TENSOR NETWORK STATES FOR LATTICE GAUGE THEORIES

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J.I. Cirac, S. Kühn (MPQ)

MPQ
Max-Planck-Institut
für Quantenoptik
(Garching b. München)

DESY 29.9.2016

In this talk...

Tensor Network States: general ideas

Matrix Product States (MPS)

Using TNS/MPS for LGT

Schwinger model as a testbench

WHAT ARE TNS?

- TNS = Tensor Network States

Context: quantum many body systems

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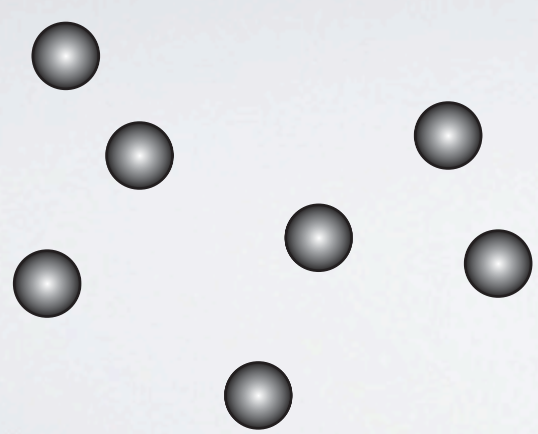
$\{|i\rangle\}_{i=0}^{d-1}$

N



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Context: quantum many body systems

interacting with each other

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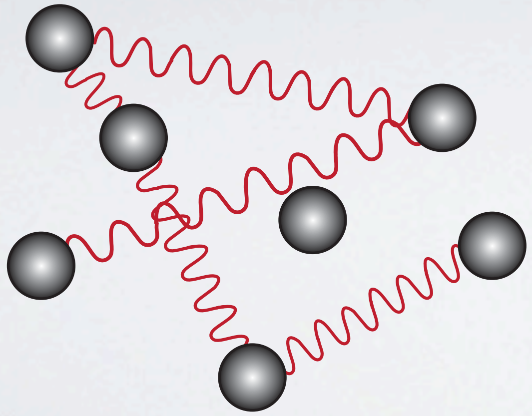

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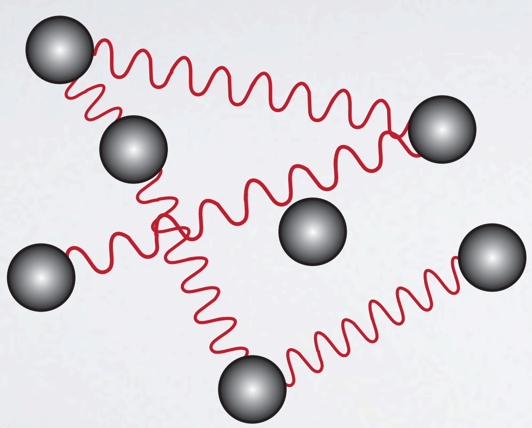

Goal: describe equilibrium states

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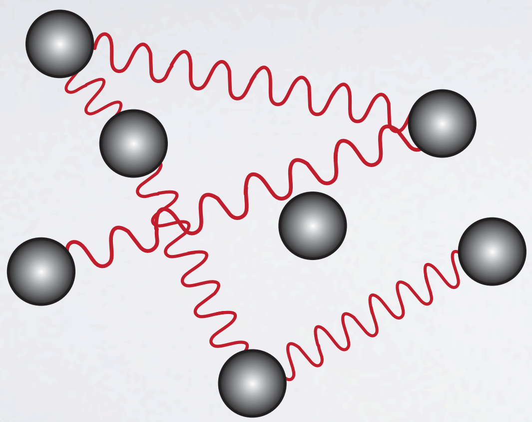


Goal: describe equilibrium states

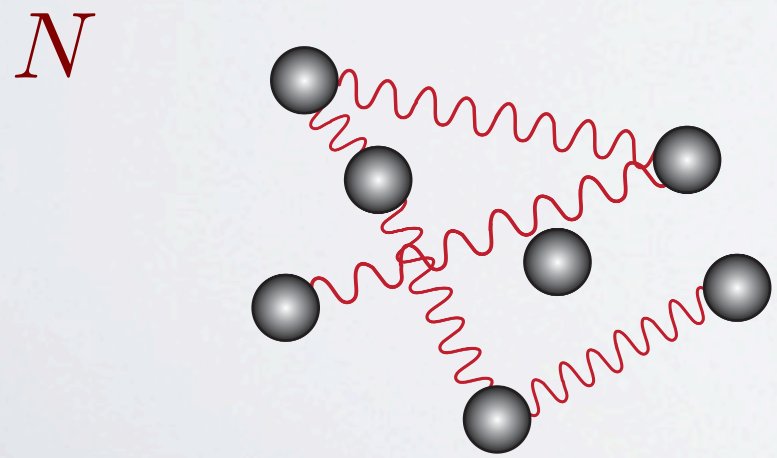
ground, thermal states

WHAT ARE TNS?

• TNS = Tensor Network States

A general state of the N-body Hilbert space has exponentially many coefficients

$$|\Psi\rangle = \sum_{i_j} c_{i_1 \ldots i_N} |i_1 \ldots i_N\rangle$$



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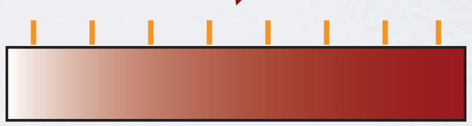
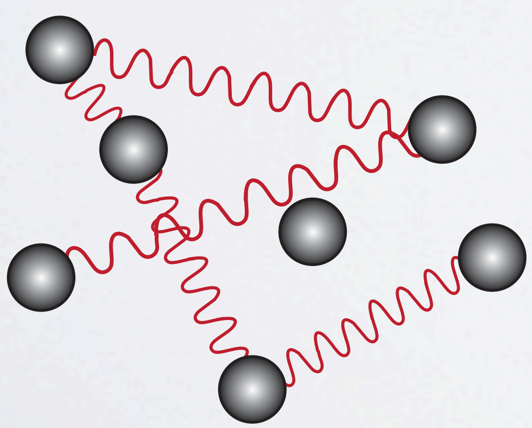


N-legged tensor

d^N

N



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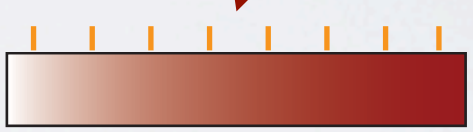

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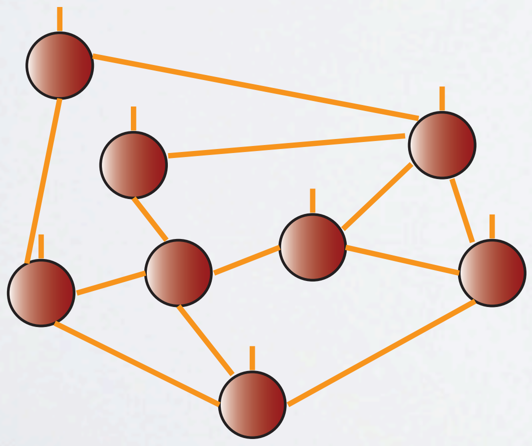

$\mathrm{poly}(N)$

A TNS has only a polynomial number of parameters

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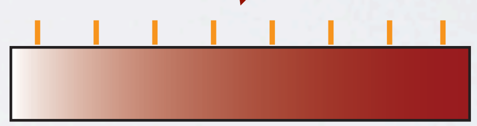

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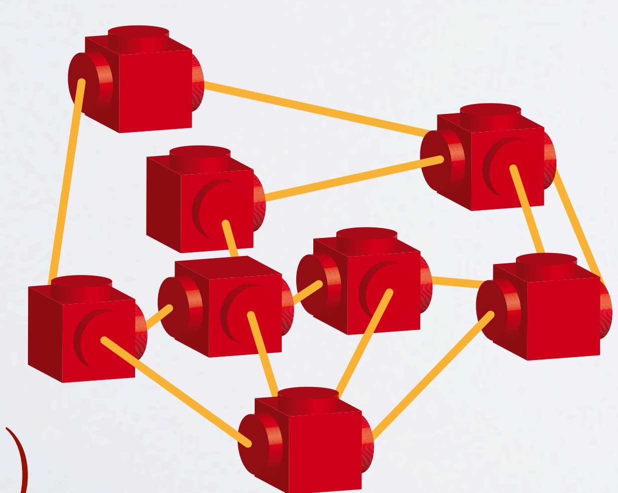

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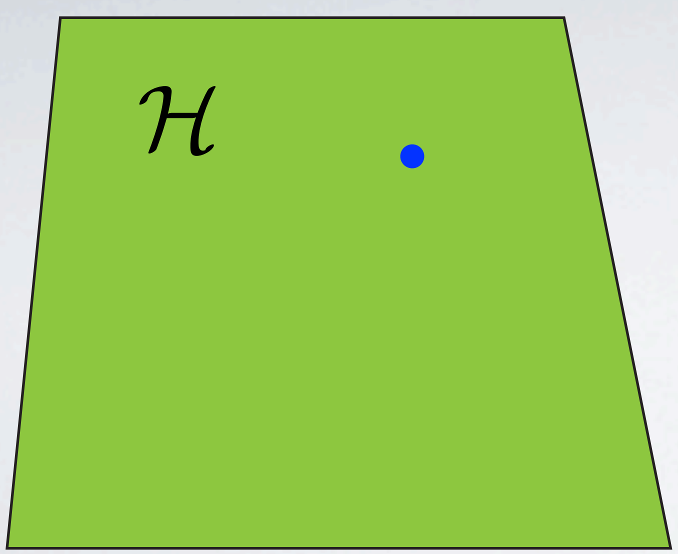
A TNS has only a polynomial number of parameters

WHY SHOULD TNS BE USEFUL?

States appearing in Nature are peculiar

State at random from Hilbert space is not close to product



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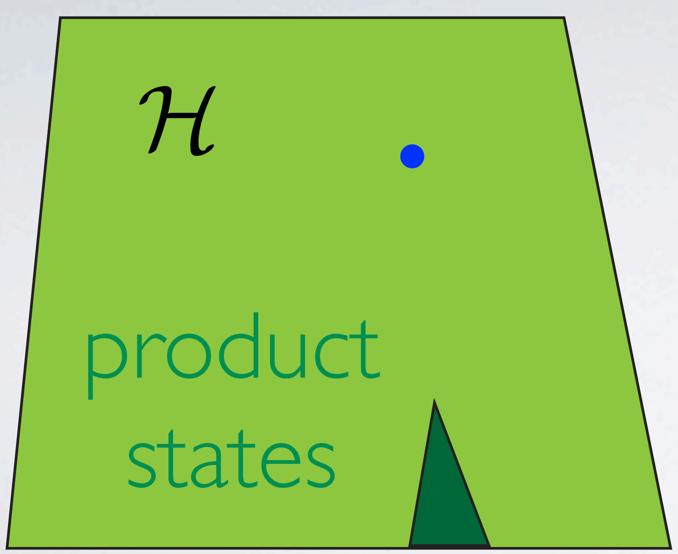

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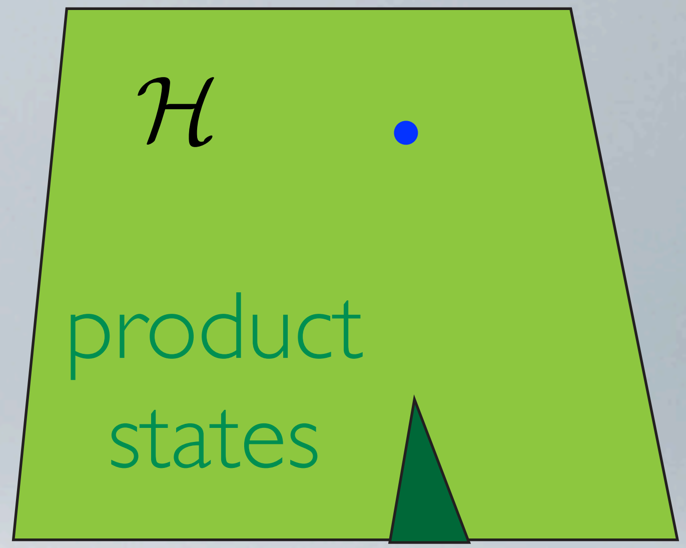

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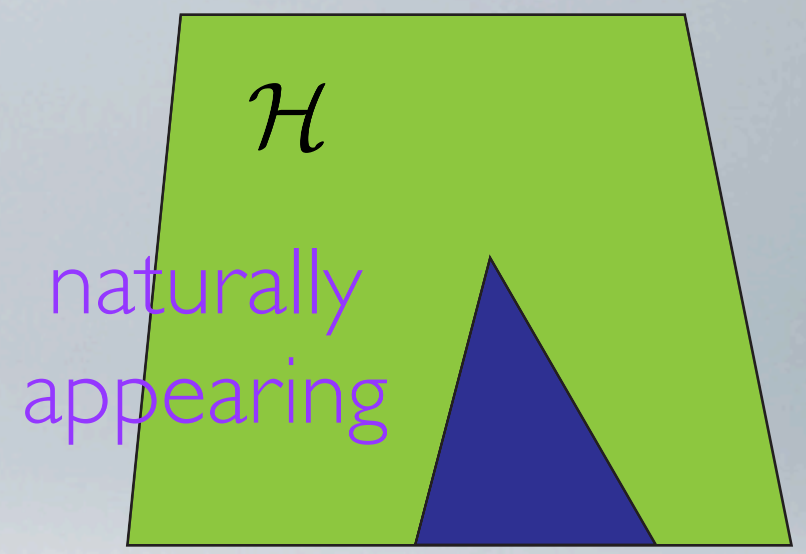

We look for the particular corner of the Hilbert space

- TNS = Tensor Network States

FINDING A GOOD ANSATZ

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Which properties characterize ground states of relevant Hamiltonians?

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ENTANGLEMENT

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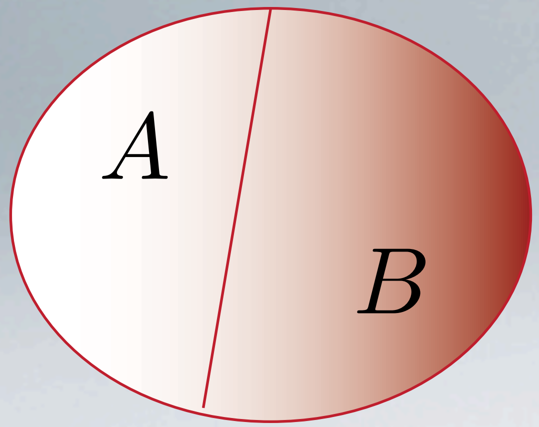

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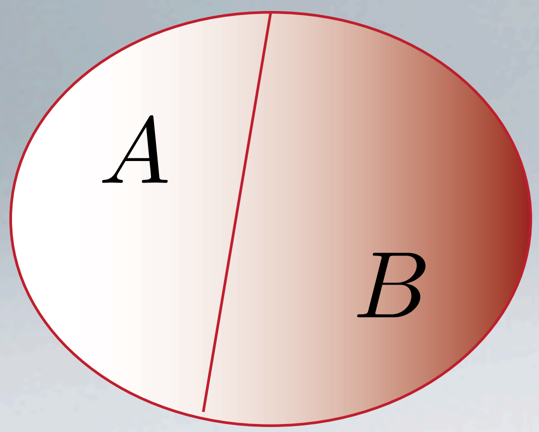

$$|a\rangle \otimes |b\rangle$$

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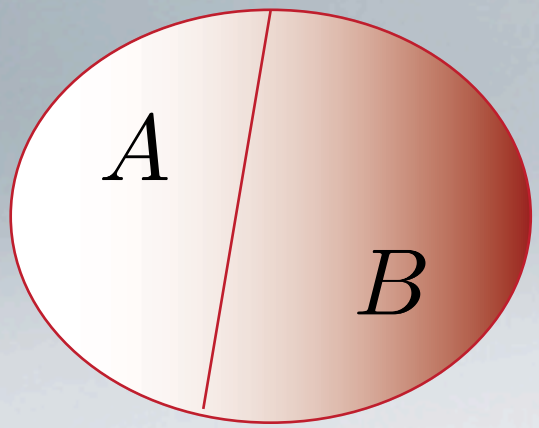

$$|a\rangle \otimes |b\rangle$$

$$|a\rangle \otimes |b\rangle + |b\rangle \otimes |a\rangle$$

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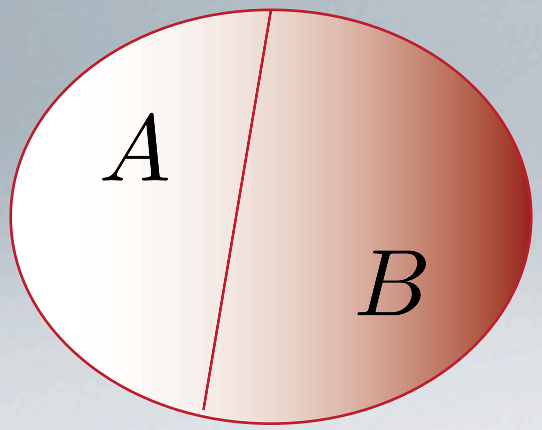

$$|a\rangle \otimes |b\rangle$$

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$$S(A) = -\mathrm{tr}(\rho_A \log(\rho_A))$$

entanglement entropy

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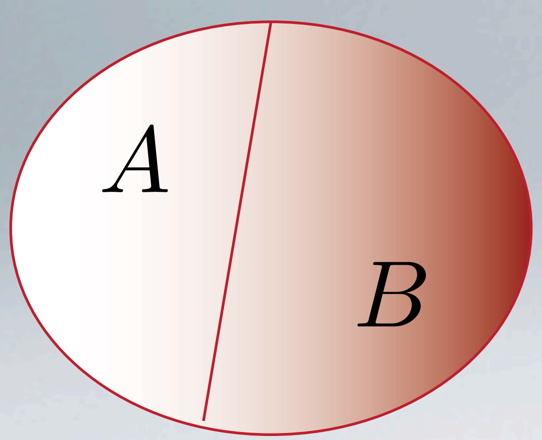

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TNS = entanglement based ansatz

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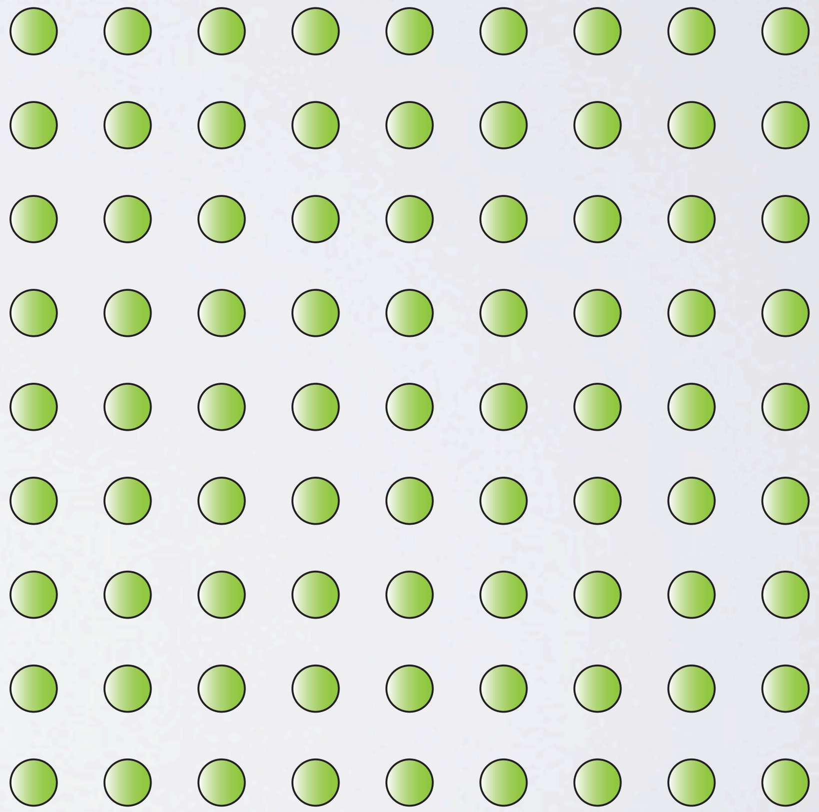

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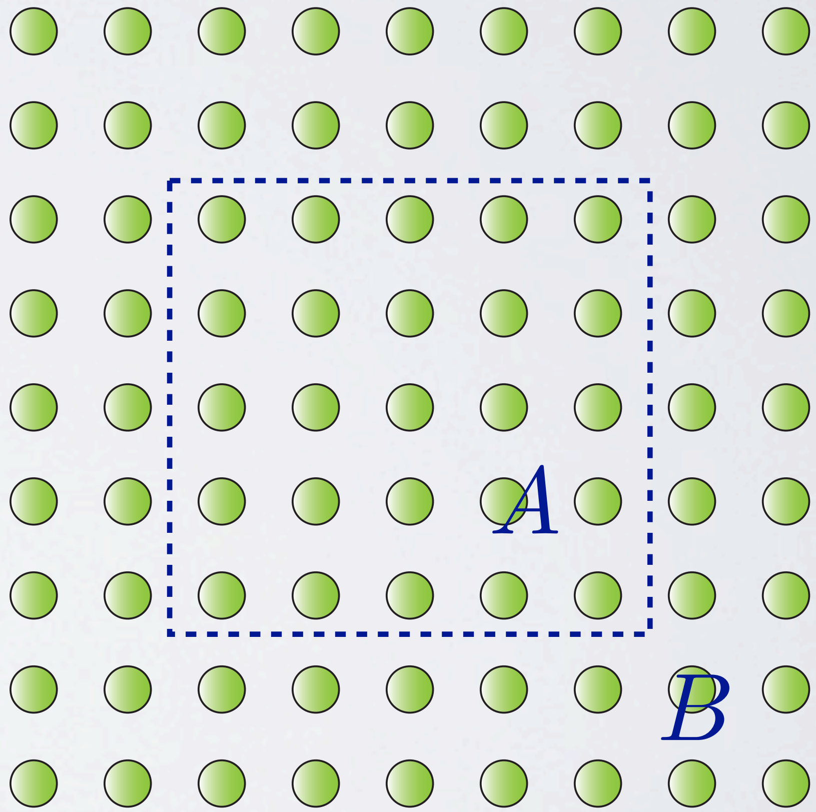

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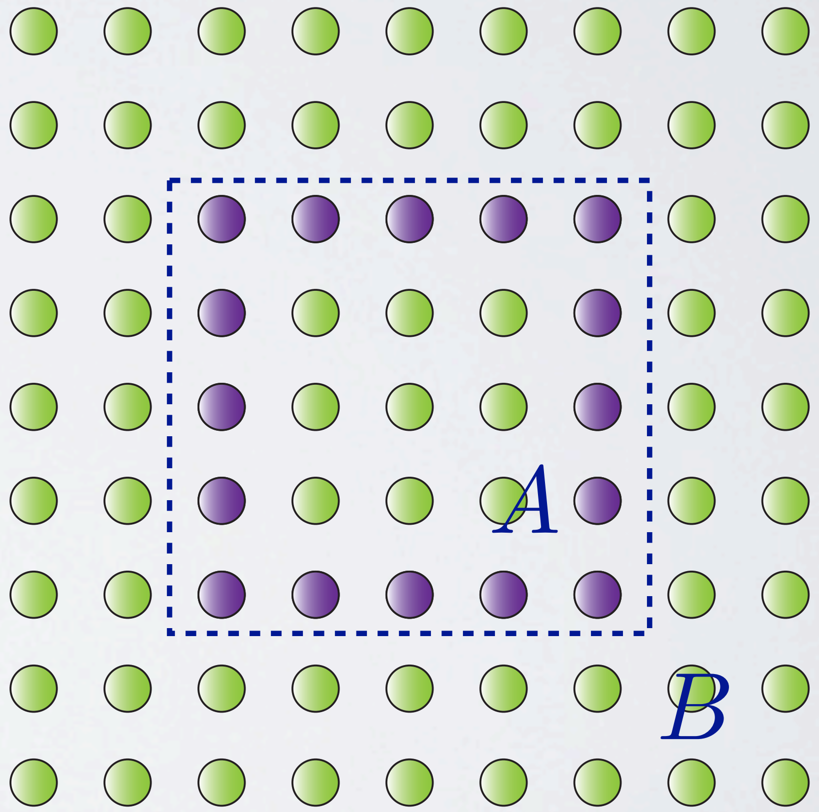

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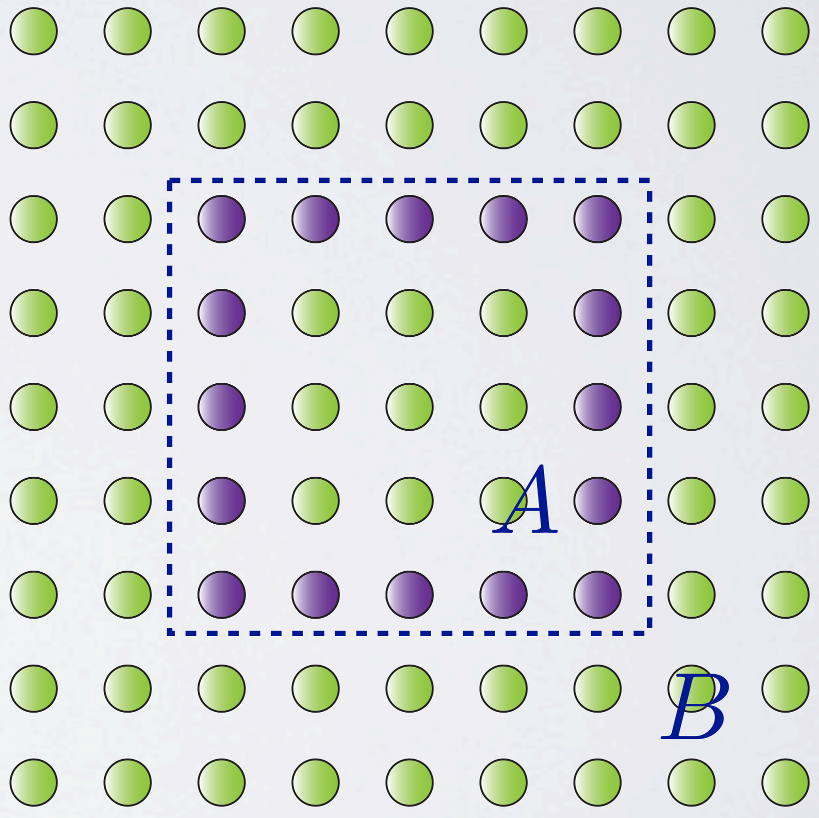
$$S_{A\,\mathrm{max}} \propto |\delta A|$$

Hastings 2007

Area law



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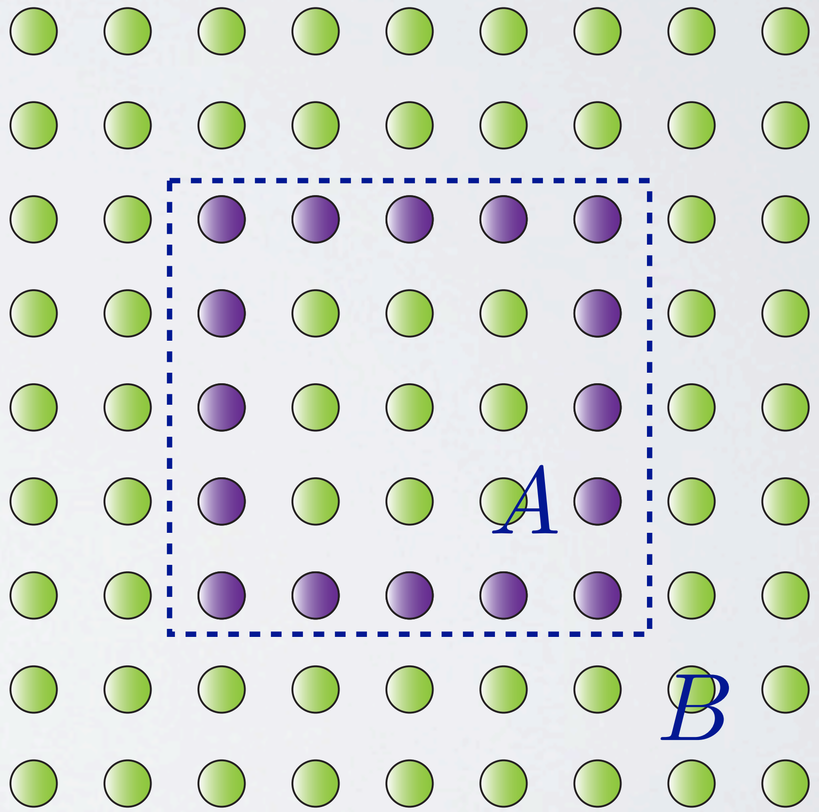
Hastings 2007

in 1D critical systems,
logarithmic corrections

$$S_{A\,\max} \propto |\delta A| \log |\delta A|$$

Calabrese, Cardy 2004

Wolf 2006



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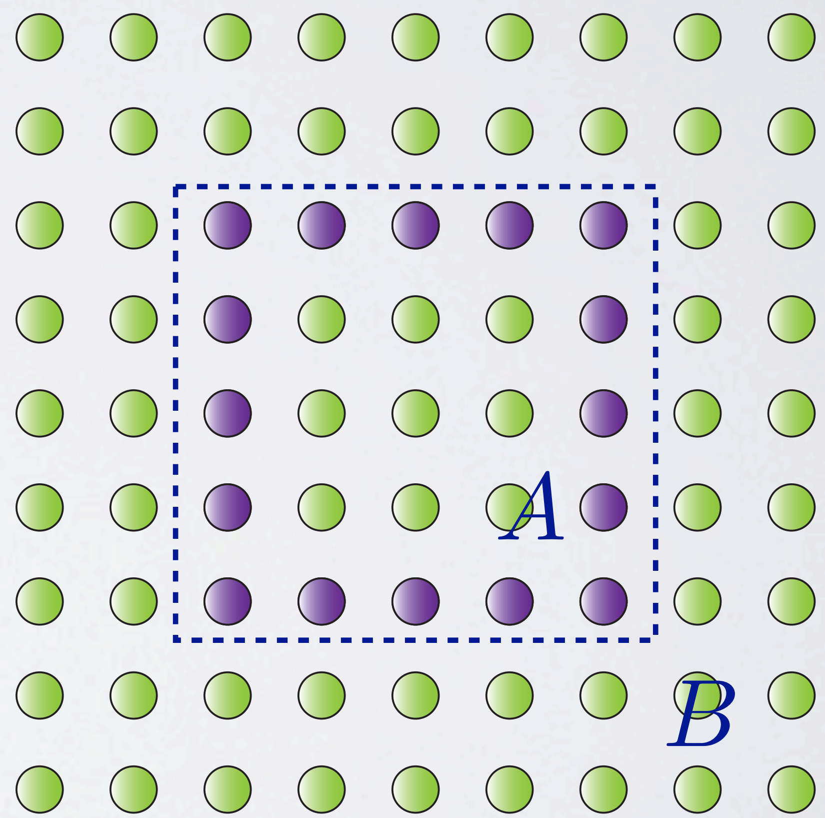

