Realizing the Relaxion with N-site Models

Nayara Fonseca

Universidade de São Paulo $\longrightarrow$ DESY Theory Group

September 28th, 2016

DESY Theory Workshop 2016

Outline

1. The Relaxion Idea

2. Realizing the Relaxion with *N*-site Models

Large field excursions from a few site relaxion model; [1601.07183]
NF, L. de Lima, C. S. Machado, R. D. Matheus; Phys.Rev. D94 (2016) 015010

3. Concluding Remarks

The Relaxion Idea

Before Relaxion:
Two different "solutions" to the SM hierarchy problem

1. **New dynamics at the weak scale:**

- **Natural solutions;**

- **We need BSM at ~ TeV scale;**

Eg.: SUSY & Composite Higgs Models & Warped Scenarios.

2. **Anthropics** !?

3. Another option: The Relaxation Mechanism

P. W. Graham, D. E. Kaplan, S. Rajendran; Phys. Rev. Lett. 115, 221801 (2015)

Warming up…

$$V(h,\phi) = \frac{1}{2} m_H^2(\phi) h^2 + \cdots = \frac{1}{2}(-\Lambda^2 + g\Lambda\phi) h^2 + \cdots$$

3. Another option: The Relaxation Mechanism

P. W. Graham, D. E. Kaplan, S. Rajendran; Phys. Rev. Lett. 115, 221801 (2015)

Warming up…

high scale

the new field

$$V(h,\phi) = \frac{1}{2} m_H^2(\phi) h^2 + \cdots = \frac{1}{2}\left(-\Lambda^2 + g\Lambda\phi\right) h^2 + \cdots$$

small coupling (spurion)

The Relaxion Idea

3. Another option: The Relaxation Mechanism

P. W. Graham, D. E. Kaplan, S. Rajendran; Phys. Rev. Lett. 115, 221801 (2015)

Warming up…

high scale

the new field

$$V(h,\phi) = \frac{1}{2}m_H^2(\phi)h^2 + \cdots = \frac{1}{2}(-\Lambda^2 + g\Lambda\phi)h^2 + \cdots$$

small coupling (spurion)

- ϕ scans $m_H^2(\phi)$ during the cosmological evolution;
- Arrange a mechanism so that ϕ stops where we want, precisely at the EW scale:

$$m_H^2(\phi_c) = -\Lambda^2 + g\Lambda\phi_c \ll \Lambda^2$$

3. Another option: The Relaxation Mechanism

P. W. Graham, D. E. Kaplan, S. Rajendran; Phys. Rev. Lett. 115, 221801 (2015)

Warming up…

$$V(h,\phi) = \frac{1}{2} m_H^2(\phi) h^2 + \cdots = \frac{1}{2}(-\Lambda^2 + g\Lambda\phi)h^2 + \cdots$$

- The initial value of ϕ is such that $m_H^2(\phi) > 0$;
- We can already see why large field excursions are crucial here: $\phi > \Lambda/g$.

The Relaxion Idea

Closer Look

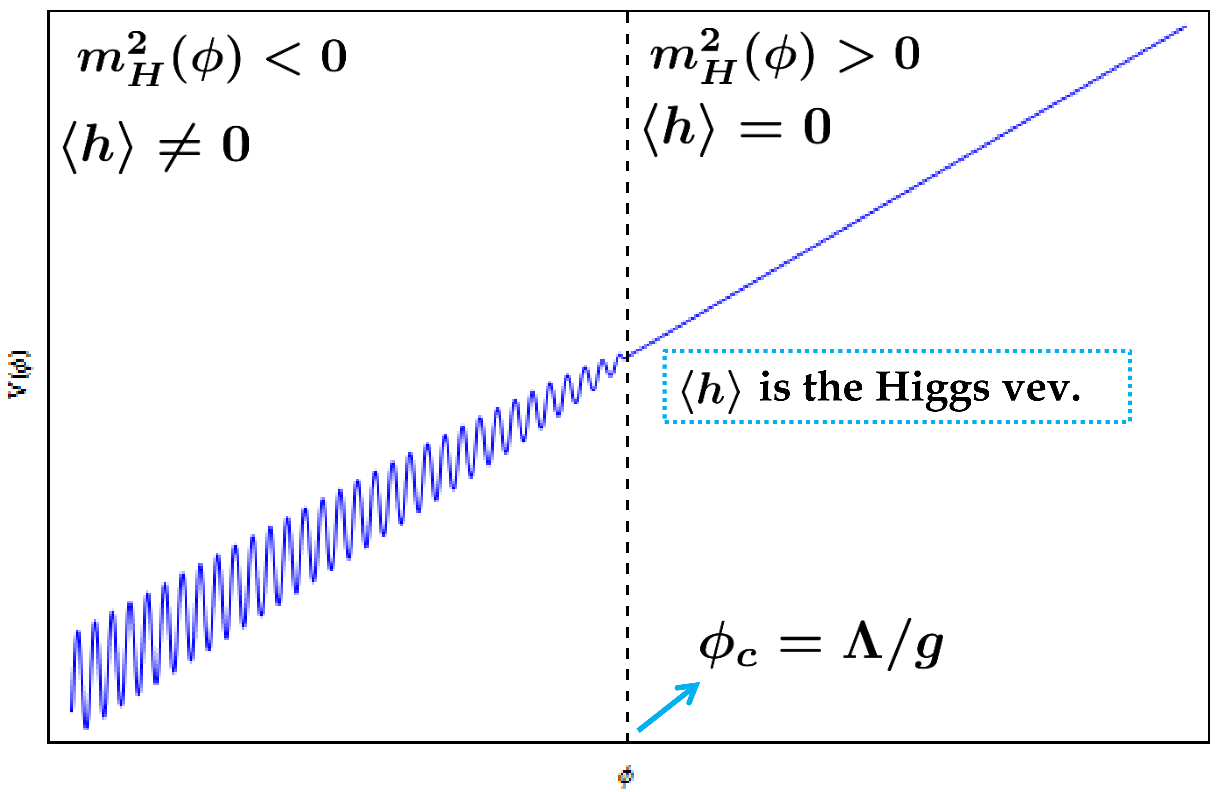
$$V(h,\phi) = \frac{1}{2}\Lambda^2\left(\frac{g\phi}{\Lambda} - 1\right)h^2 + g\Lambda^3\phi + \epsilon\Lambda_c^4\left(\frac{\langle h\rangle}{\Lambda_c}\right)^n \cos\left(\frac{\phi}{f}\right) + \cdots$$

The Relaxion Idea

Closer Look

$m_H^2(\phi)$ "Slope term" "Stopping term"

$$V(h,\phi) = \frac{1}{2}\Lambda^2\left(\frac{g\phi}{\Lambda} - 1\right)h^2 + g\Lambda^3\phi + \epsilon\Lambda_c^4\left(\frac{\langle h\rangle}{\Lambda_c}\right)^n \cos\left(\frac{\phi}{f}\right) + \cdots$$



The Relaxion Idea

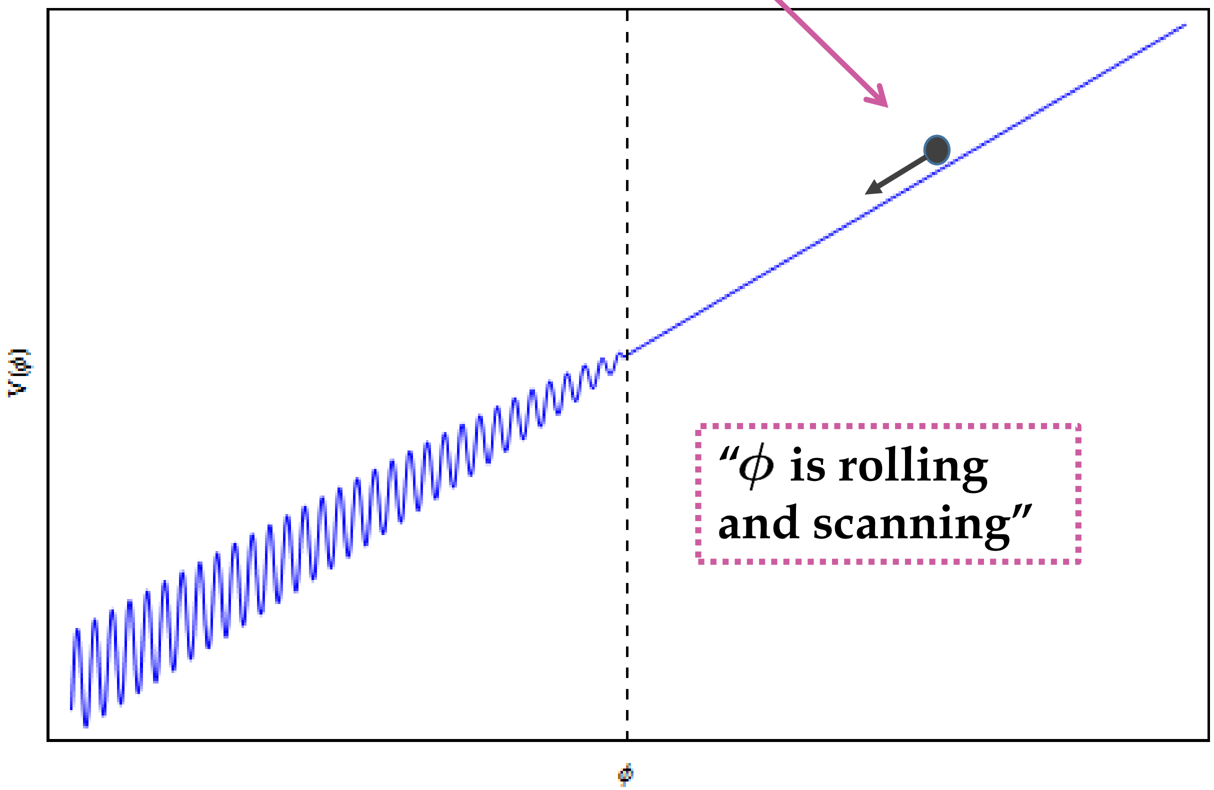
Closer Look

$m_H^2(\phi)$

"Slope term"

