Structure of Kähler potential for D-term inflationary attractor models

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Collaboration with
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Based on
K. Nakayama, KS, T. Terada, M. Yamaguchi, JHEP 1605 (2016) 067 [arXiv:1603.02557]

Abstract

- Focus on D-term dominated large field inflation models in supergravity
- Discuss general conditions of Kähler potential necessary to realize plateau-type inflationary potential
- Give a concrete example of the α-attractor models for D-term inflation

Introduction

- Inflation: attractive scenario of the early universe
  - Solution to flatness and horizon problem
  - Almost scale invariant, adiabatic, and Gaussian density fluctuations
  - The microphysical origin of inflation remains a mystery
- Supersymmetry (supergravity) is desirable, as it can limit the size of radiative corrections to the inflaton mass
- Two classes of models
  - F-term inflation
    - Typically $\mathcal{O}(H)$ contribution to the inflaton mass – "η-problem"
  - D-term inflation ← This talk
    - How to construct a successful model ? What is the difficulty ?

Large field D-term inflation

Kadota and Yamaguchi (2007), Kawano (2008), Kadota, Kawano and Yamaguchi (2008)

- Two superfields charged under U(1) gauge symmetry

$$\Phi_+ \ (q_+ = +1) \qquad \text{and} \qquad \Phi_- \ (q_- = -1)$$

inflaton $\varphi \equiv \sqrt{2}|\Phi_+|$ required by anomaly cancellation

- Introduce a gauge-singlet superfield S and superpotential

$$W = \lambda S \Phi_+ \Phi_-$$

It generates F-term mass to stabilize non-inflaton fields $\Phi_- = S = 0$

- A quartic potential for the minimal Kähler potential & gauge kinetic function

$$K = |\Phi_+|^2 + |\Phi_-|^2 + |S|^2, \quad f = 1/g^2$$

g : gauge coupling

$$\Rightarrow \quad V_D = \frac{1}{2\mathrm{Re} f}\left(\sum_{i=\pm} q_i K_i \Phi_i\right)^2 = \frac{g^2}{8}\varphi^4$$

- Note: We assume the vanishing Fayet-Iliopoulos (FI) term, which is different from the conventional "D-term inflation" model Binetruy and Dvali (1996), Halyo (1996)

Large field D-term inflation

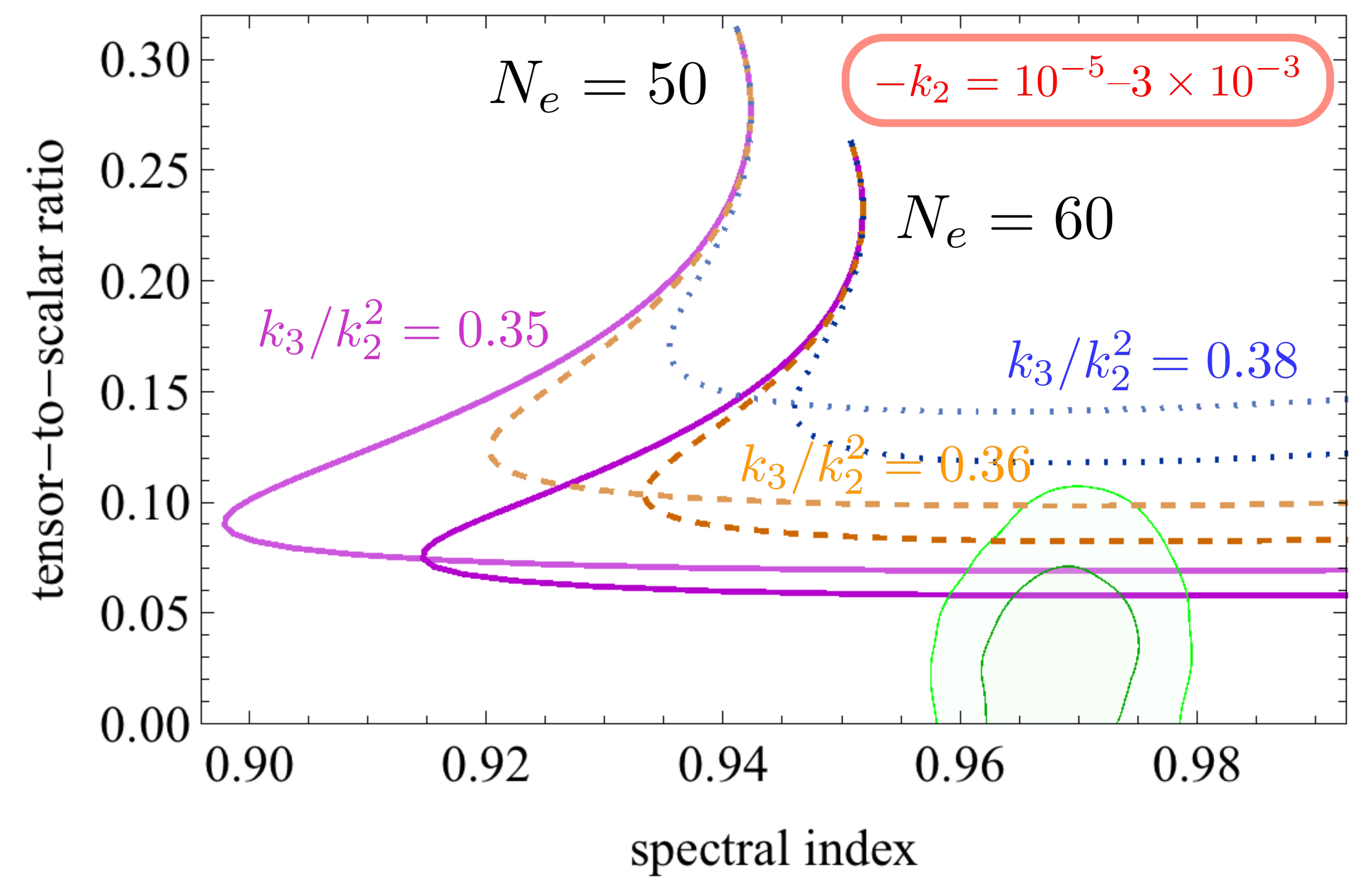
However...

