2"

String theory tree-level identity: closed string ~ (left open string) × (right open string)

$$A_n \sim \int \frac{dx_1 \cdots dx_n}{\mathcal{V}_{abc}} \prod_{1\leq i<j\leq n} |x_i - x_j|^{k_i\cdot k_j} \exp\Big[\sum_{i<j}\Big(\frac{\epsilon_i\cdot\epsilon_j}{(x_i-x_j)^2} + \frac{k_i\cdot\epsilon_j - k_j\cdot\epsilon_i}{(x_i-x_j)}\Big)\Big]\Big|_{\mathrm{multi-linear}}$$

KLT relations emerge after nontrivial world-sheet integral identities

Field theory limit ⇒ gravity theory ~ (gauge theory) × (gauge theory)

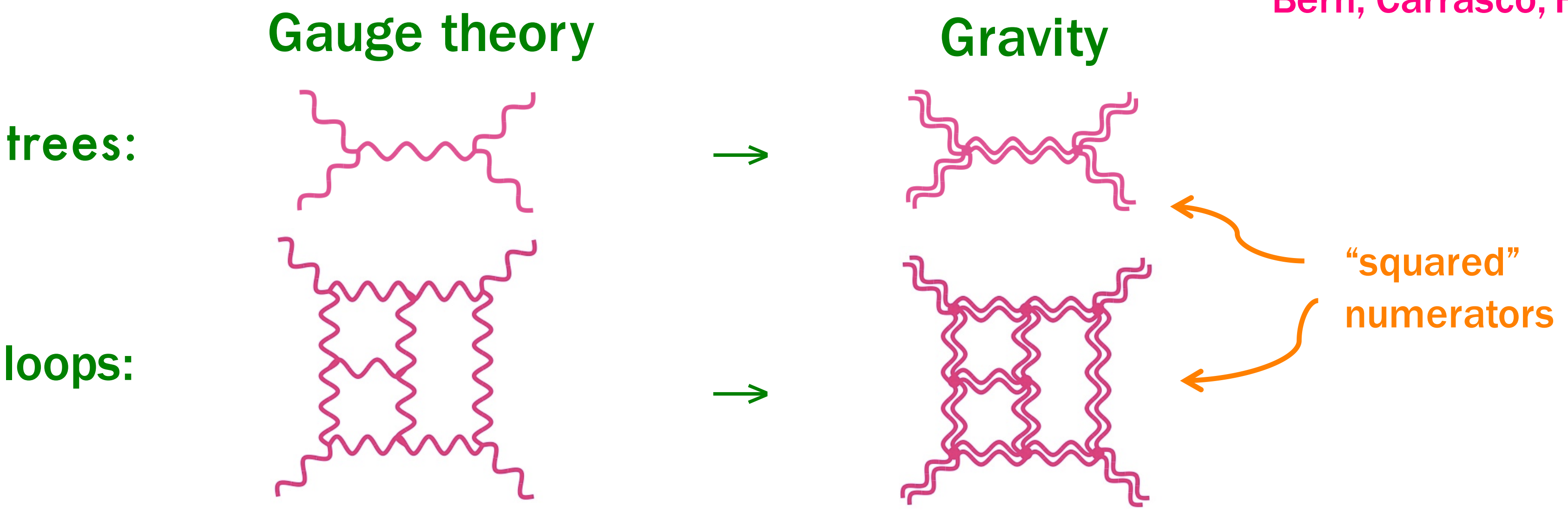
$$M_4^{\mathrm{tree}}(1,2,3,4) = -is_{12}A_4^{\mathrm{tree}}(1,2,3,4)\,\widetilde{A}_4^{\mathrm{tree}}(1,2,4,3)$$

$$M_5^{\mathrm{tree}}(1,2,3,4,5) = is_{12}s_{34}A_5^{\mathrm{tree}}(1,2,3,4,5)\,\widetilde{A}_5^{\mathrm{tree}}(2,1,4,3,5)$$
$$+is_{13}s_{24}A_5^{\mathrm{tree}}(1,3,2,4,5)\,\widetilde{A}_5^{\mathrm{tree}}(3,1,4,2,5)$$

