Locality and Quantum Physics: The Algebraic Approach to Quantum Field Theory

Klaus Fredenhagen
II. Institut für Theoretische Physik, Hamburg

Locality $\Longleftrightarrow$ Quantum Physics ?

Locality: No mutual influence between spacelike separated events

Quantum Physics: Nonlocal correlations (EPR, entanglement)

Answer: Observables are represented as elements of a non-commutative operator algebra, observables localized in spacelike separated regions commute (Einstein causality).

States are positive linear functionals on the algebra of observables yielding the expectation value.

Nonclassical features of the state space:

- There are pure states whose restrictions to subalgebras are mixed.
- There are states on a tensor product of algebras which cannot be approximated by convex combinations of product states (Bell's inequalities).

Subsystems correspond to (unital) **subalgebras**.

"Good subsystems" $\mathfrak{A}(N)$ characterized by a subregion N of spacetime M (Haag1957)

Formalization:
Embedding of spacetimes:

$$\chi : N \to M$$

induces embedding of algebras:

$$\mathfrak{A}\chi : \mathfrak{A}(N) \to \mathfrak{A}(M)$$

$\mathfrak{A}$ is a functor from the category of spacetimes (with structure preserving embeddings as morphisms) to the category of operator algebras. (Brunetti-Fredenhagen-Verch 2003)

Interpretation in terms of "fields":

Fields are natural transformations

$$\phi : \mathfrak{D} \Longrightarrow \mathfrak{A}$$

$\mathfrak{D}$ functor of test function spaces over spacetimes

i.e. (for scalar fields) there exists a family ϕ_M of operator valued distributions for spacetimes M s.t.

$$\phi_M(\chi(x)) = \mathfrak{A}\chi(\phi_N(x))$$

Construction of QFT functors

Free theories: Deformation quantization of classical field theory

$\mathfrak{E}(M)$ space of smooth classical field configurations on M

$\mathfrak{F}(M)$ space of (suitable) functionals $F : \mathfrak{E}(M) \to \mathbb{C}$

Example:

$$F(\phi) = \int L(\phi(x), \partial\phi(x)) f(x)$$

f test density on M. Normally hyperbolic field equation

$$(\Box + A)\phi = 0$$

A 1st order differential operator. Product on $\mathfrak{F}(M)$:

$$F \star G(\phi) = \sum \frac{(i\hbar)^n}{2^n n!} \langle \frac{\delta^n}{\delta\phi^n F}, \Delta^{\otimes n} \frac{\delta^n G}{\delta\phi^{\otimes n}} \rangle$$

$\Delta = \Delta_{ret} - \Delta_{adv}$ "causal propagator" for $\Box + A$

Interactions generated by local fields $L : \mathfrak{D} \longrightarrow \mathfrak{F}$ (interaction Lagrangians)

Time ordered products of local fields:

- Coincides with $\star$-product for time ordered arguments
- Uniquely determined for non-coinciding points
- Ambiguities for coinciding points correspond to renormalization conditions

Program of Causal Perturbation Theory (Stückelberg, Bogoliubov, Epstein-Glaser 1971): Base perturbation theory on the definition of time ordered products.
Can be generalized to globally hyperbolic Lorentzian spacetimes (Brunetti-Fredenhagen 2000, Hollands-Wald 2001).

Retarded Moeller map from the free to the interacting theory (Bogoliubov)

$$R_{L_M(f)}F = S(L_M(f))^{-1} \star (S(L_M(f)) \cdot_T F)$$

$\cdot_T$ time ordered product, S time ordered exponential

Crucial observation: the algebra $\mathfrak{A}(N)$, N relatively compact subregion of M, depends up to isomorphy only on the restriction of f to N

$\Longrightarrow$ the algebra of local observables of the interacting theory can be constructed locally (no infrared problems).

States of the free theory induce **states** of the interacting theory, but their physical properties have to be analyzed. A priori the existence of states with specific properties (vacuum, particle states, thermal states) is not guaranteed, in general there is even no definition of these concepts.

Gauge theories and perturbative Quantum Gravity

Use the BV-BRST formalism with a suitable gauge fixing.

One obtains an extended algebra containing auxiliary fields (ghosts etc.), together with a differential s (the BRST transformation) which acts as a derivation on the algebra.

The algebra of observables is then obtained as the cohomology of s (Hollands YM 2008, Brunetti-Fredenhagen-Rejzner QG 2016).

Subtle problem for QG: Definition of local observables.

Observables in classical gravity:

Equivariant scalar fields A_Γ (as functionals of the configuration $\Gamma = (g, \varphi, \dots)$ transform under diffeomorphisms as

$$A_{\chi^*\Gamma} = A_\Gamma \circ \chi$$

(e.g. curvature scalars).

Invariant fields (in the neighborhood of some background Γ_0) can be obtained by choosing 4 equivariant fields X^a_Γ, $a = 1, \ldots 4$ which form a coordinate system at Γ_0.
Then, for Γ near to Γ_0, consider the diffeomorphism

$$\alpha_\Gamma(x) = X_\Gamma^{-1} \circ X_{\Gamma_0}(x) = x + \delta_\Gamma(x) \ .$$

We then set for any other equivariant scalar field A_Γ

$$\mathcal{A}_\Gamma = A_\Gamma \circ \alpha_\Gamma$$

Thus we obtain gauge invariant fields

$$\mathcal{A}_\Gamma(x) := A_\Gamma(x + \delta_\Gamma(x)) \ .$$

Hence gauge invariance is obtained by evaluating the field at a point which is shifted in a Γ-dependent way. The shift δ_Γ depends on the choice of intrinsic coordinates.

In perturbation theory the observables enter only by their Taylor expansion around the background Γ_0. We obtain an expansion of $\mathcal{A}_\Gamma$ by inserting the expansions of A_Γ and δ_Γ into the Taylor series

$$\mathcal{A}_\Gamma(x) = A_\Gamma(x) + \partial_\mu A_\Gamma(x)\delta_\Gamma^\mu(x) + \dots$$

In perturbative QG this series is BRST invariant and is a bona fide observable.

The construction, however, depends on the existence of suitable coordinate functions X_Γ. They exist for generic backgrounds, but in concrete applications one usually starts from non-generic backgrounds with high symmetries.

Application to cosmology
(Brunetti-Fredenhagen-Hack-Pinamonti-Rejzner 2016)

Model: Gravity coupled to a scalar field (inflaton)

Background: spatially flat FLRW spacetime with a scalar field depending only on time.

Scalar field can be used as the time coordinate.

Nonlocal spatial coordinates can be introduced by

$$X^i = x^i + G_h \Delta_h x^i$$

h metric on equal time surfaces, Δ_h, G_h associated Laplacian and Green function.

Non-localities require extension of causal perturbation theory.

1st order: reproduces cosmological perturbation theory.

Conclusions

- Algebraic quantum field theory provides a consistent framework for fundamental physics.
- It is in agreement with other approaches.
- By separating the problem of constructing the algebra of observables from the problem of constructing states it is more flexible.
- It avoids prejudices on the existence of states, instead the structure of the state space is recognized as a prediction of the theory.
- It allows an extension of QFT to external gravitational fields and to perturbative QG.
- It provides a consistent extension of cosmological perturbation theory beyond first order.