Integrability in gauge and string theory

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Rethinking Quantum Field Theory
DESY, Hamburg 2016

Outline

Introduction and motivation

AdS/CFT and strings in $AdS_5 \times S^5$

Energy levels of a string in $AdS_5 \times S^5$

How to solve the spectral problem?

Some questions...

String interactions in $AdS_5 \times S^5$

The decompactified string vertex

The hexagon approach

Other recent lines of research

Summary

Study $\mathcal{N}=4$ Super-Yang-Mills theory — a 4D gauge theory which is a conformal theory...

Key questions:

- Find the spectrum of conformal weights
  $\equiv$ eigenvalues of the dilatation operator
  $\equiv$ (anomalous) dimensions of operators

$$\langle O(0)O(x)\rangle = \frac{1}{|x|^{2\Delta}}$$

  The dimensions are complicated functions of the coupling:

$$\Delta = \underbrace{\Delta_0(\lambda)}_{planar} + \underbrace{\frac{1}{N_c^2}\Delta_1(\lambda) + \ldots}_{nonplanar} \qquad \text{where } \lambda \equiv g_{YM}^2 N_c$$

- Find the OPE coefficients C_{ijk} defined through

$$\langle O_i(x_1)O_j(x_2)O_k(x_3)\rangle = \frac{C_{ijk}}{|x_1-x_2|^{\Delta_i+\Delta_j-\Delta_k}|x_1-x_3|^{\Delta_i+\Delta_k-\Delta_j}|x_2-x_3|^{\Delta_j+\Delta_k-\Delta_i}}$$

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1. Wilson loop observables
2. Scattering amplitudes (appropriately understood/regularized)
3. Nonplanar mixing: $1/N_c$ corrections..
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$\mathcal{N} = 4$ Super Yang-Mills theory $\equiv$ Superstrings on $AdS_5 \times S^5$

The AdS/CFT dictionary

Operators in $\mathcal{N} = 4$ SYM	$\longleftrightarrow$	(quantized) string states in $AdS_5 \times S^5$
Single trace operators	$\longleftrightarrow$	single string states
Multitrace operators	$\longleftrightarrow$	multistring states
Large N_c limit	$\longleftrightarrow$	suffices to consider single string states
Operator dimension	$\longleftrightarrow$	Energy of a string state in $AdS_5 \times S^5$
Nonplanar corrections	$\sim$	string interactions
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A historical note...

Initially most of the progress in the development of integrability was on the **gauge theory side**

- A spin chain reformulation of the gauge theory dilatation operator was performed
  Minahan, Zarembo; Staudacher; Beisert
- All-order asymptotic Bethe Ansatz was proposed
  Beisert, Staudacher
- An S-matrix for spin chain excitations was derived
  Beisert

- At a certain loop order **'wrapping'** graphs start to appear...
- In the integrable spin chain language no known way how to compute/treat...

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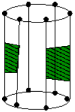

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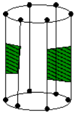

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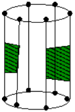

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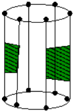

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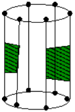

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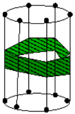

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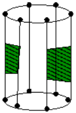

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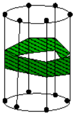

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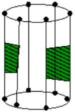

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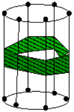

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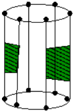

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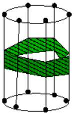

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- Consider a closed string in $AdS_5 \times S^5$:

- The embedding coordinates of the point (τ, σ) are *quantum fields* $X^\mu(\tau, \sigma)$ on the worldsheet which has the geometry of a cylinder
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  review: Arutyunov, Frolov 0901.4937

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$$E(p) = \sqrt{1 + \frac{\lambda}{\pi^2} \sin^2 \frac{p}{2}} \qquad \lambda \equiv g_{YM}^2 N_c$$

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1. **Anomalous dimensions in the planar limit:**

   ≡ energy levels of a single string in $AdS_5 \times S^5$
   ≡ energy levels of this specific 2D integrable QFT on a cylinder

2. **Nonplanar corrections to the dilatation operator or OPE coefficients:**

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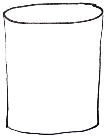

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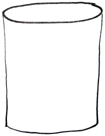

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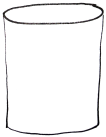

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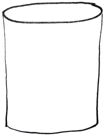