- The prediction of the simple quartic potential conflicts with observational results

$$r = \frac{16}{N_e} \simeq 0.27\text{–}0.32$$

for $N_e \simeq 50\text{–}60$

c.f. $r_{\rm obs} < 0.10$ Planck (2015)



- Introducing higher-order terms in the Kähler potential can change the prediction

$$\Delta K = \frac{k_2}{2}\left(|\Phi_+|^4 + |\Phi_-|^4 + |S|^4\right) + \frac{k_3}{3}\left(|\Phi_+|^6 + |\Phi_-|^6 + |S|^6\right) + \ldots$$

$$\Rightarrow \quad V_D = \frac{g^2\varphi^4}{8}\left(1 + \frac{k_2}{2}\varphi^2 + \frac{k_3}{4}\varphi^4\right)^2$$

with tuned k_2 and k_3

Kähler potential is not controlled by any symmetry, which poses some doubts about the predictability

α-attractor models

Consider a model with a real scalar field ρ

Ferrara, Kallosh, Linde and Porrati, 1307.7696
Kallosh, Linde and Roest, 1311.0472
Galante, Kallosh, Linde and Roest, 1412.3797

$$(\sqrt{-g})^{-1}\mathcal{L} = \frac{1}{2}R - \frac{3\alpha}{4\rho^2}(\partial_\mu \rho)^2 - V(\rho)$$

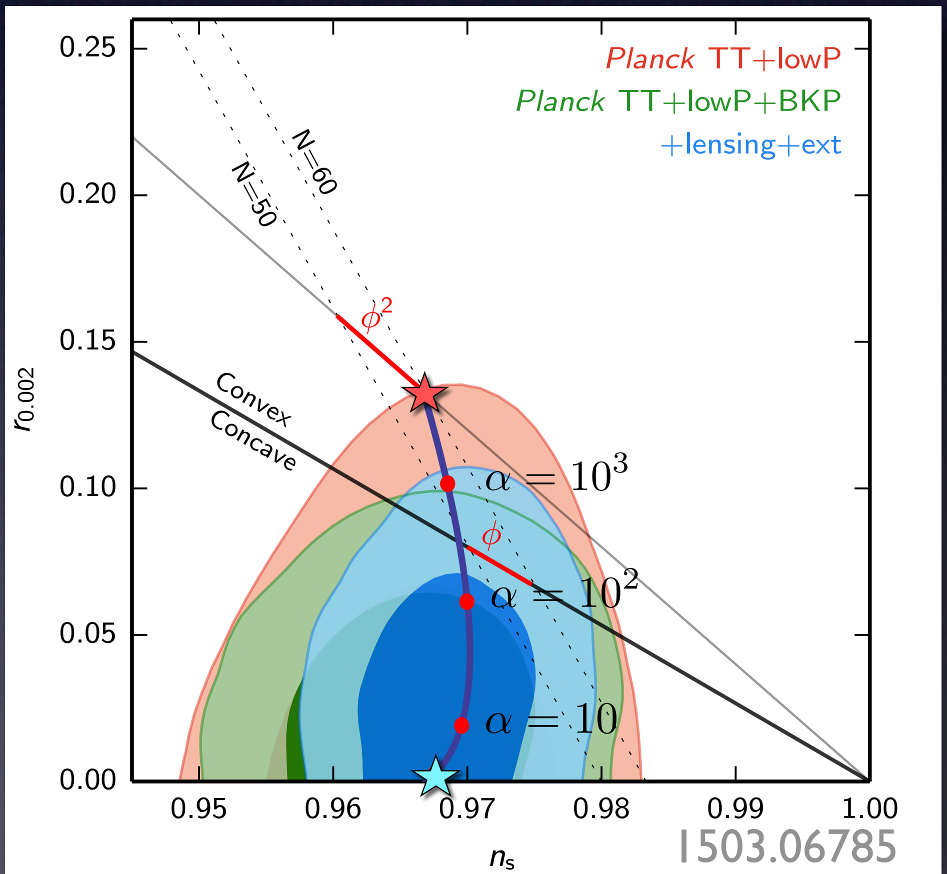
2nd order pole at $\rho = 0$

The potential becomes almost flat in canonical variables, if inflation occurs near the pole

$$\frac{1}{2}R - \frac{1}{2}(\partial_\mu \varphi)^2 - V_0\left(1 - e^{-\sqrt{\frac{2}{3\alpha}}\varphi} + \dots\right)$$

Observables approach the "attractor" regime, almost independently on $V(\rho)$

$$n_s - 1 = -\frac{2}{N_e}, \quad r = \frac{12\alpha}{N_e^2}$$



Our work: to apply this procedure to D-term inflationary models

Kähler potential for D-term attractor model

- Introduce a real variable $C = \Lambda + \bar{\Lambda}$

Ferrara, Kallosh, Linde and Porrati, 1307.7696

$\Lambda = \log(\Phi_+)$ transforms as $\Lambda \to \Lambda + iq\theta$ under U(1)

$$K = K(\Lambda + \bar{\Lambda}) = K(C)$$

- Conditions for α-attractor models

Galante, Kallosh, Linde and Roest, 1412.3797
Broy, Galante, Roest and Westphal, 1507.02277