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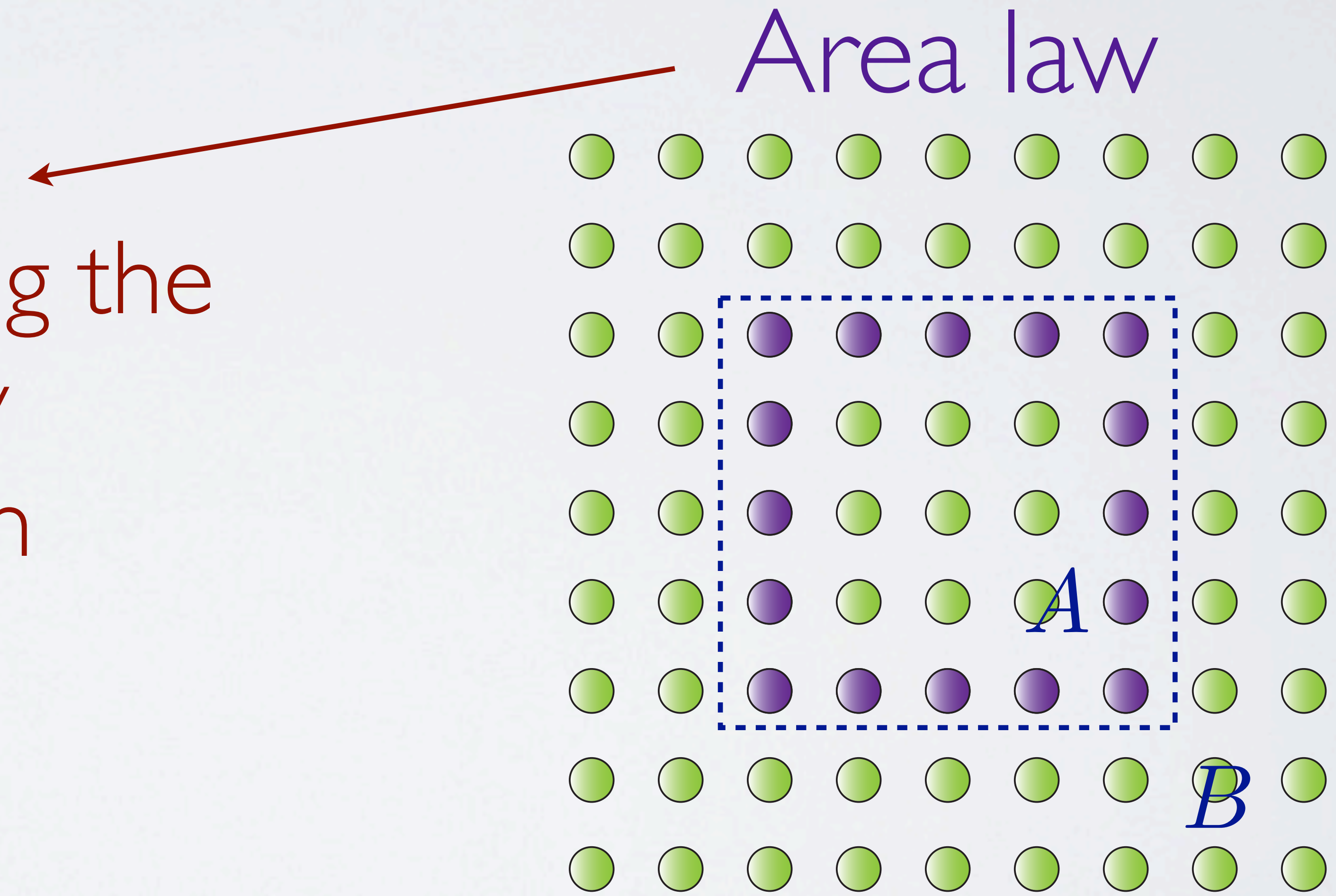
Wolf 2006

satisfied at finite temperature Wolf, Verstraete, Hastings, Cirac, PRL 2008

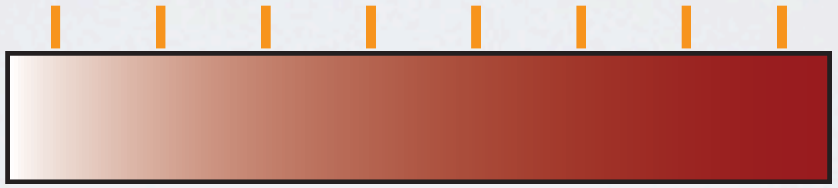
MPS & PEPS

Ansätze satisfying the area law by construction



MPS

- MPS = Matrix Product States



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Affleck, Kennedy, Lieb, Tasaki, PRL 1987

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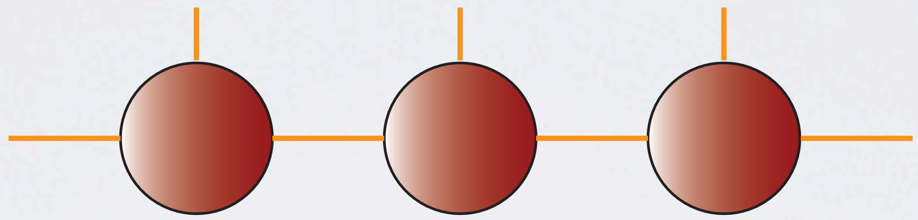

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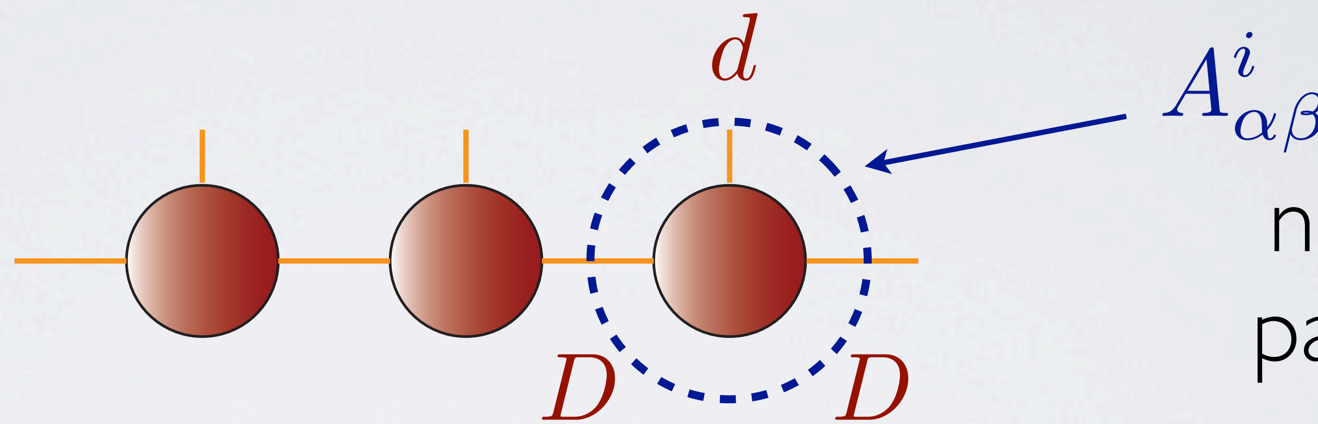

number of parameters

NdD^2

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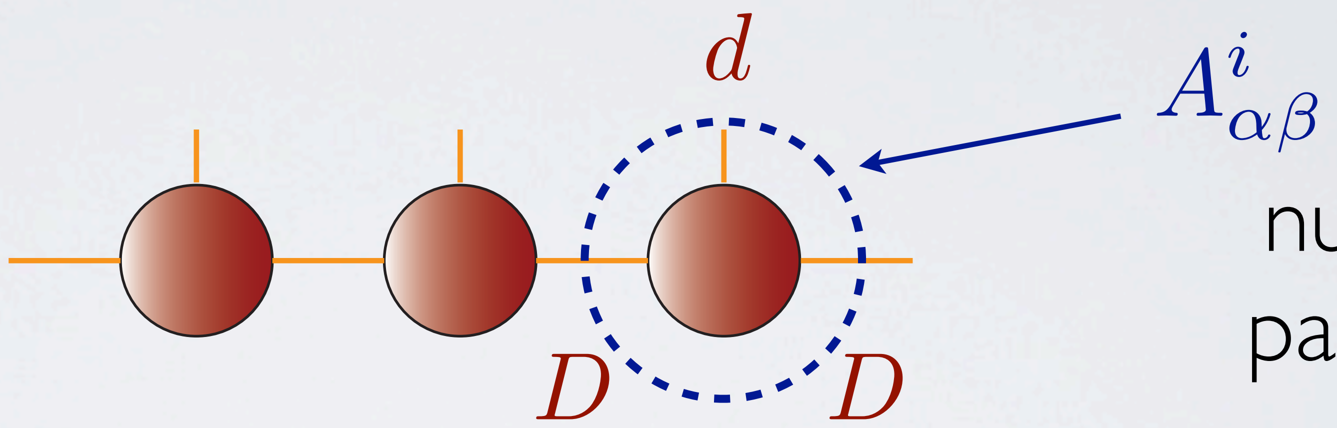

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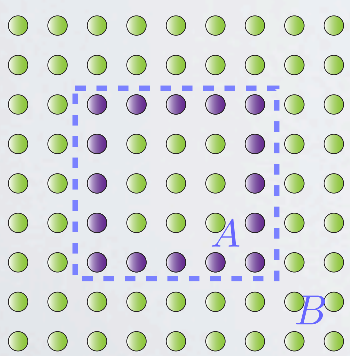

Area law by construction

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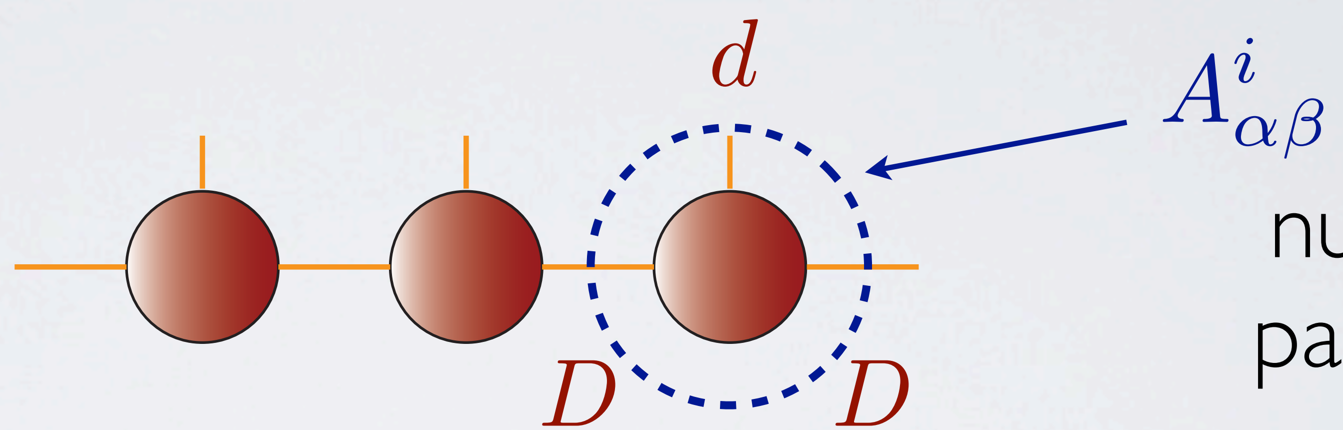

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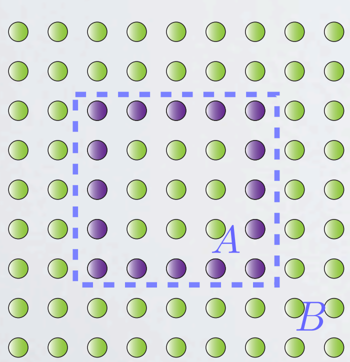

Area law by construction

Bounded entanglement

$S(L/2) \leq \log D$

Affleck, Kennedy, Lieb, Tasaki, PRL 1987

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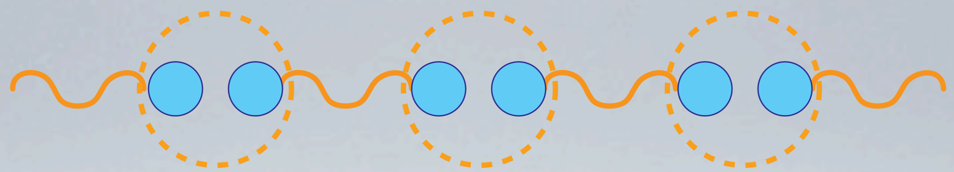
1D



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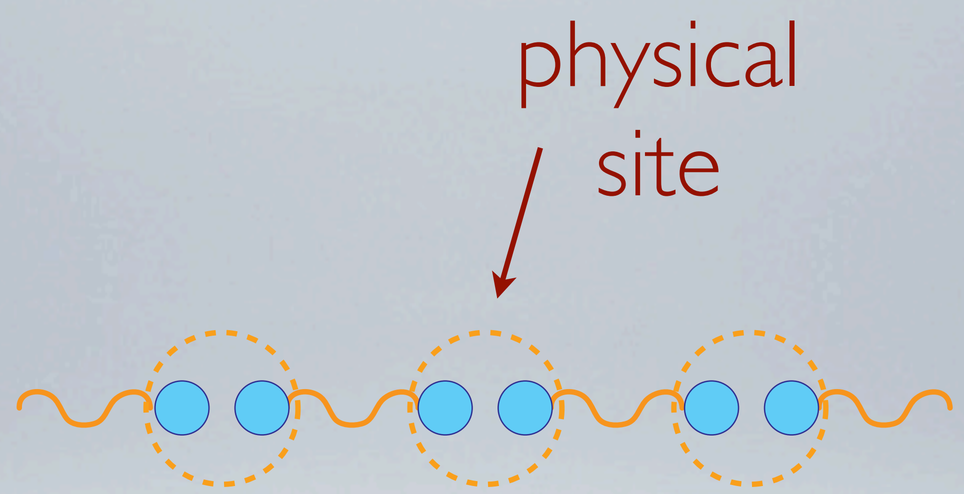

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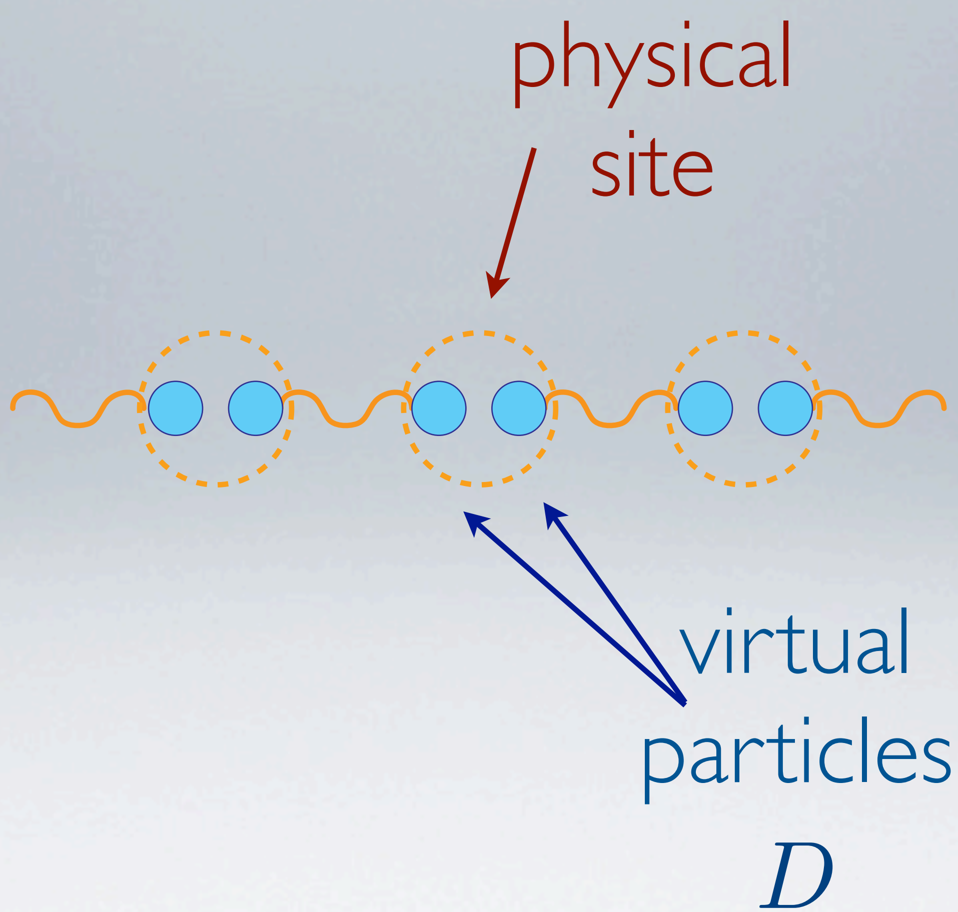

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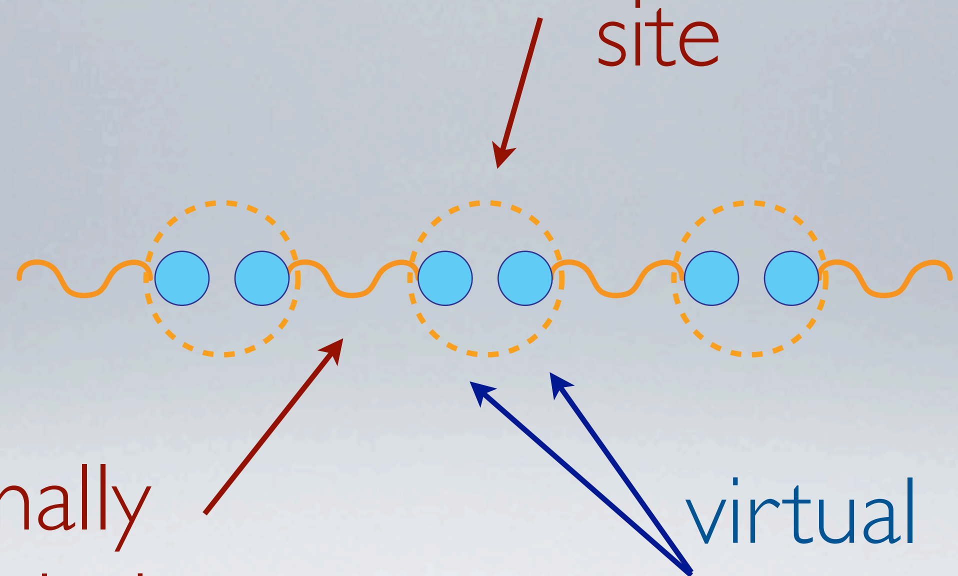

$$\sum_{\alpha=1}^{D} |\alpha\rangle|\alpha\rangle$$

D

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MPS

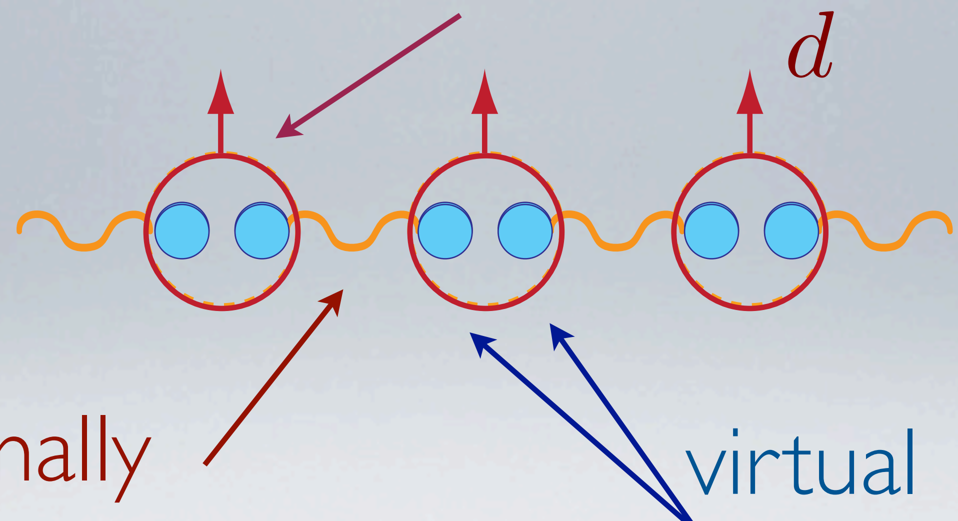
Area law by construction

- MPS = Matrix Product States

project onto the physical degrees of freedom

1D

d



maximally entangled state

$$\sum_{\alpha=1}^{D} |\alpha\rangle|\alpha\rangle$$

virtual particles

D

Affleck, Kennedy, Lieb, Tasaki, PRL 1987

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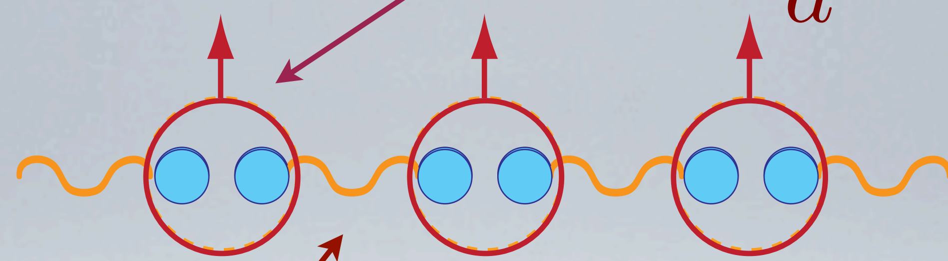

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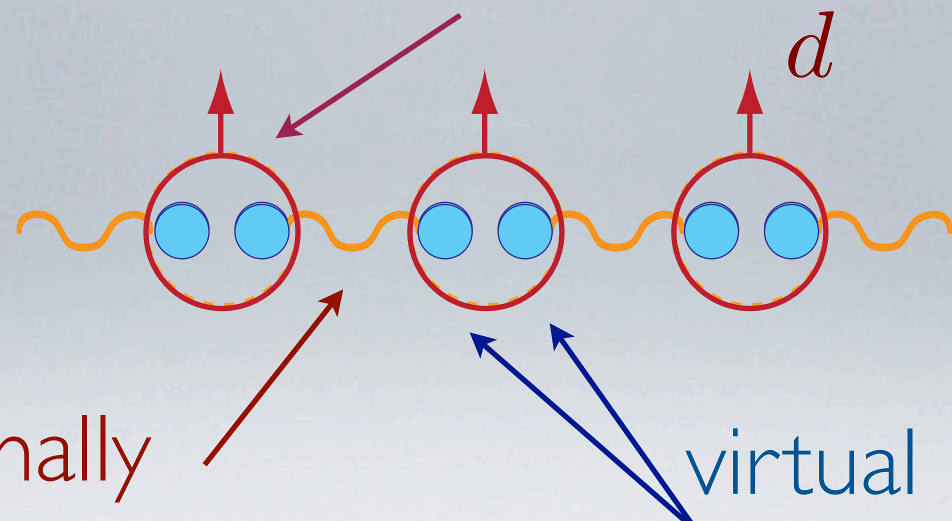

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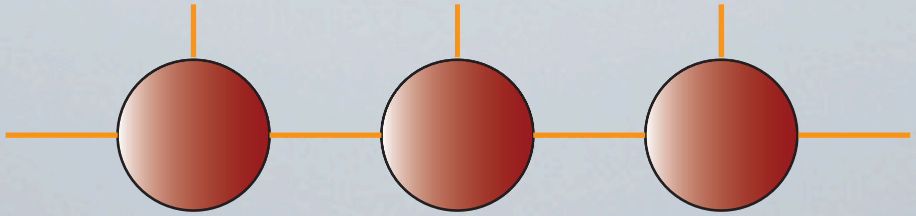
MPS EXAMPLE



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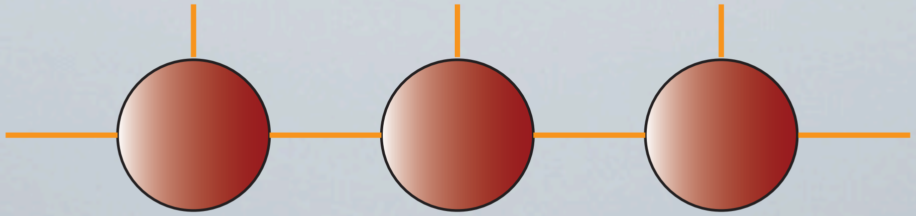

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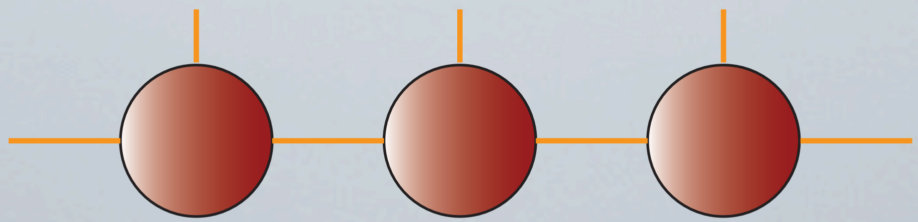
$$A^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad A^1 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

MPS EXAMPLE



$$|\Psi\rangle = \sum_{i_1 \ldots i_N} \mathrm{tr}(A_1^{i_1} A_2^{i_2} \ldots A_N^{i_N}) |i_1 \ldots i_N\rangle$$

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$$|100\ldots\rangle + |010\ldots\rangle + |001\ldots\rangle + \ldots$$

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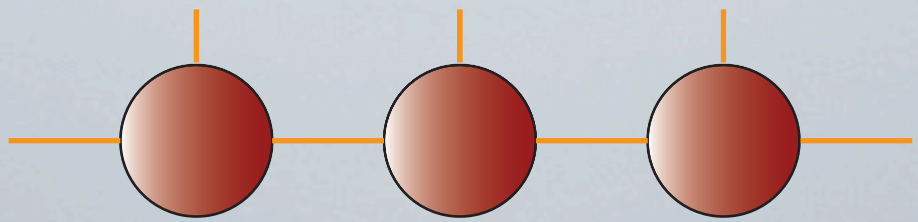

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$$|100\dots\rangle + |010\dots\rangle + |001\dots\rangle + \dots$$

$$D = 2$$

MPS PROPERTIES

- MPS = Matrix Product States

MPS

complete family $D \leq d^{N/2}$

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good approximation of ground states

Verstraete, Cirac, PRB 2006

Hastings, J. Stat. Phys 2007

gapped finite range Hamiltonian ⇒
area law (ground state)

Cramer, Eisert, Plenio, RMP 2009

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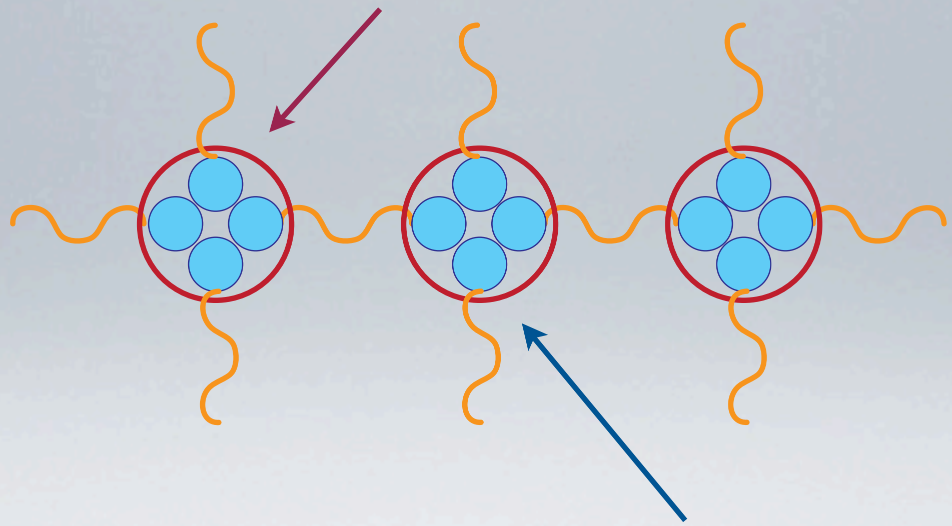
can be prepared efficiently

