The $\mathcal{N} = 2$ superconformal bootstrap

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September 29 2016

Rethinking Quantum Field Theory

The $\mathcal{N} = 2$ bootstrappers:
C. Beem, M. Lemos, W. Peelaers, I. Ramirez, L. Rastelli, J. Seo, B. van Rees.

A vast zoo of $\mathcal{N}=2$ SCFTs

$\mathcal{N}=2$ superconformal dynamics is extremely rich. We have an ample catalog of theories:

- Lagrangian theories built with vectors and hypers such that $\beta(g)=0$.
- Solutions to the Seiberg-Witten curve: some strongly interacting with no known Lagrangian.
- Low energy limit of string theory on D3-branes.
- Gaiotto theories obtained by compactifying $6d$ $(2,0)$ theories on Riemann surfaces.

Is there an underlying structure?

Having such a vast zoo, instead of solving theories one by one, maybe we should try to find the underlying principles, and attempt to classify $\mathcal{N} = 2$ SCFTs.

Work in this direction includes

- Classification of Lagrangian theories. (Bhardwaj,Tachikawa)
- Classification of solutions to the Seiberg-Witten curve. (Argyres, Lotito, Lu, Martone, Xie, Yau)
- Classification of Gaiotto theories. (Chacaltana, Distler, Tachikawa, Trimm)
- The Superconformal Bootstrap. (The $\mathcal{N} = 2$ bootstrappers)

The Conformal Group

The conformal group puts tight restrictions on correlation functions

$$\langle\phi_1(x_1)\phi_2(x_2)\rangle = \begin{cases} \frac{1}{|x_1-x_2|^{2\Delta_\phi}} & \text{if } \Delta_1 = \Delta_2 \\ 0 & \text{if } \Delta_1 \neq \Delta_2 \end{cases},$$

$$\langle\phi_1(x_1)\phi_2(x_2)\phi_3(x_3)\rangle = \frac{\lambda_{123}}{|x_{12}|^{\Delta_1+\Delta_2-\Delta_3}|x_{23}|^{\Delta_2+\Delta_3-\Delta_1}|x_{13}|^{\Delta_1+\Delta_3-\Delta_2}}.$$

The collection $\{\lambda, \Delta\}$ is the CFT data.
The four-point function is not completely fixed, for identical fields,

$$\langle\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \frac{g(u,v)}{|x_{12}|^{2\Delta_\phi}|x_{34}|^{2\Delta_\phi}},$$

with

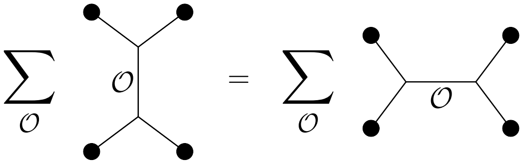
$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} \qquad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}.$$

Crossing Symmetry

Invariance under the interchange of $x_1 \leftrightarrow x_3$ gives *crossing symmetry*:

$$v^{\Delta_\phi}(1 + \sum_{\mathcal{O}} \lambda^2_{\mathcal{O}} g_{\mathcal{O}}(u, v)) = u^{\Delta_\phi}(1 + \sum_{\mathcal{O}} \lambda^2_{\mathcal{O}} g_{\mathcal{O}}(v, u))$$

It can be represented pictorially,



Overconstrained system for the CFT data, but what to do with it?

Bootstrap techniques

There are several approaches to study crossing:

- **Virasoro symmetry.** In $2d$ the enhancement to Virasoro allows for an exact solution: the celebrated minimal models.
  (Belavin, Polyakov, Zamolodchikov)
- **The numerical bootstrap.** Powerful numerical techniques that constrain the low-lying spectrum.
  (Poland, Rattazi, Rychkov, Simmons-Duffin, Tonni, Vichi)
- **The analytical bootstrap.** Analytic constraints for operators with high spin.
  (Alday, Poland, Kaplan, Komargodski, Fitzpatrick, Simmons-Duffin, Zhiboedov)
- **Solvable truncation.** In supersymmetric theories there is a solvable truncation of the crossing equations.
  (C. Beem, M. Lemos, PL, W. Peelaers, B. van Rees.)

The $\mathcal{N} = 2$ superconformal bootstrap relies only on the algebra and not on a Lagrangian! It can be formulated as a two-step process:

- **$2d$ chiral algebras**

  Any $4d$ $\mathcal{N} = 2$ SCFT contains a subset of operators described by a $2d$ chiral algebra.

  $$4d \text{ SCFT} \quad \rightarrow \quad 2d \text{ Chiral Algebra}$$

  The $2d$ chiral algebra only describes *protected* operators.