Generality of double copy

Gravity processes = product of gauge theory ones - entire S-matrix

Bern, Carrasco, HJ ('10)



Gauge theory

Gravity

trees: →

loops: →

"squared" numerators

Recent generalizations:

→ Theories that are not truncations of *N*=8 SG HJ, Ochirov; Chiodaroli, Gunaydin, Roiban

→ Theories with fundamental matter HJ, Ochirov; Chiodaroli, Gunaydin, Roiban

→ Spontaneously broken theories Chiodaroli, Gunaydin, HJ, Roiban

→ Classical (black hole) solutions Luna, Monteiro, Nicholson, O'Connell, White

→ New double copies for string theory Mafra, Schlotterer, Stieberger, Taylor, Broedel, Carrasco...

→ CHY scattering eqs, twistor strings see Geyer's talk

→ Conformal gravity HJ, Nohle

Motivation: (super)gravity UV behavior

Old results on UV properties:

- susy forbids 1,2 loop div. $\xcancel{R^2, R^3}$ Ferrara, Zumino, Deser, Kay, Stelle, Howe, Lindström, Green, Schwarz, Brink, Marcus, Sagnotti
- Pure gravity 1-loop finite, 2-loop divergent Goroff & Sagnotti, van de Ven
- With matter: 1-loop divergent ‘t Hooft & Veltman; (van Nieuwenhuizen; Fischler..)

New results on UV properties:

- $\mathcal{N}$=8 SG and $\mathcal{N}$=4 SG 3-loop finite! Bern, Carrasco, Dixon, HJ, Kosower, Roiban; Bern, Davies, Dennen, Huang
- $\mathcal{N}$=8 SG: no divergence before 7 loops Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger; Björnsson, Green, Bossard, Howe, Stelle, Vanhove Kallosh, Ramond, Lindström, Berkovits, Grisaru, Siegel, Russo, and more….
- First $\mathcal{N}$=4 SG divergence at 4 loops Bern, Davies, Dennen, Smirnov, Smirnov

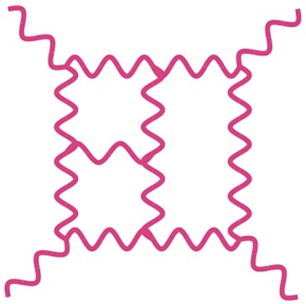
  (unclear interpretation, U(1) anomaly?)
- Evanescent effects: Einstein gravity Bern, Cheung, Chi, Davies, Dixon, Nohle

Double-copy calculations shed new light on gravity UV properties!

Einstein

Dissect Goroff & Sagnotti; van de Ven

$\mathcal{N} = 1, 2, 3$ SG



Outline

- Intro & motivation
- On-shell diffeo. sym. from gauge symmetry
- Color-kinematics duality
  - BCJ relations
  - Examples of theories
- Generalization to conformal (super-)gravity
- New dimension-six gauge theory
- Deformations and more gravities
- Conclusion

Amplitudes in a gauge theory

kinematic numerators

color factors

cubic diagram form:

$$\mathcal{A}^{\text{tree}} = \sum_{i \in \text{cubic}} \frac{n_i c_i}{D_i}$$

propagators

$n_i \equiv \varepsilon_\mu(p)\, n_i^\mu$ Consider a gauge transformation $\varepsilon \to \varepsilon + \alpha p$

$$n_i \to n_i + \Delta_i \qquad \Delta_i = \alpha\, p_\mu n_i^\mu$$

Invariance of $\mathcal{A}^{\text{tree}}$ requires that c_i are linearly dependent

$$c_i - c_j = c_k$$

[Jacobi id. or Lie algebra]

thus the combination $\displaystyle\sum_{i \in \text{cubic}} \frac{\Delta_i c_i}{D_i} = 0$ vanishes.