"Stopping term"

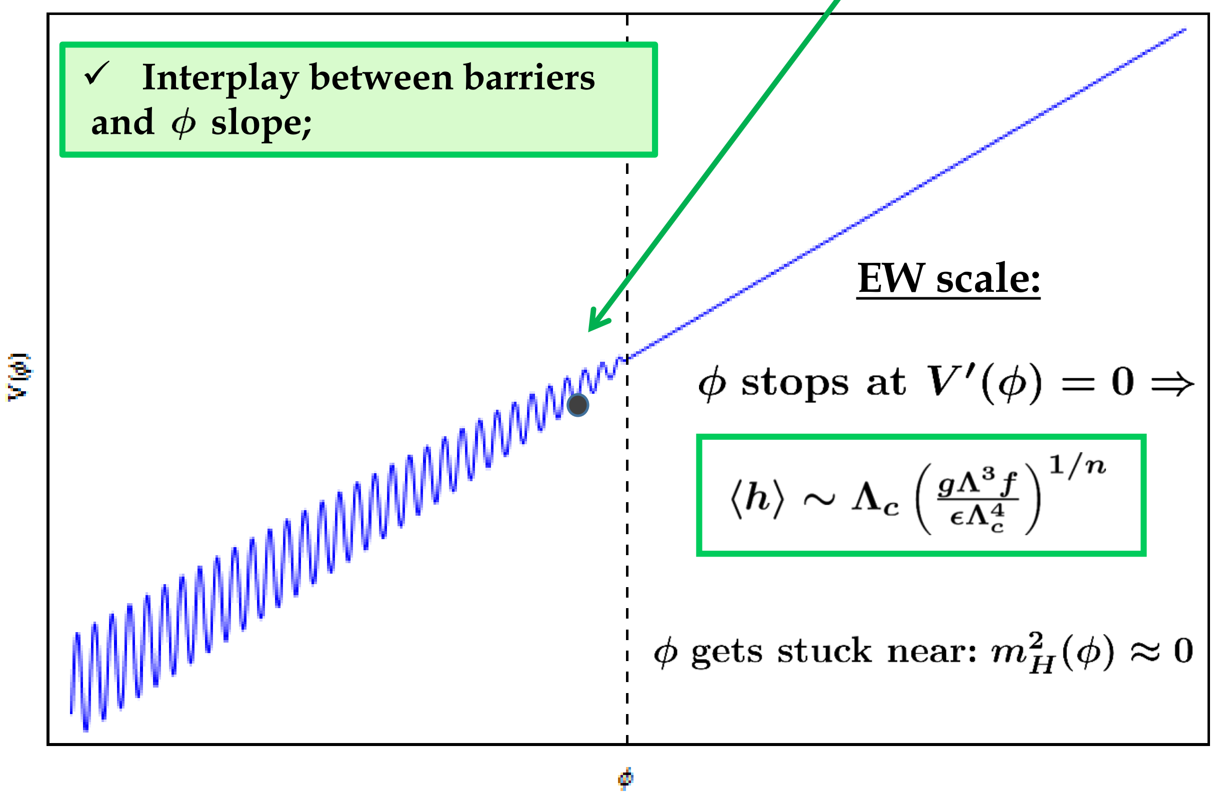
$$V(h,\phi) = \frac{1}{2}\Lambda^2\left(\frac{g\phi}{\Lambda} - 1\right)h^2 + g\Lambda^3\phi + \epsilon\Lambda_c^4\left(\frac{\langle h\rangle}{\Lambda_c}\right)^n \cos\left(\frac{\phi}{f}\right) + \cdots$$



Closer Look

$m_H^2(\phi)$ "Slope term" "Stopping term"

$$V(h,\phi) = \frac{1}{2}\Lambda^2\left(\frac{g\phi}{\Lambda}-1\right)h^2 + g\Lambda^3\phi + \epsilon\Lambda_c^4\left(\frac{\langle h\rangle}{\Lambda_c}\right)^n \cos\left(\frac{\phi}{f}\right) + \cdots$$

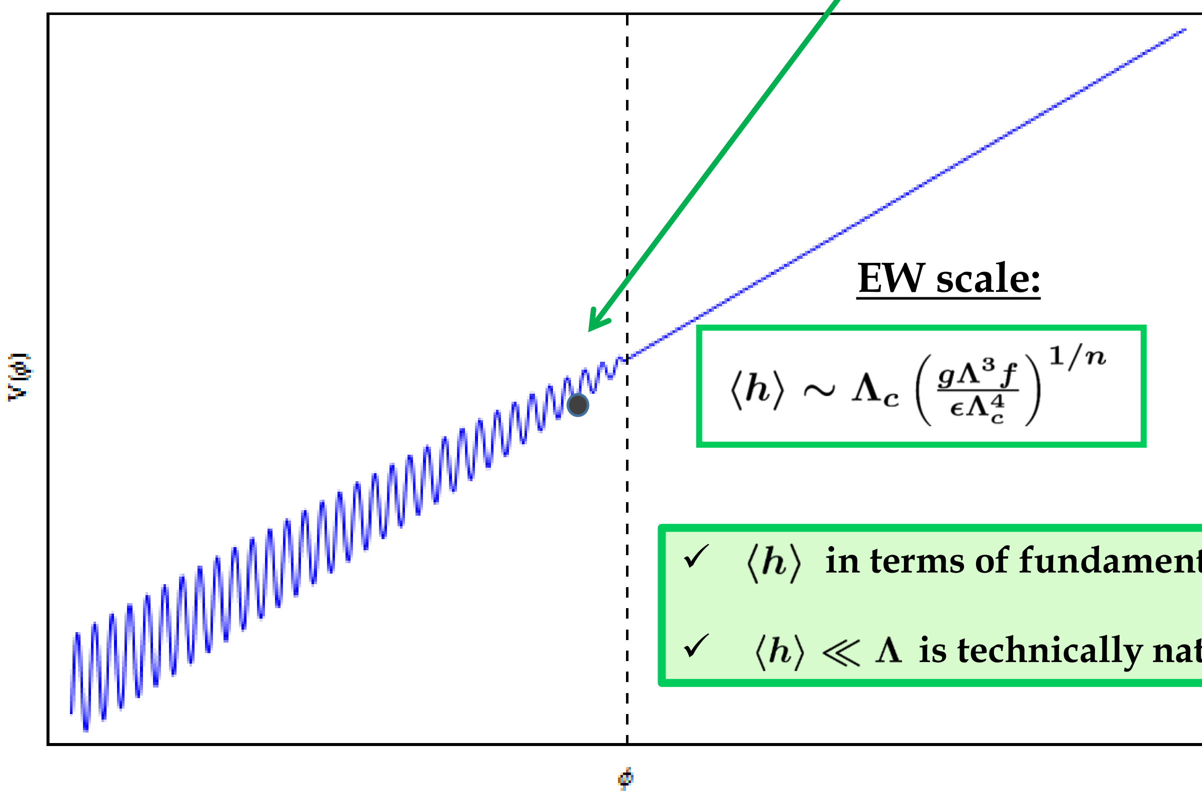


The Relaxion Idea

Closer Look

$m_H^2(\phi)$ | "Slope term" | "Stopping term"

$$V(h,\phi) = \frac{1}{2}\Lambda^2\left(\frac{g\phi}{\Lambda} - 1\right)h^2 + g\Lambda^3\phi + \epsilon\Lambda_c^4\left(\frac{\langle h\rangle}{\Lambda_c}\right)^n \cos\left(\frac{\phi}{f}\right) + \cdots$$



EW scale:

$$\langle h\rangle \sim \Lambda_c\left(\frac{g\Lambda^3 f}{\epsilon\Lambda_c^4}\right)^{1/n}$$