1. Kinetic term has 2nd order pole

$$\frac{K''}{2}(\partial C)^2 = \frac{3\alpha}{2\tilde{C}^2}(\partial \tilde{C})^2 \quad \text{for} \quad \tilde{C} = \tilde{C}(C)$$

2. D-term potential is "smooth" around the pole

$$V_D = \frac{g^2}{2}(K')^2, \qquad K' = b + a\tilde{C} + \ldots$$

- The form of the Kähler potential is determined

$$K = -3\alpha \log |C| + 3\beta C + (\text{const.}) \qquad C = \Lambda + \bar{\Lambda} = \log |\Phi_+|^2$$
$$= -3\alpha \log\left(\log |\Phi_+|^2\right) + 3\beta \log |\Phi_+|^2 + (\text{const.})$$

$$\Rightarrow \quad V_D = \frac{g^2}{2}\left(3\beta - \frac{3\alpha}{\log |\Phi_+|^2}\right)^2 = \frac{9g^2\beta^2}{2}\left(1 - e^{-\sqrt{\frac{2}{3\alpha}}\tilde{\varphi}}\right)^2$$

- Canonical inflaton field $\tilde{\varphi} = \sqrt{3\alpha/2}\log\left[(\beta/\alpha)(\Lambda + \bar{\Lambda})\right]$

  has an approximate shift symmetry $\tilde{\varphi} \to \tilde{\varphi}' = \tilde{\varphi} - c$

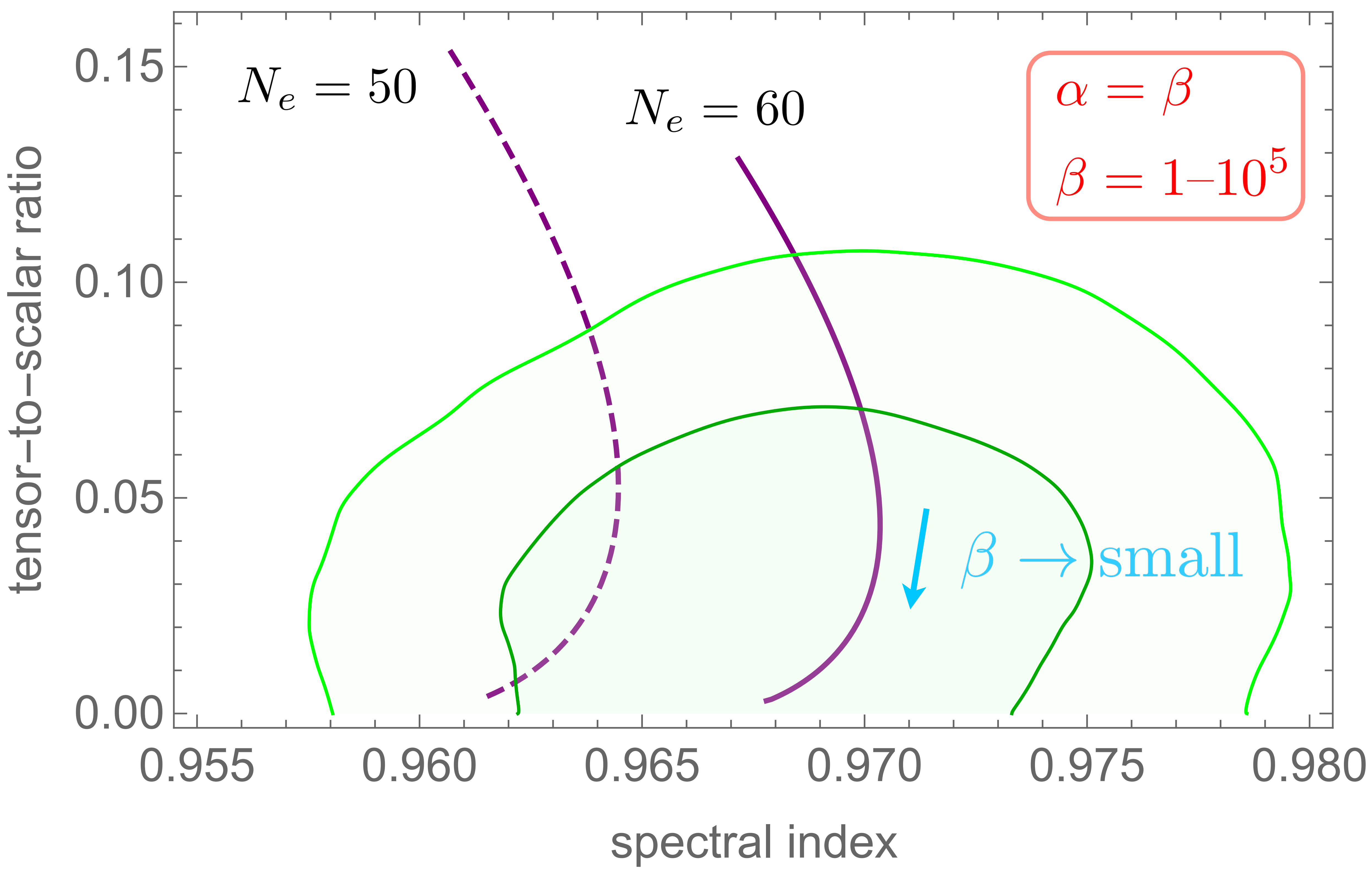
- This corresponds to scale transformation $\Lambda \to \Lambda' = e^{-\sqrt{2/3\alpha}c}\Lambda$

  K is not invariant but Kähler metric is invariant
  (pole structure is unchanged)

$$K \to -3\alpha \log(\log |\Phi'|^2) - \sqrt{6\alpha}c + 3e^{\sqrt{2/3\alpha}c}\beta \log |\Phi'|^2, \qquad K_{\Phi\bar{\Phi}} \to K_{\Phi\bar{\Phi}}$$