Schön et al PRL 2005

MPS AND PEPS

- PEPS = Projected Entangled Pairs States

PEPS= generalization to higher dimensions



Area law by construction

Verstraete, Cirac, 2004

MPS AND PEPS

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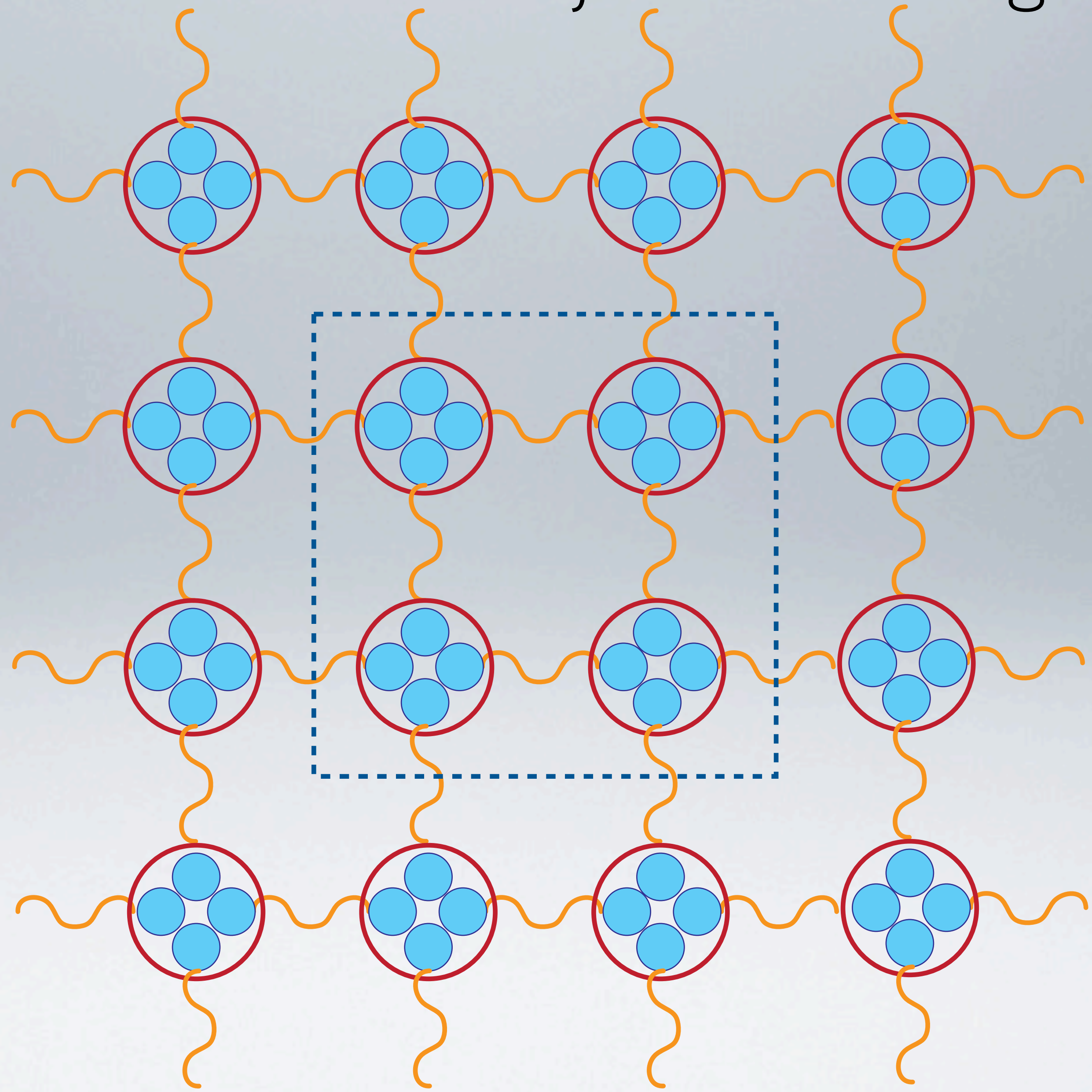
Entropy of a region bounded by the number of cut bonds



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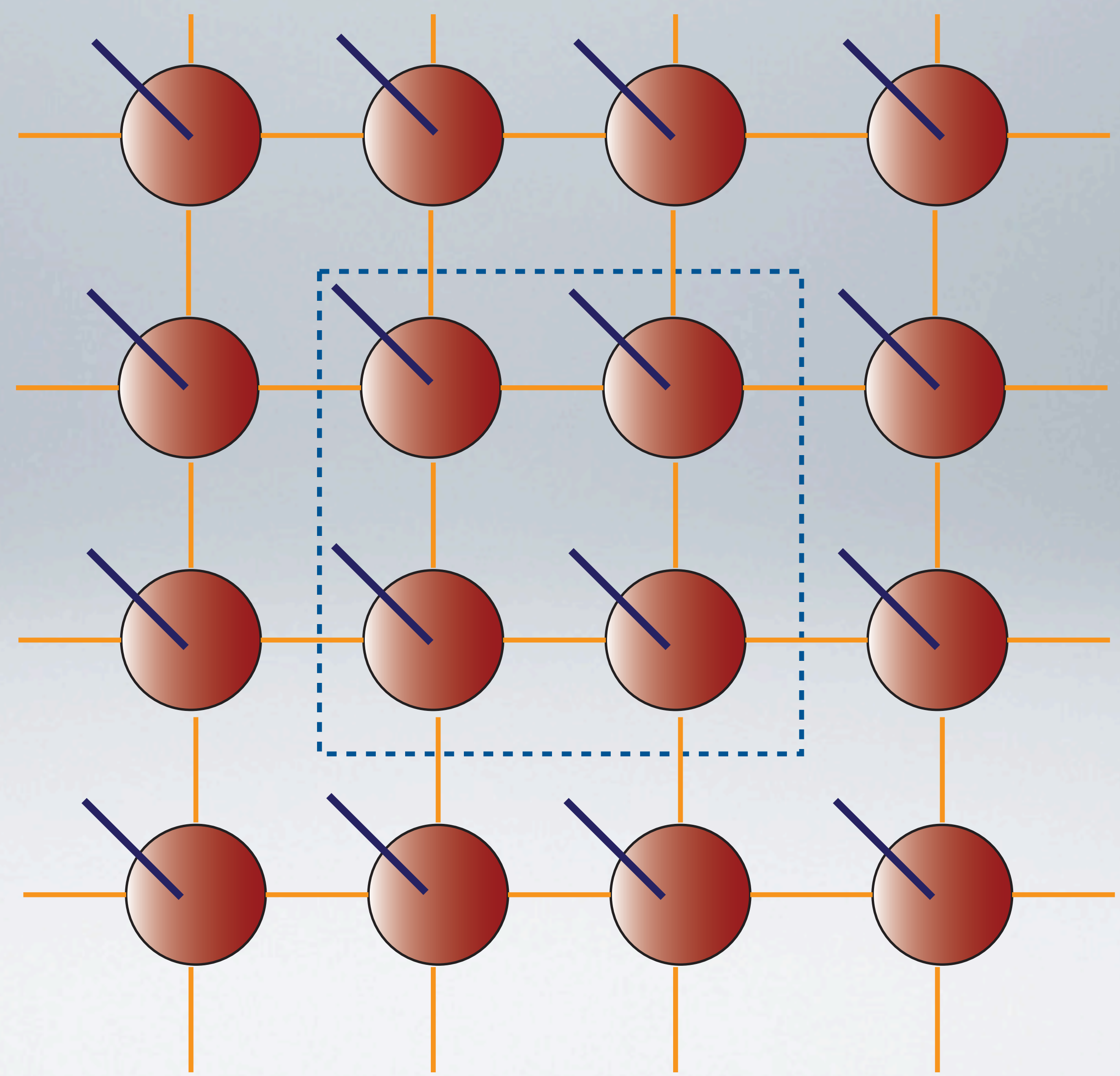


Area law by construction

Verstraete, Cirac, 2004

PEPS PROPERTIES

- PEPS = Projected Entangled Pairs States

PEPS

complete family

good approximation of thermal states

Hastings PRB 2006

Molnar et al PRB 2015

no efficient calculation of expectation values

but approximate contractions possible

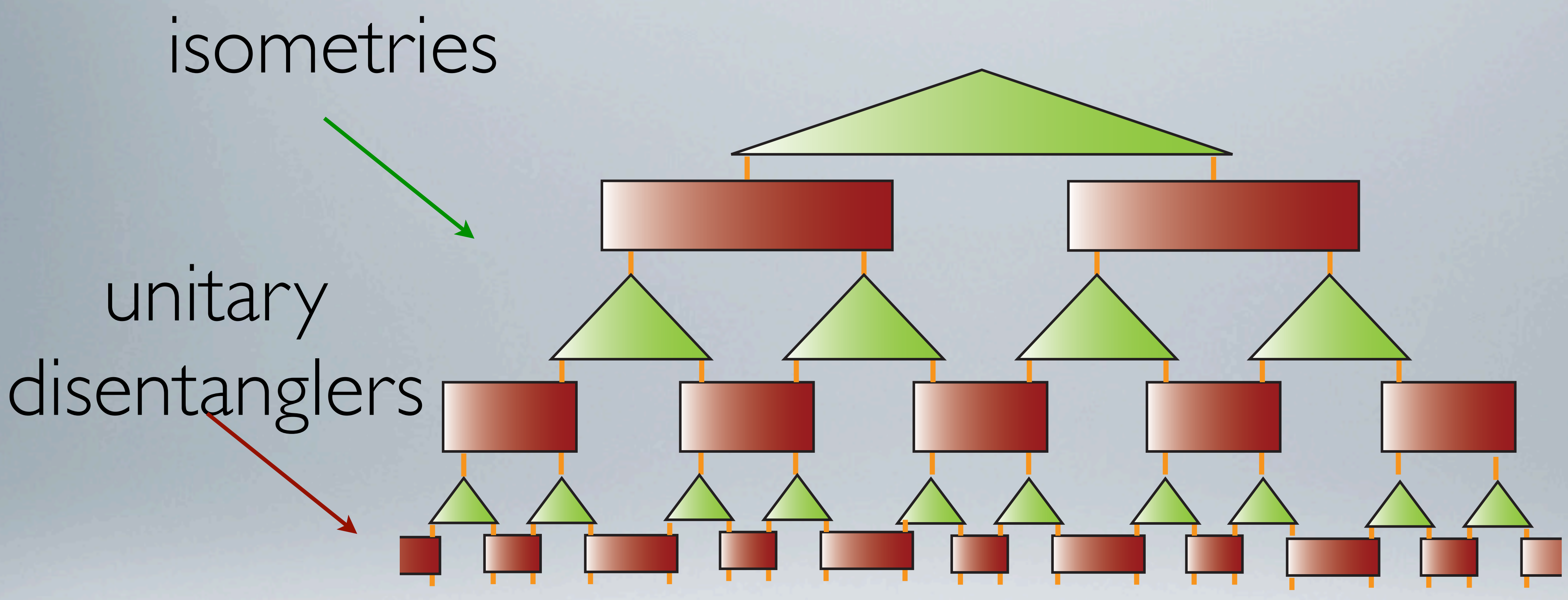
can hold algebraically decaying correlations

cannot be prepared efficiently

Schuch et al PRL 2007

OTHER TNS

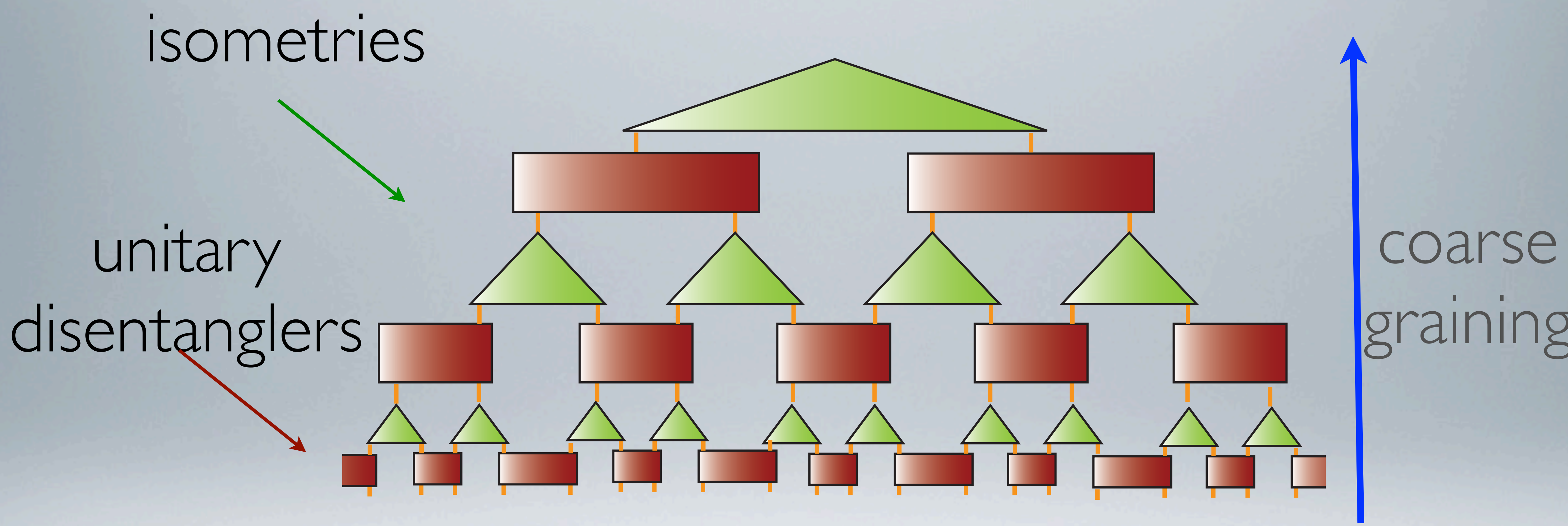
MERA



Violate area law logarithmically (in 1D)

Vidal PRL 2007
Evenbly, Vidal, PRB2009

MERA



Violate area law logarithmically (in 1D)

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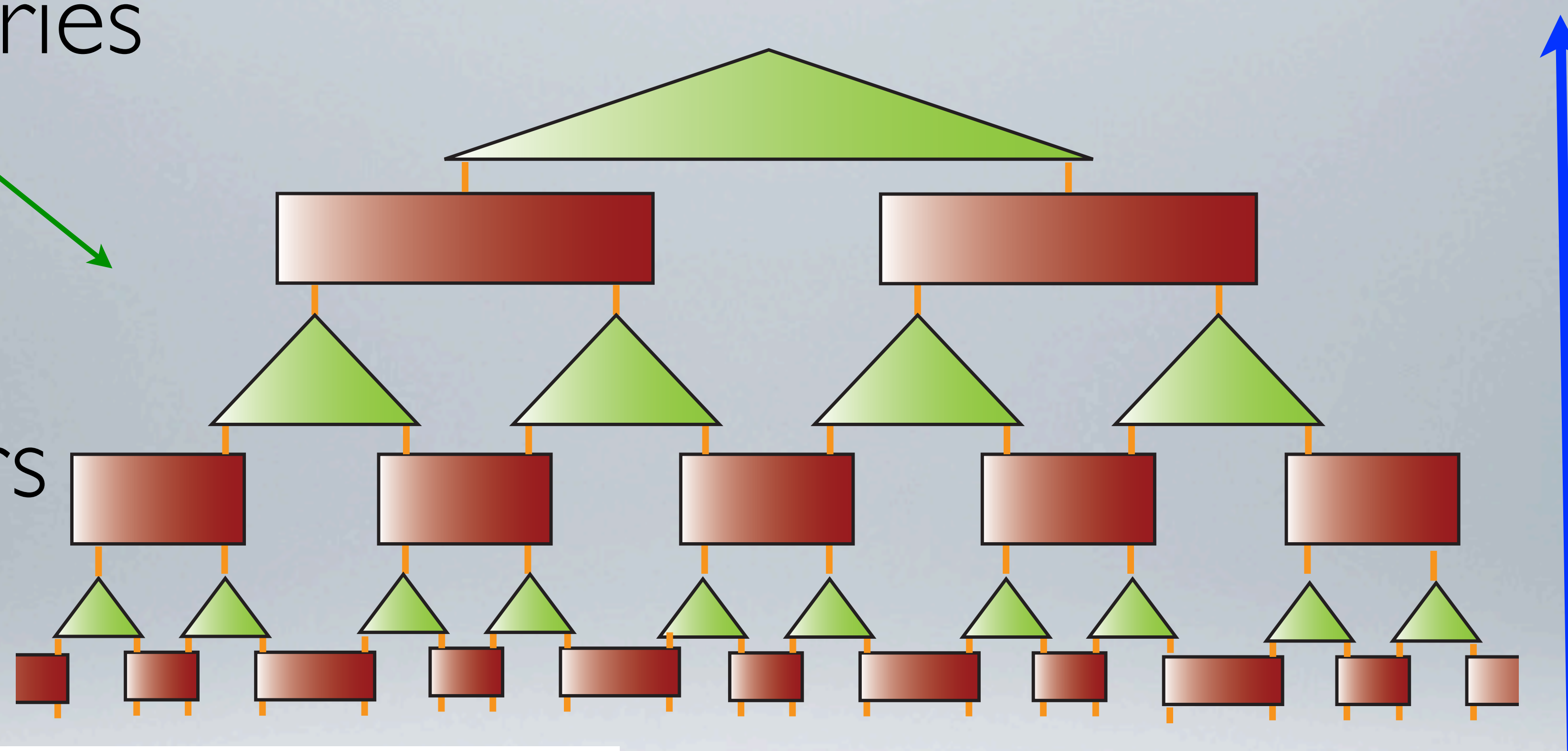
MERA

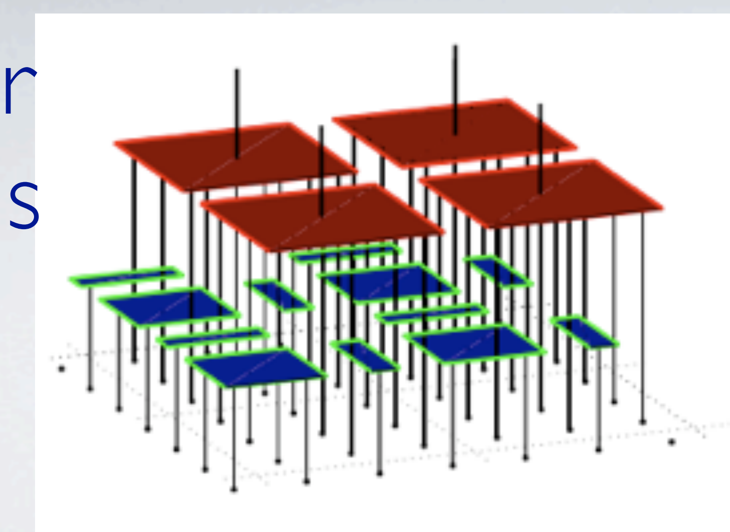
isometries

unitary disentanglers



coarse graining

also higher dimensions



several configurations possible

Violate area law logarithmically (in 1D)

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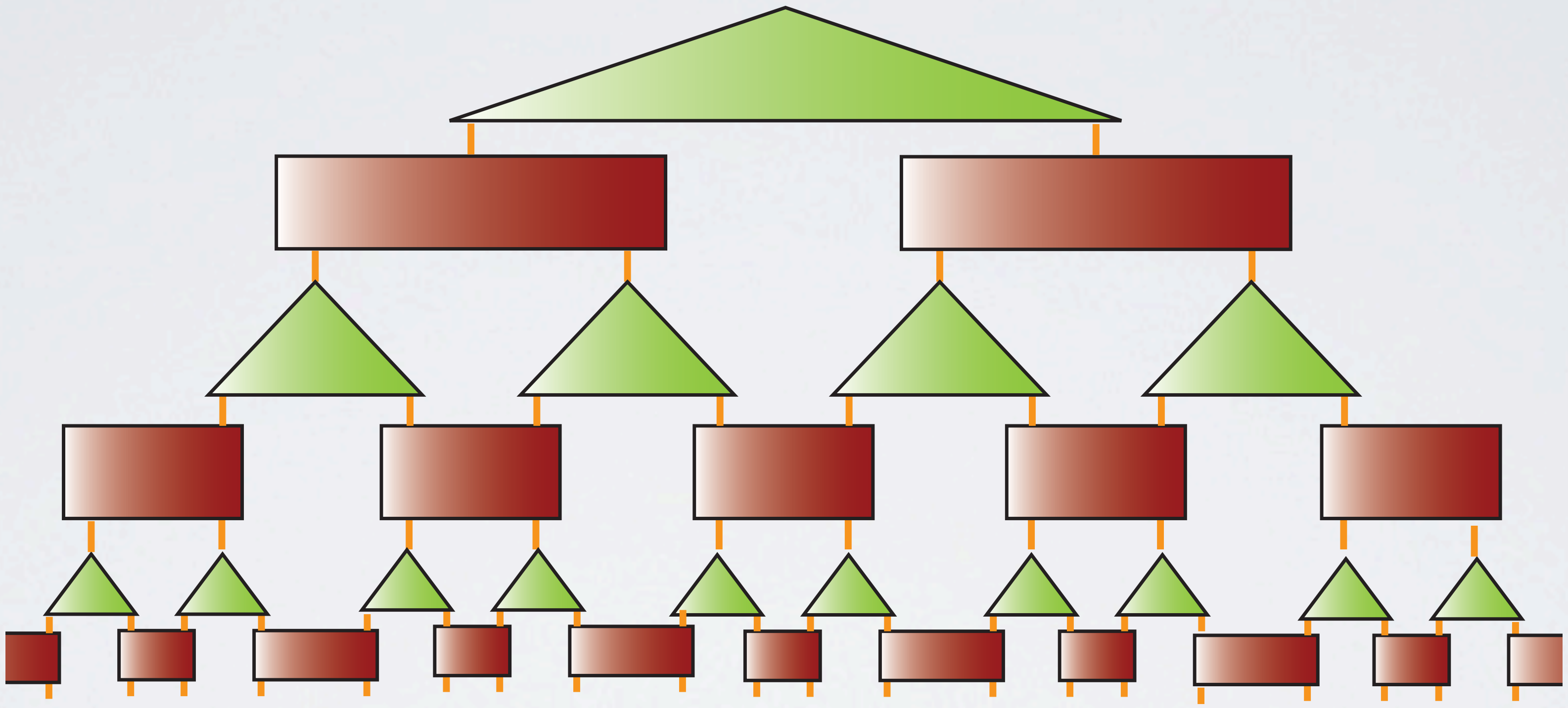

Swingle PRD 2012
Molina JHEP 2013
Nozaki et al JHEP 2012
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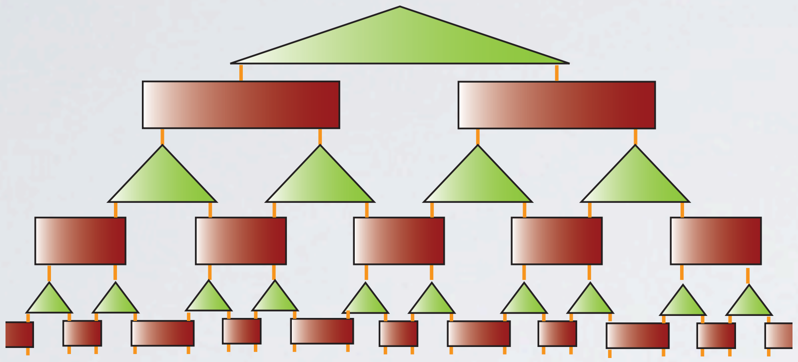
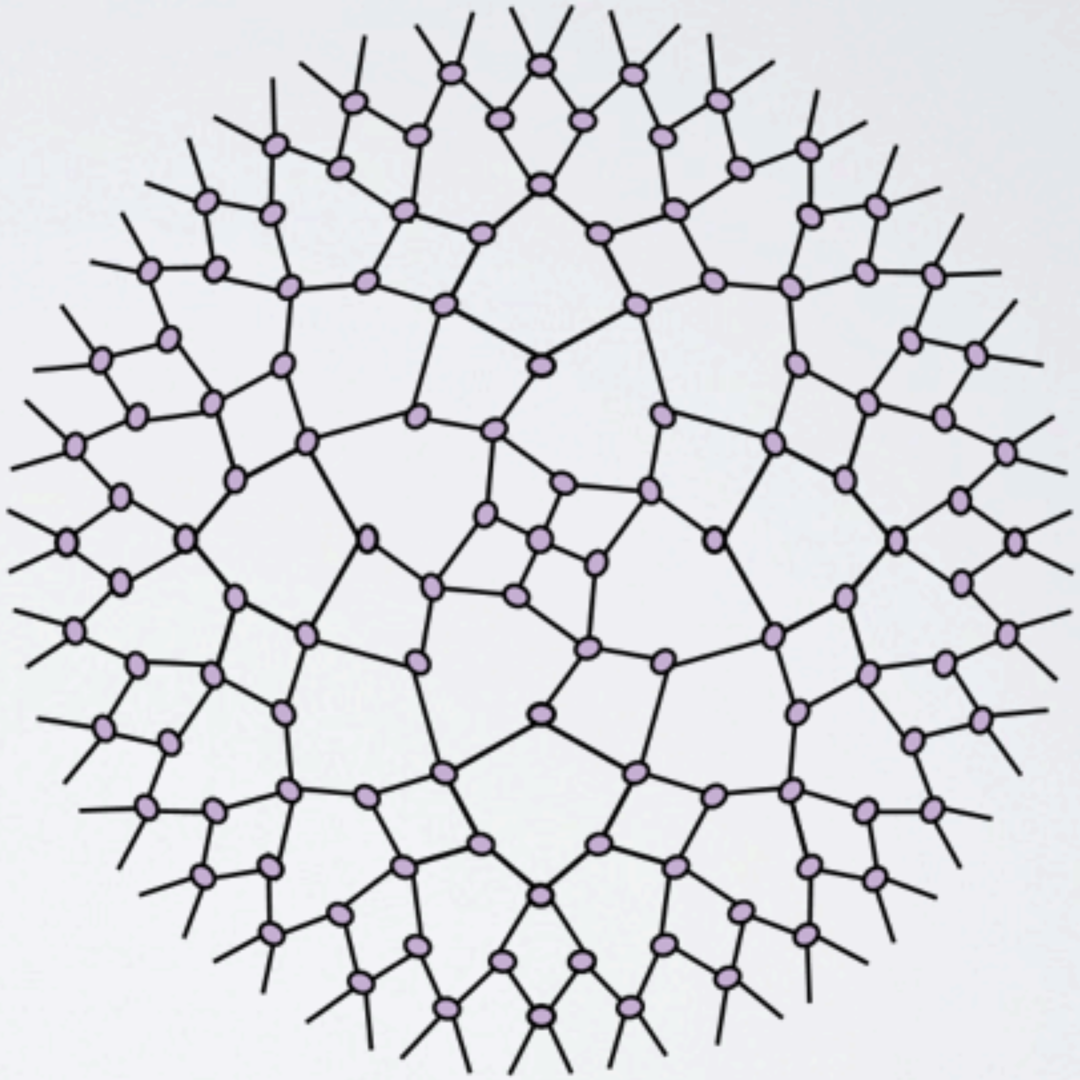
MERA





suggested connection
to AdS/CFT

geometry from
entanglement: discrete AdS

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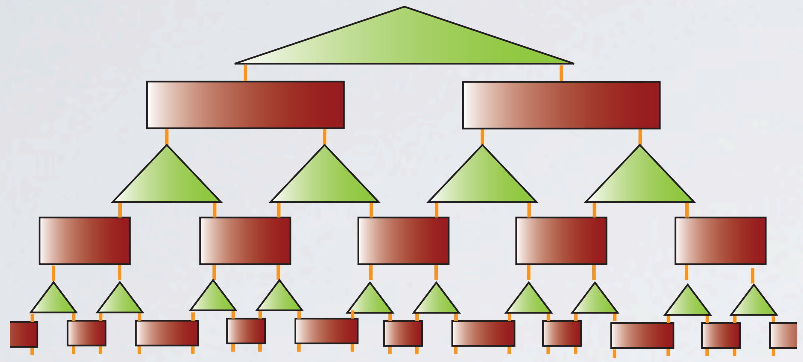

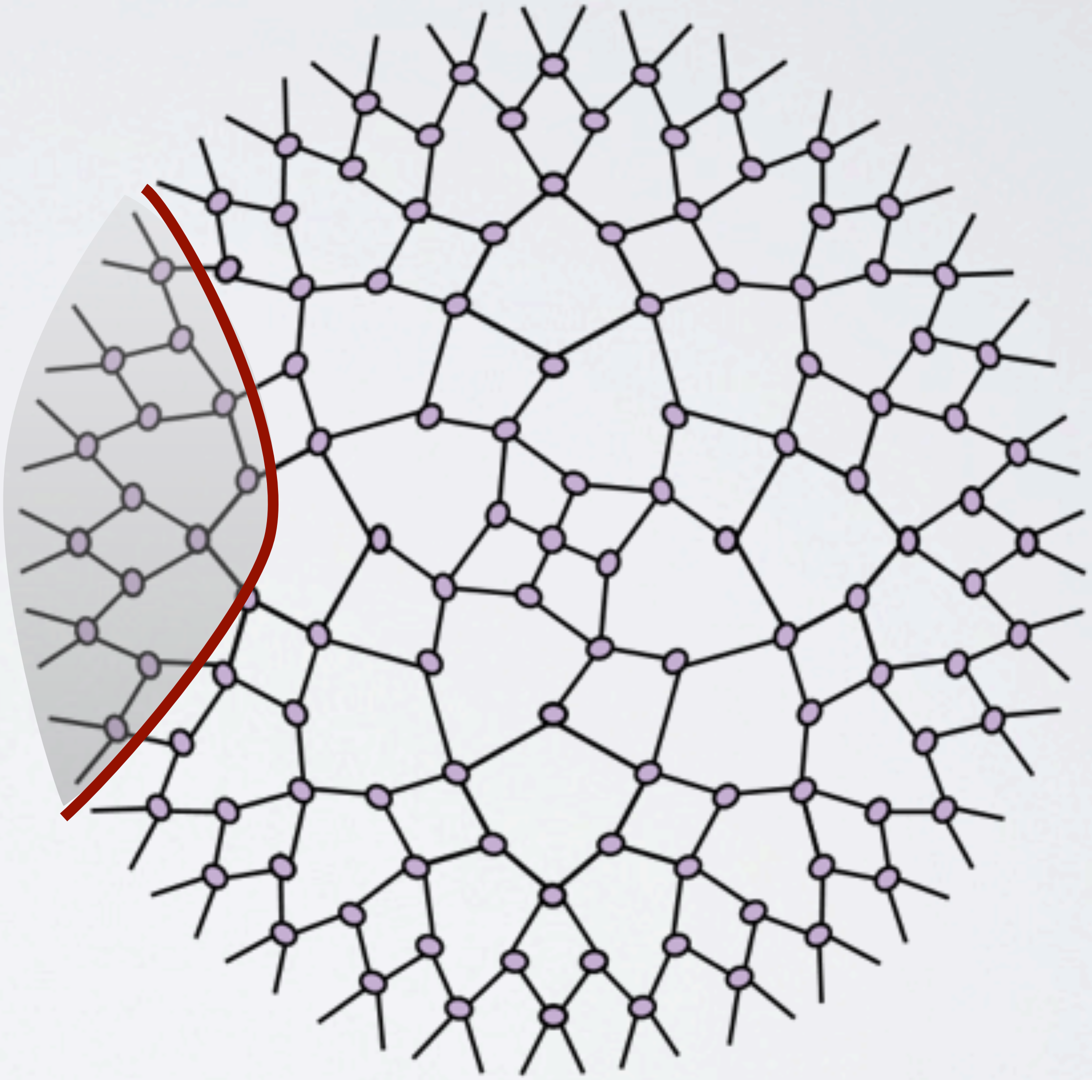

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minimal curves give entropy

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TNS AND SYMMETRY

invariant Hamiltonian → symmetric eigenstates

$$UHU^\dagger = H$$

Symmetries can also act only on virtual level=> related to topological properties
Symmetry can be gauged!!! REFS!