Build gravity amplitudes

Assume the gauge freedom can be exploited to find numerators

$$c_i - c_j = c_k \quad \Leftrightarrow \quad n_i - n_j = n_k$$

dual to the color factors

Then the double copy $\mathcal{M}^{\text{tree}} = \sum_{i\in\text{cubic}} \frac{n_i \tilde{n}_i}{D_i}$ → Gravity

describes a spin-2 theory $\varepsilon_{\mu\nu} = \varepsilon_\mu \varepsilon_\nu$

invariant under (linear) diffeos $\varepsilon_{\mu\nu} \to \varepsilon_{\mu\nu} + p_\mu \xi_\nu + \xi_\mu p_\nu$

$$\mathcal{M}^{\text{tree}} \to \mathcal{M}^{\text{tree}} + \underbrace{\sum_{i\in\text{cubic}} \frac{\Delta_i \tilde{n}_i}{D_i} + \sum_{i\in\text{cubic}} \frac{n_i \tilde{\Delta}_i}{D_i}}_{=\,0}$$

Color-kinematics duality

Color-kinematics duality for pure (S)YM

YM theories are controlled by a hidden kinematic Lie algebra

- Amplitude expanded in terms of cubic graphs:

kinematic numerators

color factors

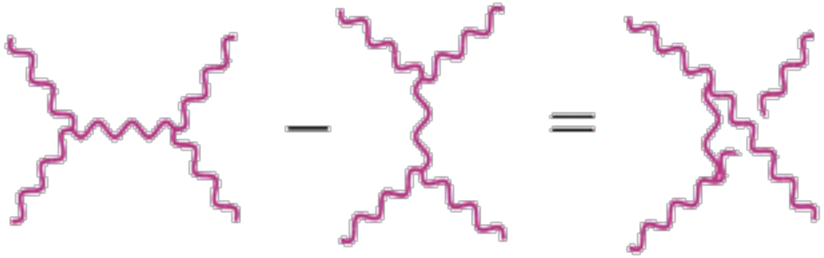
$$\mathcal{A}_n^{(L)} = \sum_{i\in\Gamma_3} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i c_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2}$$

propagators

Color & kinematic numerators satisfy same relations:

$$n_i - n_j = n_k \quad \Leftrightarrow \quad c_i - c_j = c_k$$

Bern, Carrasco, HJ



Jacobi identity

$$f^{dac} f^{cbe} - f^{dbc} f^{cae} = f^{abc} f^{dce} \quad \Rightarrow \quad c_i - c_j = c_k$$

Gauge-invariant relations (pure glue)

$$A(1,2,\ldots,n-1,n) = A(n,1,2,\ldots,n-1)$$ cyclicity → (*n*-1)! basis

$$\sum_{i=1}^{n-1} A(1,2,\ldots,i,n,i+1,\ldots,n-1) = 0$$ U(1) decoupling

$$A(1,\beta,2,\alpha) = (-1)^{|\beta|} \sum_{\sigma\in\alpha\sqcup\!\sqcup\beta^T} A(1,2,\sigma)$$ Kleiss-Kuijf relations (‘89)

$(n\text{-}2)!$ basis

$$\sum_{i=2}^{n-1} \Big(\sum_{j=2}^{i} s_{jn}\Big) A(1,2,\ldots i,n,i+1,\ldots,n-1) = 0$$

$$A(1,2,\alpha,3,\beta) = \sum_{\sigma\in S(\alpha)\sqcup\!\sqcup\beta} A(1,2,3,\sigma) \prod_{i=1}^{|\alpha|} \frac{\mathcal{F}(3,\sigma,1|i)}{s_{2,\alpha_1,\ldots,\alpha_i}}$$

BCJ relations (‘08)
$(n\text{-}3)!$ basis

BCJ rels. proven via string theory by Bjerrum-Bohr, Damgaard, Vanhove; Stieberger (’09)

and field theory proofs through BCFW: Feng, Huang, Jia; Chen, Du, Feng (’10 -’11)