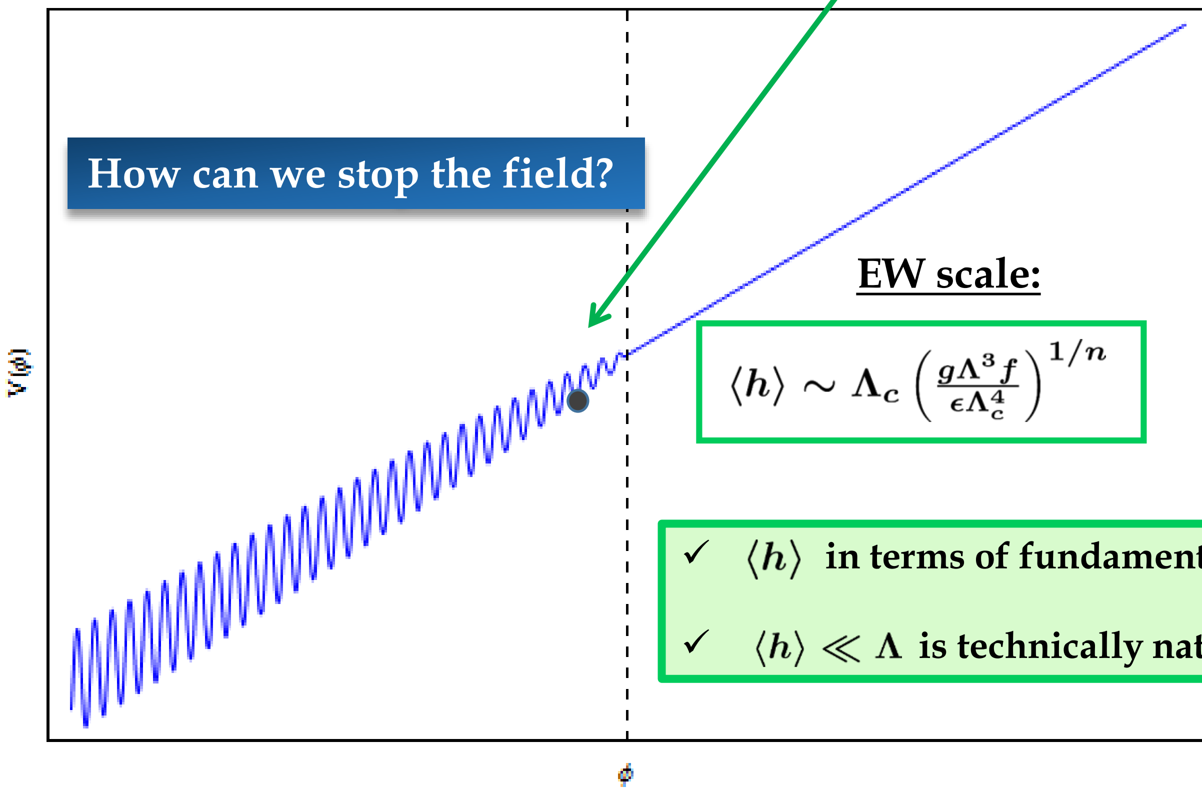
- ✓ $\langle h\rangle$ in terms of fundamental parameters;
- ✓ $\langle h\rangle \ll \Lambda$ is technically natural.

The Relaxion Idea

Closer Look

$m_H^2(\phi)$ — "Slope term" — "Stopping term"

$$V(h,\phi) = \frac{1}{2}\Lambda^2\left(\frac{g\phi}{\Lambda}-1\right)h^2 + g\Lambda^3\phi + \epsilon\Lambda_c^4\left(\frac{\langle h\rangle}{\Lambda_c}\right)^n\cos\left(\frac{\phi}{f}\right)+\cdots$$



How can we stop the field?

EW scale:

$$\langle h\rangle \sim \Lambda_c\left(\frac{g\Lambda^3 f}{\epsilon\Lambda_c^4}\right)^{1/n}$$

- ✓ $\langle h\rangle$ in terms of fundamental parameters;
- ✓ $\langle h\rangle \ll \Lambda$ is technically natural.

The Relaxion Idea

P. W. Graham, D. E. Kaplan, S. Rajendran; Phys. Rev. Lett. 115, 221801 (2015)

How can we stop the field? ⇨ **We need dissipation!**

The Relaxion Idea

P. W. Graham, D. E. Kaplan, S. Rajendran; Phys. Rev. Lett. 115, 221801 (2015)

How can we stop the field? ⇨ **We need dissipation!**

Slow-roll during inflation
(Hubble friction provides the dissipation)

- **The relaxion dynamical evolution requires energy transfer that should be dissipated, so the field can stop.**

Outline

1. The Relaxion Idea

2. Realizing the Relaxion with *N*-site Models (‘*N*-Relaxion’)

Large field excursions from a few site relaxion model; [1601.07183]
NF, L. de Lima, C. S. Machado, R. D. Matheus; Phys.Rev. D94 (2016) 015010

3. Concluding Remarks

Realizing the Relaxion with *N*-site Models (‘*N*-Relaxion’)

Some concerns about the original idea ⇨ **Let’s count symmetries**

Where do we find shift symmetries? UV completion?
Spontaneous breaking of a Global Symmetry!

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\Lambda^2\left(\frac{g\phi}{\Lambda} - 1\right)h^2 - g\Lambda^3\phi - \epsilon\Lambda_c^4\left(\frac{\langle h\rangle}{\Lambda_c}\right)^n \cos\left(\frac{\phi}{f}\right)$$

Realizing the Relaxion with N-site Models ('N-Relaxion')

Some concerns about the original idea ⇨ **Let's count symmetries**

Where do we find shift symmetries? UV completion?
Spontaneous breaking of a Global Symmetry!

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\Lambda^2 \left(\frac{g\phi}{\Lambda} - 1\right) h^2 - g\Lambda^3\phi - \epsilon\Lambda_c^4 \left(\frac{\langle h \rangle}{\Lambda_c}\right)^n \cos\left(\frac{\phi}{f}\right)$$

○ $g = 0, \epsilon = 0$ ⇨ $\mathcal{L}$ is invariant under a continuous shift symmetry: $\phi \to \phi + c$

Nambu-Goldstone boson

$$\mathcal{L}(\phi) = \mathcal{L}(\phi + c)$$
$$V(\phi) = 0$$

○ $\epsilon \neq 0$ ⇨ it breaks the continuous shift symmetry to: $\phi \to \phi + 2\pi n f$

Pseudo Nambu-Goldstone boson

Realizing the Relaxion with N-site Models ('N-Relaxion')

Some concerns about the original idea ⇨ **Let's count symmetries**

Where do we find shift symmetries? UV completion?
<u>Spontaneous breaking of a Global Symmetry!</u>

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\Lambda^2 \left(\frac{g\phi}{\Lambda} - 1\right) h^2 - g\Lambda^3 \phi - \epsilon \Lambda_c^4 \left(\frac{\langle h \rangle}{\Lambda_c}\right)^n \cos\left(\frac{\phi}{f}\right)$$

- $g = 0, \epsilon = 0$ ⇨ $\mathcal{L}$ is invariant under a continuous shift symmetry: $\phi \to \phi + c$

  Nambu-Goldstone boson

  $\mathcal{L}(\phi) = \mathcal{L}(\phi + c)$
  $V(\phi) = 0$

- $\epsilon \neq 0$ ⇨ it breaks the continuous shift symmetry to: $\phi \to \phi + 2\pi n f$

  Pseudo Nambu-Goldstone boson

- $g \neq 0$ ⇨ they break the discrete shift symmetry!

Realizing the Relaxion with *N*-site Models ('*N*-Relaxion')

Some concerns about the original idea

- ϵ **term:** $V \supset \epsilon \Lambda_c^4 \cos\left(\frac{\phi}{f}\right)$ ⇨ **Breaks the continuous shift symmetry to:** $\phi \to \phi + 2\pi n f$
- g **terms:** $V \supset g\Lambda^3 \phi + \frac{1}{2} g \Lambda \phi h^2$ ⇨ g **is the spurion that breaks the discrete shift symmetry!**

⇨ ϕ **is a pNGB (i.e., it has compact field range), and its periodicity;**

⇨ g **cannot break a gauge symmetry (the discrete shift symmetry is a redundancy)**

Is the Relaxion an Axion? ; R. S. Gupta, Z. Komargodski, G. Perez, L. Ubaldi. JHEP 1602 (2016) 166

Realizing the Relaxion with *N*-site Models ('*N*-Relaxion')

Some concerns about the original idea

- ϵ **term:** $V \supset \epsilon \Lambda_c^4 \cos\left(\frac{\phi}{f}\right)$ ⇨ **Breaks the continuous shift symmetry to:** $\phi \to \phi + 2\pi n f$

- g **terms:** $V \supset g\Lambda^3\phi + \frac{1}{2} g\Lambda\phi h^2$ ⇨ g **is the spurion that breaks the discrete shift symmetry!**

⇨ ϕ **is a pNGB (i.e., it has compact field range), and its periodicity;**

⇨ g **cannot break a gauge symmetry (the discrete shift symmetry is a redundancy)**

Is the Relaxion an Axion? ; R. S. Gupta, Z. Komargodski, G. Perez, L. Ubaldi. JHEP 1602 (2016) 166

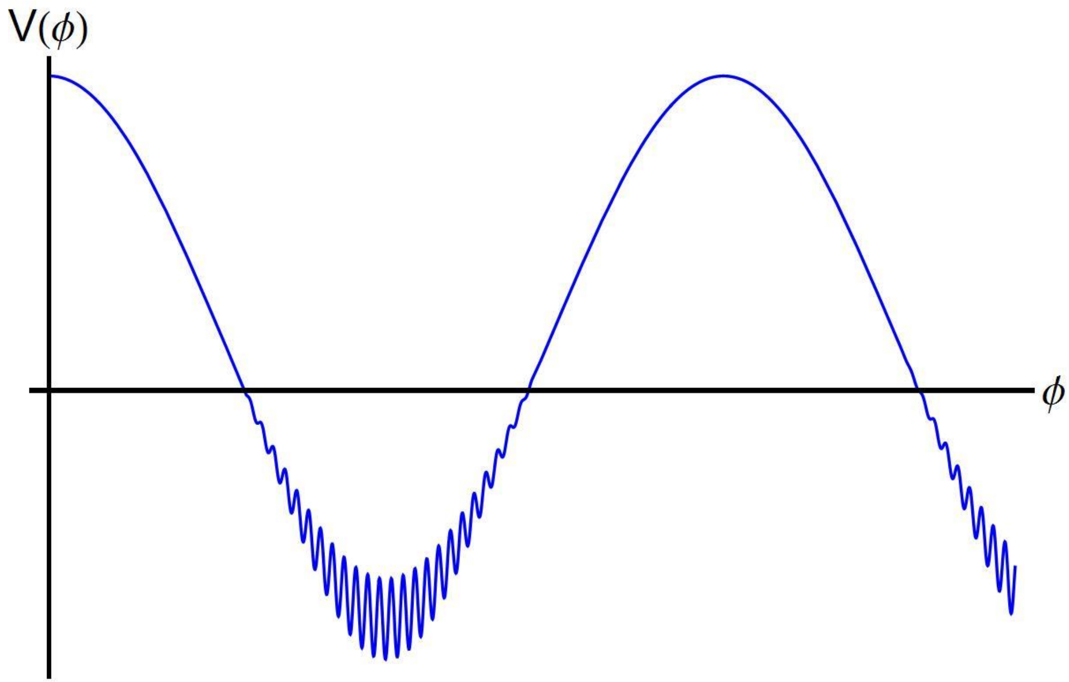
The ϕ operators must be periodic!