Pérez-García et al., PRL 2008
Sanz et al., PRA 2009
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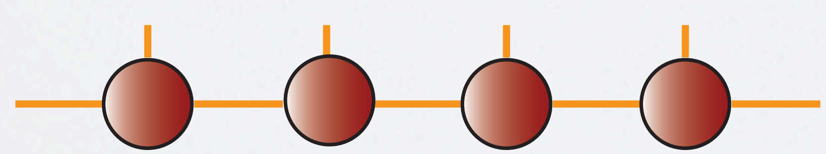

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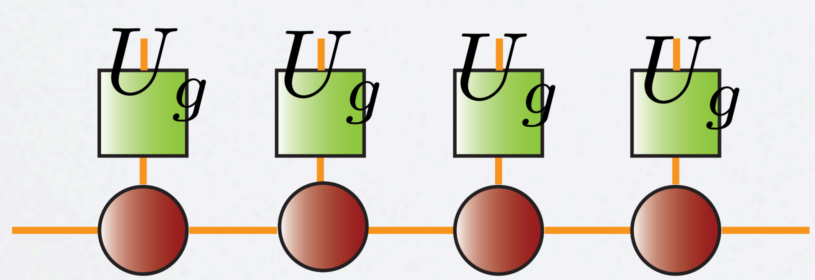

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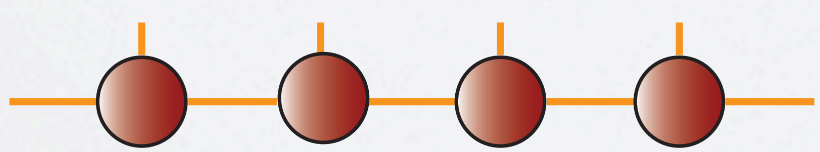

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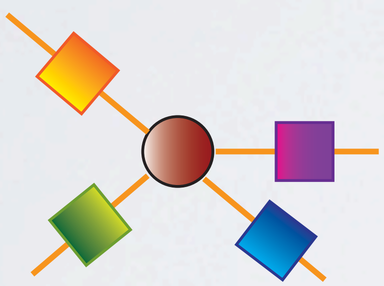

invaria

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nt

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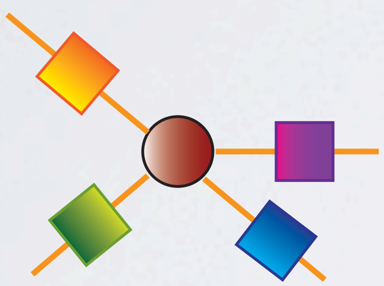

invaria

nt

MPS & PEPS   state invariant ⇔

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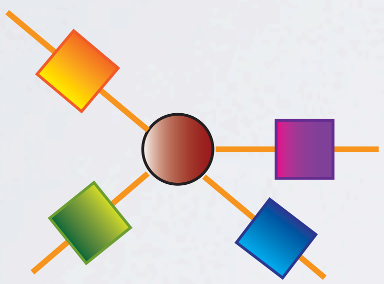

invaria ... nt

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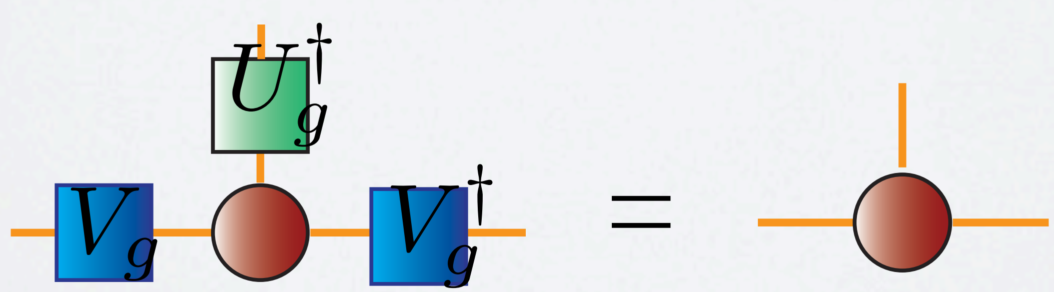

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USING TNS

USING TNS FOR QMB

a formal approach

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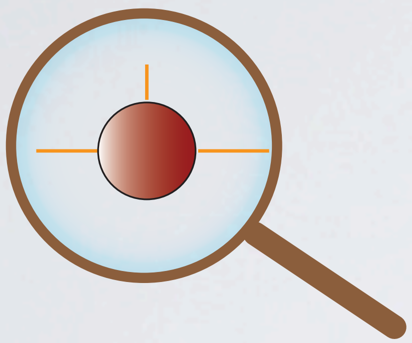

classifying tensors

constructing states

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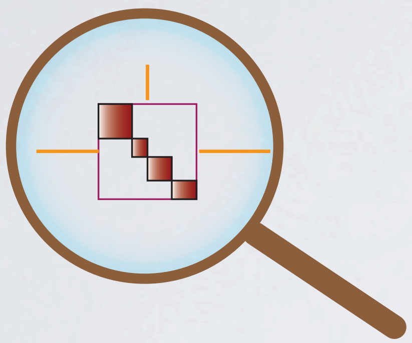

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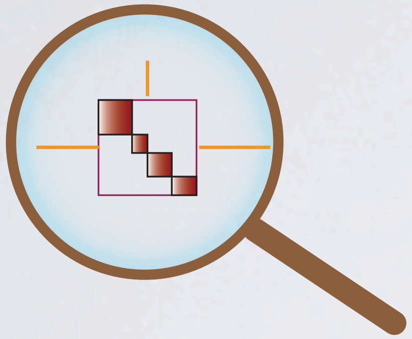

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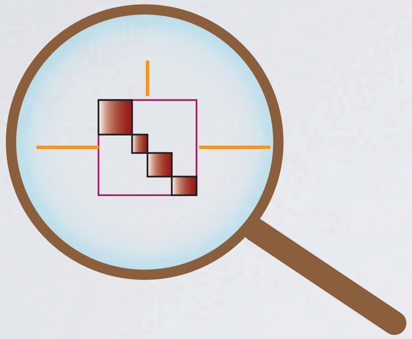

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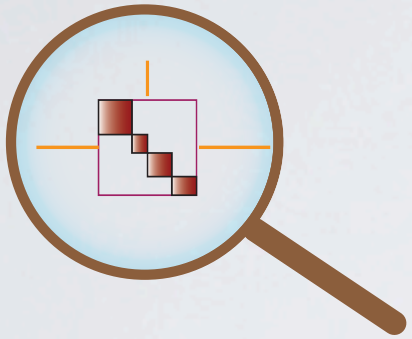

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tensor networks describe partition functions (observables)

need to contract a TN
TRG approaches

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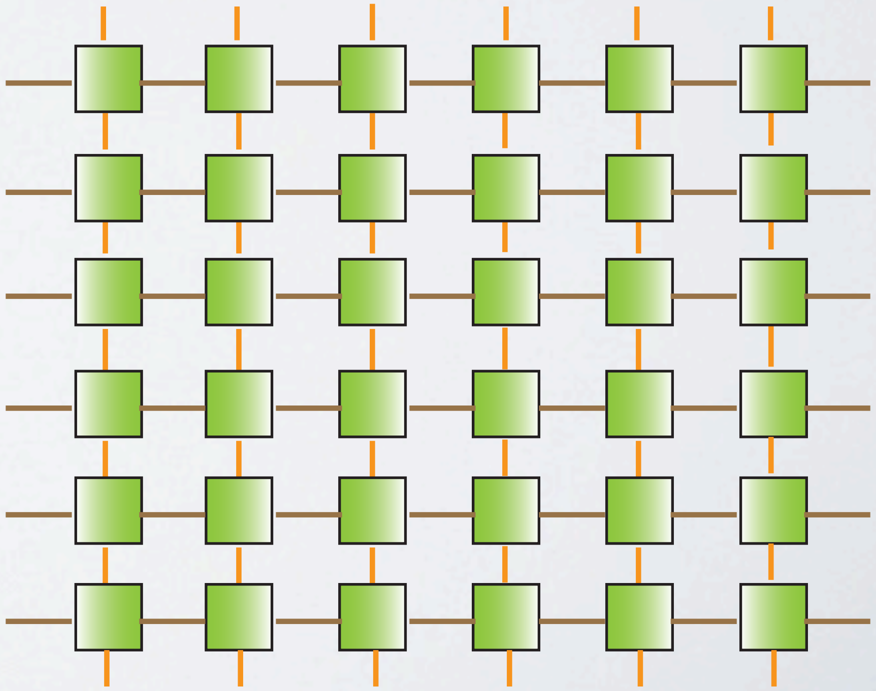

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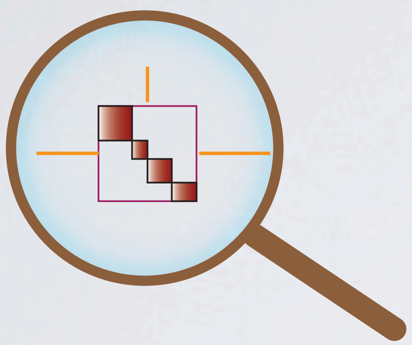

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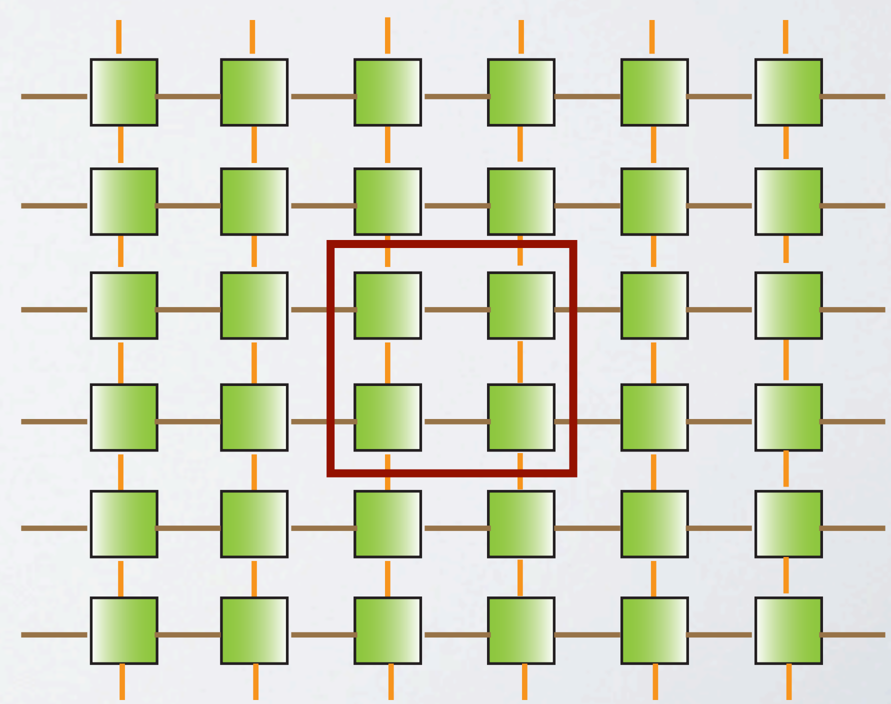

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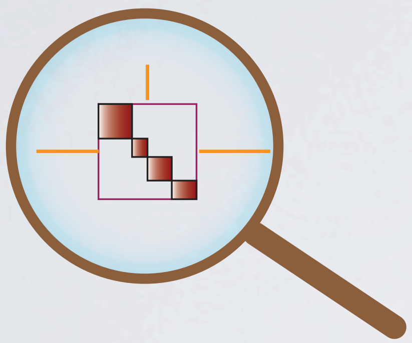

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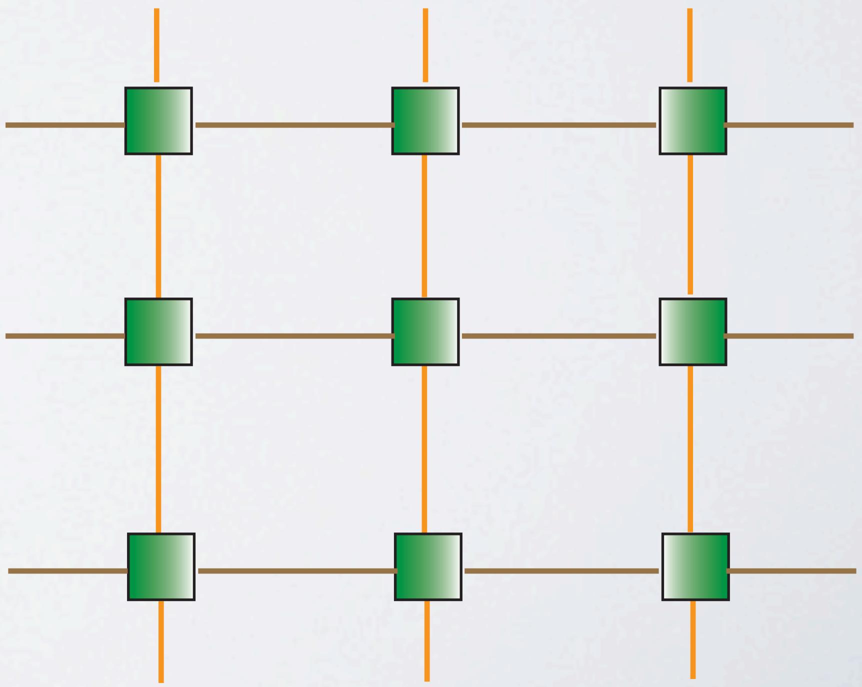

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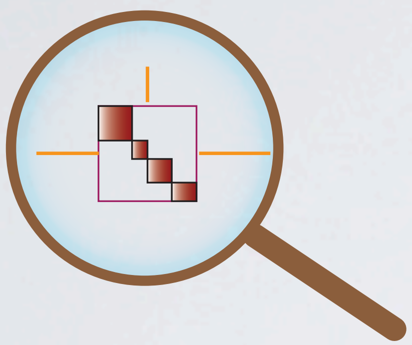

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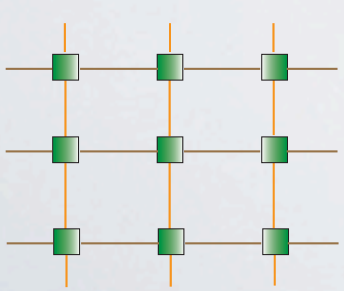

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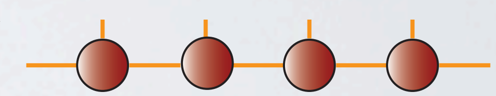

TNS as ansatz for the state

efficient algorithms for GS, low excited states, thermal, dynamics

White PRL 1992; Schollwöck RMP 2011
Vidal PRL 2003; Verstraete et al PRL 2004
Verstraete et al Adv Phys 2008; Orus Ann Phys 2014

USING TNS FOR LGT

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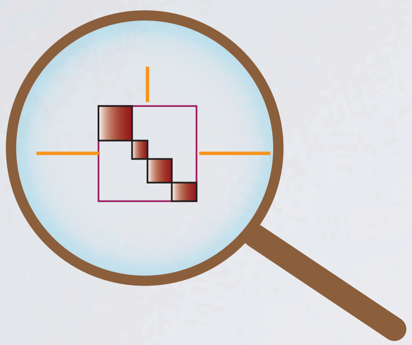

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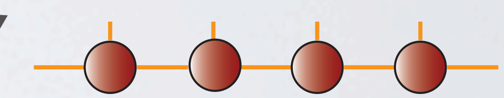

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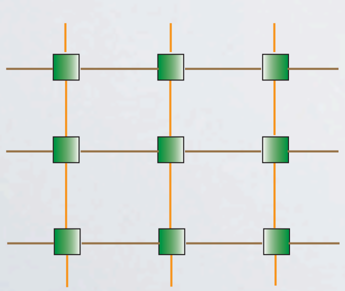

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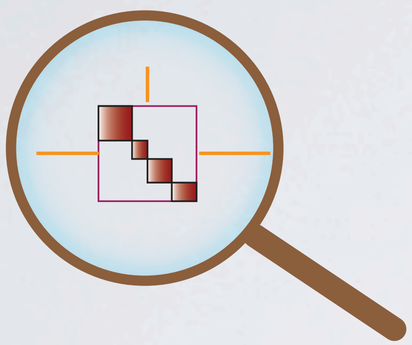

gauging the symmetry

explicitly invariant states

Tagliacozzo et al PRX 2014
Haegeman et al PRX 2014
Zohar et al Ann Phys 2015

general prescriptions, U(1), SU(2)

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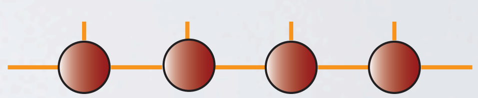

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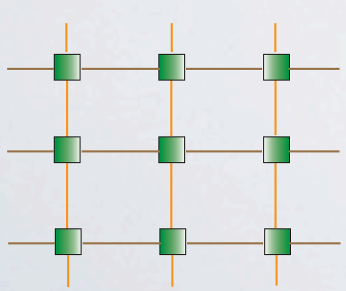

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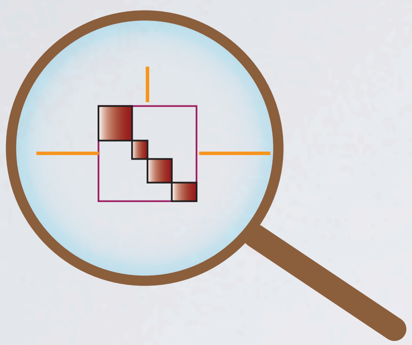

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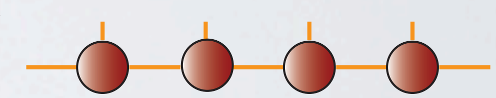

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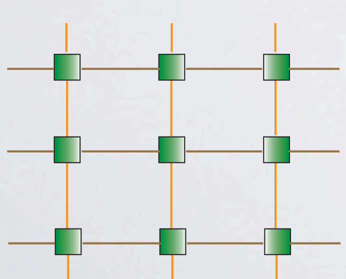

TRG approaches to classical
and quantum models

Liu et al PRD 2013
Shimizu, Kuramashi, PRD 2014
Kawauchi, Takeda 2015

USING TNS FOR LGT

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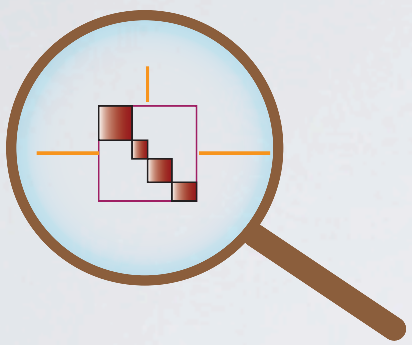

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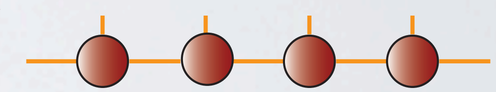
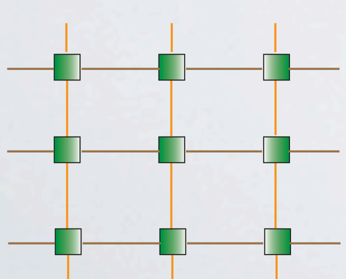


TNS as ansatz for the state



TRG approaches to classical
and quantum models

Liu et al PRD 2013
Shimizu, Kuramashi, PRD 2014
Kawauchi, Takeda 2015

next...

TNS FOR LGT early approaches

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TNS for classical gauge models

Meurice et al. 2013

TESTBENCH: SCHWINGER MODEL

Relevant states can be described as MPS

TN allow reliable continuum limit

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- Mass spectrum
- Chiral condensate (order parameter of chiral symmetry breaking)

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MCB, Cichy, Jansen, Cirac, JHEP11(2013)158
PoS 2014 arXiv:1412.0596
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Thermal equilibrium states well approximated by MPO

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Multiflavour Schwinger model

also Buyens et al., arXiv:1606.03385

Phase diagram at finite density: no sign problem

S. Kühn et al., in preparation

SCHWINGER MODEL

continuum

$$H = \int dx \left[-i\bar{\Psi}\gamma^1\partial_1\Psi + g\bar{\Psi}\gamma^1 A_1\Psi + m\bar{\Psi}\Psi + \tfrac{1}{2}E^2 \right]$$

plus constraint: Gauss' Law

$$\partial_1 E = g\bar{\Psi}\gamma^0\Psi$$

SCHWINGER MODEL

discretized

$$H = -\frac{i}{2a}\sum_n \left(\phi_n^\dagger e^{i\theta_n}\phi_{n+1} - \text{h.c.}\right) + m\sum_n (-1)^n \phi_n^\dagger \phi_n + \frac{ag^2}{2}\sum_n L_n^2$$

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MPS representation with OPEN BOUNDARIES

basis $|\ldots s_e \, \ell \, s_o \, \ell \, s_e \, \ell \, s_o \ldots\rangle$

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can be implemented with explicitly gauge invariant tensors

Buyens et al., PRL 2014

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$$|\ell_0 \ldots s_e\; s_o\; s_e\; s_o \ldots\rangle$$ non-local terms

COMPUTING THE SPECTRUM WITH MPS

JHEP11(2013)158

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Scan parameters

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m/g mass gaps and GS energy density in the continuum $x \to \infty$

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N $N \propto x$ (up to ~850)

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m/g	mass gaps and GS energy density in the continuum $x \to \infty$
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JHEP11(2013)158

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$x \to \infty$

x — $x \in [5,\ 600]$

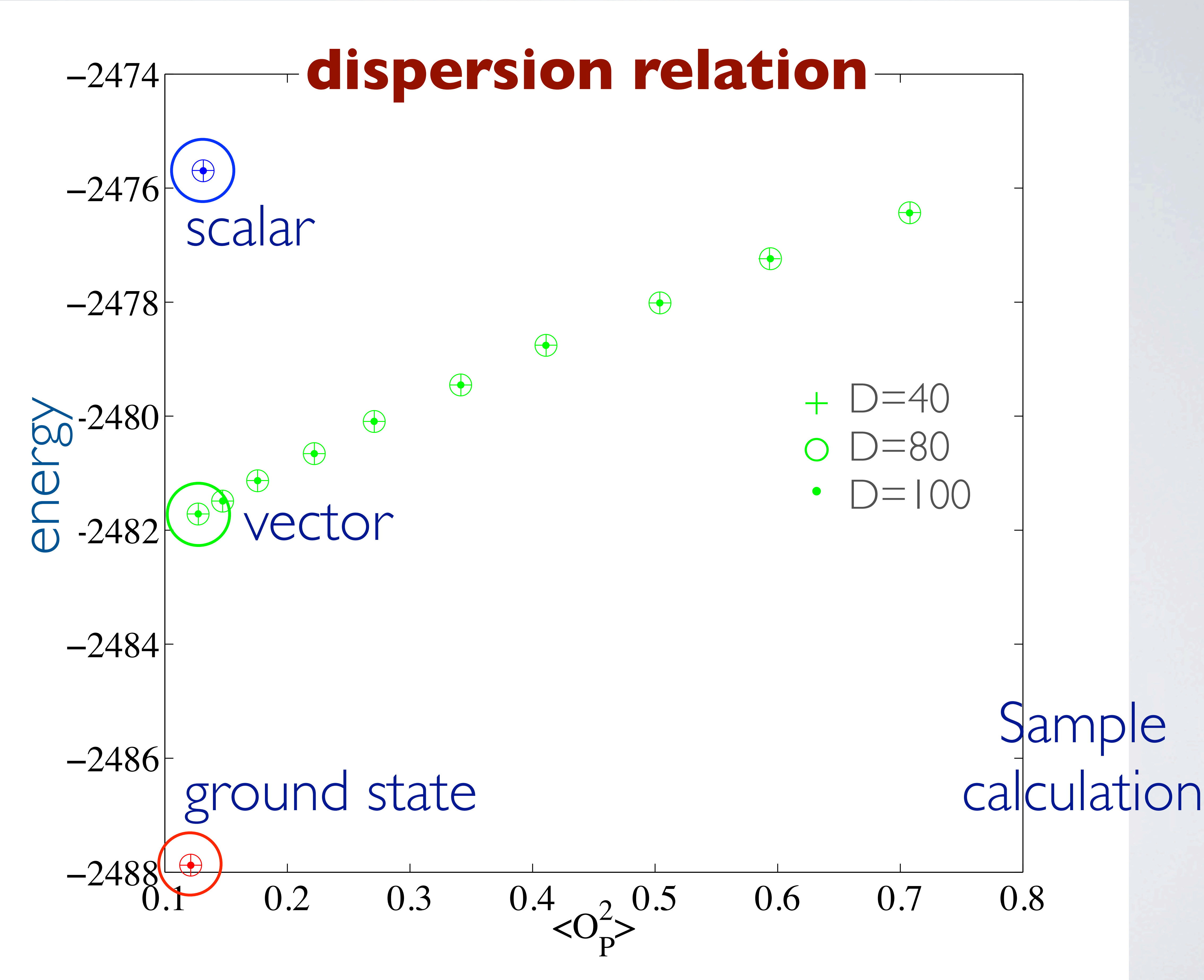
finite-size

N — $N \propto x$ (up to ~850)

convergence

D — $D \in [20,\ 120]$

JHEP11(2013)158

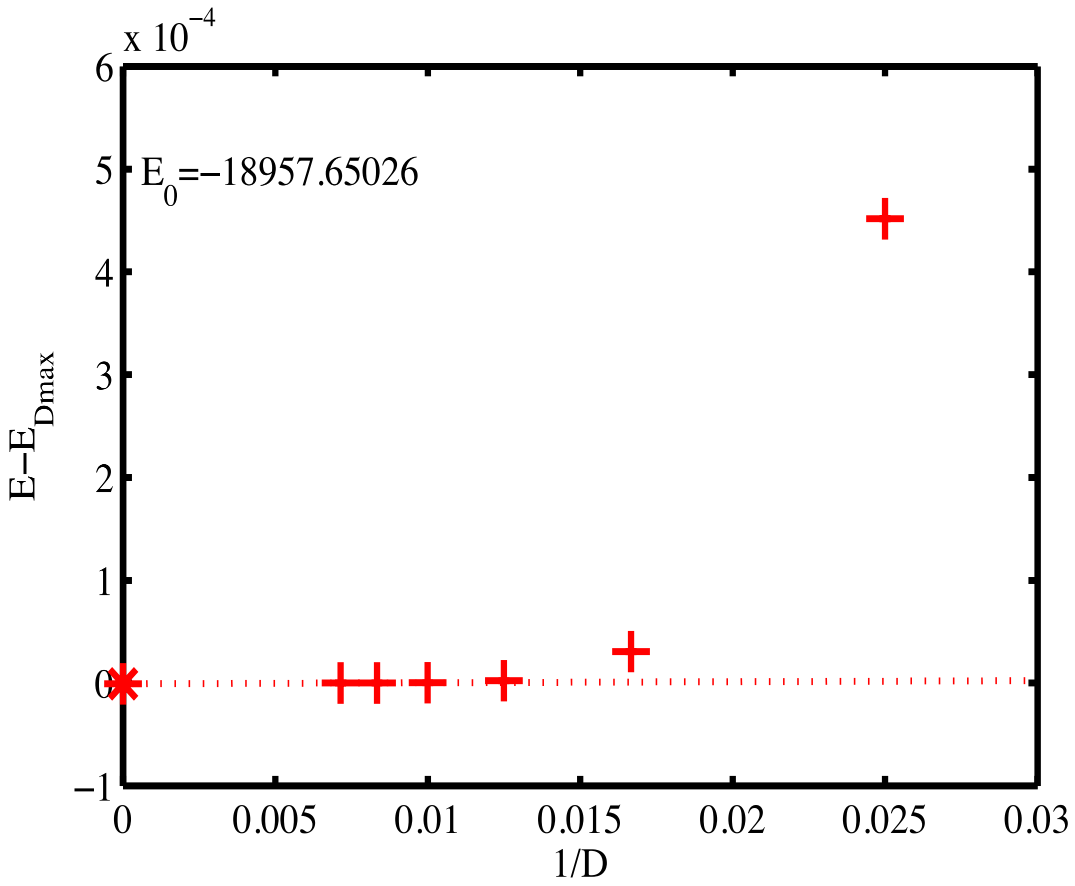


dispersion relation
energy
–2474
–2476
–2478
-2480
-2482
–2484
–2486
–2488
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
<O$_P^2$>
scalar
vector
ground state
D=40
D=80
D=100
Sample calculation