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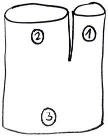

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   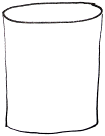

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   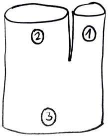

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I) solve the theory on an infinite plane

1. Particle momenta are completely unconstrained!
2. We may perform analytic continuation into the complex plane (of appropriate rapidities)
3. We get **crossing** equation
4. This together with unitarity...
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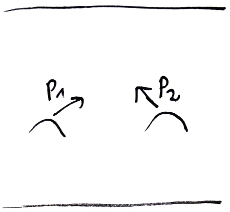

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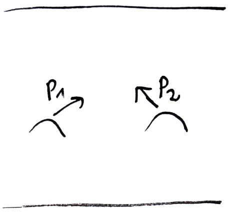

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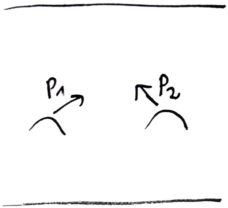

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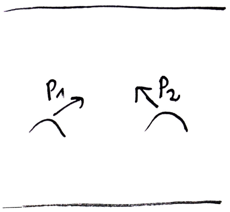

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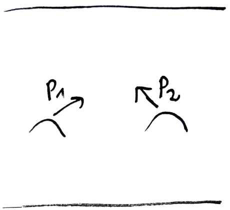

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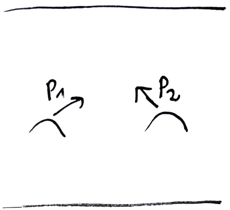

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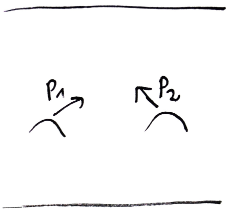

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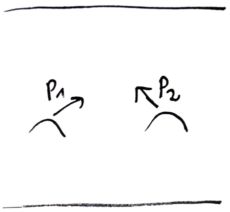

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$$e^{ip_k L} \prod_{l \neq k} S(p_k, p_l) = 1$$

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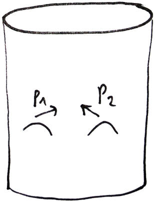

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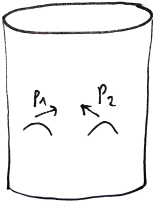

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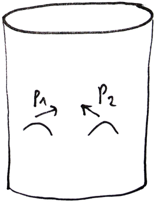

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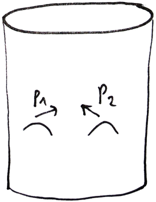

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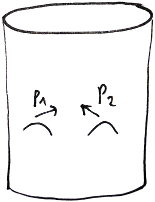

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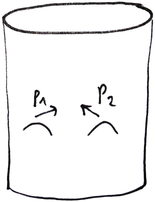

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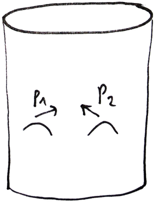

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— generalized Lüscher formulas

- lack of relativistic invariance

RJ, Lukowski

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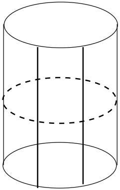

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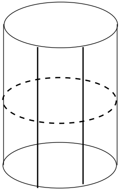

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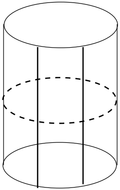

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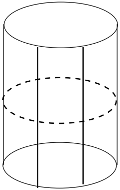

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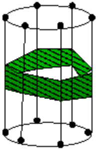

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IV) Exact answer: Resum all wrapping corrections

Arutyunov, Frolov; Bombardelli, Fiorvanati, Tateo; Gromov, Kazakov, Vieira

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... can be generalized to excited states...

- TBA equations can be reformulated in terms of systems of functional equations with some analyticity conditions (here e.g. Y-system):

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Gromov, Kazakov, Leurent, Volin

How to solve the spectral problem?