It might be interpreted as a geometric property of Kähler manifold

Carrasco, Kallosh, Linde and Roest, 1504.05557



$N_e = 50$
$N_e = 60$
$\alpha = \beta$
$\beta = 1$–10^5
$\beta \to$ small
0.15
0.10
0.05
0.00
0.955
0.960
0.965
0.970
0.975
0.980
tensor–to–scalar ratio
spectral index

Stabilization of other fields and their masses

- Non-inflaton fields (Φ_-, S) must be stabilized during inflation

$$W = \lambda S \Phi_+ \Phi_-$$

$$\Rightarrow \quad m^2_{\Phi_-} = m^2_S \simeq e^K K^{-1}_{S\bar{S}} K^{-1}_{\Phi_-\bar{\Phi}_-} \frac{\lambda^2 \varphi^2}{2} \gtrsim \mathcal{O}(H^2)$$

- But, their masses tend to be large for naively extended Kähler potential

$$K = -3\alpha \log\left[\log\left(|\Phi_+|^2 + |\Phi_-|^2 + |S|^2\right)\right] + 3\beta \log\left(|\Phi_+|^2 + |\Phi_-|^2 + |S|^2\right)$$

$$\Rightarrow \quad m^2_{\Phi_-,S} \propto \varphi^{6\beta+4} \sim e^{\mathcal{O}(N_e)} \quad \text{since} \quad \varphi \simeq \sqrt{2} e^{\frac{2N_e}{3\beta}}$$

Masses are typically much larger than the Planck scale
Validity of the calculation is lost

- This difficulty can be alleviated if we introduce an additional term

$$\Delta K = h|S|^2\left(|\Phi_+|^{6\beta+4} + |\Phi_-|^{6\beta+4}\right)$$

The exponentially large factor is canceled by $K^{-1}_{S\bar{S}} \propto \varphi^{-(6\beta+4)}$

Summary

- Structure of Kähler potential is limited in the context of D-term inflationary attractor models

$$K = -3\alpha \log(\Lambda + \bar{\Lambda}) + 3\beta(\Lambda + \bar{\Lambda})$$
$$= -3\alpha \log\left[\log\left(|\Phi|^2\right)\right] + 3\beta \log\left(|\Phi|^2\right)$$

- The origin of (approximate) shift symmetry for canonical inflaton is identified as scale transformation of Λ

- Masses of the non-inflaton fields (Φ_-, S) tend to exceed the Planck mass, which might be alleviated by introducing a specific coupling to the singlet field S

Backup

Symmetry of the potential

For the transformation $\Lambda \to \Lambda' = e^{-\sqrt{2/3\alpha c}}\Lambda$

(or $\Phi = e^{\Lambda} \to \Phi' = \Phi^{1/\hat{c}}$ with $\hat{c} \equiv e^{\sqrt{2/3\alpha c}}$)

- One can define the transformation of vector superfield

$$V \to V' = V/\hat{c}$$

such that the gauge invariant combination transforms covariantly

$$(\bar{\Phi} e^{V} \Phi) = (\bar{\Phi}' e^{V'} \Phi')^{\hat{c}}$$

- Accordingly, the normalization of gauge kinetic term changes as

$$\frac{1}{g^2}\mathcal{W}\mathcal{W} = \frac{\hat{c}^2}{g^2}\mathcal{W}'\mathcal{W}'$$

- The coefficient β in Kähler potential is also rescaled

$$K \to -3\alpha \log(\log|\Phi'|^2) - 3\alpha \log\hat{c} + 3\hat{c}\beta \log|\Phi'|^2$$

- These facts make the D-term potential $V_D \propto g^2\beta^2$ almost invariant

Comments on the reheating

- The inflaton and gauge boson masses around the potential minimum are given by

$$m_\varphi = m_A \simeq \frac{\sqrt{6} g \beta}{\sqrt{\alpha}} \sim 2\times 10^{12}\,\mathrm{GeV}\sqrt{\frac{\beta}{\alpha}}\left(\frac{g}{10^{-6}}\right)\left(\frac{\sqrt{\beta}}{10^{18}\mathrm{GeV}}\right)$$

- To reheat the universe, we may introduce a kinetic mixing between the U(1) gauge boson and the standard model gauge boson

$$\mathcal{L} \supset -\frac{\chi}{2} F_{\mu\nu} X^{\mu\nu}$$

- There are no bounds on the mixing parameter χ above $m_A \gtrsim \mathrm{MeV}$

Redondo and Postma, 0811.0326

- Decay late

$$\Gamma \gtrsim \frac{\alpha\chi^2}{2} m_A \sim 4\times 10^5\,\mathrm{GeV}\left(\frac{\chi}{10^{-2}}\right)^2\left(\frac{m_A}{10^{12}\,\mathrm{GeV}}\right)$$

- Reheating temperature

$$T_R \sim 10^{11}\,\mathrm{GeV}\left(\frac{\chi}{10^{-2}}\right)\left(\frac{m_A}{10^{12}\,\mathrm{GeV}}\right)^{1/2}$$

Modification of D-term attractor model

- The Kähler potential

$$K = -3\alpha \log\left(\log|\Phi_+|^2\right) + 3\beta \log|\Phi_+|^2$$

is defined only in the domain $|\Phi_+| > 1$:

It cannot describe the Coulomb phase $\Phi_+ = 0$

(U(1) symmetry is unbroken)

- We enlarge the field space to include the Coulomb phase

$$K = -3\alpha \log\left[\log\left(\gamma + |\Phi_+|^2\right)\right] + 3\beta \log\left(\gamma + |\Phi_+|^2\right)$$

by introducing an additional parameter γ

Properties of D-term attractor models

$$K = -3\alpha \log\left[\log\left(\gamma + |\Phi_+|^2\right)\right] + 3\beta \log\left(\gamma + |\Phi_+|^2\right)$$