- **Numerics**

  For operators outside the $2d$ chiral algebra, we have less analytic control and we have to resort to numerics. In general *unprotected* quantities.

Meromorphy in $\mathcal{N} = 2$ SCFTs

First, we fix a plane $\mathbb{R}^2 \in \mathbb{R}^4$.

Claim: There is a subsector with meromorphic correlators.

$$\langle \mathcal{O}_1(z_1, \bar{z}_1)\mathcal{O}_2(z_2, \bar{z}_2) \cdots \mathcal{O}_n(z_n, \bar{z}_n)\rangle = f(z_i)$$

We consider operators killed by a particular combination of supercharges:

$$[\mathbb{Q}, \mathcal{O}(z, \bar{z})] = 0 \qquad \mathbb{Q} = \mathcal{Q} + \mathcal{S}$$

The $\bar{z}$-dependence is $\mathbb{Q}$-exact. A standard argument implies the correlator is independent of $\bar{z}$

The R-symmetry current

The R-symmetry current is a Schur operator:

$$J^{j}_{i\mu}(z,\bar{z}) \to T(z)$$

The $4d$ OPE implies

$$T(z)T(0) \sim -\frac{6\,c_{4d}}{z^4} + 2\frac{T(0)}{z^2} + \frac{\partial T(0)}{z} + \ldots .$$

we have Virasoro symmetry with central charge

$$c_{2d} = -12\,c_{4d}\,.$$

The R-symmetry current is related to the $4d$ stress tensor $T_{\mu\nu} \sim (\mathcal{Q}\bar{\mathcal{Q}}J)_{\mu\nu}$.

The moment map operator

The moment map operator M^{ij} is the highest weight of a 1/2 BPS multiplet and is a generator in the Higgs branch chiral ring.

$$M^{ij}(z,\bar{z}) \to J(z)$$

The $4d$ OPE implies

$$J^a(z)J^b(0) \sim \frac{k_{2d}}{z^2}\delta^{ab} + if^{abc}\frac{J^c(0)}{z} + \dots \,.$$

we have an AKM algebra at level

$$k_{2d} = -\frac{1}{2}\, k_{4d}\,.$$

The moment map is related to the flavor current $J_{F\,\mu} \sim (\mathcal{Q}\bar{\mathcal{Q}}M)_\mu$.

The truncated four-point function for $T(z)$ is in textbooks...

$$\langle T(0)T(z)T(1)T(\infty)\rangle \sim 1 + \frac{z^4}{(1-z)^4} + \frac{8}{2c_{2d}}z^2 + \dots$$

A truncation of the full correlator is solvable.
This allows us to solve for certain OPE coefficients

$$\lambda^2_{\mathcal{O}} = \left(2 - \frac{11}{15c_{4d}}\right)$$

Unitarity implies

$$c_{4d} \geqslant \frac{11}{30}$$

The assumptions were

- $\mathcal{N}=2$ superconformal symmetry.
- A stress-tensor or locality.
- That the theory is interacting.

Let's review the literature...

The simplest solutions to the Seiberg-Witten curve are

G	A_0	A_1	A_2	D_4	E_6	E_7	E_8
c	$\frac{11}{30}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{7}{6}$	$\frac{13}{6}$	$\frac{19}{6}$	$\frac{31}{6}$
k	—	$\frac{8}{3}$	3	4	6	8	12

Table: Central charges for canonical $\mathcal{N} = 2$ theories.

These are theories of rank one: only one Coulmob branch generator.

Combining all the bounds

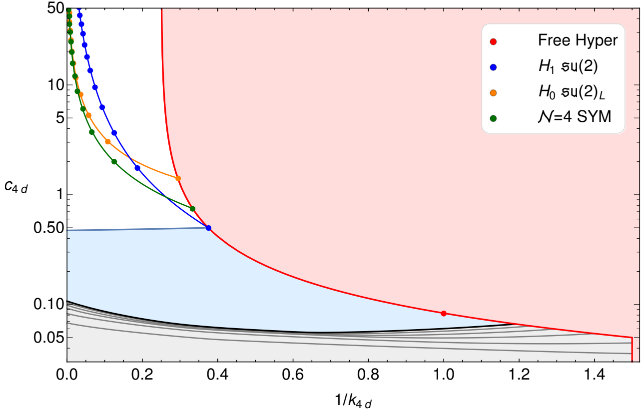


Figure: The landscape of $\mathcal{N} = 2$ SCFTs with flavor group $SU(2)$.

The corner is at $(c, k) = (1/2, 8/3)$.

Thank you.