IV) Exact answer: Resum all wrapping corrections

Arutyunov, Frolov; Bombardelli, Fiorvanati, Tateo; Gromov, Kazakov, Vieira

- Thermodynamic Bethe Ansatz (TBA) Zamolodchikov

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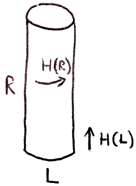

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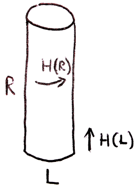

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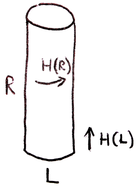

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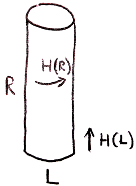

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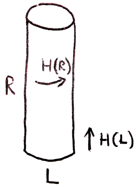

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- **What happens in other gauges?** (e.g. AdS light cone gauge used by Roiban, Tsytlin)
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Question #3

- In a covariant gauge the worldsheet theory of a string is a 2D relativistic conformal field theory (CFT)
- We have a good understanding of what CFT's could lead to consistent (super)string theory (like $c = 15$)
- In specific light cone gauges in $AdS_5 \times S^5$ the worldsheet theory is **not conformal** but massive and integrable..
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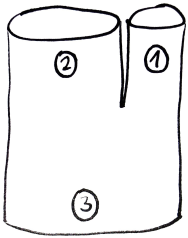

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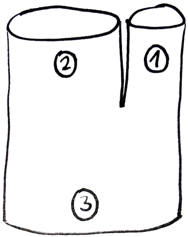

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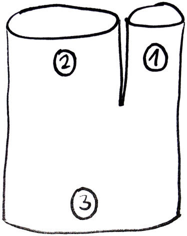

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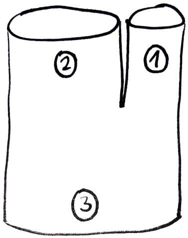

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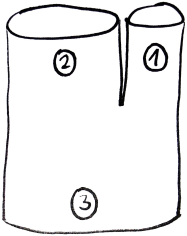

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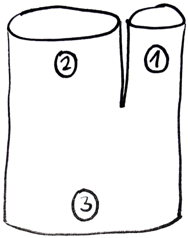

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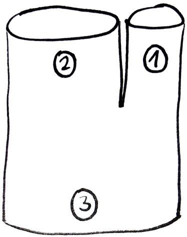

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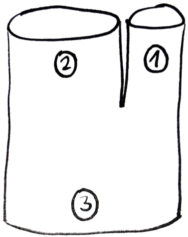

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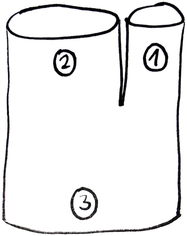

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- The theory was first solved on the infinite plane..
- The infinite plane was crucial to have crossing equations and analyticity..
- We need a 'decompactified' version of the string pants diagram...

- The infinite past is string $\#3$, infinite future is string $\#2$ while the emission of string $\#1$ looks like a nonlocal operator
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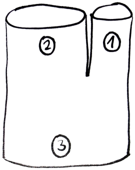

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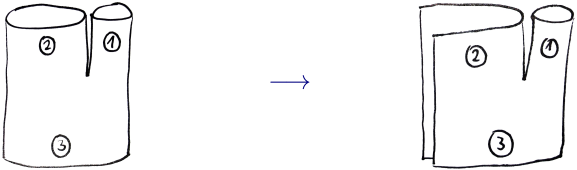

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The decompactified string vertex

Bajnok, RJ

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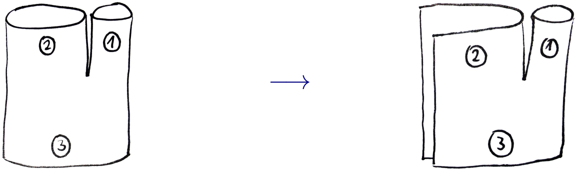

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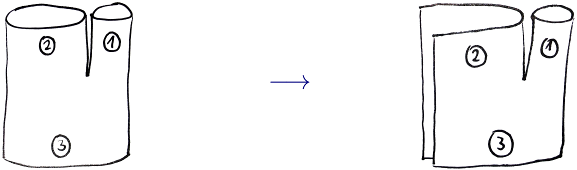

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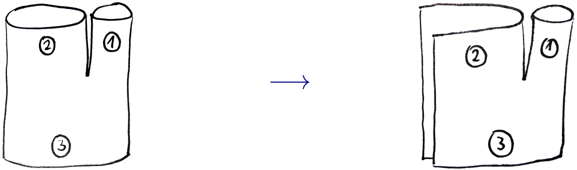

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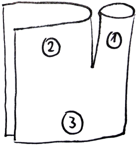
Functional equations for the (decompactified) string vertex

written here for two incoming particles and, for the moment, free theory



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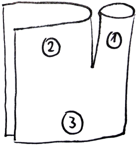