Relations used in string calcs: Mafra, Stieberger, Schlotterer, et al. (’11 -’15)

Relations used by Cachazo, He, Yuan to motivate CHY and scattering eqns (’13)

Gravity is a double copy of YM

Gravity amplitudes obtained by replacing color with kinematics

$$\mathcal{A}_m^{(L)} = \sum_{i\in\Gamma_3}\int \frac{d^{LD}\ell}{(2\pi)^{LD}}\frac{1}{S_i}\frac{n_i c_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2\cdots p_{i_l}^2}$$

$$\mathcal{M}_m^{(L)} = \sum_{i\in\Gamma_3}\int \frac{d^{LD}\ell}{(2\pi)^{LD}}\frac{1}{S_i}\frac{n_i \tilde{n}_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2\cdots p_{i_l}^2}$$

double copy

Bern, Carrasco, HJ

- The two numerators can differ by a generalized gauge transformation
  → only one copy needs to satisfy the kinematic algebra
- The two numerators can differ by the external/internal states
  → graviton, dilaton, axion (*B*-tensor), matter amplitudes
- The two numerators can belong to different theories
  → give a host of different gravitational theories

Equivalent to
KLT at tree level
for adj. rep.
and CHY, twistor str
→ Geyer's talk

Which "gauge" theories obey C-K duality

- Pure $\mathcal{N}$=0,1,2,4 super-Yang-Mills (any dimension) Bern, Carrasco, HJ ('08) Bjerrum-Bohr, Damgaard, Vanhove; Stieberger; Feng et al. Mafra, Schlotterer, etc ('08-'11)
- Self-dual Yang-Mills theory O'Connell, Monteiro ('11)
- Heterotic string theory Stieberger, Taylor ('14)
- Yang-Mills + F^3 theory Broedel, Dixon ('12)
- QCD, super-QCD, higher-dim QCD HJ, Ochirov ('15)
- Generic matter coupled to $\mathcal{N}$= 0,1,2,4 super-Yang-Mills Chiodaroli, Gunaydin, Roiban; HJ, Ochirov ('14)
- Spontaneously broken $\mathcal{N}$= 0,2,4 SYM Chiodaroli, Gunaydin, HJ, Roiban ('15)
- Yang-Mills + scalar ϕ^3 theory Chiodaroli, Gunaydin, HJ, Roiban ('14)
- Bi-adjoint scalar ϕ^3 theory Bern, de Freitas, Wong ('99), Bern, Dennen, Huang; Du, Feng, Fu; Bjerrum-Bohr, Damgaard, Monteiro, O'Connell
- NLSM/Chiral Lagrangian Chen, Du ('13)
- *D*=3 Bagger-Lambert-Gustavsson theory (Chern-Simons-matter) Bargheer, He, McLoughlin; Huang, HJ, Lee ('12 -'13)
- (Non-)Abelian Z-theory Carrasco, Mafra, Schlotterer

see Schlotterer's talk

Which “gravity” theories are double copies

- Pure $\mathcal{N}$=4,5,6,8 supergravity (2 < D < 11) KLT (‘86), Bern, Carrasco, HJ (’08 -’10)
- Einstein gravity and pure $\mathcal{N}$=1,2,3 supergravity HJ, Ochirov (’14)
- Self-dual gravity O’Connell, Monteiro (’11)
- Closed string theories Mafra, Schlotterer, Stieberger (’11); Stieberger, Taylor (’14)
- Einstein + R^3 theory Broedel, Dixon (’12)
- Abelian matter coupled to supergravity { Carrasco, Chiodaroli, Gunaydin, Roiban (‘12) HJ, Ochirov (’14 - ’15)
- Magical sugra, homogeneous sugra Chiodaroli, Gunaydin, HJ, Roiban (’15)
- SYM coupled to supergravity Chiodaroli, Gunaydin, HJ, Roiban (’14)
- Spontaneously broken YM-Einstein gravity Chiodaroli, Gunaydin, HJ, Roiban (’15)
- D=3 supergravity (BLG Chern-Simons-matter theory)^2 { Bargheer, He, McLoughlin; Huang, HJ, Lee (’12 -’13)
- Born-Infeld, DBI, Galileon theories (CHY form) Cachazo, He, Yuan (’14)
- Conformal gravity HJ Nohle (‘16)