Kinetic term

$$\mathcal{L}_K = k(\varphi)\frac{(\partial_\mu \varphi)^2}{2} \qquad \varphi \equiv \sqrt{2}|\Phi_+|$$

$$k(\varphi) = \frac{3[\alpha\varphi^2/2 + \gamma\log(\gamma+\varphi^2/2)(\beta\log(\gamma+\varphi^2/2) - \alpha)]}{(\gamma+\varphi^2/2)^2[\log(\gamma+\varphi^2/2)]^2}$$

Potential

$$V(\varphi) = \frac{9g^2}{2}\left(\frac{\varphi^2/2}{\gamma+\varphi^2/2}\right)^2\left(\beta - \frac{\alpha}{\log(\gamma+\varphi^2/2)}\right)^2$$

There are several different regimes according to the value of γ

$\gamma < 1$

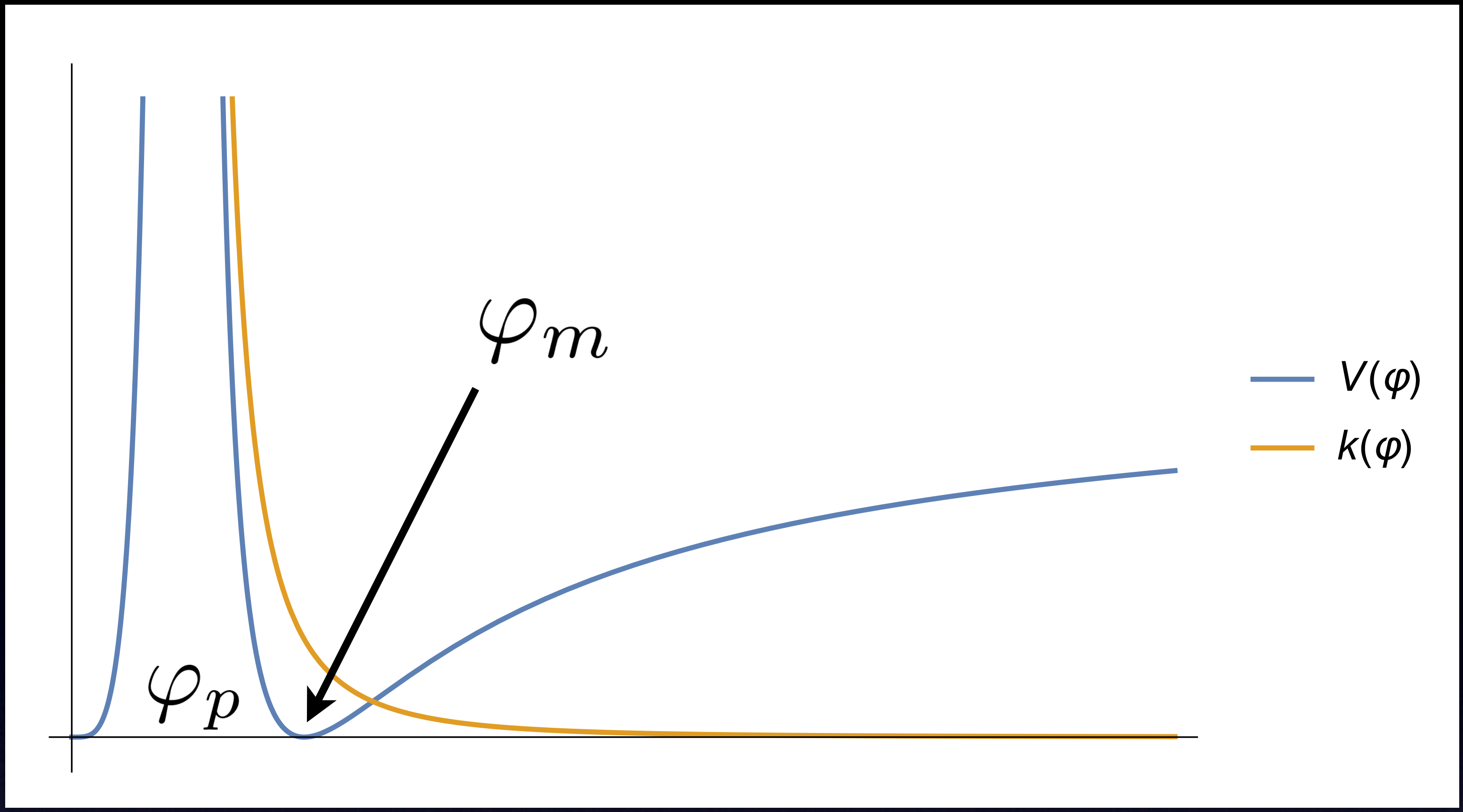
$k(\varphi)$ and $V(\varphi)$

diverge at

$$\varphi_p = \sqrt{2(1-\gamma)}$$

$V(\varphi)$ has a minimum at