Realizing the Relaxion with *N*-site Models ('*N*-Relaxion')

The ϕ operators must be periodic!

Effectively, it is enough to have a hierarchy of decay constants: $F = nf \gg f$

$$V \sim A\cos\left(\frac{\phi}{F}\right) + B(h)\cos\left(\frac{\phi}{f}\right)$$

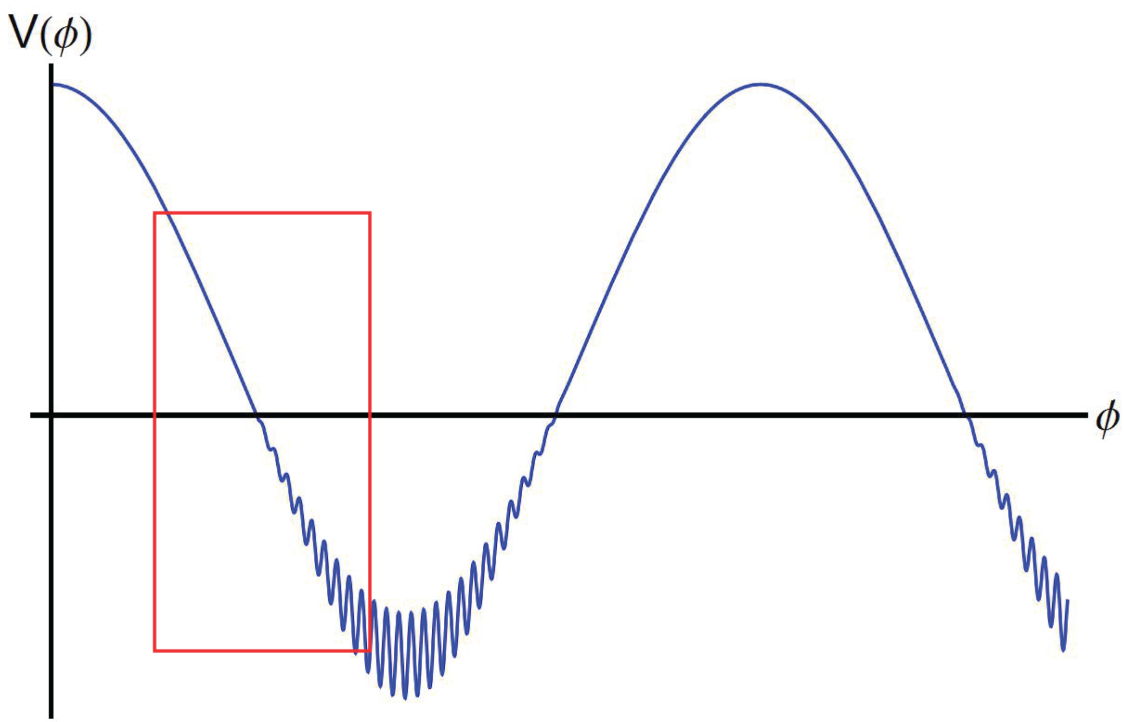


Realizing the Relaxion with *N*-site Models ('*N*-Relaxion')

The ϕ operators must be periodic!

Effectively, it is enough to have a hierarchy of decay constants: $F = nf \gg f$

$$V \sim A\cos\left(\frac{\phi}{F}\right) + B(h)\cos\left(\frac{\phi}{f}\right)$$



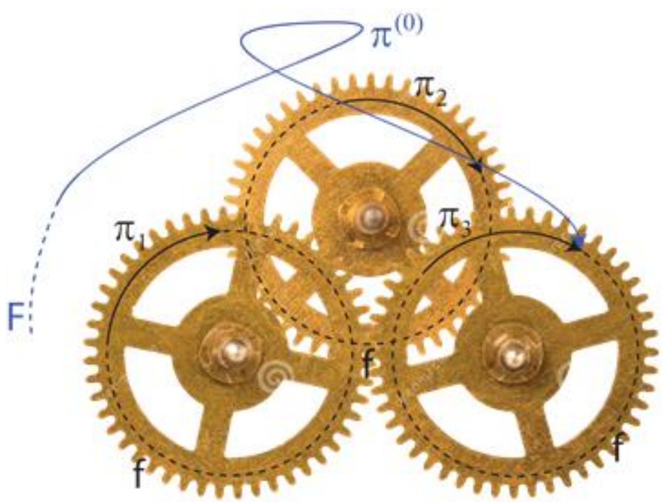
Realizing the Relaxion with *N*-site Models ('*N*-Relaxion')

It seems not reasonable to get an exponential hierarchy $F \gg f$!

Realizing the Relaxion with N-site Models ('N-Relaxion')

It seems not reasonable to get an exponential hierarchy $F \gg f$!

Clockwork Realizations



R. Matheus, PPC'16

N **pNGBs with the same decay constant** ***f*****:**

$$F \gg e^{cN} f,\ c \sim \mathcal{O}(1)$$

Eg.: D. E. Kaplan, R. Rattazzi; Phys.Rev. D93 (2016) no.8, 085007
K. Choi, S. H. Im; JHEP 1601 (2016) 149

Realizing the Relaxion with *N*-site Models (‘*N*-Relaxion’)

NF, L. de Lima, C. S. Machado, R. D. Matheus; Phys.Rev. D94 (2016) no.1, 015010

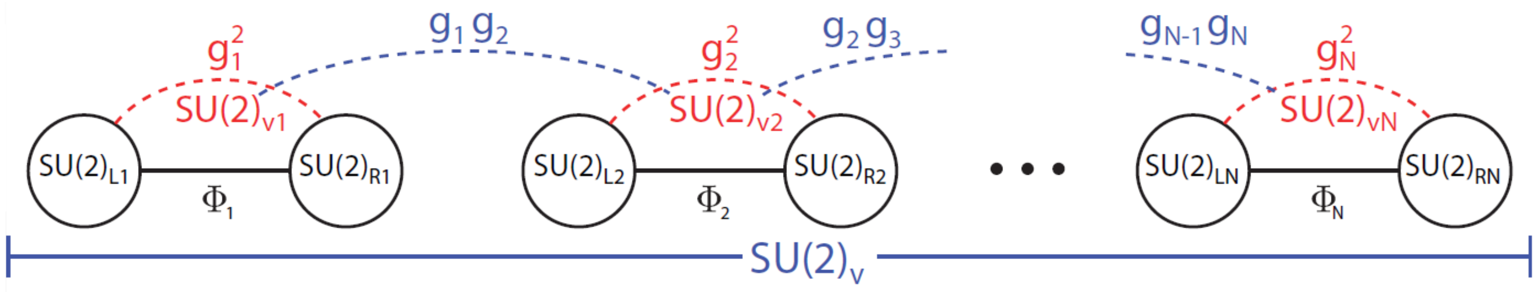
Main goals

- **Obtain a realization from an *N*-site model (find a model close to a deconstructed extra-dimension);**
- **pNGB as the relaxion;**
- **Generate an effective scale *F* much larger than *f*;**
- **Generalize to non-abelian symmetries.**

Realizing the Relaxion with *N*-site Models ('*N*-Relaxion')

NF, L. de Lima, C. S. Machado, R. D. Matheus; Phys.Rev. D94 (2016) no.1, 015010

Minimum Model

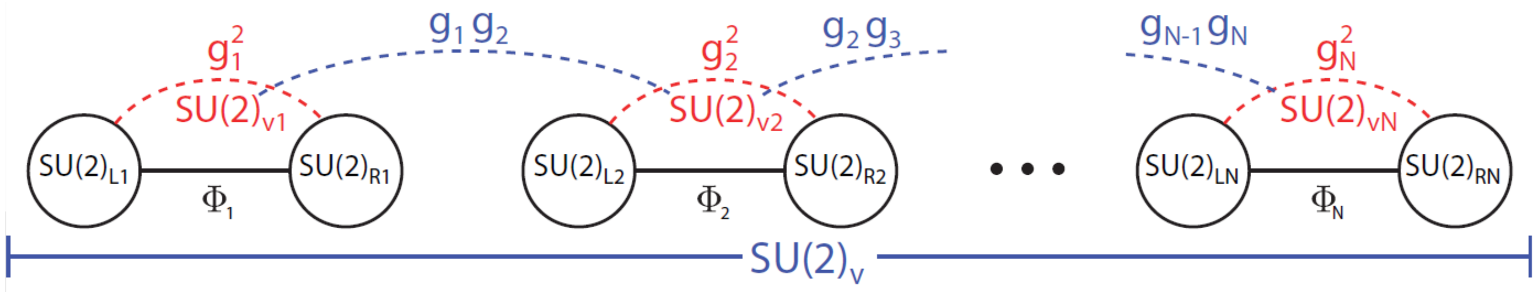


- $\Phi_j \to L_j \Phi_j R_j^\dagger$;
- Φ_j gets a vev $\langle \Phi_j \rangle = \frac{f}{2}$, spontaneously breaking $SU(2)_{L_j} \times SU(2)_{R_j} \to SU(2)_{V_j}$;
- $g_j g_{j+1}$ terms break $SU(2)_{V_j} \times SU(2)_{V_{j+1}} \to SU(2)_{V_{j,j+1}}$;