I truncation error

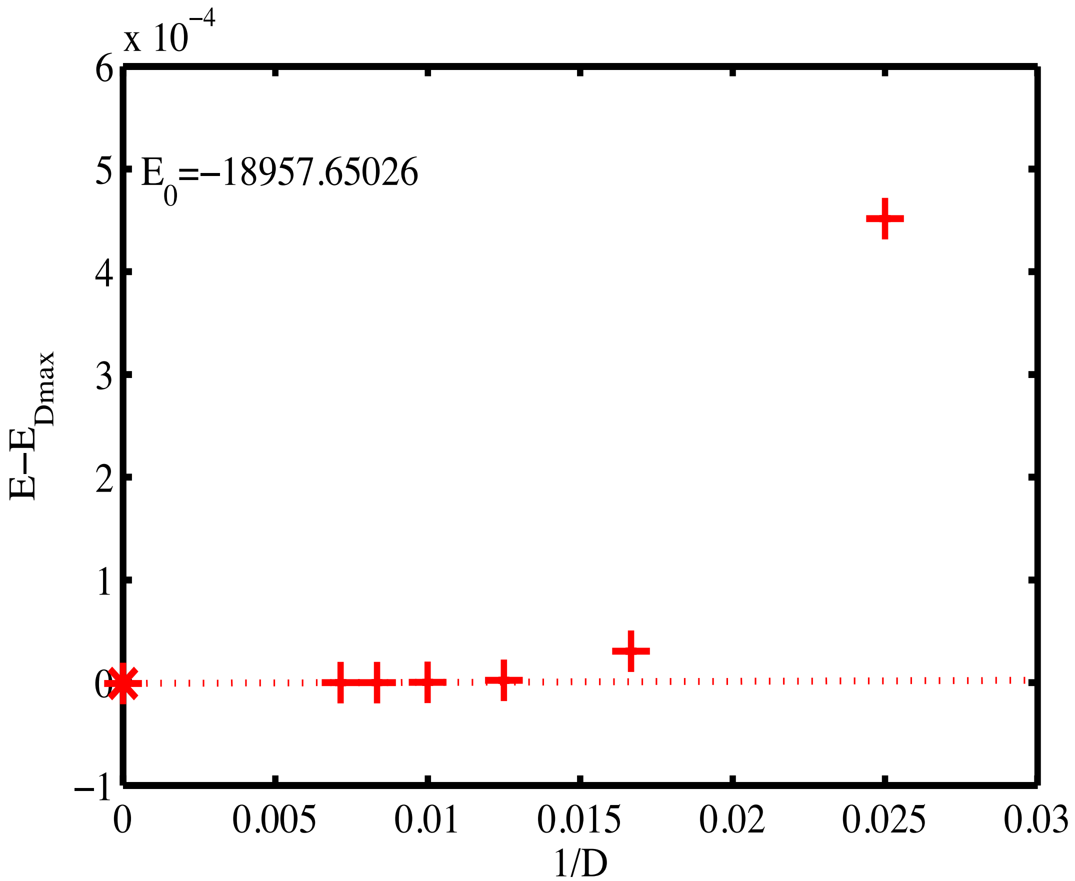
$m/g = 0 \quad x = 100$

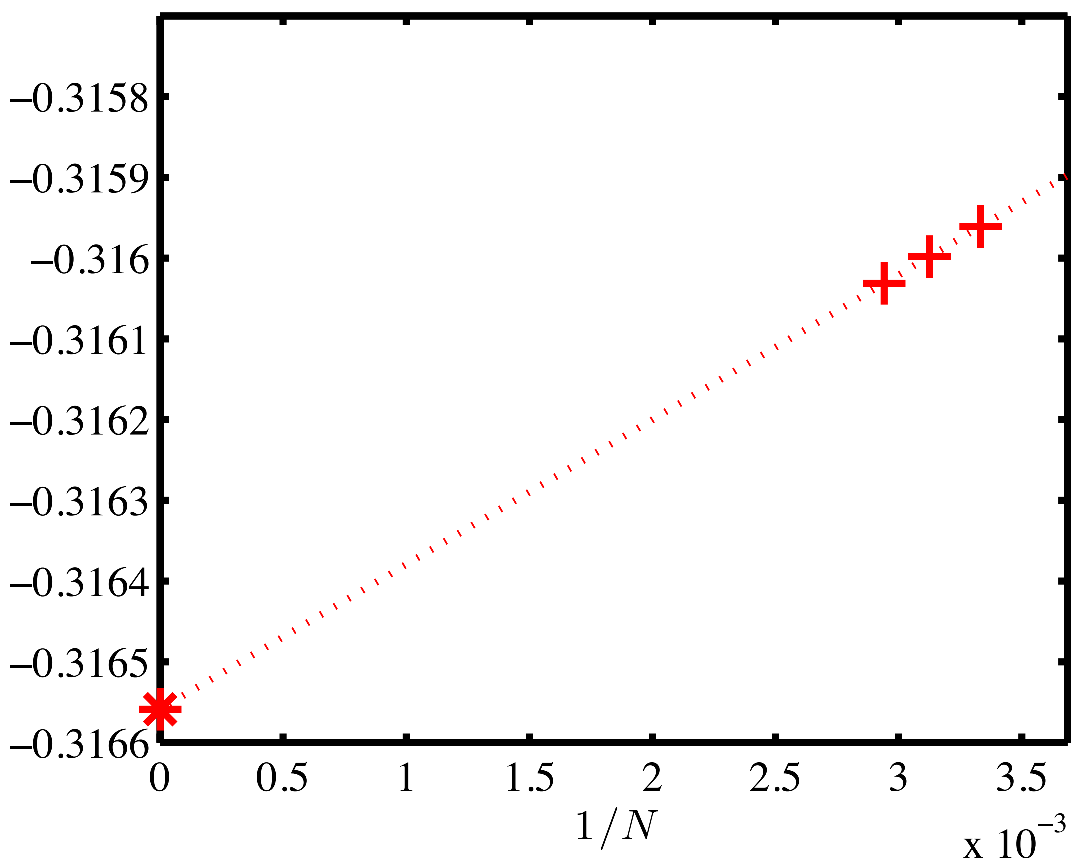
$N = 300$



JHEP11(2013)158

2 finite-size scaling $m/g = 0 \quad x = 100$





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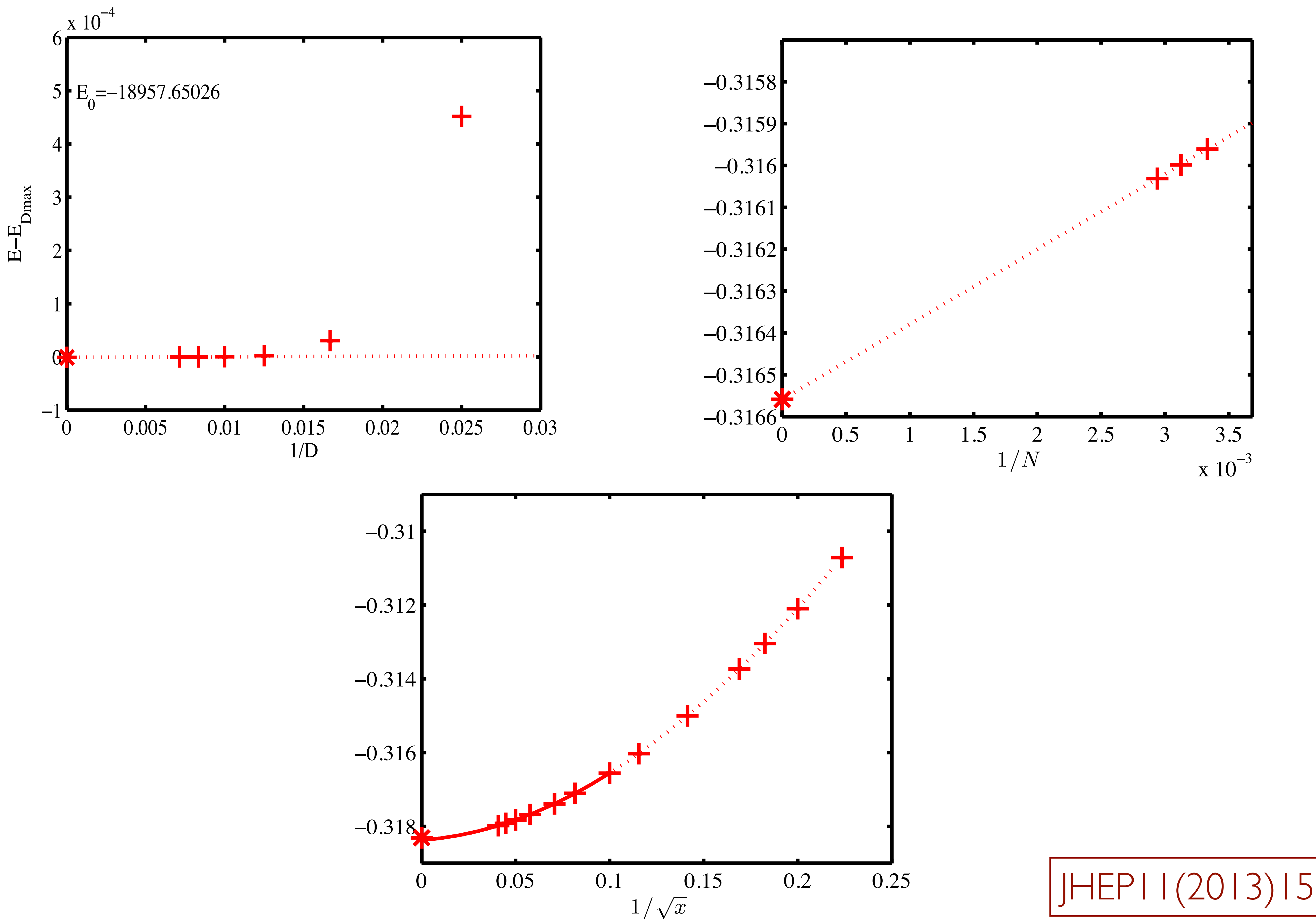
3 continuum limit $m/g = 0$



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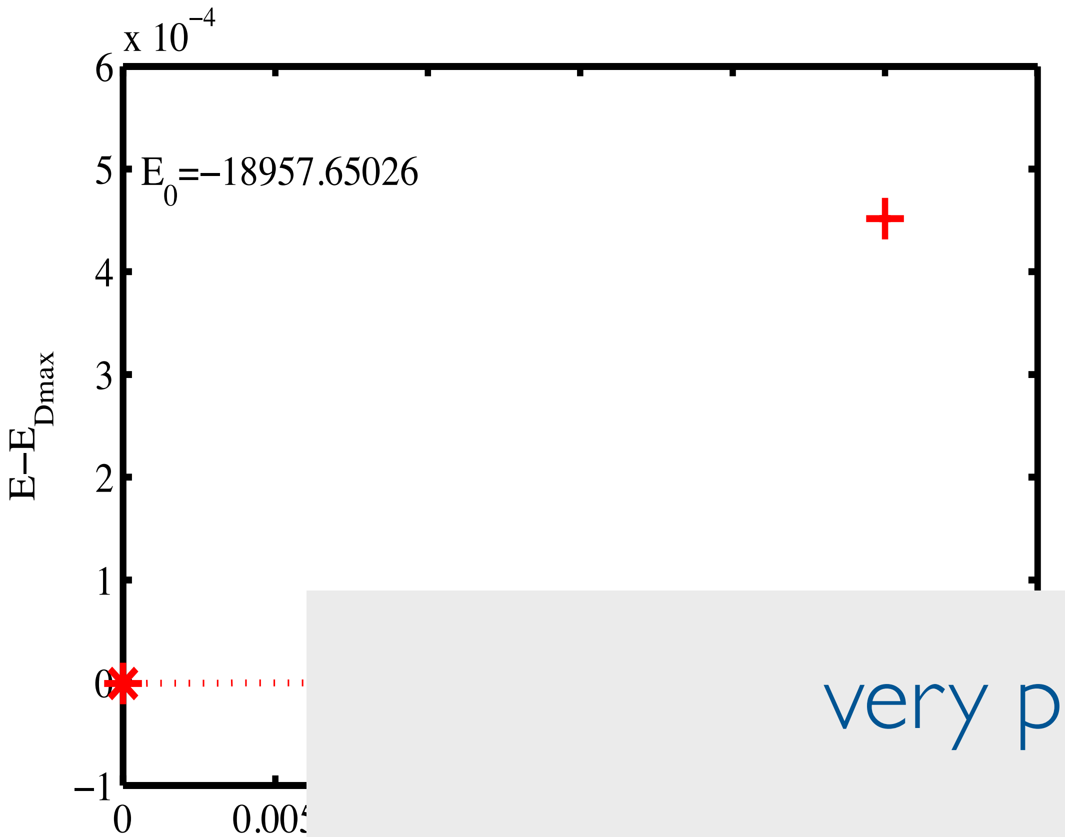
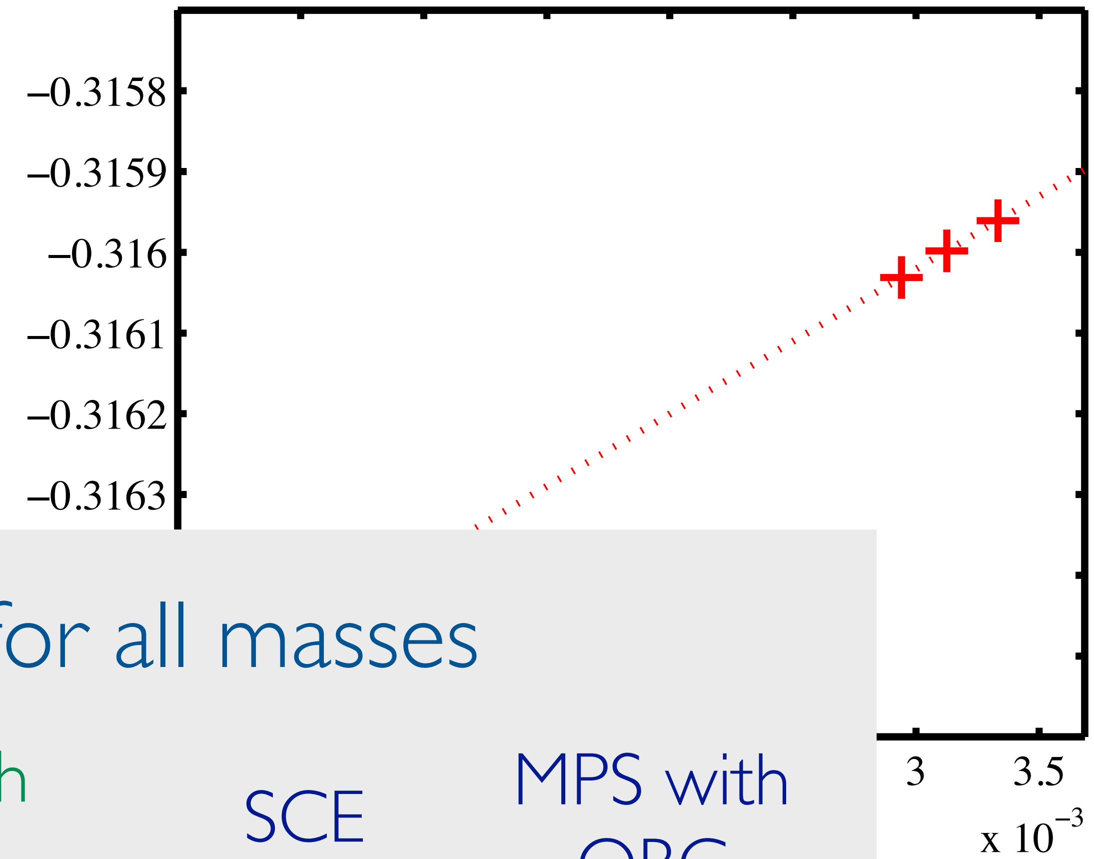
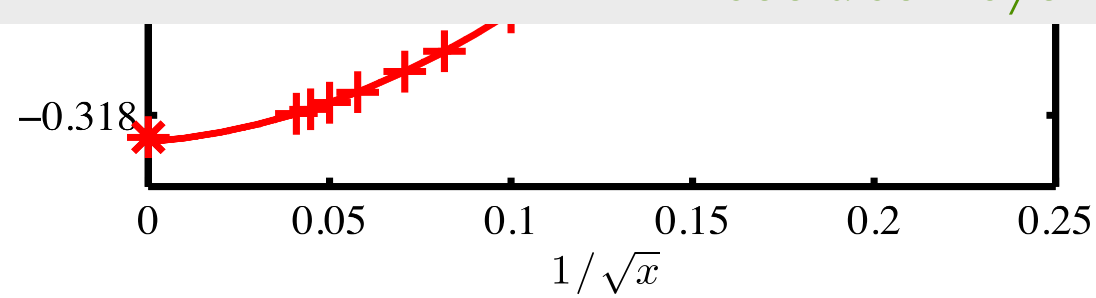




very precise for all masses

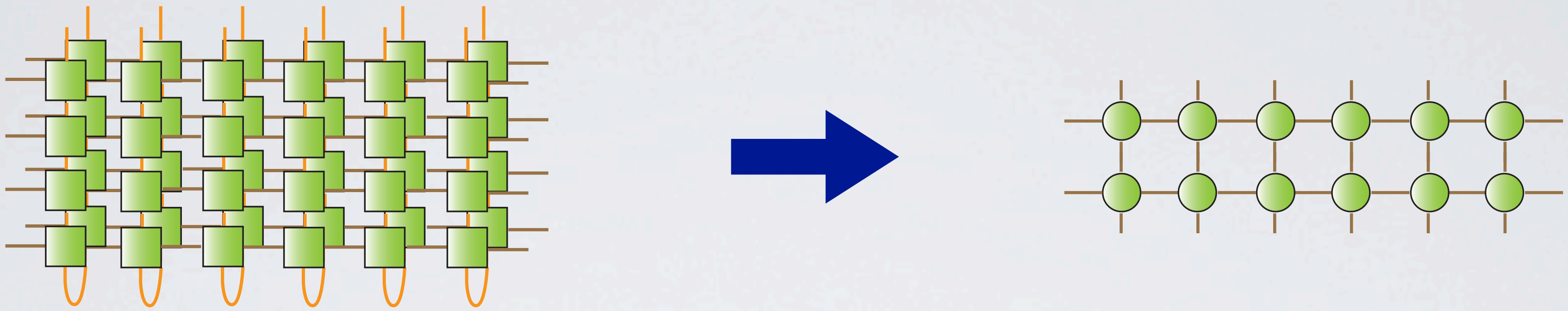
m/g	DMRG	MPS with OBC	SCE	MPS with OBC
0	0,5641859	0.56414(26)	1,128379	1.1283(10)
0,125	0.53950(7)	0.53946(20)	1.22(2)	1.221(2)
0,25	0.51918(5)	0.51915(14)	1.24(3)	1.239(6)
0,5	0.48747(2)	0.48748(6)	1.20(3)	1.231(5)

see also Buyens et al., PRL 2014



JHEP11(2013)158

THERMAL PROPERTIES WITH MPO



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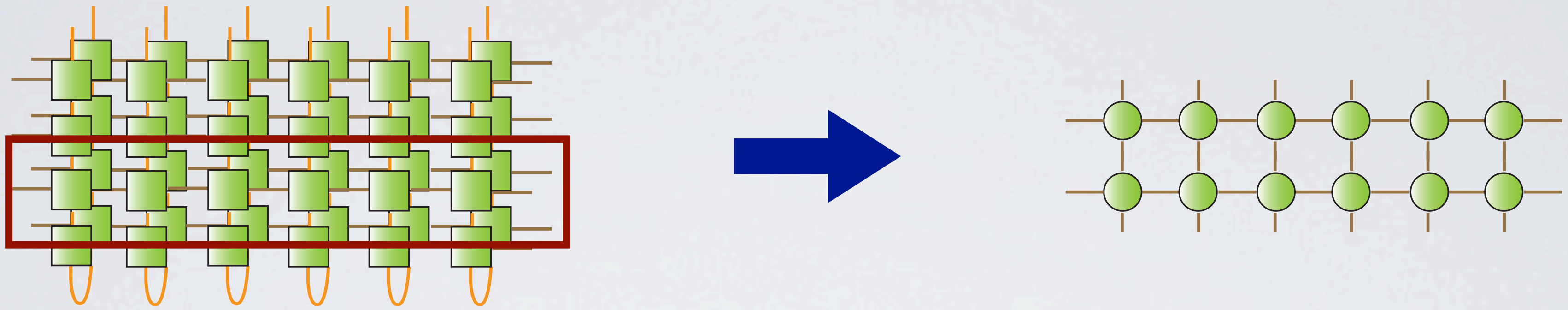

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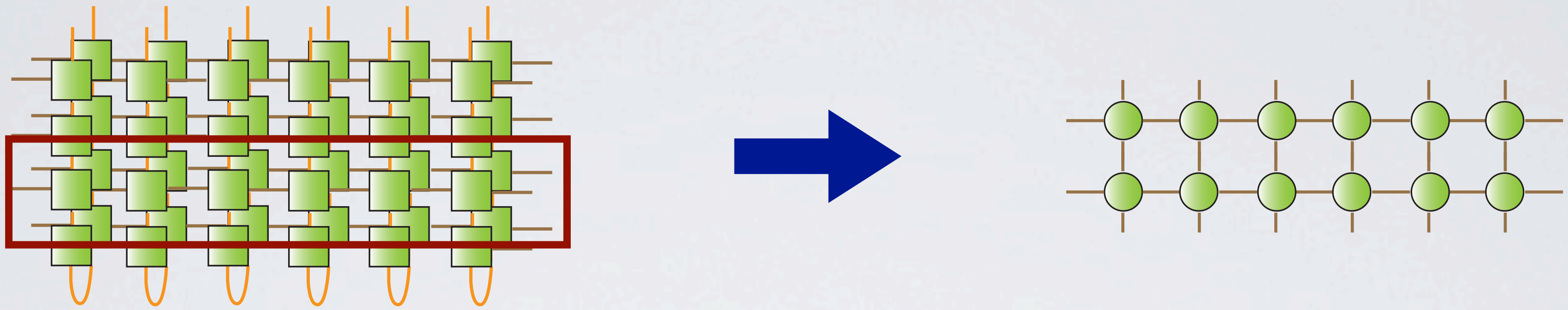

$$H = x \sum_{n=0}^{N-2} \left[\sigma_n^+ \sigma_{n+1}^- + \sigma_n^- \sigma_{n+1}^+\right] + \frac{\mu}{2} \sum_{n=0}^{N-1} \left[1 + (-1)^n \sigma_n^z\right] + \sum_{n=0}^{N-2} (L_n + \alpha)^2$$

hopping → even-odd

diagonal terms

$$L_n = \ell_0 + \frac{1}{2} \sum_{k \leq n} \sigma_n^3 + \dots$$

long range

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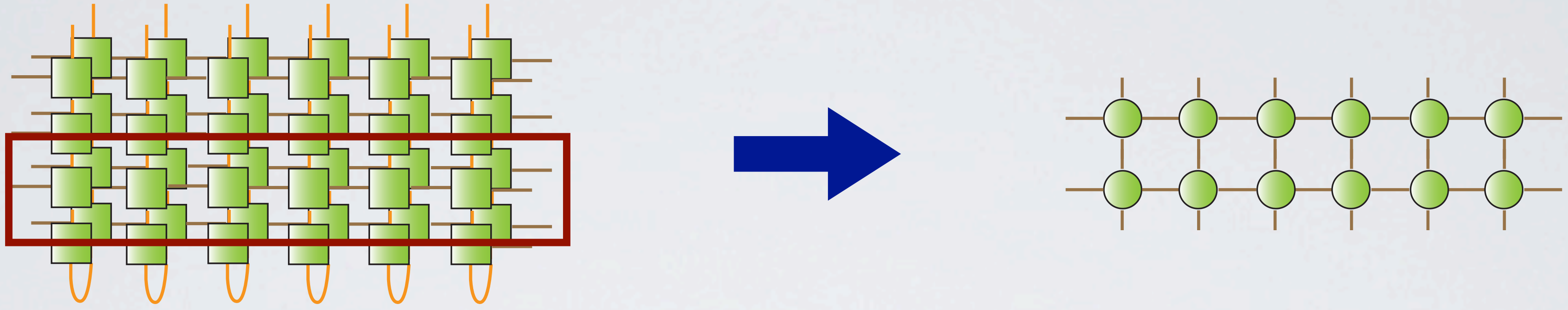

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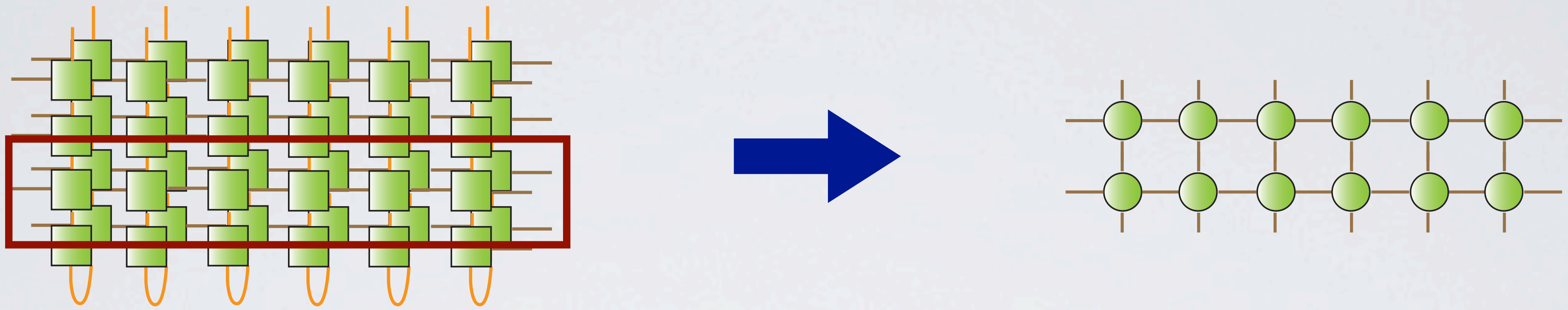

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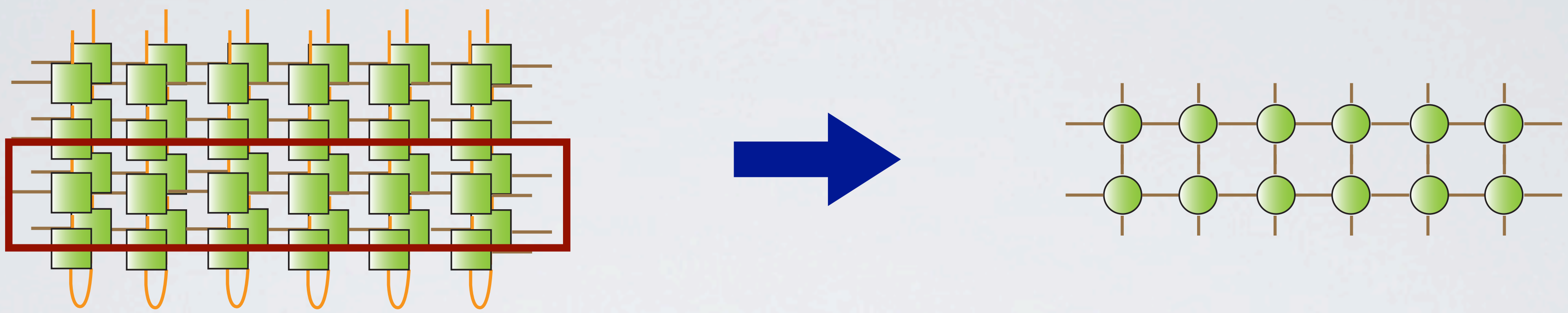

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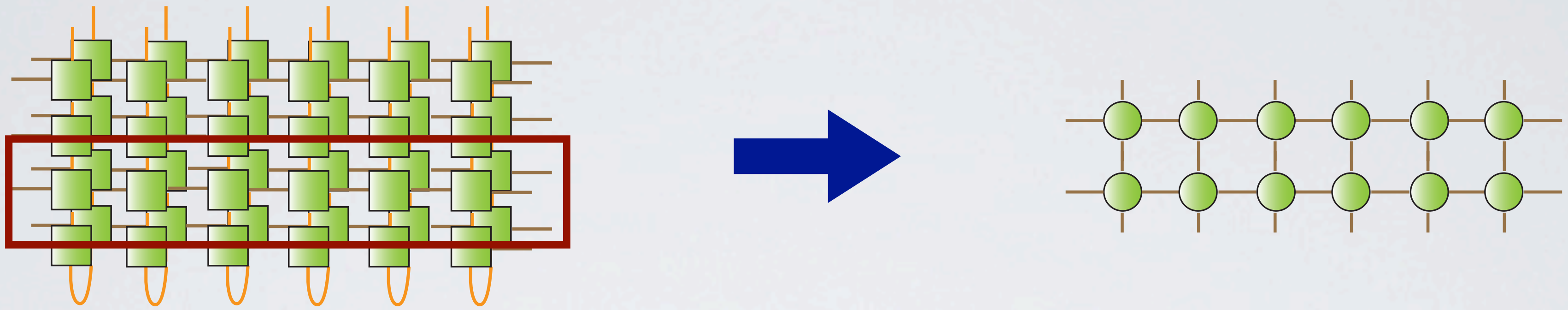

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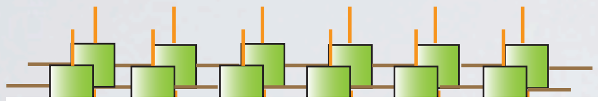

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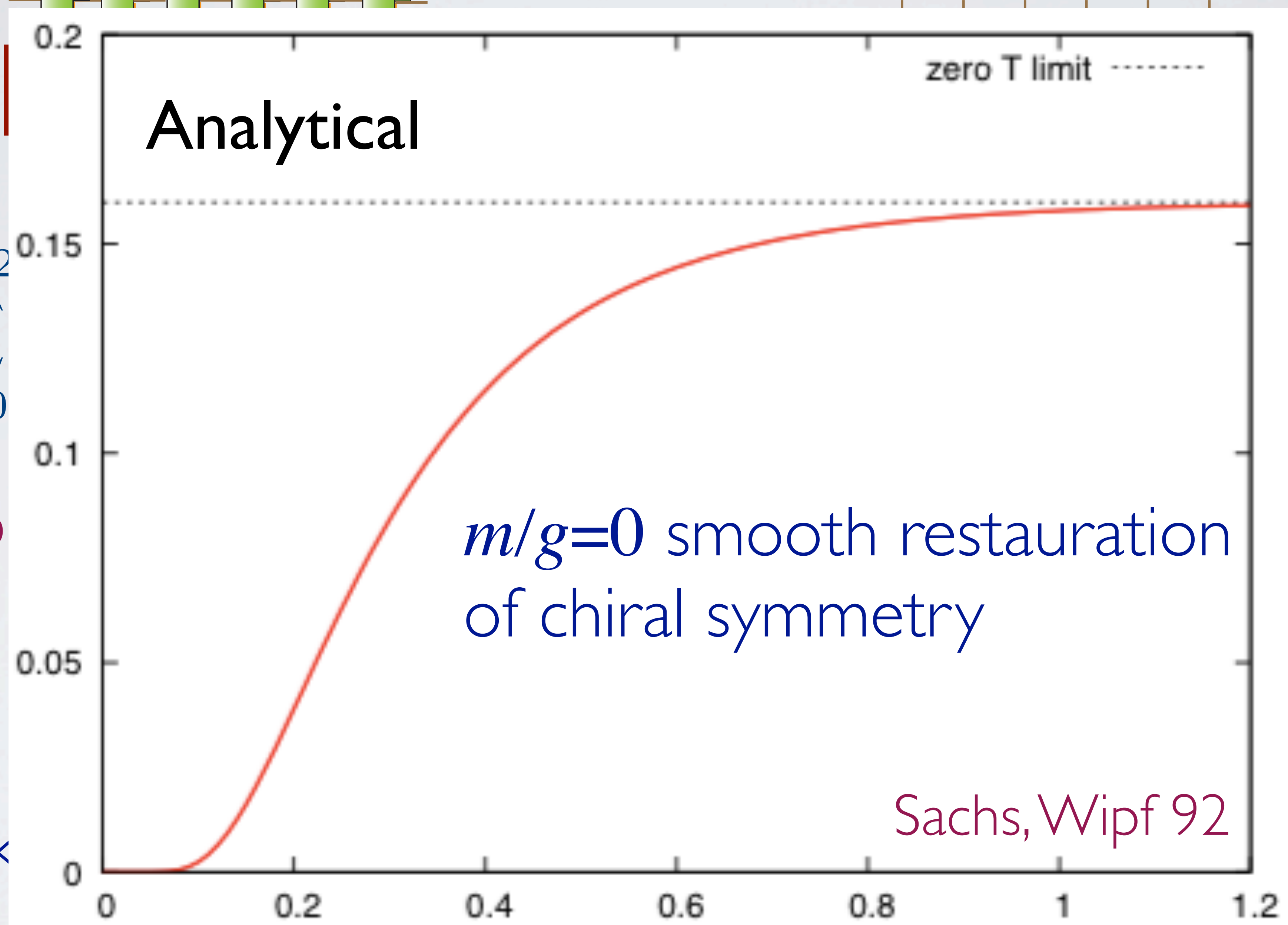
hopp

range



Suzuk

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Scan parameters; perform extrapolations for each β