Magical and homogeneous SUGRAs

Maxwell-Einstein 5d supergravity theories

Gunaydin, Sierra, Townsend

$$e^{-1}\mathcal{L} = -\frac{R}{2} - \frac{1}{4}\mathring{a}_{IJ}F^{I}_{\mu\nu}F^{J\mu\nu} - \frac{1}{2}g_{xy}\partial_\mu\varphi^x\partial^\mu\varphi^y + \frac{e^{-1}}{6\sqrt{6}}C_{IJK}\epsilon^{\mu\nu\rho\sigma\lambda}F^I_{\mu\nu}F^J_{\rho\sigma}A^K_\lambda$$

Everything is determined by a prepotential parameterized by C_{IJK}

Describe scalar manifold $\mathcal{M}$ as hypersurface in ambient space

$$N(\xi) = \left(\frac{2}{3}\right)^{3/2} C_{IJK}\xi^I\xi^J\xi^K \qquad \xi = (\phi^x, \rho) \qquad a_{IJ} = -\frac{1}{2}\partial_I\partial_J \ln N(\xi)$$

$$\mathring{a}_{IJ}(\varphi) = a_{IJ}\big|_{N(\xi)=1} \qquad g_{xy}(\varphi) = \mathring{a}_{IJ}\partial_x\xi^I\partial_y\xi^J$$

$$(\mathcal{N} = 2 \text{ SQCD}) \otimes (D = 7, 8, 10, 14 \text{ QCD})$$
$$= \text{Magical } \mathcal{N} = 2 \text{ Supergravity}$$
$$(\mathbb{R}, \mathbb{C}, \mathbb{H}, \mathbb{O} \text{ type})$$

Chiodaroli, Gunaydin,
HJ, Roiban ('15)

Conformal Gravity

Double copy for conformal gravity?

How to obtain conformal gravity and supersymmetric extensions?

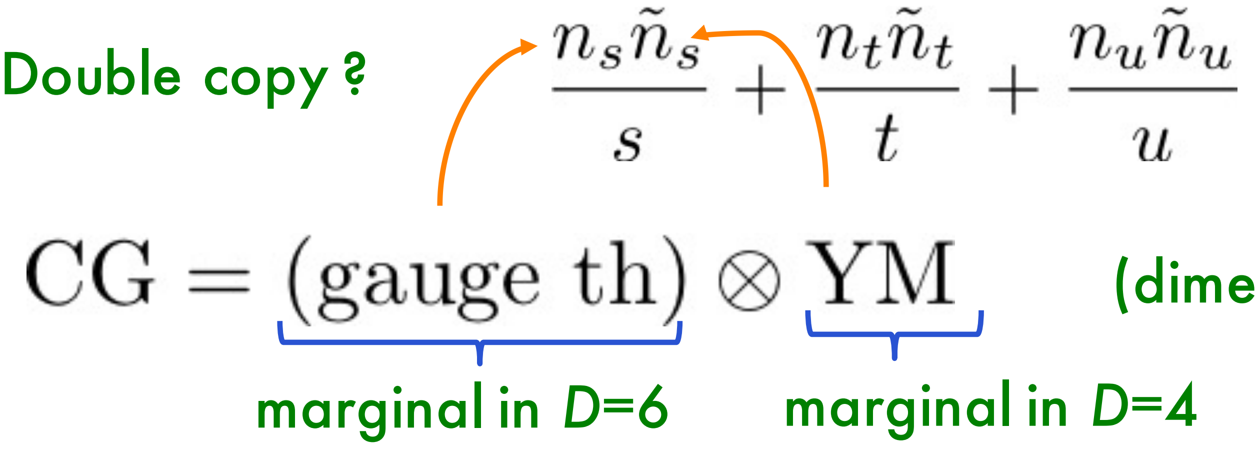
4-derivative action
$$\int d^4x\sqrt{-g}\,W^2$$