Realizing the Relaxion with *N*-site Models ('*N*-Relaxion')

NF, L. de Lima, C. S. Machado, R. D. Matheus; Phys.Rev. D94 (2016) no.1, 015010

Minimum Model



- $\Phi_j \to L_j \Phi_j R_j^\dagger$;
- Φ_j gets a vev $\langle \Phi_j \rangle = \frac{f}{2}$, spontaneously breaking $SU(2)_{L_j} \times SU(2)_{R_j} \to SU(2)_{V_j}$;
- $g_j g_{j+1}$ terms break $SU(2)_{V_j} \times SU(2)_{V_{j+1}} \to SU(2)_{V_{j,j+1}}$;

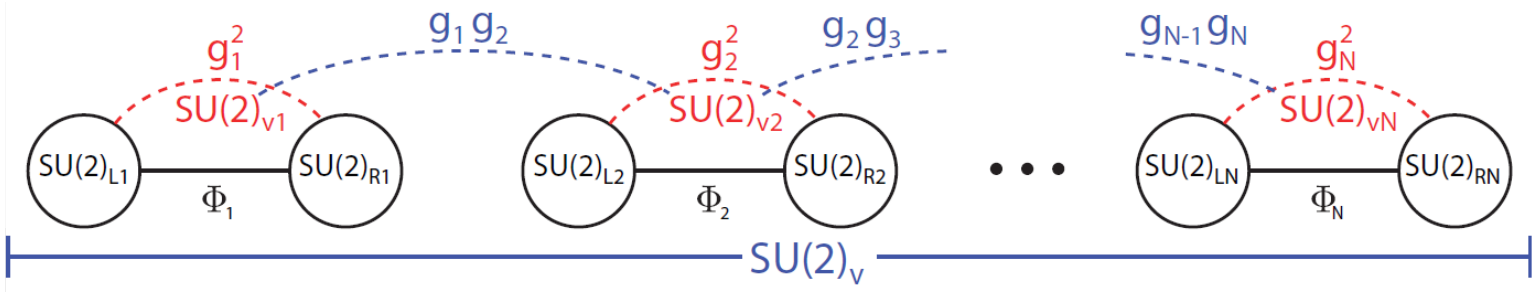
Small symmetry breaking terms

$$\mathcal{L}_\Phi = \sum_{j=1}^N \mathrm{Tr}\left[\partial_\mu \Phi_j^\dagger \partial^\mu \Phi_j + \frac{f^3}{2}(2-\delta_{j,1}-\delta_{j,N}) g_j^2 \left(\Phi_j + \Phi_j^\dagger\right)\right] - \frac{f^2}{2} \sum_{j=1}^{N-1} g_j g_{j+1} \mathrm{Tr}\left[(\Phi_j - \Phi_j^\dagger)(\Phi_{j+1} - \Phi_{j+1}^\dagger)\right]$$

Realizing the Relaxion with *N*-site Models ('*N*-Relaxion')

NF, L. de Lima, C. S. Machado, R. D. Matheus; Phys.Rev. D94 (2016) no.1, 015010

Minimum Model



- $\Phi_j \to L_j \Phi_j R_j^\dagger$;
- Φ_j gets a vev $\langle \Phi_j \rangle = \frac{f}{2}$, spontaneously breaking $SU(2)_{L_j} \times SU(2)_{R_j} \to SU(2)_{V_j}$;
- $g_j g_{j+1}$ terms break $SU(2)_{V_j} \times SU(2)_{V_{j+1}} \to SU(2)_{V_{j,j+1}}$;
- **In the low energy limit, these fields are non-linearly realized:**

$$\Phi_j = \frac{f}{2} e^{i\vec{\pi}_j \cdot \vec{\sigma}/f}, \quad \vec{\pi}_j \text{ are the Nambu-Goldstone bosons.}$$

Realizing the Relaxion with *N*-site Models (‘*N*-Relaxion’)

NF, L. de Lima, C. S. Machado, R. D. Matheus; Phys.Rev. D94 (2016) no.1, 015010

Writing in terms of the zero mode η_0 (the relaxion):

$g_j \to q^j,\ 0 < q < 1$

$f_j \equiv f q^{j-N} \mathcal{C}_N$

$$\mathcal{L}_\eta = \sum_{j=1}^{N} \left[\frac{1}{2}\partial_\mu \vec{\eta}_0 \cdot \partial^\mu \vec{\eta}_0 + f^4(2-\delta_{j,1}-\delta_{j,N})q^{2j}\cos\frac{\eta_0}{f_j}\right] + \sum_{j=1}^{N-1} f^4 q^{2j+1} \sin\frac{\eta_0}{f_j}\sin\frac{\eta_0}{f_{j+1}}$$

Realizing the Relaxion with *N*-site Models ('*N*-Relaxion')

NF, L. de Lima, C. S. Machado, R. D. Matheus; Phys.Rev. D94 (2016) no.1, 015010

Writing in terms of the zero mode η_0 (the relaxion):

$g_j \rightarrow q^j, \; 0 < q < 1$

$f_j \equiv f q^{j-N} \mathcal{C}_N$

$\eta_0 \equiv \sqrt{\vec{\eta}_0 \cdot \vec{\eta}_0}$

$$\mathcal{L}_\eta = \sum_{j=1}^{N} \left[\frac{1}{2} \partial_\mu \vec{\eta}_0 \cdot \partial^\mu \vec{\eta}_0 + f^4 (2 - \delta_{j,1} - \delta_{j,N}) q^{2j} \cos \frac{\eta_0}{f_j} \right] + \sum_{j=1}^{N-1} f^4 q^{2j+1} \sin \frac{\eta_0}{f_j} \sin \frac{\eta_0}{f_{j+1}}$$

Oscillating with different scales

Realizing the Relaxion with N-site Models (‘N-Relaxion’)

NF, L. de Lima, C. S. Machado, R. D. Matheus; Phys.Rev. D94 (2016) no.1, 015010

Writing in terms of the zero mode η_0 (the relaxion):

$$g_j \to q^j, \; 0 < q < 1$$

$$f_j \equiv f q^{j-N} \mathcal{C}_N$$

$$\eta_0 \equiv \sqrt{\vec{\eta}_0 \cdot \vec{\eta}_0}$$

$$\mathcal{L}_\eta = \sum_{j=1}^{N} \left[\frac{1}{2} \partial_\mu \vec{\eta}_0 \cdot \partial^\mu \vec{\eta}_0 + f^4 (2 - \delta_{j,1} - \delta_{j,N}) q^{2j} \cos \frac{\eta_0}{f_j} \right] + \sum_{j=1}^{N-1} f^4 q^{2j+1} \sin \frac{\eta_0}{f_j} \sin \frac{\eta_0}{f_{j+1}}$$

q also controls the amplitudes

Oscillating with different scales

$$f_j \equiv f q^{j-N} \mathcal{C}_N$$

$$f_{\max} = f_1 \approx f / q^{N-1} \; ✓$$

$$f_{\min} = f_N \approx f \; ✓$$

Realizing the Relaxion with *N*-site Models ('*N*-Relaxion')

NF, L. de Lima, C. S. Machado, R. D. Matheus; Phys.Rev. D94 (2016) no.1, 015010

Typical Predictions

- Natural model with $\Lambda \approx 10^8$ GeV;
- No collider signals (no new physics at $\sim$ TeV);
- Extremely light axion-like states.

Realizing the Relaxion with *N*-site Models (‘*N*-Relaxion’)

NF, L. de Lima, C. S. Machado, R. D. Matheus; Phys.Rev. D94 (2016) no.1, 015010

Typical Predictions

- Natural model with $\Lambda \approx 10^8$ GeV;
- No collider signals (no new physics at $\sim$ TeV);
- Extremely light axion-like states.

$q^{N+1} \to g$

$V \supset \epsilon \, \Lambda_c^4 \cos\left(\frac{\eta_0}{f}\right)$

- $f \approx 10^8 \text{ GeV} \to q^{N+1} \approx 10^{-24}; \; \epsilon \approx 10^{-12}$

$$N = 2 \to m_{\eta_0} \approx 10^{-8} \text{ GeV}$$
$$N = 3 \to m_{\eta_0} \approx 10^{-10} \text{ GeV}$$

q^{N+1} fixed

$m_{\eta_0}^2 \approx q^{2N} f^2 \approx 10^{-24} \, q^{N-1} f^2$ (at loop)

- Interactions with the SM through the mixing with the Higgs, suppressed by $\frac{1}{f}$ and/or q^{N+1}, ϵ. Large parameter space allowed.