JHEP11(2013)158
PRD 93, 094512 (2016)

THERMAL PROPERTIES WITH MPO

Scan parameters; perform extrapolations for each β

m/g chiral condensate as a function of temperature, in the continuum $x \to \infty$

x $x \in [9,\ 1024]$

N $N \propto \sqrt{x}$ (up to ~800)

δ sufficiently small for resolution

D $D \in [80,\ 160]$

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finite-size

N $\quad N \propto \sqrt{x}$ (up to ~800)

extrapolation

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convergence

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$x \to \infty$

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plus L_{cut}

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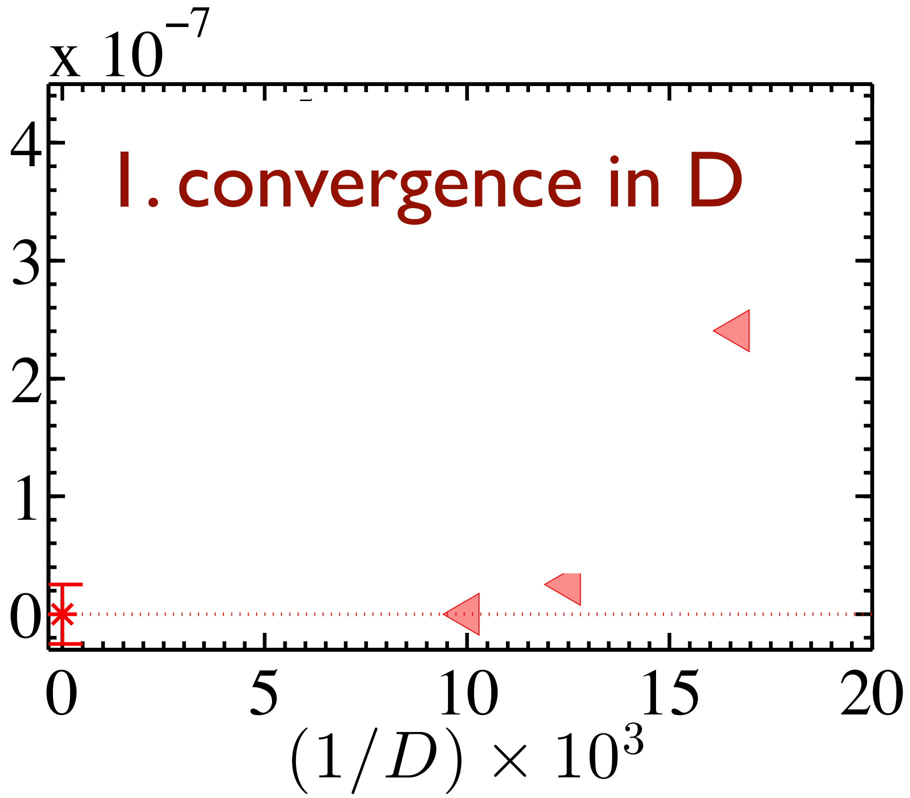

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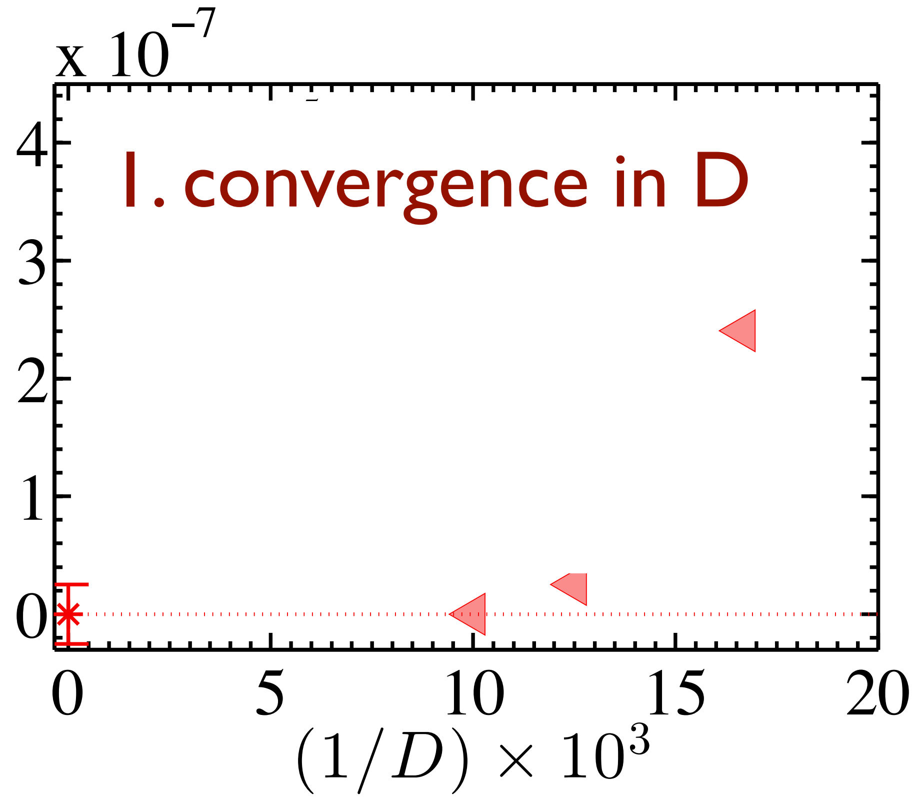
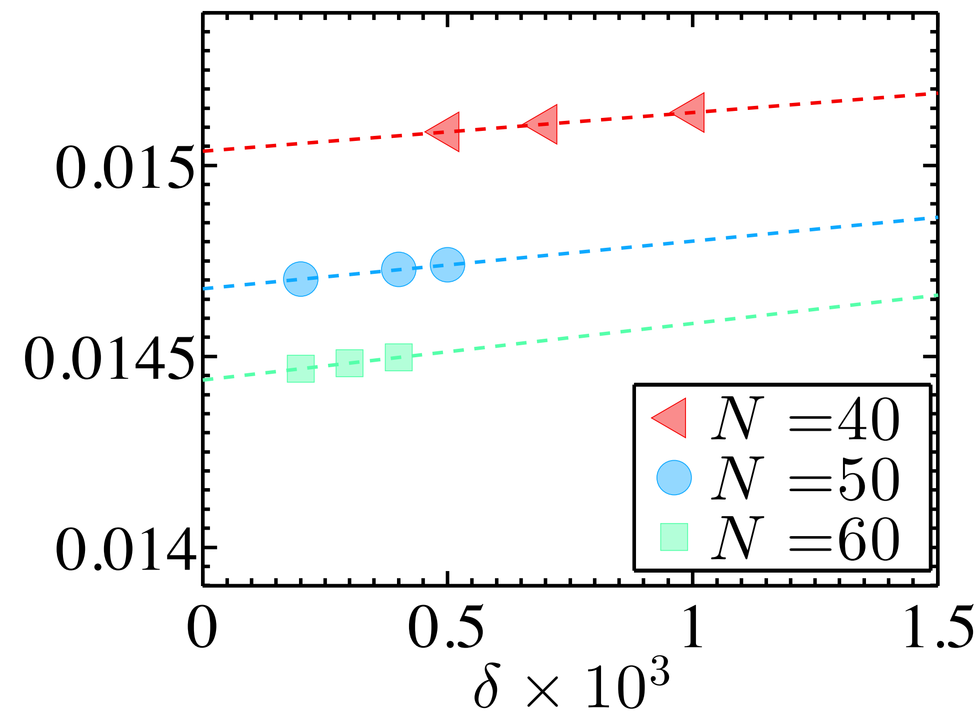


2. convergence in δ



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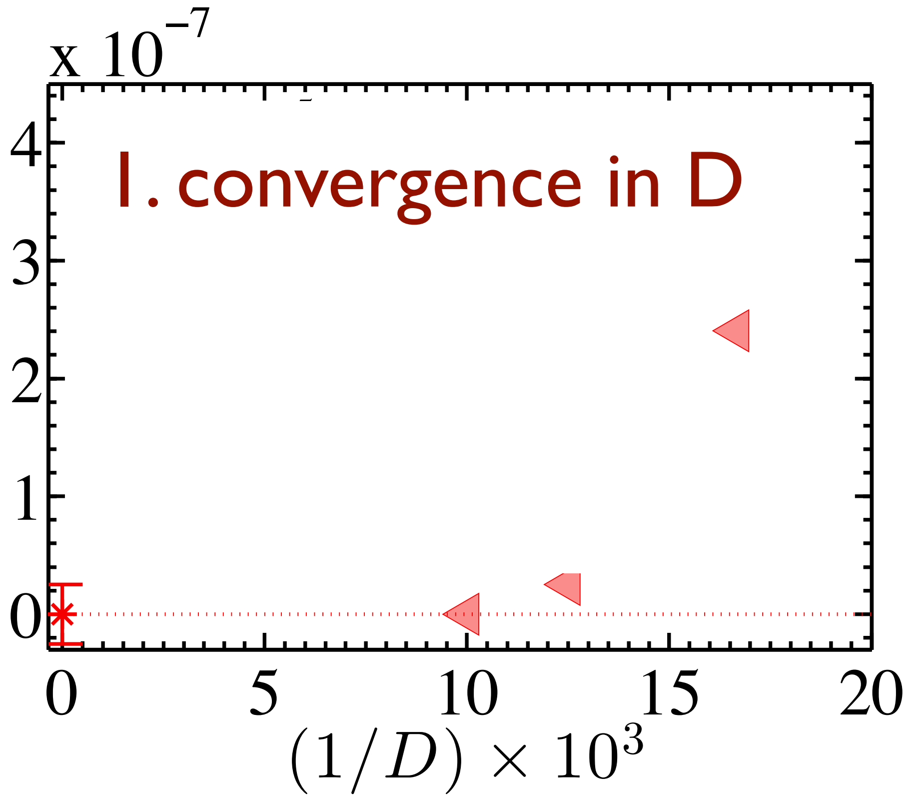
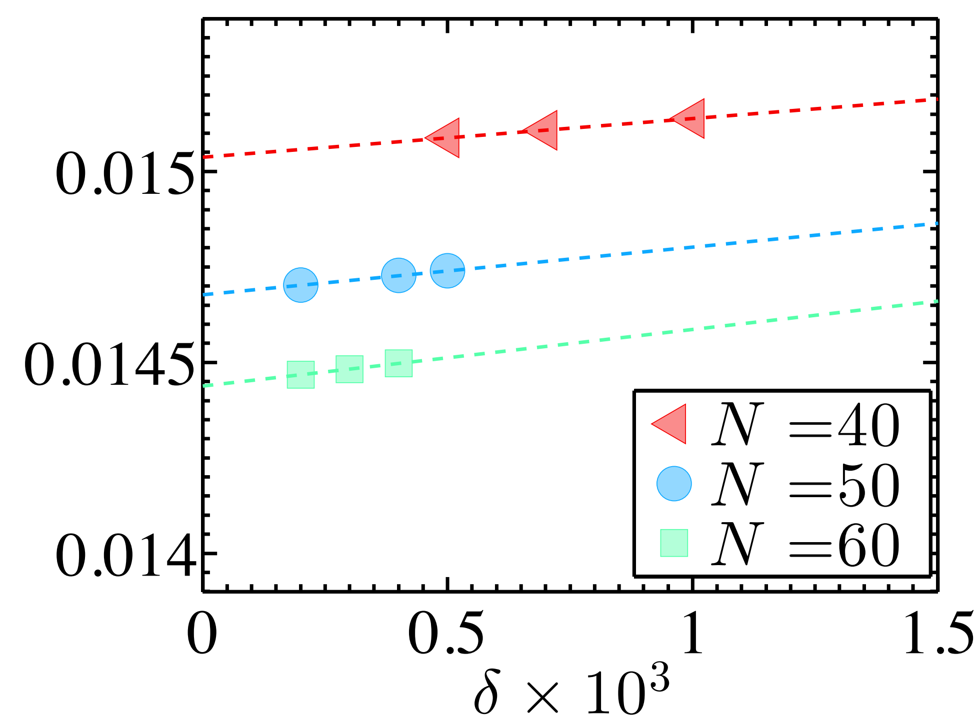
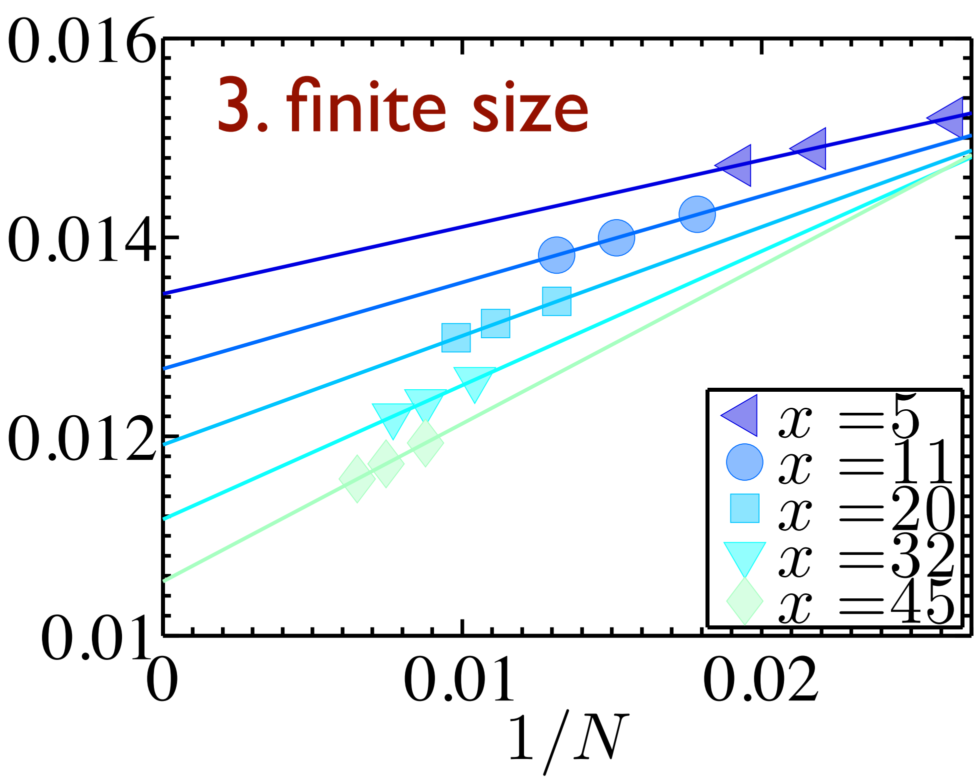
2. convergence in δ

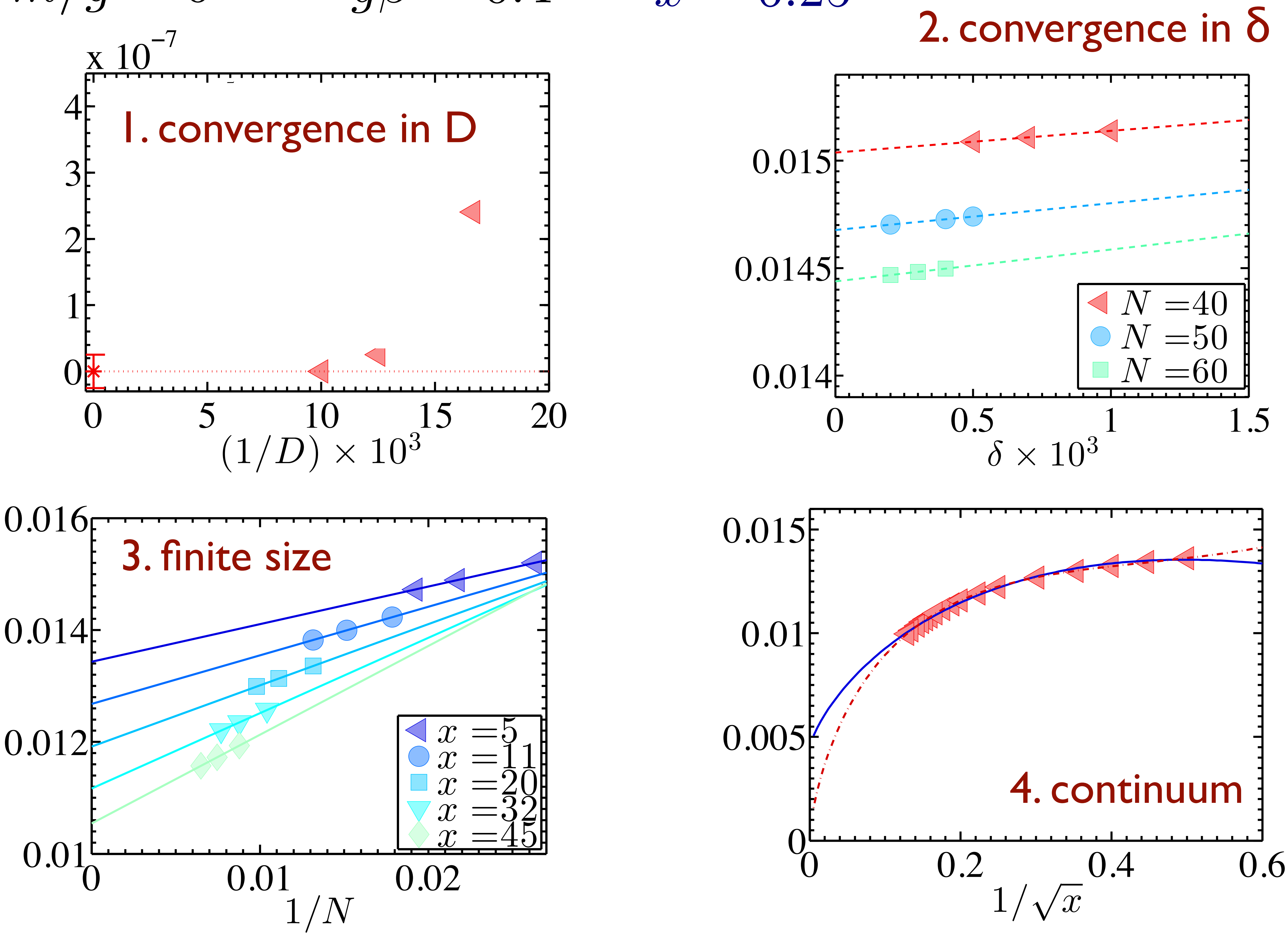






THERMAL PROPERTIES WITH MPO

$m/g = 0$ $g\beta = 0.4$ $x = 6.25$



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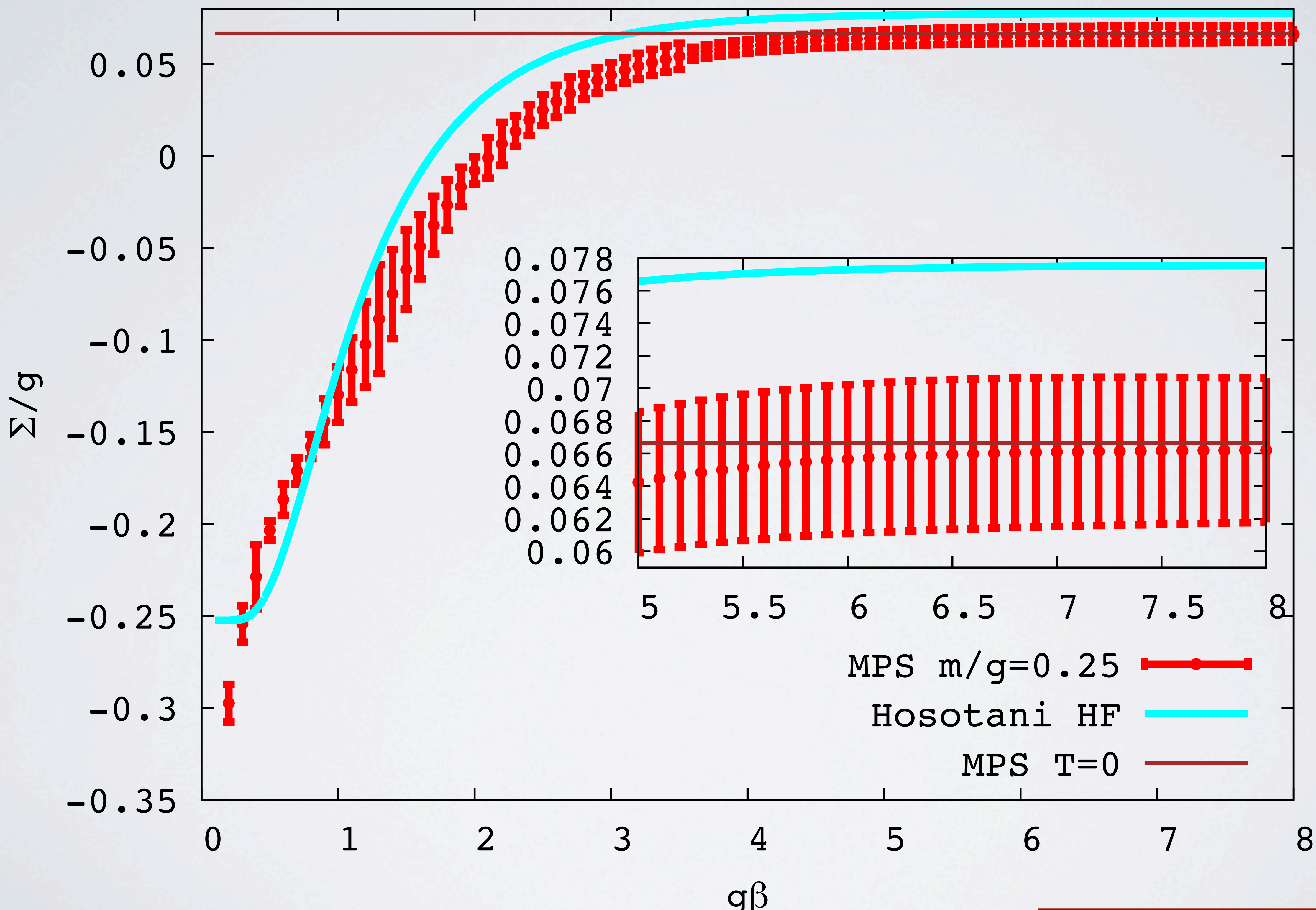
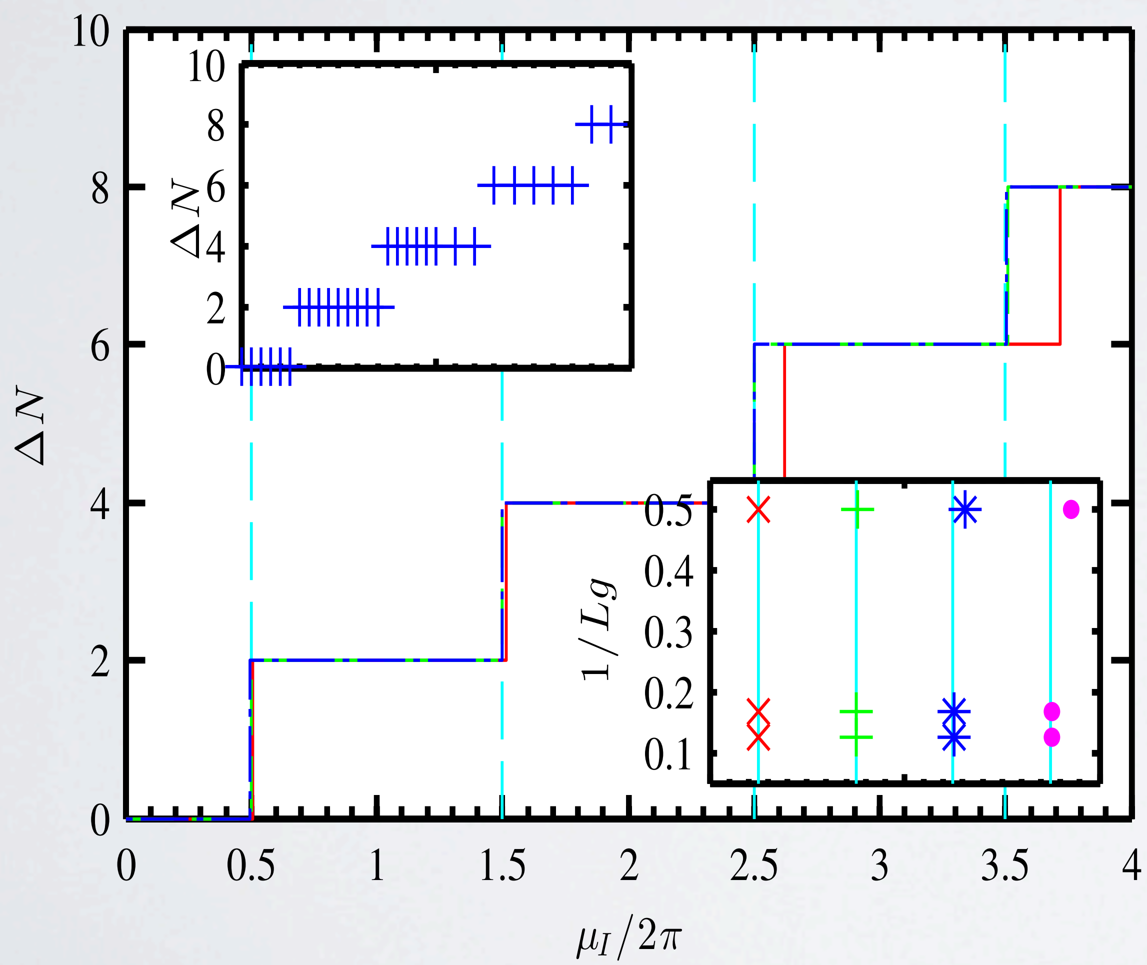


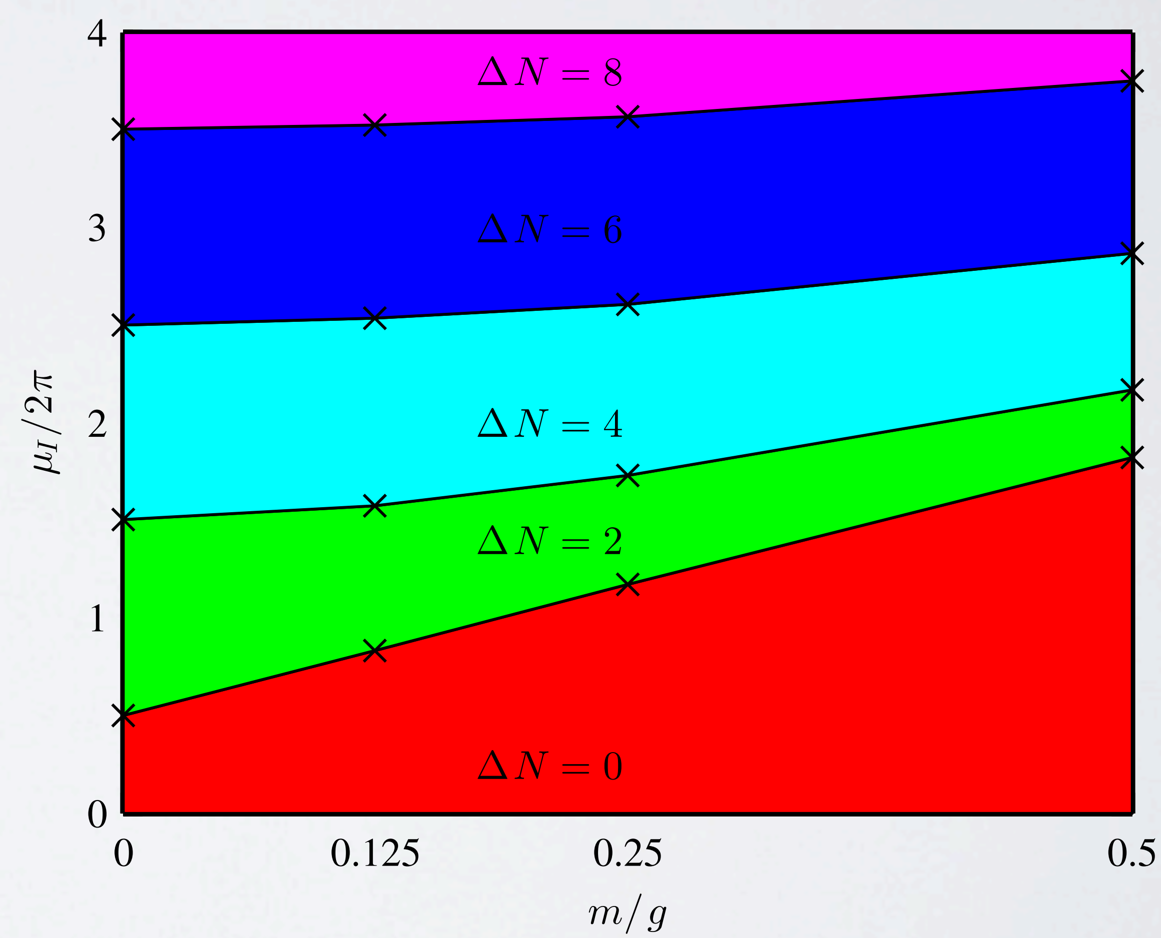
PRD 93, 094512 (2016)

FINITE DENSITY WITH MPS

Several fermion flavors, chemical potentials

ground state density changes (first order PT)





S. Kühn et al, in preparation

In this talk...

TNS = entanglement based ansatz

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Feasibility for LQFT

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high numerical precision attainable (controlled errors)

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- spectrum, thermal equilibrium, finite density, (some) dynamics

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Not in this talk...