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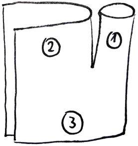

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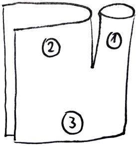

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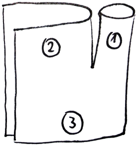

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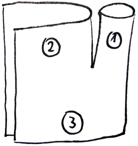

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- Functional equations difficult to solve in the $AdS_5 \times S^5$ case
- Currently only partial solution for two particles (a minimal L-dependent part of the full solution)

(Still) open problem:

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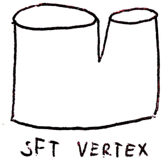

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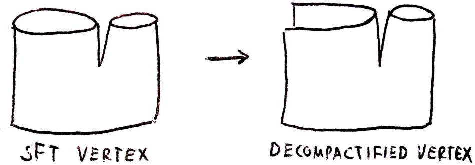

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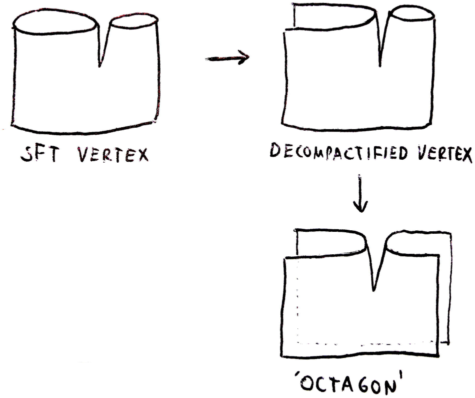

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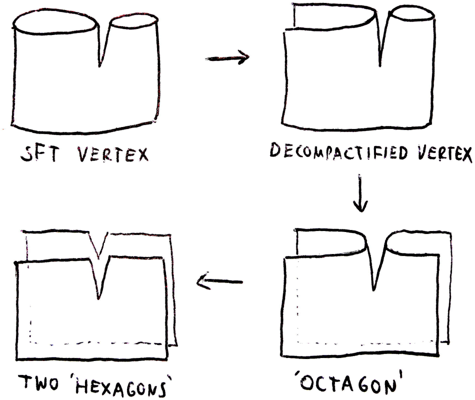

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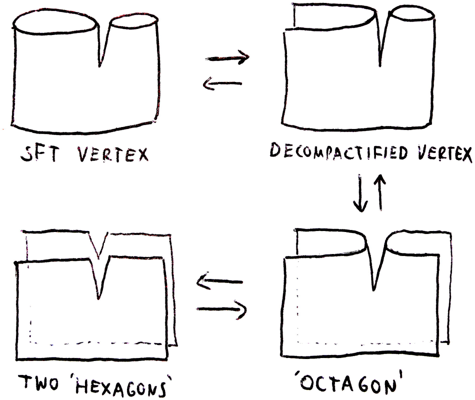

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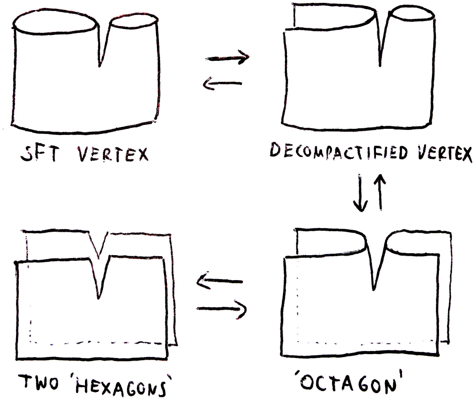

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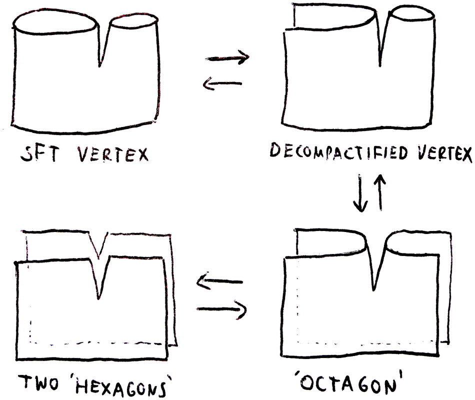

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- We have a very good understanding of the spectrum of $\mathcal{N} = 4$ SYM conformal dimensions
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