$q^{N+1} < \epsilon < 1$

Outline

1. The Relaxion Idea

2. Realizing the Relaxion with *N*-site Models

3. Concluding Remarks

Concluding Remarks & Outlook

- **UV sensibility to the Higgs mass: one of the leading motivation for new physics at the LHC;**
- **No compelling evidence of BSM at the LHC current data!** 

Concluding Remarks & Outlook

- **UV** **sensibility** **to the Higgs mass: one of the leading motivation for new physics at the LHC;**
- **No compelling evidence of BSM at the LHC current data!** 
- **Relaxation models: proof of concept. If self-consistent, then the hierarchy problem cannot be an argument for new physics at the TeV scale.**

Chiricahua National Monument, Arizona

Concluding Remarks & Outlook

- *N*-site model generating a large-scale hierarchy; ✓

- **Many possible directions:**

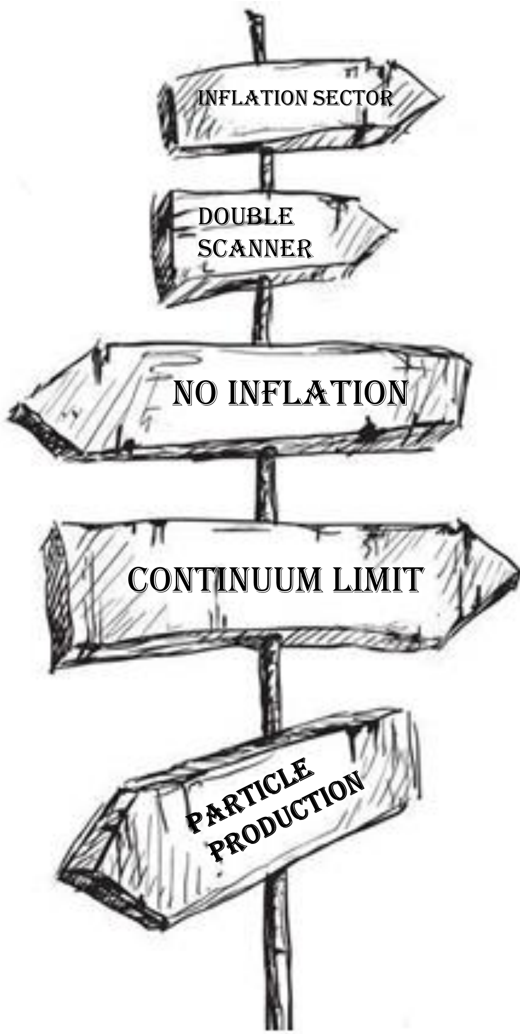
  - **Alternatives to inflation? How can we generate the <u>friction term</u>?**

    Eg.: Relaxation from particle production; A. Hook & G. Marques-Tavares; 1607.01786
    Dissipative Axial Inflation; Notari & Tywoniuk; 1608.06223

  - **<u>Continuum limit</u>? Which theory do we get in AdS_5?**

  - **Can we apply this mechanism to <u>solve other problems</u>?**

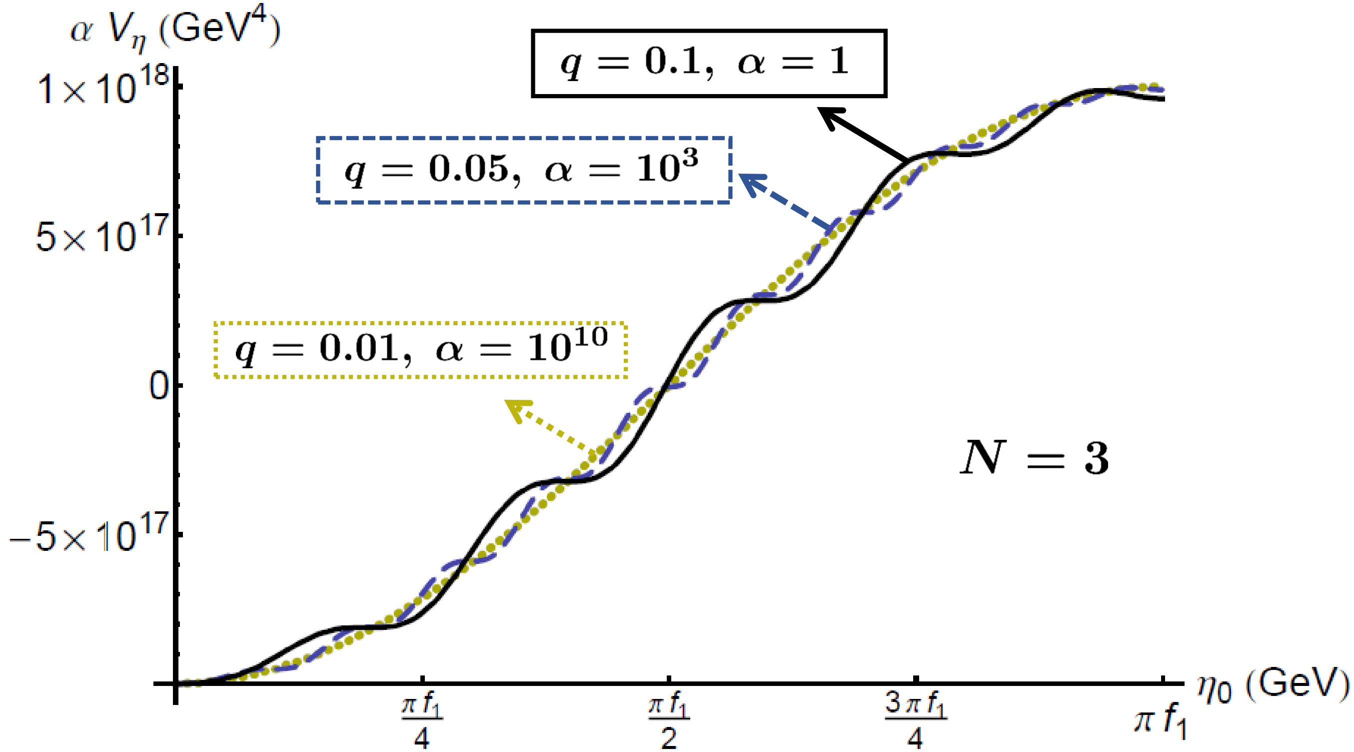
    Eg.: Relaxing the Cosmological Constant: a Proof of Concept; Creminelli et al.; 1608.05715



Thanks!

Realizing the Relaxion with *N*-site Models (‘*N*-Relaxion’)

NF, L. de Lima, C. S. Machado, R. D. Matheus; Phys.Rev. D94 (2016) no.1, 015010



- Quite large values of *q* (to exaggerate the features of the potential);
- the slope quickly gets smooth;
- before the inclusion of the Higgs the relaxion is able to roll down.

Fermionic UV Model

NF, L. de Lima, C. S. Machado, R. D. Matheus; Phys.Rev. D94 (2016) no.1, 015010

$$\mathcal{L}_{UV} = \sum_{j=1}^{N} \{\bar{\psi}_j \not{p} \psi_j + \bar{\chi}_j \not{p} \chi_j\} + \sum_{j=1}^{N-1} \{\bar{\psi}_{Lj} \left[\lambda_j \phi_j + \lambda_{j+1}\phi_{j+1} - \lambda'_j f\right] \psi_{Rj} + \bar{\chi}_{Lj} \left[\tilde{\lambda}_j \phi_j - \tilde{\lambda}_{j+1}\phi^\dagger_{j+1} - \tilde{\lambda}'_j f\right] \chi_{Rj} + \text{h.c.}\}$$

$$\Downarrow$$

$$\mathcal{L}_\Phi = \sum_{j=1}^{N} \text{Tr}\left[\partial_\mu \Phi_j^\dagger \partial^\mu \Phi_j + \frac{f^3}{2}(2-\delta_{j,1}-\delta_{j,N}) g_j^2 \left(\Phi_j + \Phi_j^\dagger\right)\right] - \frac{f^2}{2}\sum_{j=1}^{N-1} g_j g_{j+1} \text{Tr}\left[(\Phi_j - \Phi_j^\dagger)(\Phi_{j+1} - \Phi_{j+1}^\dagger)\right]$$

$$\mathcal{L}'_{UV} = \xi^\dagger \not{p} \xi + \zeta \not{p} \zeta^\dagger + \xi(\epsilon\phi_N - m)\zeta + \text{h.c.},$$

$$\Downarrow$$

$$\mathcal{L}_{\eta,H} \to \mathcal{L}_{\eta,H} + \epsilon \frac{\Lambda_c}{16\pi} \text{Tr}[\Phi_N + \Phi_N^\dagger]|H|^2$$

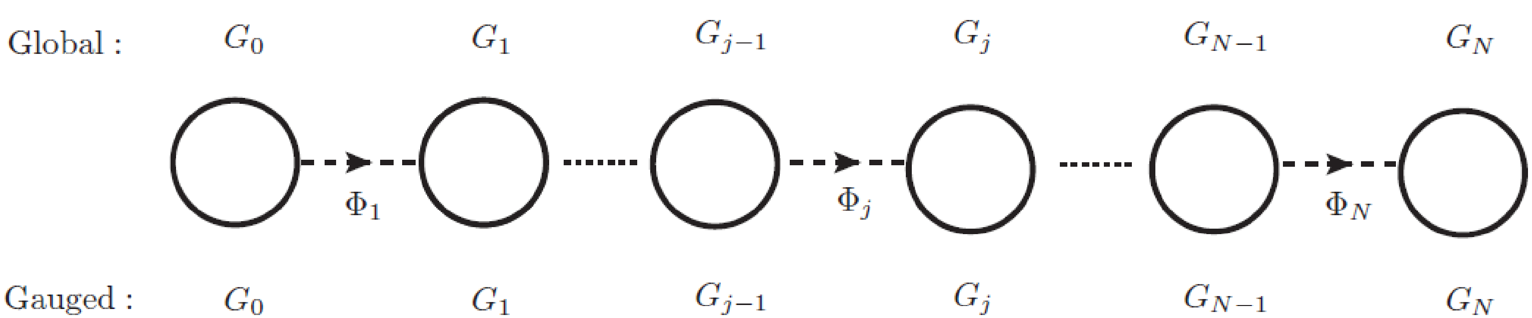
Dimensional Deconstruction

Arkani-Hamed et al. (2001), Hill et al. (2001), Randall et al. (2002), Falkowski et al. (2002)