$$\varphi_m = \sqrt{2(e^{\alpha/\beta} - \gamma)}$$

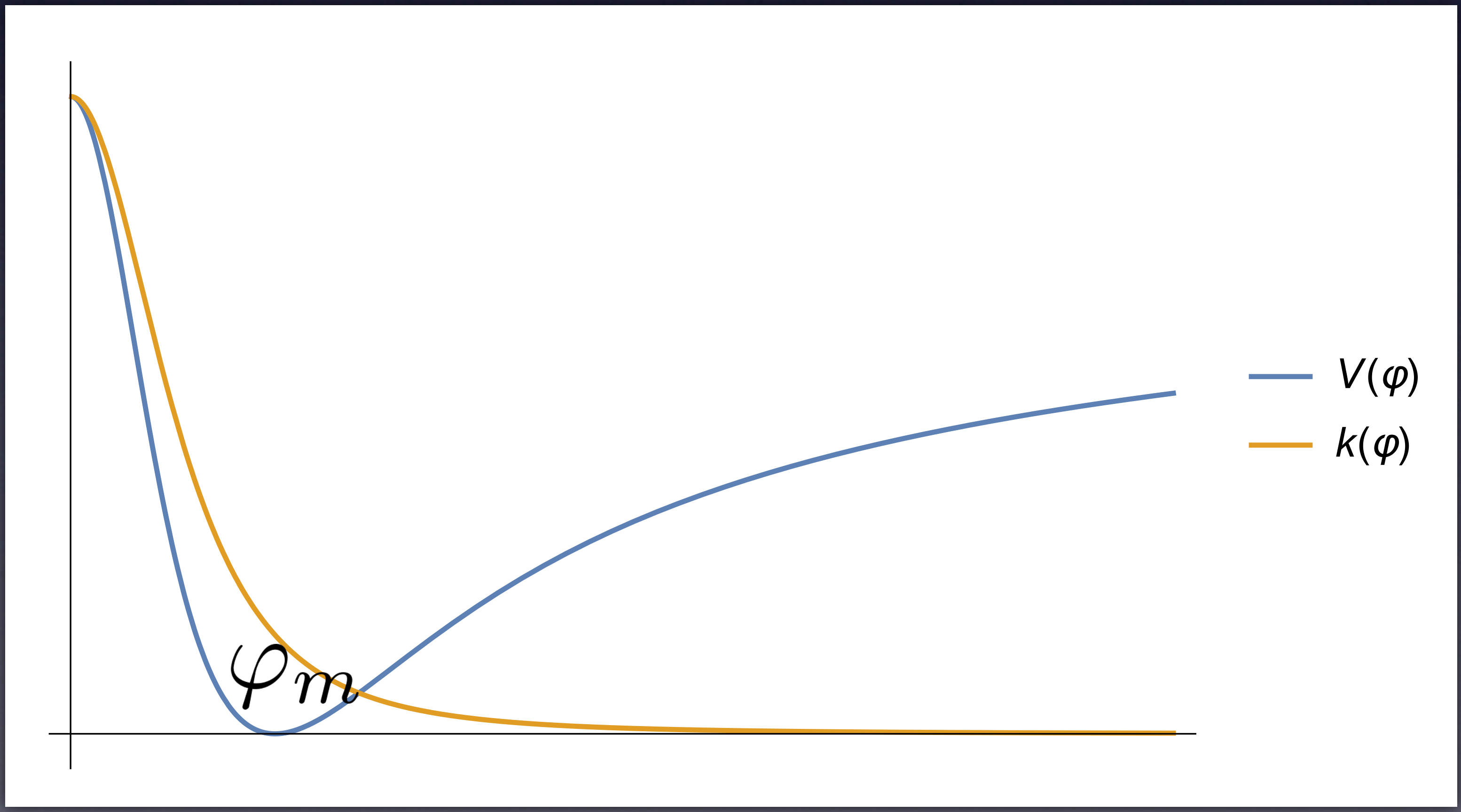


$\gamma = 1$

Potential at $\varphi_p = 0$

has a finite value

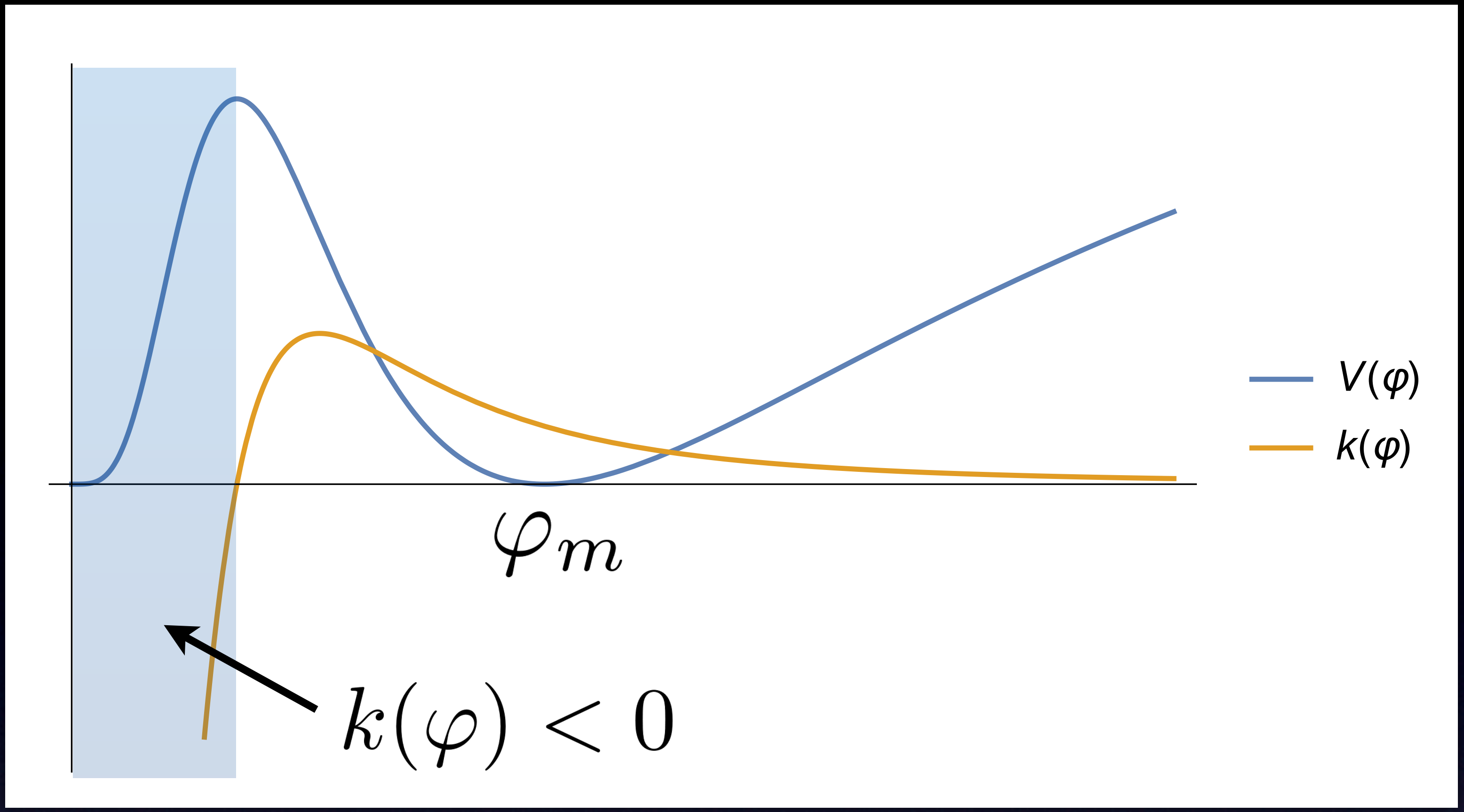
$$V(0) = 9g^2\alpha^2/2$$



$1 < \gamma < \exp(\alpha/\beta)$

Two minima at

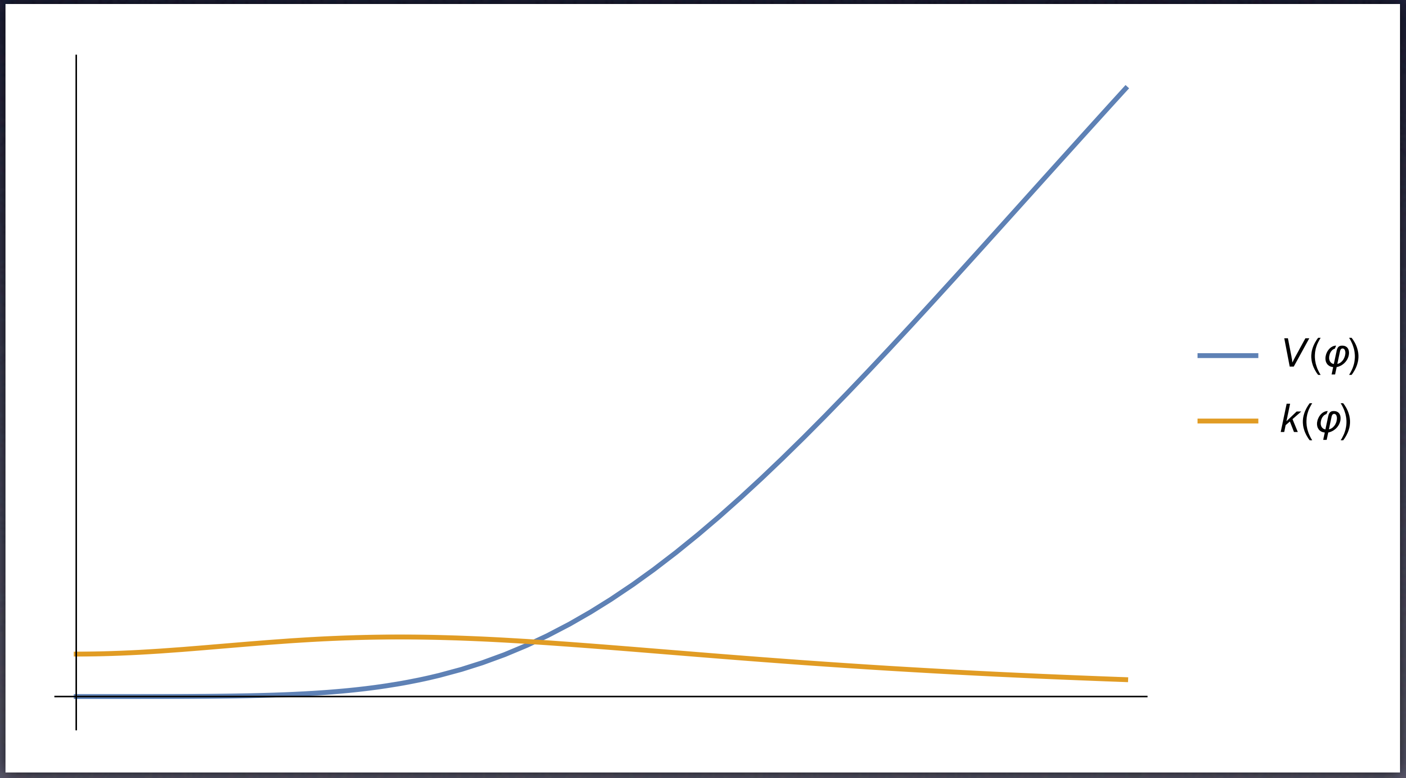
$$\varphi = 0 \text{ and } \varphi = \varphi_m$$