(Weyl)^2
$$(W_{\mu\nu\rho\sigma})^2 = (R_{\mu\nu\rho\sigma})^2 - 2(R_{\mu\nu})^2 + \frac{1}{3}R^2$$

propagator $\sim \dfrac{1}{k^4} \quad \sim \quad \dfrac{1}{k^2} - \dfrac{1}{k^2 - m^2}$

2 + 5 – 1 states:
- graviton: 2+2
- vector: 2

→ negative-norm states → non-unitary
(but renormalizable) K. Stelle ('77)



Double copy ?
$$\frac{n_s\tilde{n}_s}{s} + \frac{n_t\tilde{n}_t}{t} + \frac{n_u\tilde{n}_u}{u}$$

$$\mathrm{CG} = \underbrace{(\mathrm{gauge\ th})}_{\text{marginal in } D=6} \otimes \underbrace{\mathrm{YM}}_{\text{marginal in } D=4}$$

(dimensional analysis)

HJ, Nohle

Dimension-six gauge theory

HJ, Nohle

Two dim-6 operators:

$$\underbrace{\underbrace{\frac{1}{2}(D_\mu F^{\mu\nu})^2}_{\substack{\text{correct } 1/k^4 \text{ propagator} \\ \text{but trivial S-matrix}}} - \frac{1}{3} g F^3}_{A_3 = \langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,1\rangle}$$

3pt double copy is promising:

$$M_3 = \frac{\langle i\,j\rangle^4}{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,1\rangle} \times \langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,1\rangle = \langle i\,j\rangle^4 \sim \phi R^2$$

(3-graviton amplitude vanish = Gauss-Bonnet term)

Dimension-six gauge theory

HJ, Nohle

Candidate theory: $\frac{1}{2}(D_\mu F^{\mu\nu})^2 - \frac{1}{3}gF^3$

4pt ampl: $A_4(1^-,2^-,3^+,4^+) = \frac{\langle 1\,2\rangle^2}{\langle 3\,4\rangle^2}(u-t)$

Check color-kinematics duality (BCJ relation):

$$0 \overset{?}{=} tA_4(1,2,3,4) - uA_4(2,1,3,4) = \frac{\langle 1\,2\rangle^2}{\langle 3\,4\rangle^2}s(t-u)$$

Missing contribution: $\Delta = \frac{\langle 1\,2\rangle^2[3\,4]^2}{s} \quad \to \quad \varphi F^2$

new dim-6 operator!

Add scalar, new operators: $\left\{(D_\mu\varphi)^2,\, \varphi F^2,\, \varphi^3\right\}$

Ansatz for dimension-six theory

HJ, Nohle

$$\mathcal{L} = \frac{1}{2}(D_\mu F^{a\,\mu\nu})^2 - \frac{1}{3}gF^3 + \frac{1}{2}(D_\mu\varphi^\alpha)^2 + \frac{1}{2}g\,C^{\alpha ab}\varphi^\alpha F^a_{\mu\nu}F^{b\,\mu\nu} + \frac{1}{3!}g\,d^{\alpha\beta\gamma}\varphi^\alpha\varphi^\beta\varphi^\gamma$$

scalar in some real representation of gauge group (not adjoint)

unknown Clebsh-Gordan coeff: $C^{\alpha ab}$, $d^{\alpha\beta\gamma}$ (symmetric)

Assume: diagrams with internal scalars reduce to $\sim f^{abc}f^{cde}\ldots$

4pt BCJ relation → $C^{\alpha ab}C^{\alpha cd} = f^{ace}f^{edb} + f^{ade}f^{ecb}$

6pt BCJ relation → $C^{\alpha ab}d^{\alpha\beta\gamma} = (T^a)^{\beta\alpha}(T^b)^{\alpha\gamma} + C^{\beta ac}C^{\gamma cb} + (a \leftrightarrow b)$

sufficient to compute any tree amplitude with external vectors!