- generalizations (continuous TNS), ...
- applications to quantum simulation settings

U. Wiese's talk

THANKS

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U. Wiese's talk

QUANTUM SIMULATIONS

MPS (TNS) tool for classical simulation

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Ultimately wanted: quantum simulator of HEP models

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Several proposals exist using ultracold atoms

finite dimensional dof

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Banerjee et al., PRL 2012

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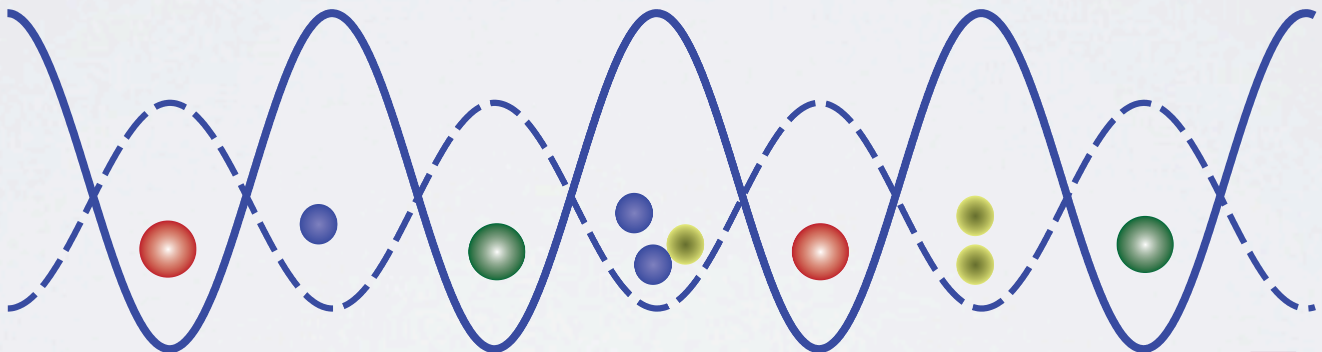
Banerjee et al., PRL 2012

MPS can be very good to validate such schemes

Rico et al. PRL 2014
Pichler et al, PRX 2016

QUANTUM SIMULATION OF SCHWINGER MODEL



Zohar et al. PRA 2013

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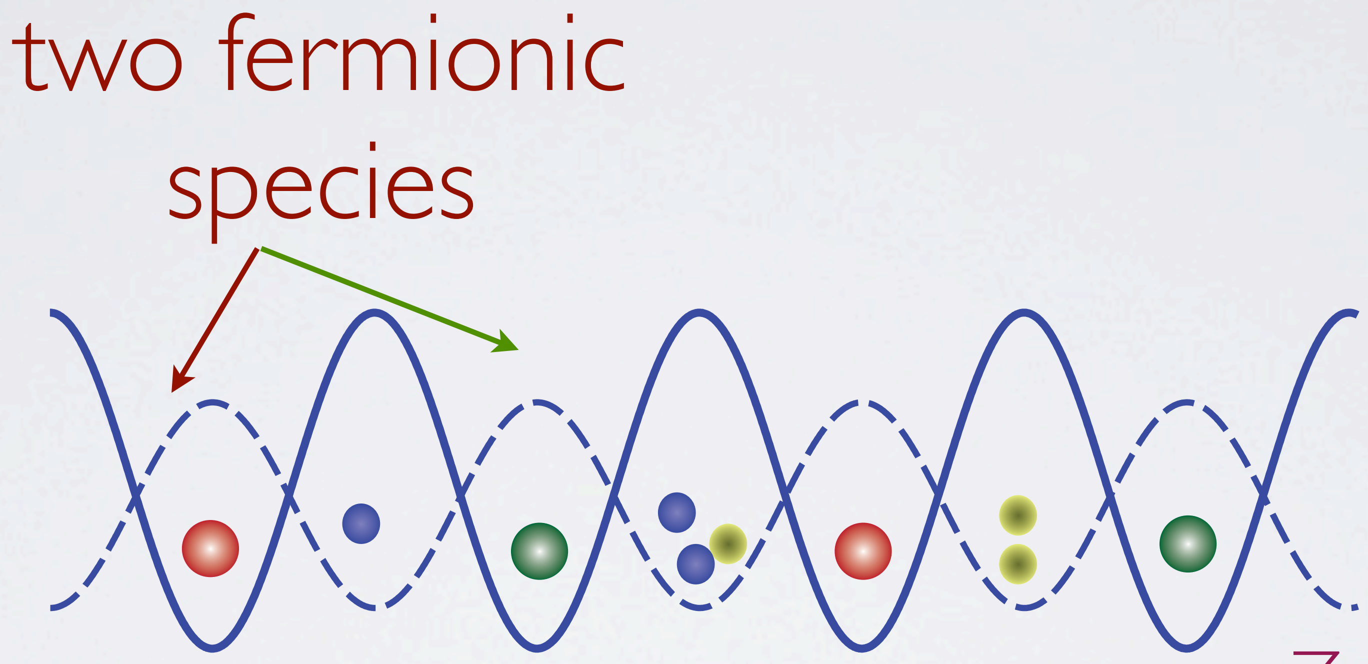

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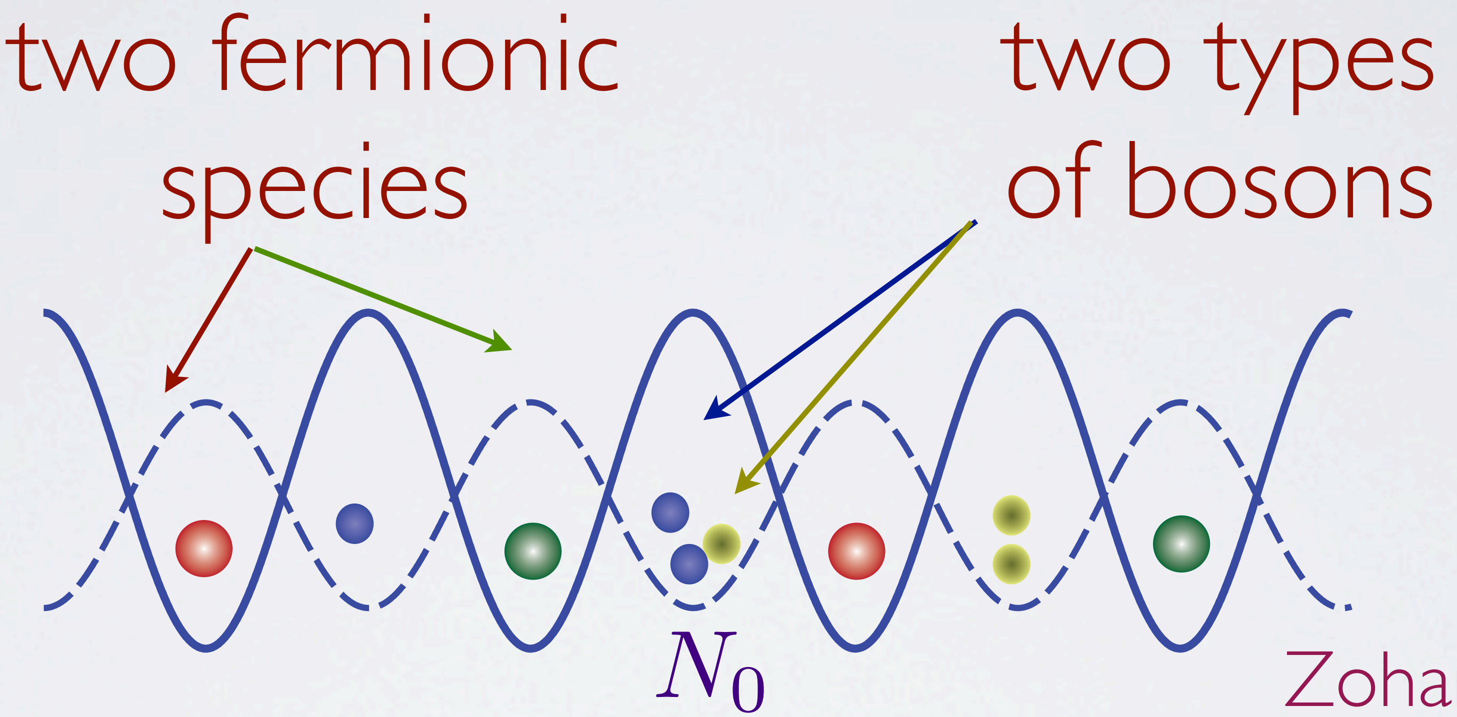

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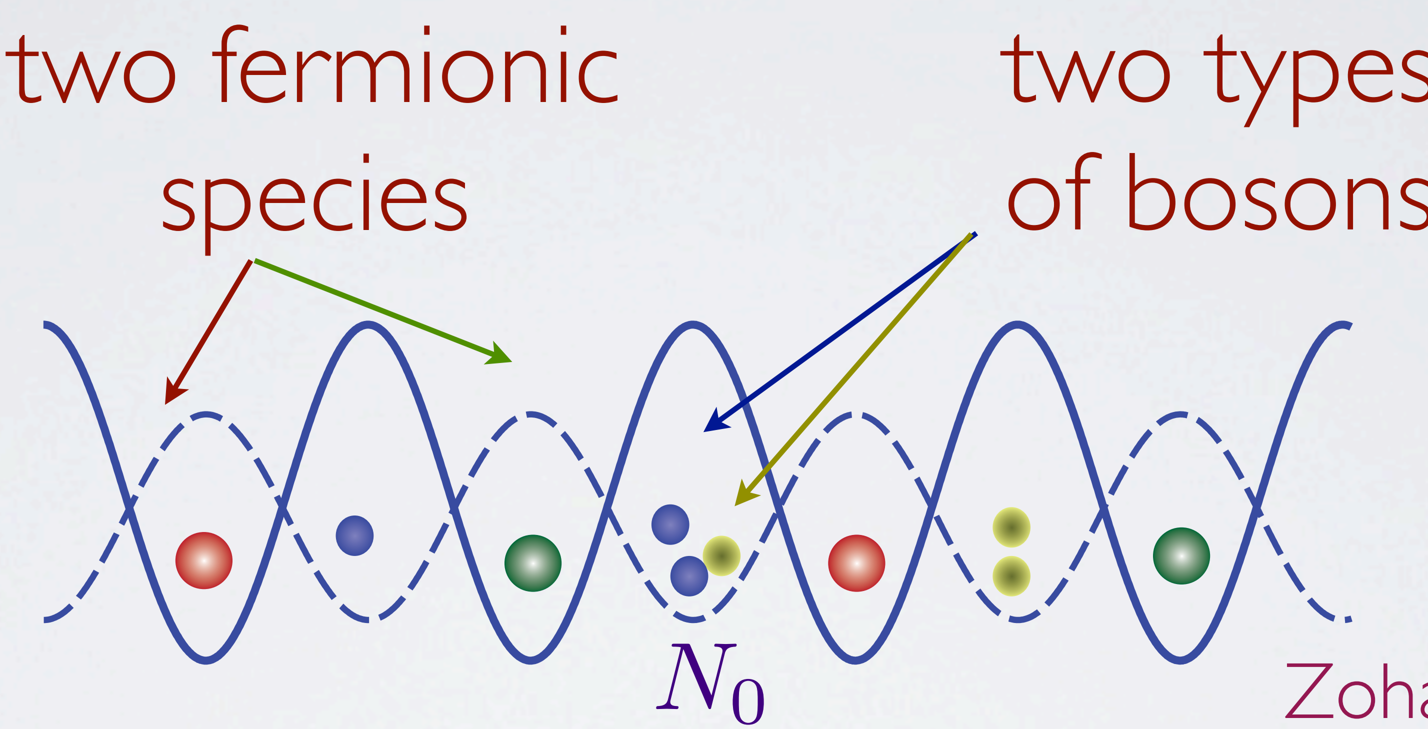

Zohar et al. PRA 2013

Gauge invariance from angular momentum conservation

$$\frac{1}{\sqrt{\ell(\ell+1)}}\Psi_n^\dagger a_n^\dagger b_n \Psi_{n+1} \xrightarrow[N_0 \gg]{} \Psi_n^\dagger e^{i\phi_n}\Psi_{n+1}$$

MPS: FEASIBILITY STUDY

Questions that MPS can answer

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Questions that MPS can answer

Approaching the continuum limit

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Questions that MPS can answer

- Approaching the continuum limit
- Effect of small N_0, errors...

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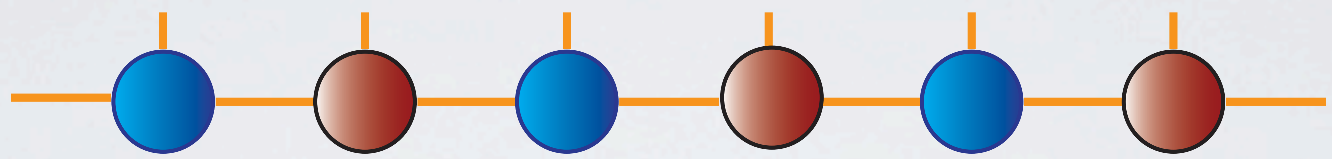
Questions that MPS can answer

Approaching the continuum limit

Effect of small N_0, errors...

Adiabatic preparation procedure: scaling

MPS: FEASIBILITY STUDY



S. Kühn et al., Phys. Rev. A 90, 042305 (2014)

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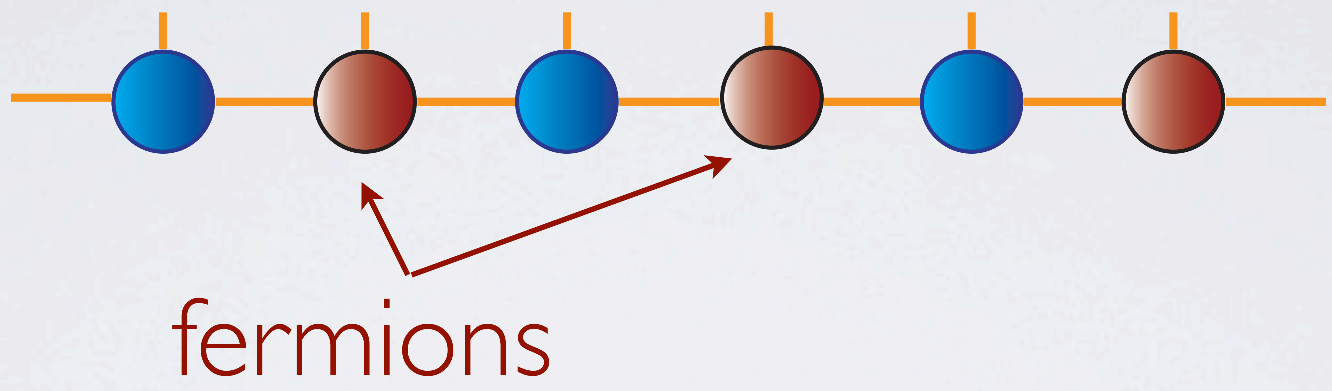

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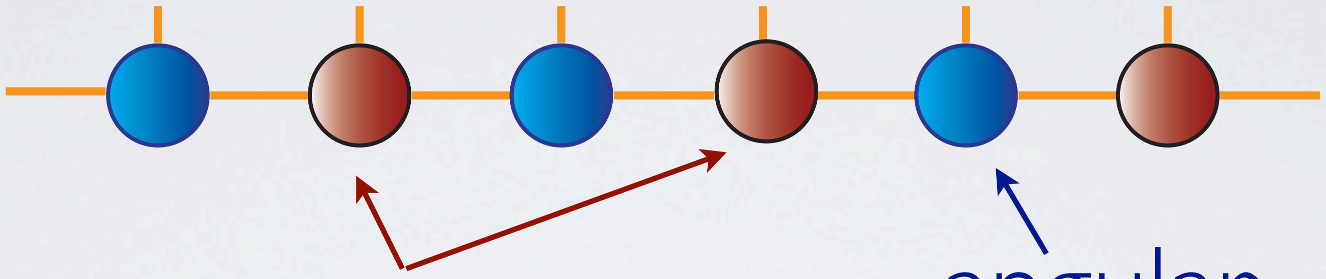

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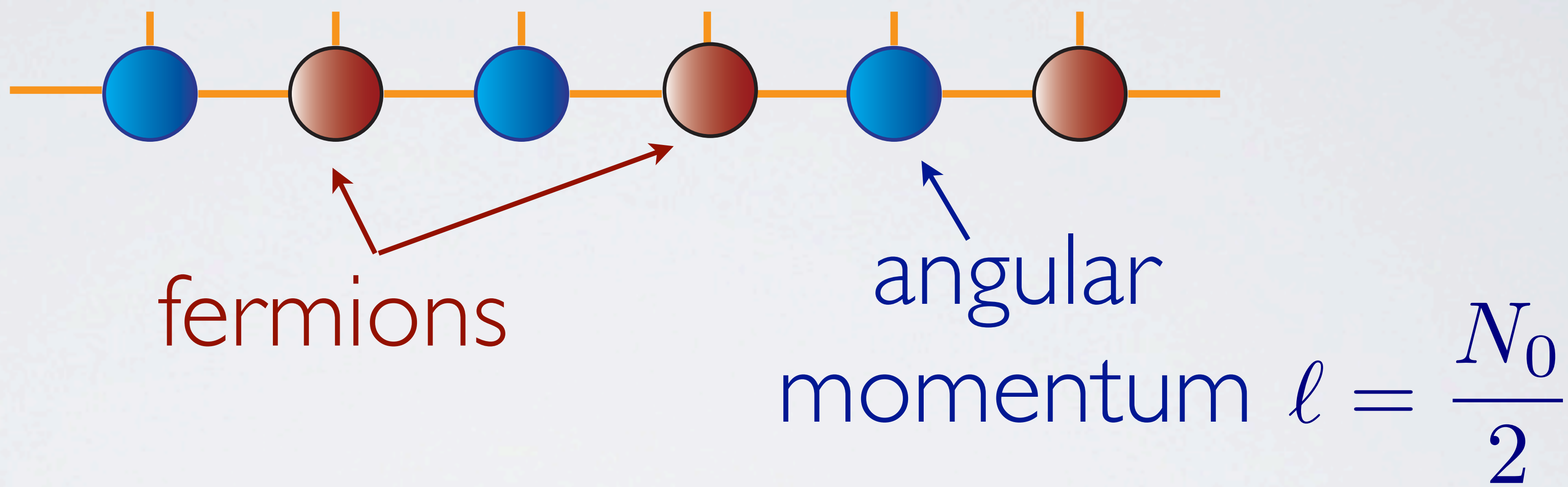


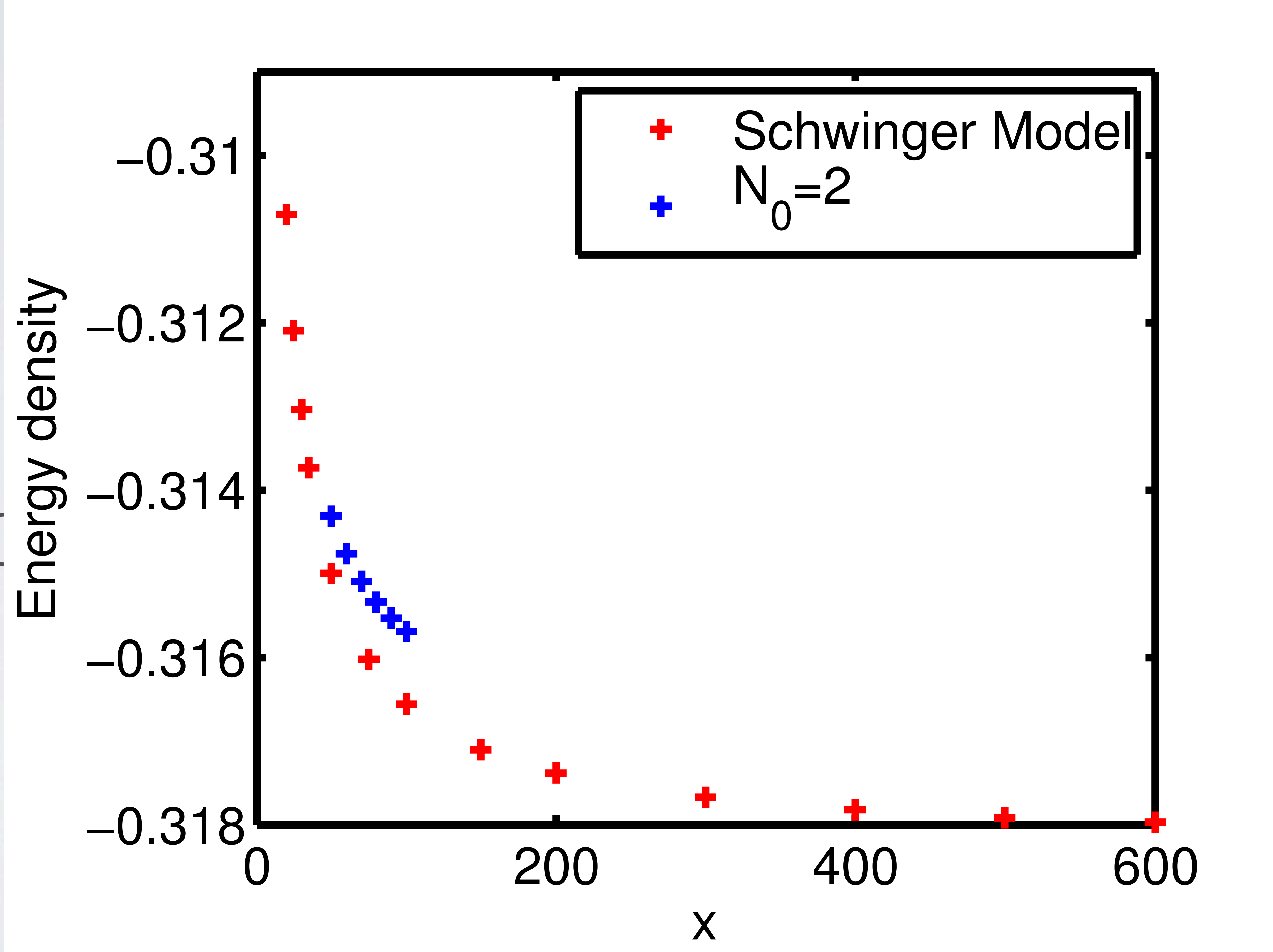
Continuum limit

As learned from the MPS simulations

Study convergence of the GS

S. Kühn et al., Phys. Rev. A 90, 042305 (2014)

MPS: FEASIBILITY STUDY



$\frac{N_0}{2}$

ons

S. Kühn et al., Phys. Rev. A 90, 042305 (2014)

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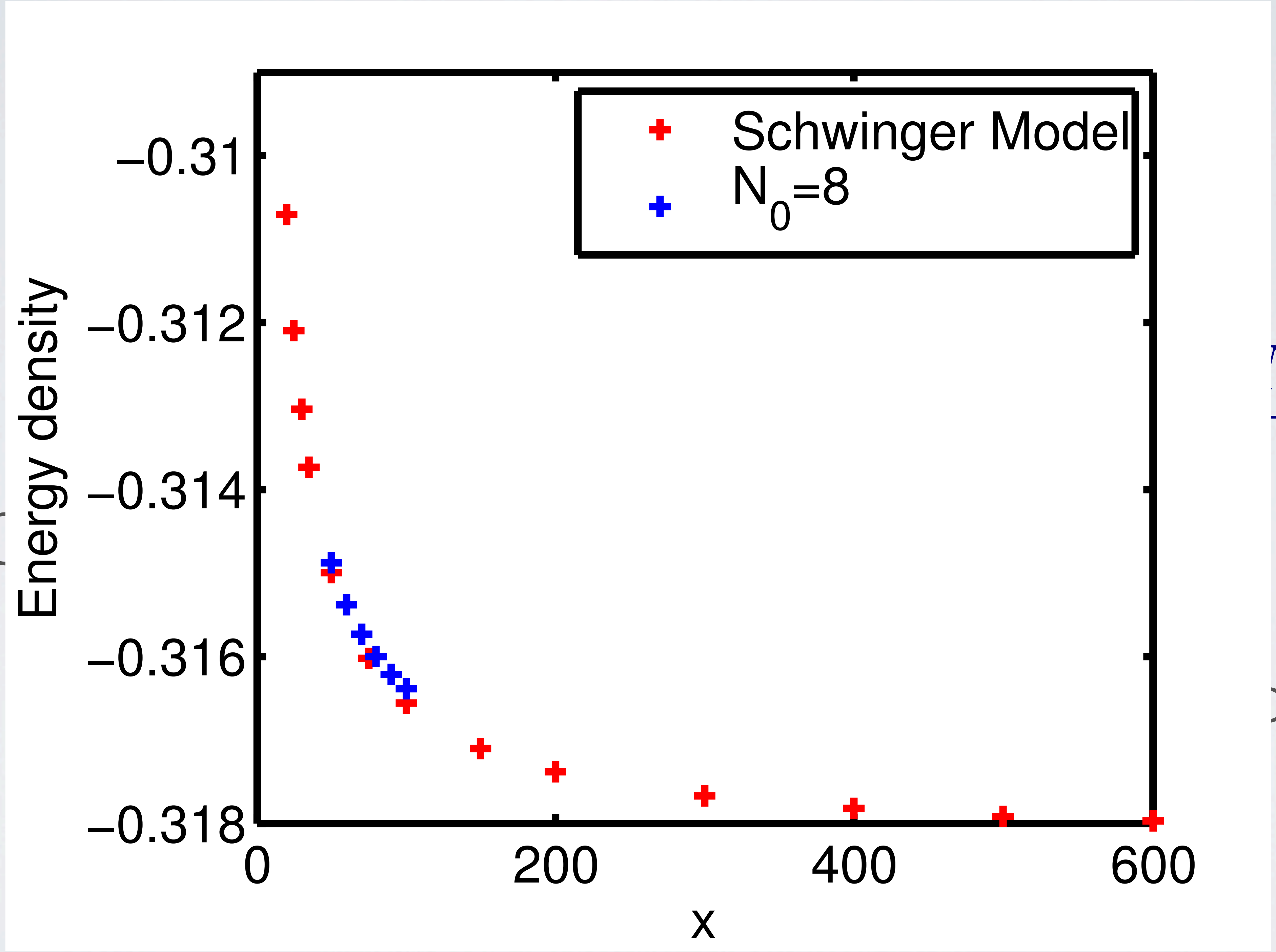

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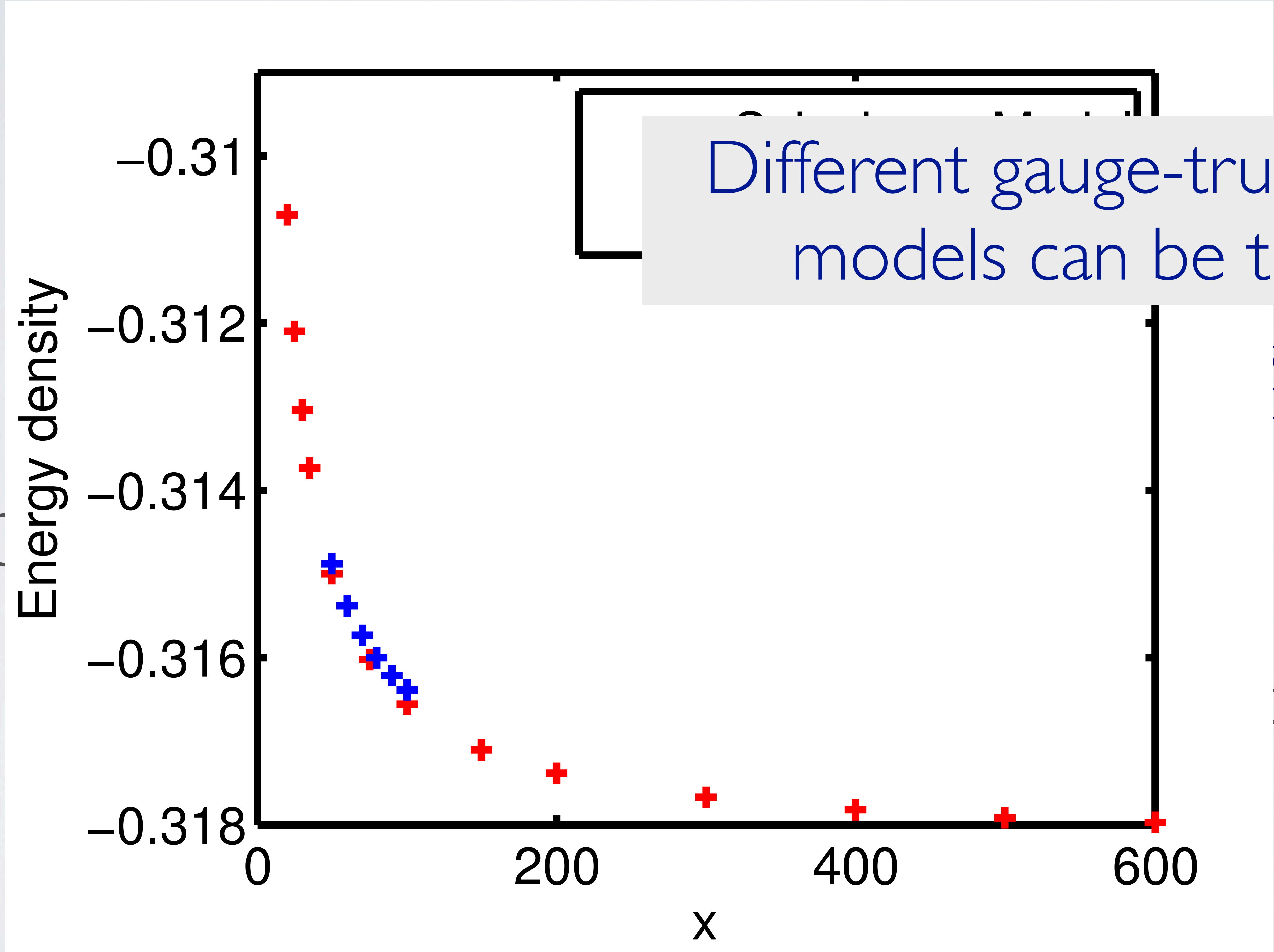