$\exp(\alpha/\beta) \leq \gamma$

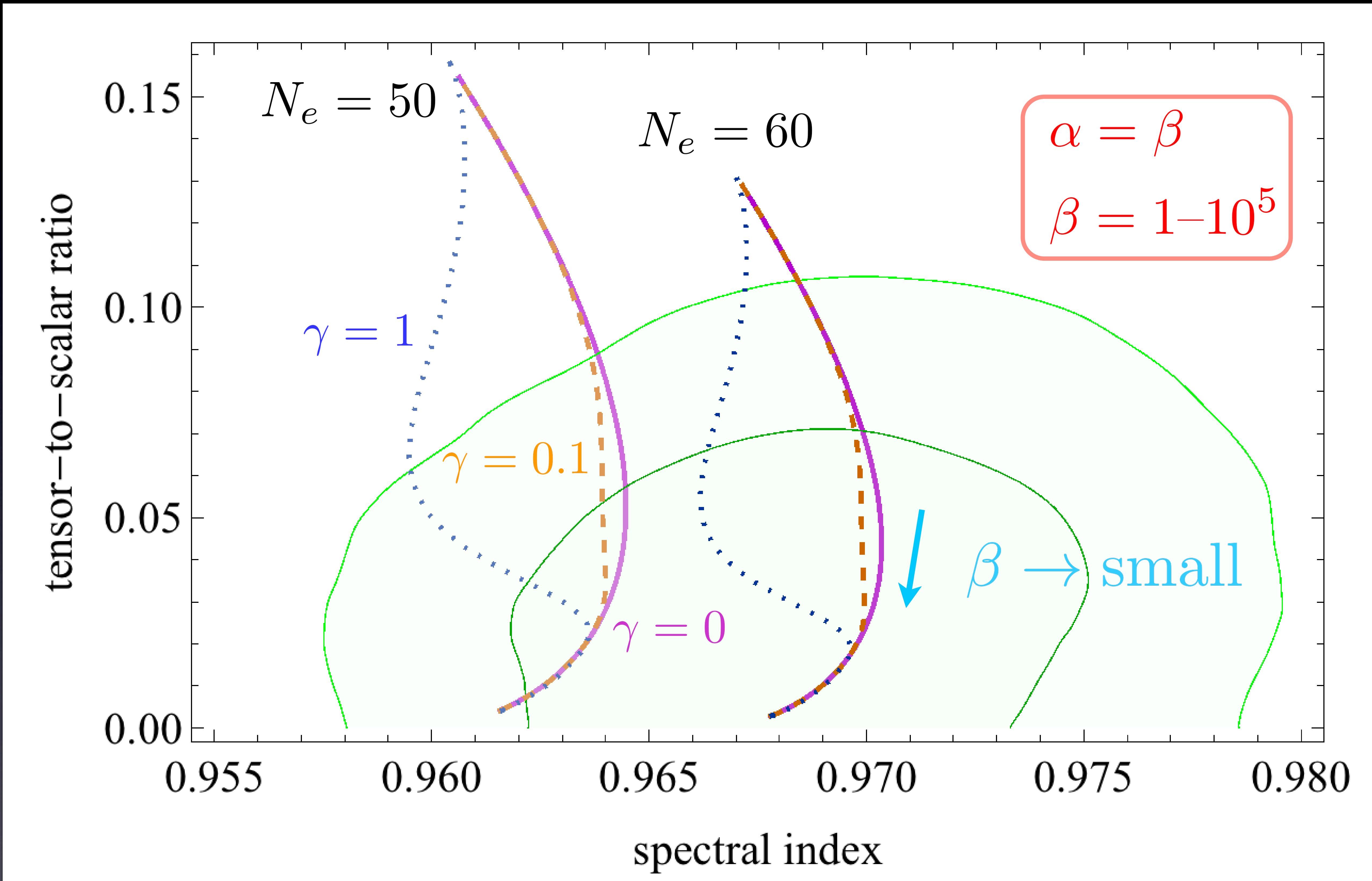
Only one minimum at

$$\varphi = 0$$



In the large γ limit, it approaches the minimal model $V \simeq g^2\tilde{\varphi}^4/8$

Predictions of modified D-term attractor models



Kähler potential for D-term pole inflation (1)

When the inflaton field $\tilde{C}$ has a pole of order p in the kinetic term

$$(\sqrt{-g})^{-1}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\frac{a_p}{\tilde{C}^p}(\partial_\mu \tilde{C})^2 - V(\tilde{C})$$

- Condition for the kinetic term

$$\frac{K''}{2}\left(\frac{dC}{d\tilde{C}}\right)^2 = \frac{a_p}{\tilde{C}^p}$$

- Condition for the D-term potential

$$V_D = \frac{g^2}{2}\left(K'\right)^2, \qquad K' = b + a\tilde{C}$$

- From these conditions, we obtain

$$K = \begin{cases} K_0 + bC - 2a_2 \log|C| & (p=2) \\ K_0 + bC - \frac{p-1}{p-2}\left(\frac{2a_p}{p-1}\right)^{\frac{1}{p-1}}(-aC)^{\frac{p-2}{p-1}} & (p \neq 2) \end{cases}$$

K_0 : integration constant

Kähler potential for D-term pole inflation (2)

- Higher order poles in the kinetic term are regarded as symmetry breaking terms
  Broy, Galante, Roest and Westphal, 1507.02277

$$(\sqrt{-g})^{-1}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\left(\frac{a_p}{\tilde{C}^p} + \frac{a_q}{\tilde{C}^q}\right)(\partial_\mu \tilde{C})^2 - V(\tilde{C})$$

where $q > p$ and $a_p/\tilde{C}^p \gg a_q/\tilde{C}^q$ is assumed

- Condition for the kinetic term is modified as

$$\frac{K''}{2}\left(\frac{dC}{d\tilde{C}}\right)^2 = \frac{a_p}{\tilde{C}^p} + \frac{a_q}{\tilde{C}^q}$$

- Additional term in the Kähler potential

$$\Delta K = -\frac{(p-1)a_q}{(q-1)(q-2)a_p}\left(\frac{2a_p}{p-1}\right)^{\frac{p-q+1}{p-1}}(-aC)^{\frac{q-2}{p-1}}$$

Higher order terms $\propto C^n$ $(n > 1)$ in the Käheler potential are interpreted as shift symmetry breaking effect