- *4D* gauge theory with *N+1* gauge groups;
- $G = G_0 \times G_1 \times \ldots \times G_{N-1} \times G_N$;
- The link fields are bi-fundamental: $\Phi_j \to U_{j-1}\Phi_j U_j^\dagger$;

$$S_4 = \int d^4x \left\{ -\frac{1}{2} \sum_{j=0}^{N} \mathrm{Tr}\left[F_{\mu\nu,j}F_j^{\mu\nu}\right] + \sum_{j=1}^{N} \mathrm{Tr}\left[(D_\mu\Phi_j)^\dagger D^\mu\Phi_j\right] - V(\Phi_j) \right\}$$

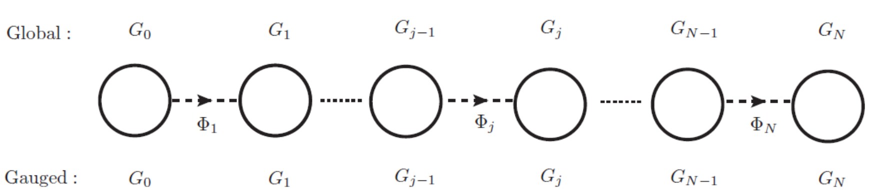
- $\Phi_j = \frac{f_j}{\sqrt{2}} e^{i\sqrt{2}\pi_j^a \hat{T}^a/f_j}$, the vevs break $SU(m)_{j-1} \times SU(m)_j$ down to the diagonal .



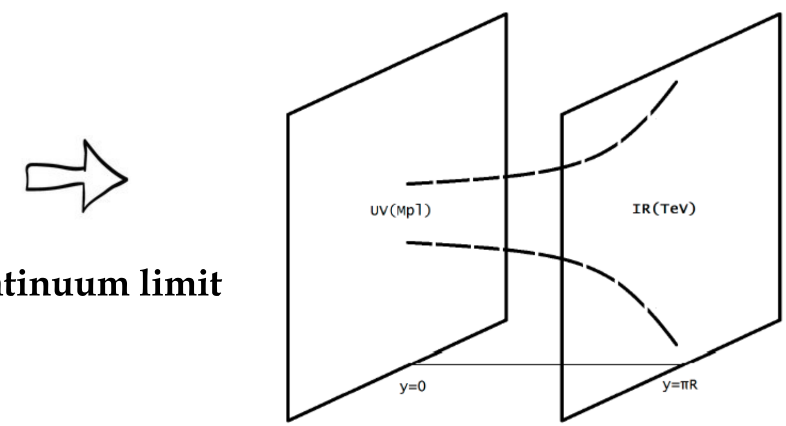
Dimensional Deconstruction

- AdS_5 metric can be mimicked: $f_j = q^j f, \; 0 < q < 1$
- The $f_j's$ progressively decrease from the *zeroth* to the *N-th* site

We compare the actions in order to match the Randall-Sundrum model in the continuum limit of the Deconstructed Theory.



Continuum limit



Dimensional Deconstruction: Dictionary

- ***N*-site action in 4D**

$$S_4 = \frac{1}{\tilde{g}^2}\int d^4x \left\{ -\frac{1}{2}\sum_{j=0}^{N} \mathrm{Tr}\left[F_{\mu\nu,j}F_j^{\mu\nu}\right] + \sum_{j=1}^{N} f_j^2 \tilde{g}^2 \mathrm{Tr}\left[A_{\mu,j} - A_{\mu,j-1}\right]^2 \right\}$$

- **Discretized AdS$_5$**

$$S_5 = \frac{a}{g_5^2}\int d^4x \left\{ -\frac{1}{2}\sum_{j=0}^{N} \mathrm{Tr}\left[F_{\mu\nu,j}F_j^{\mu\nu}\right] + \sum_{j=1}^{N} e^{-2kaj}\mathrm{Tr}\left[\frac{A_{\mu,j} - A_{\mu,j-1}}{a}\right]^2 \right\}$$

$$\int_0^{\pi R} dy \to \sum_{j=0}^{N} a, \;\; \partial_5 A_\mu \to \frac{A_{\mu,j} - A_{\mu,j-1}}{a}$$

Dimensional Deconstruction: Dictionary

- ***N*-site action in 4D**

$$S_4 = \frac{1}{\tilde{g}^2}\int d^4x \left\{ -\frac{1}{2}\sum_{j=0}^{N} \mathrm{Tr}\left[F_{\mu\nu,j}F_j^{\mu\nu}\right] + \sum_{j=1}^{N} f_j^2\tilde{g}^2 \mathrm{Tr}\left[A_{\mu,j} - A_{\mu,j-1}\right]^2 \right\}$$

- **Discretized AdS_5**

$$S_5 = \frac{a}{g_5^2}\int d^4x \left\{ -\frac{1}{2}\sum_{j=0}^{N} \mathrm{Tr}\left[F_{\mu\nu,j}F_j^{\mu\nu}\right] + \sum_{j=1}^{N} e^{-2kaj} \mathrm{Tr}\left[\frac{A_{\mu,j} - A_{\mu,j-1}}{a}\right]^2 \right\}$$

Both agree in the continuum limit if:

$$\frac{1}{\tilde{g}^2} \leftrightarrow \frac{a}{g_5^2}$$

$$f_j \equiv f q^j \leftrightarrow \frac{e^{-kaj}}{a\tilde{g}}$$

The continuum is obtained in the limit $a \to 0$, $N \to \infty$, keeping fixed $Na = L$, with $L = \pi R$ the size of the extra dimension.

Realizing the Relaxion with *N*-site Models ('*N*-Relaxion')

NF, L. de Lima, C. S. Machado, R. D. Matheus; Phys.Rev. D94 (2016) no.1, 015010

Higgs-Axion Interplay

$$\mathcal{L}_{\eta,H} = \left(1 + \frac{|H|^2}{\Lambda^2}\right)\mathcal{L}_\eta + |D_\mu H|^2 + \frac{\Lambda^2}{2}|H|^2 - \frac{\lambda_H}{4}|H|^4 + \epsilon\frac{\Lambda_c}{16\pi}\mathrm{Tr}[\Phi_N + \Phi_N^\dagger]|H|^2$$

Realizing the Relaxion with N-site Models (‘N-Relaxion’)

NF, L. de Lima, C. S. Machado, R. D. Matheus; Phys.Rev. D94 (2016) no.1, 015010

Higgs-Axion Interplay

SM

New breaking at site N

$$\mathcal{L}_{\eta,H} = \left(1+\frac{|H|^2}{\Lambda^2}\right)\mathcal{L}_\eta + |D_\mu H|^2 + \frac{\Lambda^2}{2}|H|^2 - \frac{\lambda_H}{4}|H|^4 + \epsilon\frac{\Lambda_c}{16\pi}\mathrm{Tr}[\Phi_N + \Phi_N^\dagger]|H|^2$$

$\Lambda_c \sim \Lambda \sim f$

Linear terms:
$-g\eta_0\Lambda^3 - \frac{1}{2}g\eta_0\Lambda H^2$

When $\langle h\rangle \neq 0$, it generates: $\epsilon f^2|H|^2\cos\frac{\eta_0}{f_N}$

Closing the Higgs loop: $\epsilon f^4\cos\frac{\eta_0}{f_N}$

Realizing the Relaxion with N-site Models ('N-Relaxion')

NF, L. de Lima, C. S. Machado, R. D. Matheus; Phys.Rev. D94 (2016) no.1, 015010

Higgs-Axion Interplay

SM

New breaking at site N

$$\mathcal{L}_{\eta,H} = \left(1 + \frac{|H|^2}{\Lambda^2}\right)\mathcal{L}_\eta + |D_\mu H|^2 + \frac{\Lambda^2}{2}|H|^2 - \frac{\lambda_H}{4}|H|^4 + \epsilon\frac{\Lambda_c}{16\pi}\mathrm{Tr}[\Phi_N + \Phi_N^\dagger]|H|^2$$

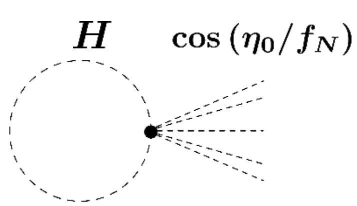
Linear terms:
$-g\eta_0\Lambda^3 - \frac{1}{2}g\eta_0\Lambda H^2$

$\Lambda_c \sim \Lambda \sim f$

When $\langle h\rangle \neq 0$, it generates: $\epsilon f^2|H|^2\cos\frac{\eta_0}{f_N}$

Closing the Higgs loop: $\epsilon f^4\cos\frac{\eta_0}{f_N}$

High barriers everywhere!



Realizing the Relaxion with N-site Models (‘N-Relaxion’)

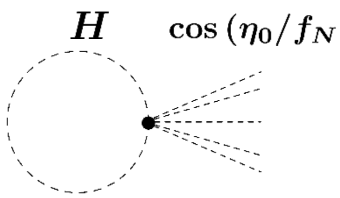
NF, L. de Lima, C. S. Machado, R. D. Matheus; Phys.Rev. D94 (2016) no.1, 015010

Higgs-Axion Interplay

SM — **New breaking at site N**

$$\mathcal{L}_{\eta,H} = \left(1 + \frac{|H|^2}{\Lambda^2}\right)\mathcal{L}_\eta + |D_\mu H|^2 + \frac{\Lambda^2}{2}|H|^2 - \frac{\lambda_H}{4}|H|^4 + \epsilon\frac{\Lambda_c}{16\pi}\mathrm{Tr}[\Phi_N + \Phi_N^\dagger]|H|^2$$

High barriers everywhere!