Which representation for scalar ? "Bi-adjoint", "auxiliary" rep.

Construction works!

$$\mathcal{L} = \frac{1}{2}(D_\mu F^{a\,\mu\nu})^2 - \frac{1}{3}gF^3 + \frac{1}{2}(D_\mu\varphi^\alpha)^2 + \frac{1}{2}g\,C^{\alpha ab}\varphi^\alpha F^a_{\mu\nu}F^{b\,\mu\nu} + \frac{1}{3!}g\,d^{\alpha\beta\gamma}\varphi^\alpha\varphi^\beta\varphi^\gamma$$

Color-kinematics duality checked up to 8 pts ! HJ, Nohle

(no new Feynman vertices beyond 6pt)

Double copy with YM agrees with conformal gravity: (Berkovits, Witten)

$$M^{\rm CG}(1^-,2^-,3^+,\ldots,n^+) = \langle 1\,2\rangle^4 \prod_{i=3}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{[i\,j]\,\langle j\,q\rangle^2}{\langle i\,j\rangle\,\langle i\,q\rangle^2}$$

All-plus amplitude is non-zero → no susy extension of $\mathcal{L}$

$$A(1^+,2^+,3^+,4^+) = u\frac{[1\,2]\,[3\,4]}{\langle 1\,2\rangle\,\langle 3\,4\rangle}$$

Supersymmetry of conformal supergravity sits on the YM side:

$$\text{CSG} = (\text{dim-6 theory}) \otimes (\mathcal{N} = 1, 2, 4\ \text{SYM})$$

Generalizations and deformations

Curiously no interacting scalars are obtained from dimensional reduction

Instead add regular scalars in adjoint…

HJ, Nohle

$$\mathcal{L} = \frac{1}{2}(D_\mu F^{a\,\mu\nu})^2 - \frac{1}{3}gF^3 + \frac{1}{2}(D_\mu\varphi^\alpha)^2 + \frac{1}{2}g\,C^{\alpha ab}\varphi^\alpha F^a_{\mu\nu}F^{b\,\mu\nu} + \frac{1}{3!}g\,d^{\alpha\beta\gamma}\varphi^\alpha\varphi^\beta\varphi^\gamma$$

$$\underbrace{+(D_\mu\phi^{aA})^2 + \frac{1}{2}gC^{\alpha ab}\phi^{aA}\phi^{bA}\varphi^\alpha}_{} \quad \underbrace{+\frac{1}{3!}g\lambda f^{abc}\tilde{f}^{ABC}\phi^{aA}\phi^{bB}\phi^{cC}}_{}$$

color-kinematics fixes interactions

Double copy: Maxwell-Weyl gravity:

$$\sqrt{-g}(W^2 + f(\phi)F^2 + \ldots)$$

Bi-adjoint ϕ^3

Double copy: Yang-Mills-Weyl

N=4 case: Witten's twistor string!

finally, deform with dim-4 operators: → Yang-Mills-Einstein-Weyl gravity

$$-\frac{1}{4}m^2F^2 - \frac{1}{2}m^2(\varphi^\alpha)^2 \quad \rightarrow \quad \sqrt{-g}(R + W^2 + f(\phi)F^2 + \ldots)$$

Summary

- Powerful framework for constructing scattering amplitudes in various gravitational theories – well suited for multi-loop UV calculations
- Color-kinematics duality and gauge symmetry underlies consistency of construction. (Kinematic Lie algebra ubiquitous in gauge theory.)
- Constructed new dim-6 theory using color-kinematics duality - theory has several unusual features.
- First construction of conformal gravity as a double copy – may simplify analysis of any unresolved questions regarding unitarity of theory.
- Checks: Explicitly up to 8pts tree level (loop level analysis remains…)
- An increasing number of gravitational theories exhibit double-copy structure (some in surprising ways) – more are likely to be found!

Extra Slides