Different gauge-truncated models can be tried

S. Kühn et al., Phys. Rev. A 90, 042305 (2014)

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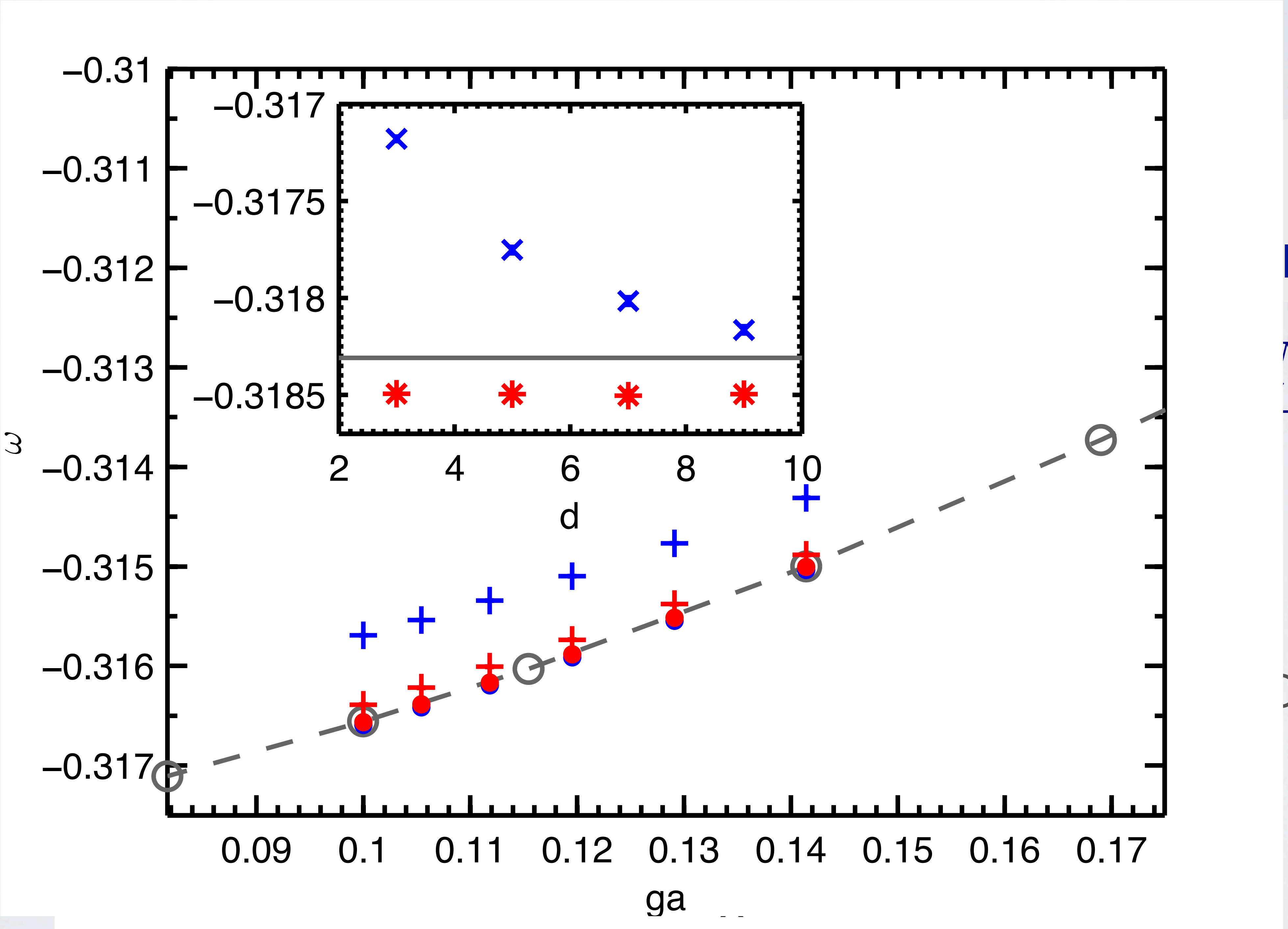

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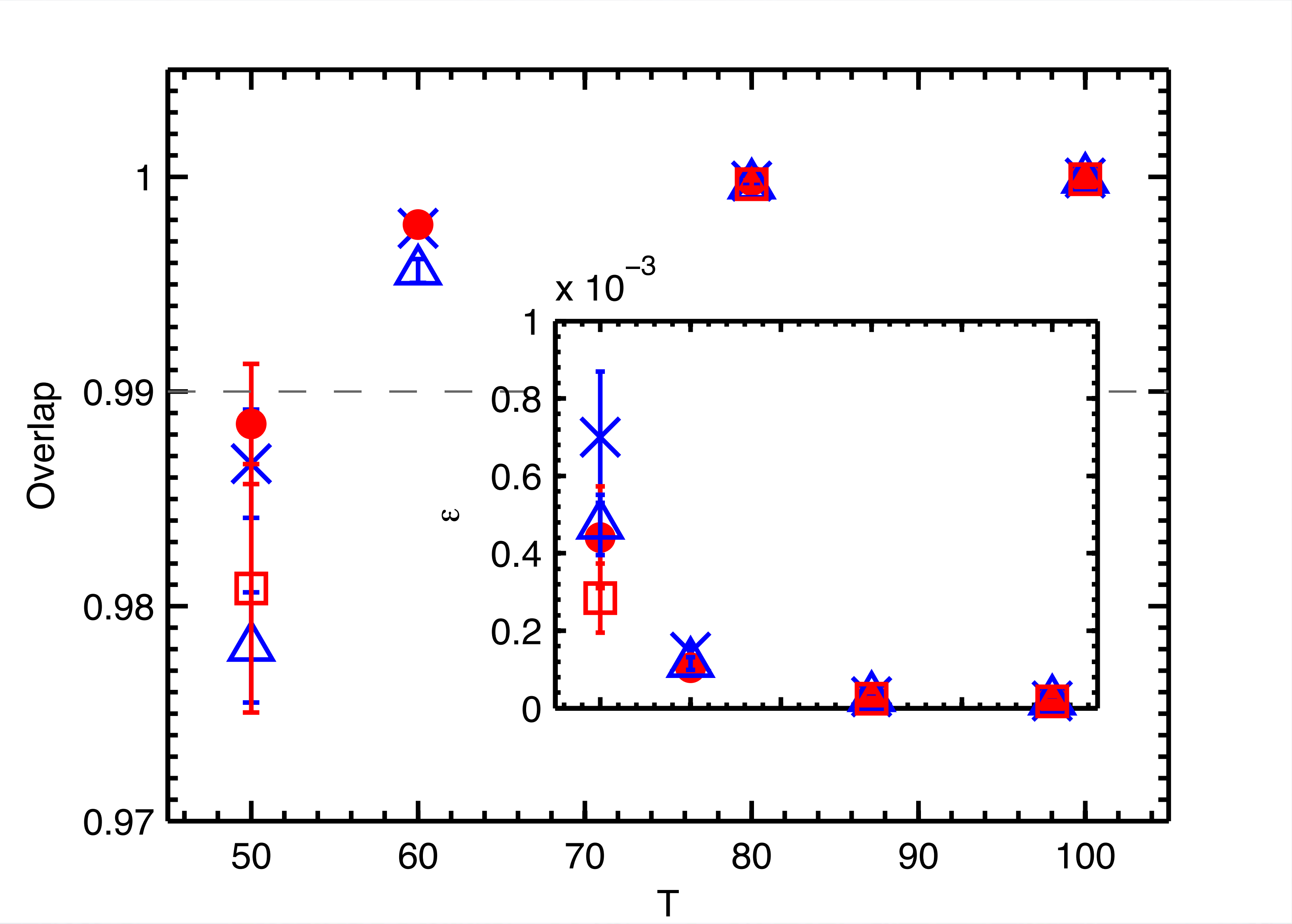
MPS: FEASIBILITY STUDY

Also adiabatic preparation procedure



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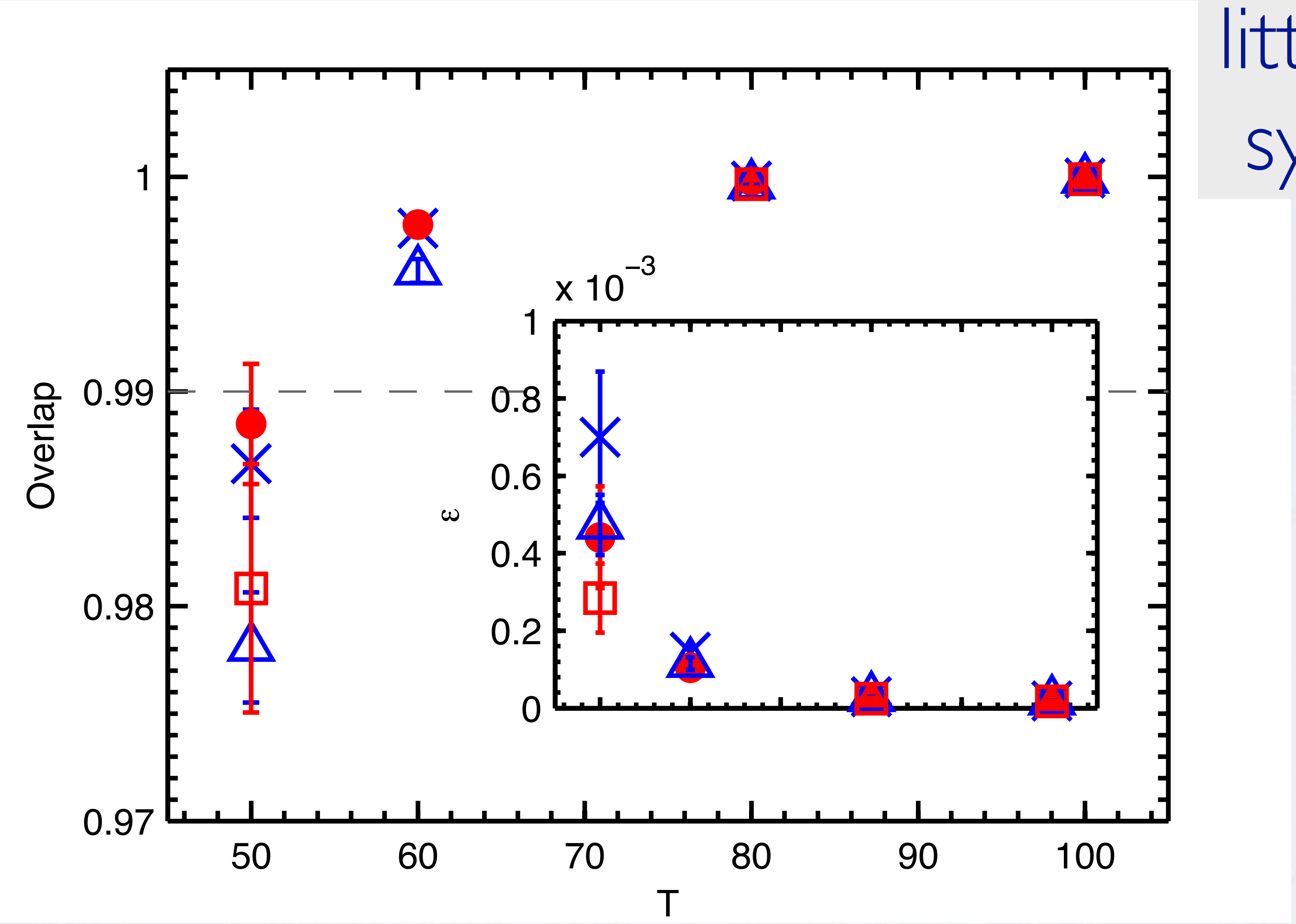

little effect of system size!

S. Kühn et al., Phys. Rev. A 90, 042305 (2014)

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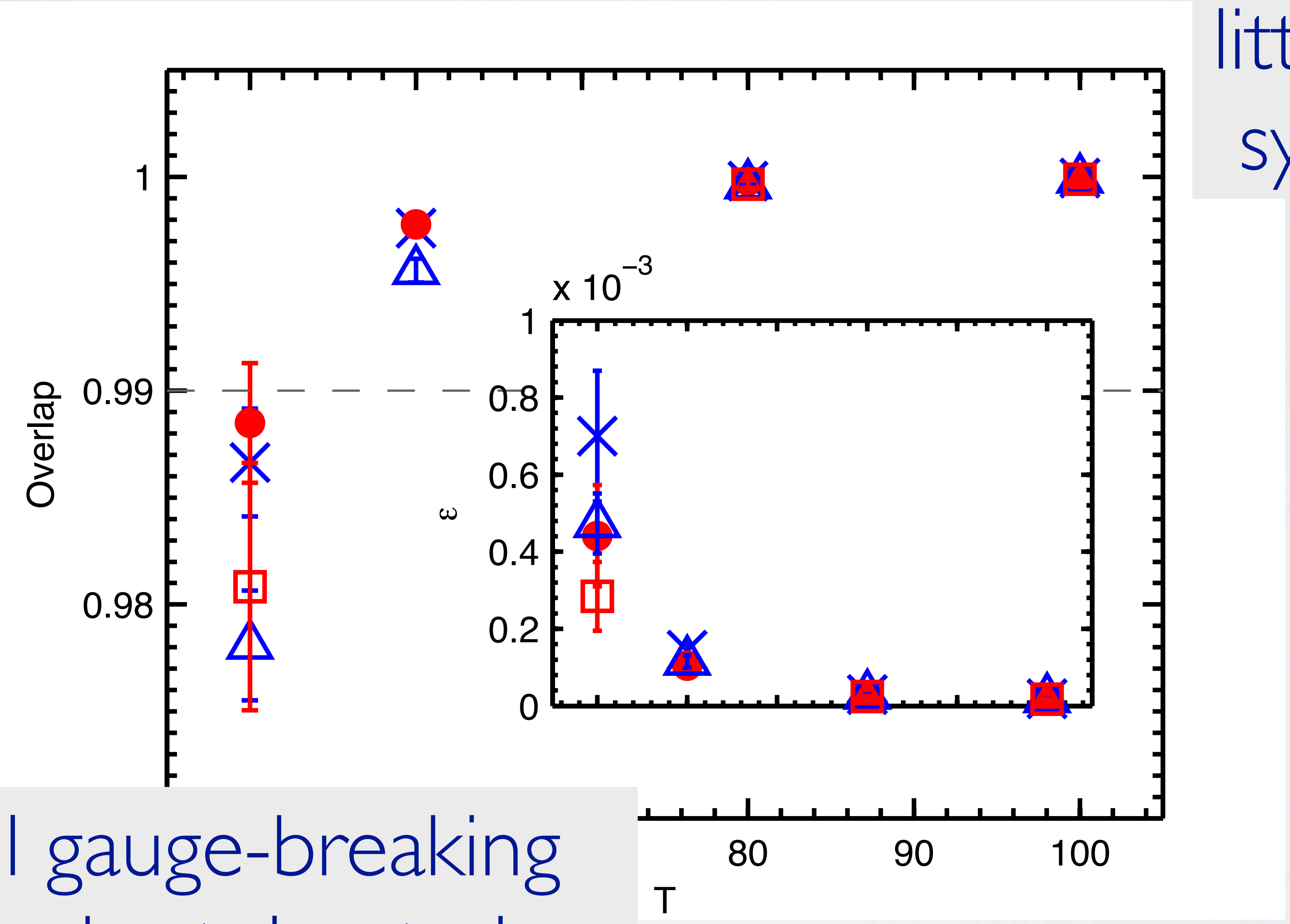

little effect of system size!

And small gauge-breaking noise can be tolerated

S. Kühn et al., Phys. Rev. A 90, 042305 (2014)

QUANTUM SIMULATION OF NON-ABELIAN MODELS

$$H = \sum \left(\Psi_n^\dagger U_n \Psi_{n+1} + h.c.\right) + m \sum (-1)^n \Psi_n^\dagger \Psi_n + \frac{g^2}{2} \sum J_n^2$$

S. Kühn et al., JHEP 07 (2015) 130

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Truncated model with exact SU(2) symmetry

Zohar, Burrello 2015

S. Kühn et al., JHEP 07 (2015) 130

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Simplest case: link variables with dimension 5

Staggered fermions: two colors per site

S. Kühn et al., JHEP 07 (2015) 130

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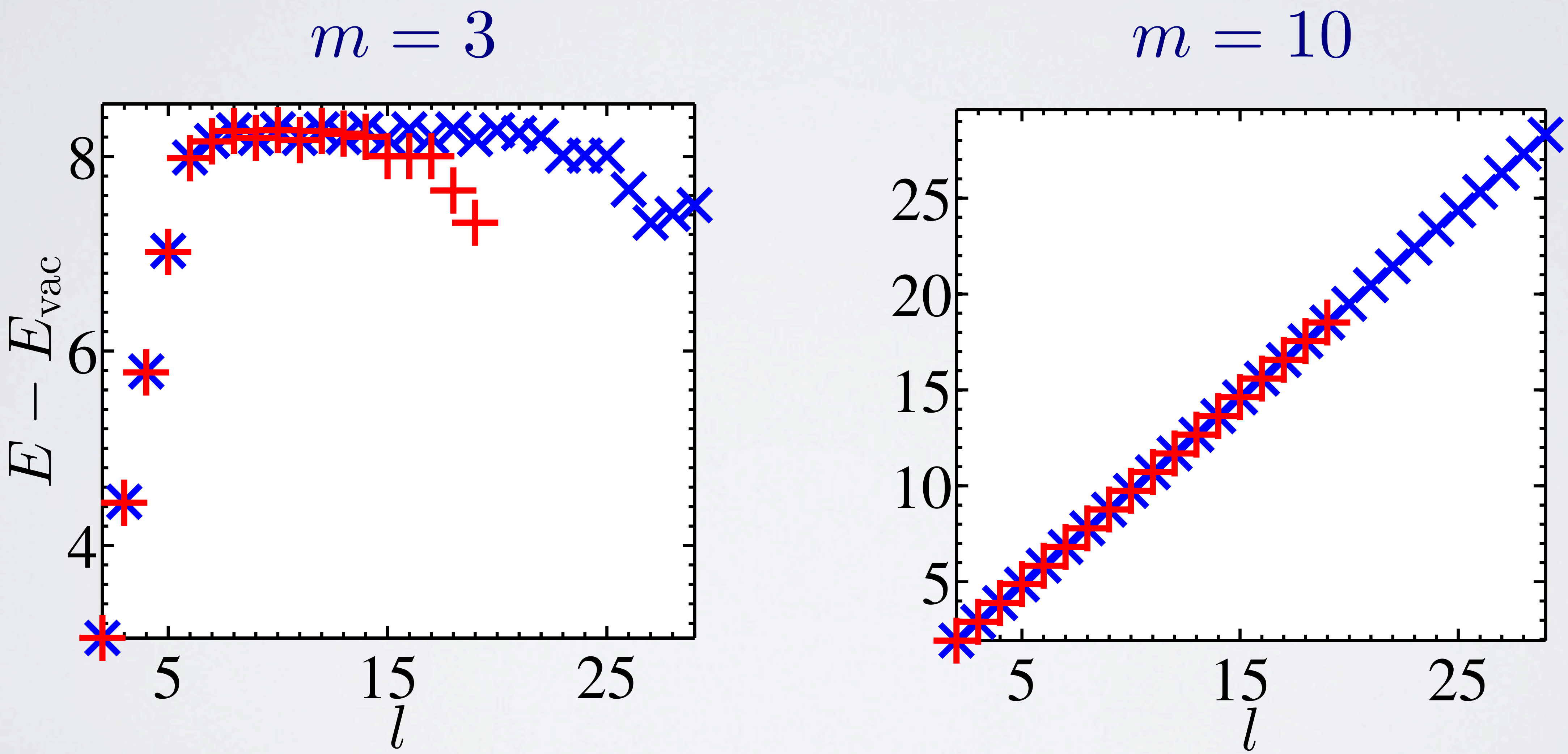
Simulating **statical and dynamical** properties

S. Kühn et al., JHEP 07 (2015) 130

SU(2) STRING BREAKING

Ground state energy with external charges



S. Kühn et al., JHEP 07 (2015) 130

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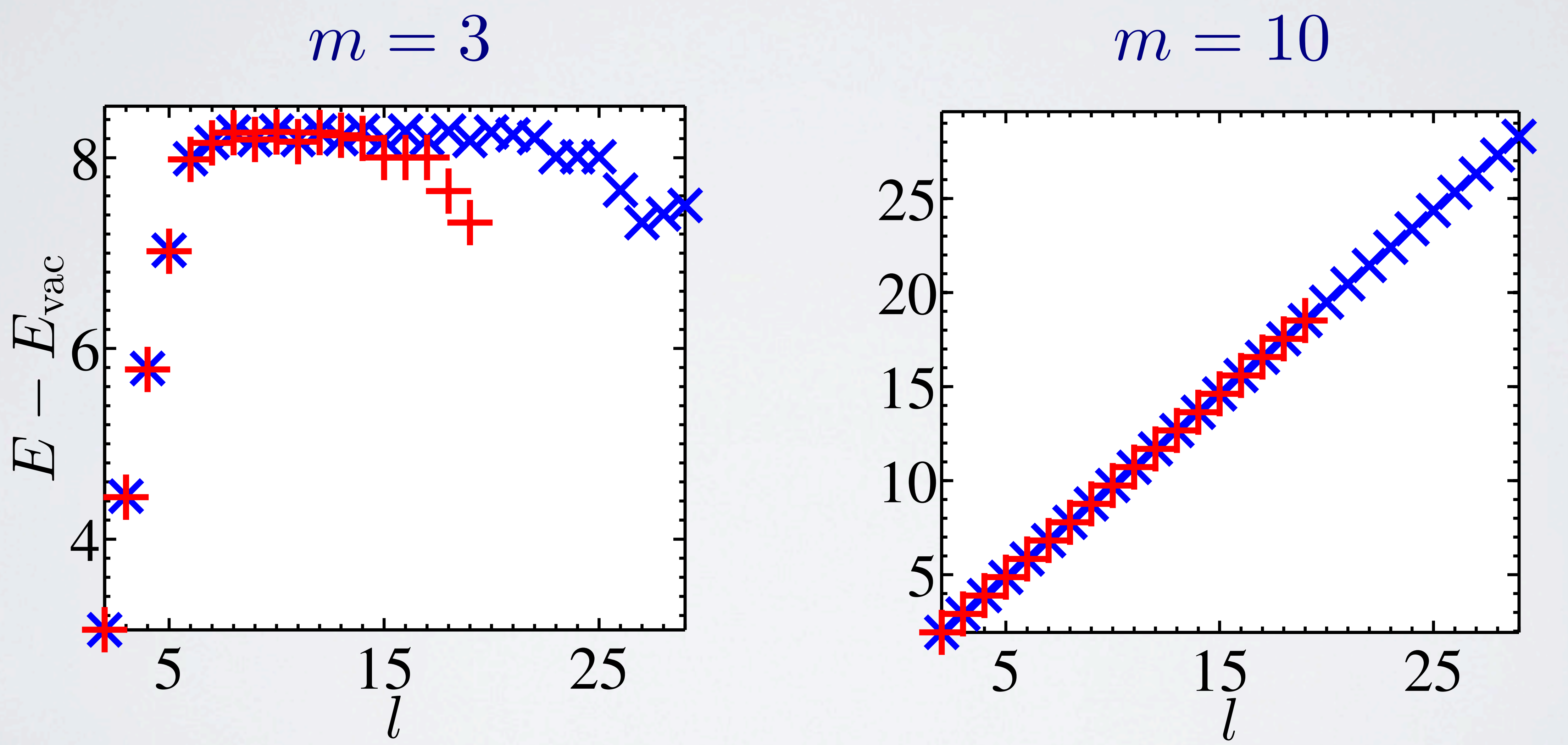


Proposed observables to detect string

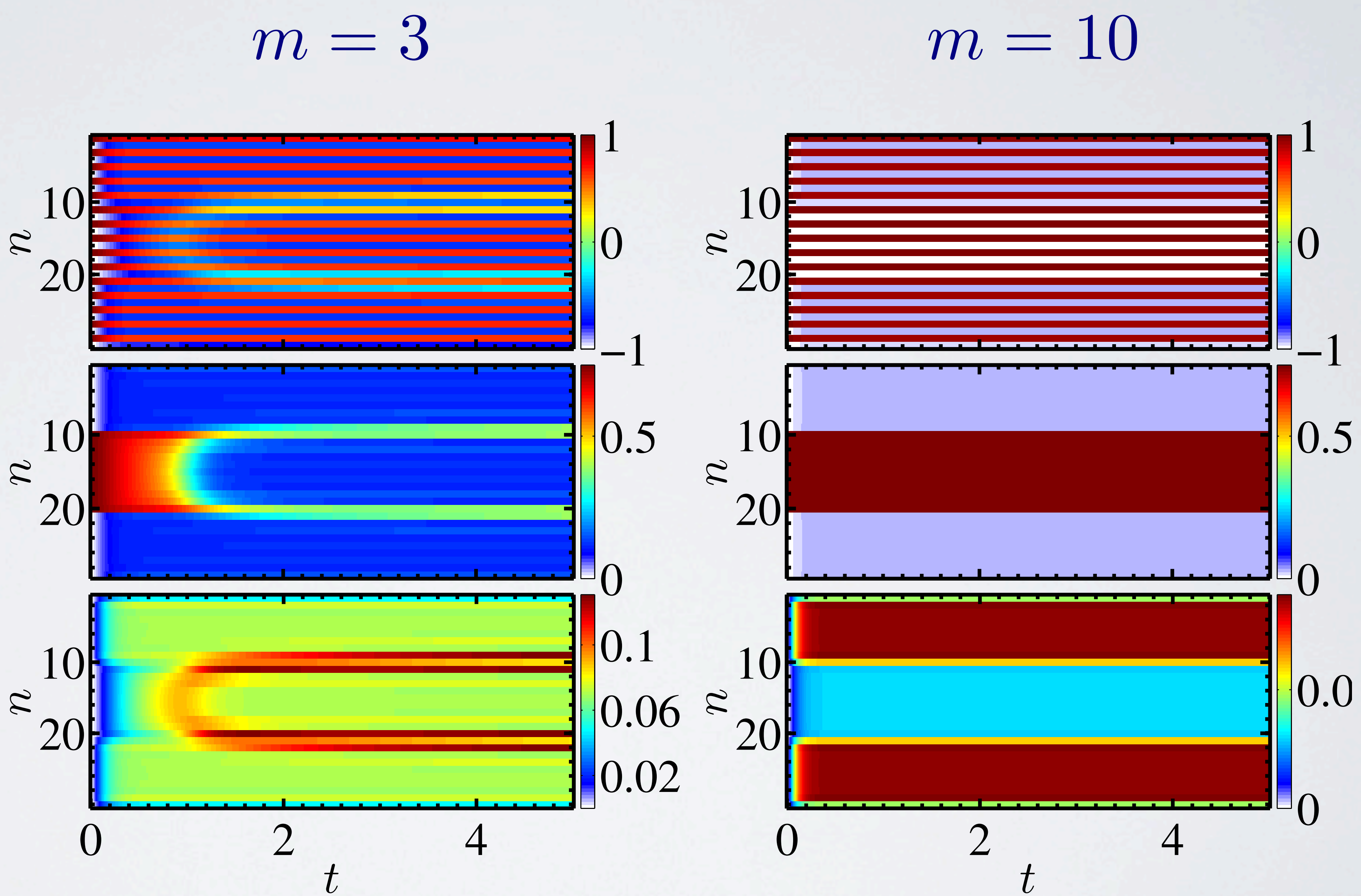
S. Kühn et al., JHEP 07 (2015) 130

SU(2) STRING BREAKING

fermion occupancy

color flux

charge



external charges,
imaginary time

S. Kühn et al., JHEP 07 (2015) 130

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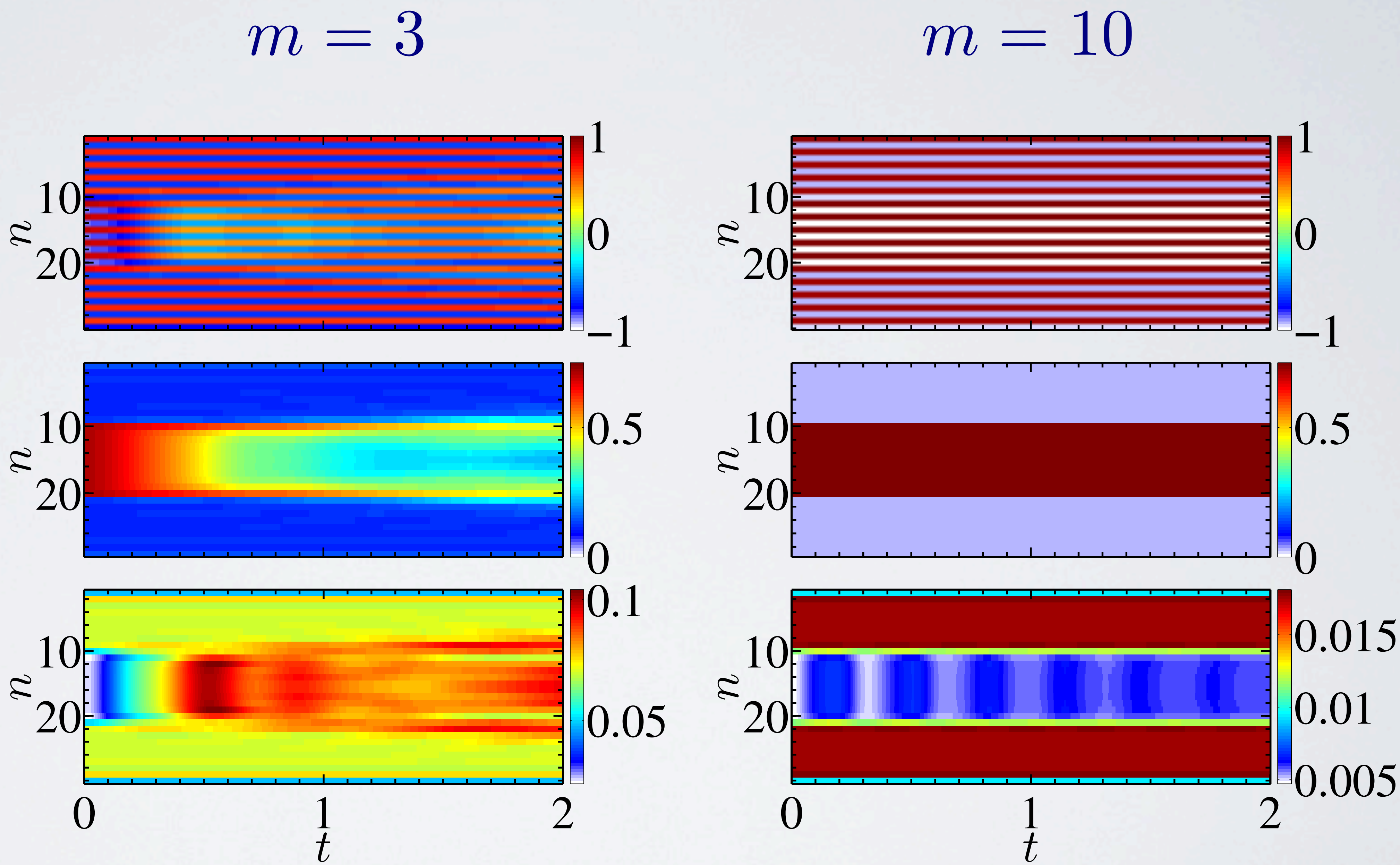
color flux

charge



external charges,
real time

S. Kühn et al., JHEP 07 (2015) 130