Double scanner mechanism
(scan also the amplitude of the $\cos(\eta_0/f_N)$ term)

J.R. Espinosa, C. Grojean, G. Panico, A. Pomarol O. Pujolàs, G. Servant; Phys.Rev.Lett. 115 (2015) no.25, 251803

Adding a new scalar singlet field (amplitude scanner)

Realizing the Relaxion with *N*-site Models (‘*N*-Relaxion’)

NF, L. de Lima, C. S. Machado, R. D. Matheus; Phys.Rev. D94 (2016) no.1, 015010

Mass Matrix

$$\mathcal{L}_\Phi = \sum_{j=1}^{N} \mathrm{Tr}\left[\partial_\mu \Phi_j^\dagger \partial^\mu \Phi_j + \frac{f^3}{2}(2-\delta_{j,1}-\delta_{j,N}) g_j^2 \left(\Phi_j + \Phi_j^\dagger\right)\right] - \frac{f^2}{2} \sum_{j=1}^{N-1} g_j g_{j+1} \mathrm{Tr}\left[(\Phi_j - \Phi_j^\dagger)(\Phi_{j+1} - \Phi_{j+1}^\dagger)\right]$$

Writing in terms of the $\vec{\pi}_j{}'s$, we obtain the mass matrix for the pNGBs:

$$\vec{\pi}^T \cdot M_\pi^2 \cdot \vec{\pi} \equiv \sum_{j=1}^{N-1} f^2 (g_j \vec{\pi}_j - g_{j+1} \vec{\pi}_{j+1})^2$$

$$\vec{\pi}^T \equiv \{\vec{\pi}_1, \cdots, \vec{\pi}_N\}$$

$$M_\pi^2 = f^2 \begin{pmatrix} q^2 & -q^3 & 0 & \ldots & 0 & 0 \\ -q^3 & 2q^4 & -q^5 & \ldots & 0 & 0 \\ 0 & -q^5 & 2q^6 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \ldots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & 2q^{2(N-1)} & -q^{2N-1} \\ 0 & 0 & 0 & \ldots & -q^{2N-1} & q^{2N} \end{pmatrix}$$

✓ The parametrization $g_j \to q^j,\ 0 < q < 1$ results in a mass matrix that is identical to the one obtained for a pNGB Wilson line in the deconstruction of AdS_5.

Realizing the Relaxion with N-site Models ('N-Relaxion')

NF, L. de Lima, C. S. Machado, R. D. Matheus; Phys.Rev. D94 (2016) no.1, 015010

This matrix has a zero mode (at tree level), and its profile is:

$$m_{\eta_0}^2 \approx q^{2N} f^2 \text{ (at loop)}$$

$$\vec{\eta}_0 = \sum_{j=1}^{N} \frac{q^{N-j}}{\sqrt{\sum_{k=1}^{N} q^{2(k-1)}}} \vec{\pi}_j$$

Exponentially localized in the last site

✓ **Our scenario does admit a continuum limit, which should correspond to a pNGB in AdS_5.**

$$\mathcal{L}_\eta = \sum_{j=1}^{N} \left[\frac{1}{2} \partial_\mu \vec{\eta}_0 \cdot \partial^\mu \vec{\eta}_0 + f^4 (2 - \delta_{j,1} - \delta_{j,N}) q^{2j} \cos \frac{\eta_0}{f_j} \right] + \sum_{j=1}^{N-1} f^4 q^{2j+1} \sin \frac{\eta_0}{f_j} \sin \frac{\eta_0}{f_{j+1}}$$

Realizing the Relaxion with N-site Models ('N-Relaxion')

NF, L. de Lima, C. S. Machado, R. D. Matheus; Phys.Rev. D94 (2016) no.1, 015010

This matrix has a zero mode (at tree level), and its profile is:

$$m_{\eta_0}^2 \approx q^{2N} f^2 \text{ (at loop)}$$

$$\vec{\eta}_0 = \sum_{j=1}^{N} \frac{q^{N-j}}{\sqrt{\sum_{k=1}^{N} q^{2(k-1)}}} \vec{\pi}_j$$

Exponentially localized in the last site

✓ Our scenario does admit a continuum limit, which should correspond to a pNGB in AdS_5.

$$\eta_0 \equiv \sqrt{\vec{\eta}_0 \cdot \vec{\eta}_0}$$

$$\mathcal{L}_\eta = \sum_{j=1}^{N} \left[\frac{1}{2} \partial_\mu \vec{\eta}_0 \cdot \partial^\mu \vec{\eta}_0 + f^4 (2 - \delta_{j,1} - \delta_{j,N}) q^{2j} \cos \frac{\eta_0}{f_j} \right] + \sum_{j=1}^{N-1} f^4 q^{2j+1} \sin \frac{\eta_0}{f_j} \sin \frac{\eta_0}{f_{j+1}}$$

q also controls the amplitudes

Oscillating with different scales

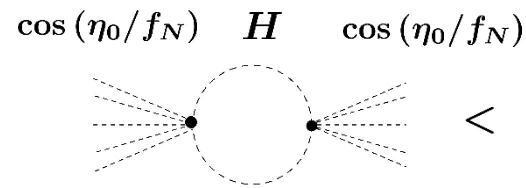
$$f_j \equiv f q^{j-N} \mathcal{C}_N$$

Realizing the Relaxion with *N*-site Models (‘*N*-Relaxion’)

NF, L. de Lima, C. S. Machado, R. D. Matheus; Phys.Rev. D94 (2016) no.1, 015010

Requirements to get the electroweak scale

- The relaxion does not drive inflation;
- Classical rolling > quantum fluctuations;



$$< \epsilon f^2 v^2 \cos\frac{\eta_0}{f_N}$$

- Suppressing higher order terms $\sim \epsilon^2 f^4 \cos^2(\eta_0/f_N) \Rightarrow \epsilon < (v/f)^2.$

- <u>EW scale:</u> the slope should be zero when $\langle h \rangle \approx 246$ GeV;
- Solving classical rolling stop $\frac{\partial V_{\eta,H}}{\partial \eta_0} = 0 \Rightarrow \langle h \rangle^2 \approx \frac{f^2}{\epsilon} q^{N+1}.$

$q^{N+1} \to g$

$$q^{N+1} < \epsilon < 1$$

$$\frac{\Lambda^6}{f^3 M_{\rm Pl}^3} \lesssim q^{N+1} \lesssim \frac{v^4}{f^4}$$

$$f \lesssim 10^8 \text{ GeV}$$

$$q \lesssim 10^{-23/(N+1)}$$

Realizing the Relaxion with *N*-site Models (‘*N*-Relaxion’)

NF, L. de Lima, C. S. Machado, R. D. Matheus; Phys.Rev. D94 (2016) no.1, 015010

Distinctive features:

- ***N*-site model generating a large-scale hierarchy;**

  ⇨ **The *N* fields are bi-fundamentals of 2*N* SU(2) groups (can be generalized to other non-abelian groups);**

  ⇨ **The mass matrix for the pNGBs is exactly the one obtained from a pNGB Wilson line in the deconstruction of AdS_5;**

$$v_{\rm EW}^2 \approx \frac{f^2}{\epsilon} q^{N+1}$$

  ⇨ **The relation between the $v_{\rm EW}$ and f is maintained in the continuum limit ($N \to \infty;\ q \to 1;\ q^{N+1}$ fixed):**

$$f^2 q^{N+1} \to \frac{M}{g_5^2} e^{-kL}$$

- *L* is the size of the extra dimension;
- *k* is the curvature, g_5 is the 5D gauge coupling;
- *M* is the cutoff of the UV theory.

$$\frac{f_1}{f_N} \to e^{kL}$$

related by the AdS_5 warp factor.

QCD Relaxion

P. W. Graham, D. E. Kaplan, S. Rajendran; Phys. Rev. Lett. 115, 221801 (2015)

ϕ **is the QCD axion,** $\mathcal{L} \supset \frac{g_s^2}{32\pi^2}\frac{\phi}{f} G_{\mu\nu}\tilde{G}^{\mu\nu}$

Instanton effects generate: $V(\phi, H) \sim m_u(H)\langle q\bar{q}\rangle \cos(\phi/f)$

$$\Lambda_c = \Lambda_{QCD} \qquad \epsilon = Y_u \qquad V_{\text{stop}} = \epsilon\Lambda_c^4 \left(\frac{\langle h\rangle}{\Lambda_c}\right)^n \cos\frac{\phi}{f} \qquad n = 1$$

$$\Lambda < 10^7 \text{ GeV} \left(\frac{10^9 \text{ GeV}}{f}\right)^{1/6}$$

$$10^9 \text{ GeV} < f < 10^{12} \text{ GeV}$$

Star cooling — DM abundance

- **<u>But</u> this model is ruled out by the strong CP problem (**$\theta_{\text{QCD}} < 10^{-10}$**)**

- **If the relaxion is the QCD axion, its vev determines the QCD theta parameter.**

$$\Rightarrow \theta_{\text{QCD}} = \langle \frac{\Delta\phi}{f} \rangle \sim \mathcal{O}(1)$$!? **(Due to the tilt of the potential)**

Ways to solve this: result in new physics close to the TeV (even if it is not there to solve the hierarchy problem)