# Efficient calculation of cosmological neutrino clustering

MARIA ARCHIDIACONO
RWTH AACHEN UNIVERSITY

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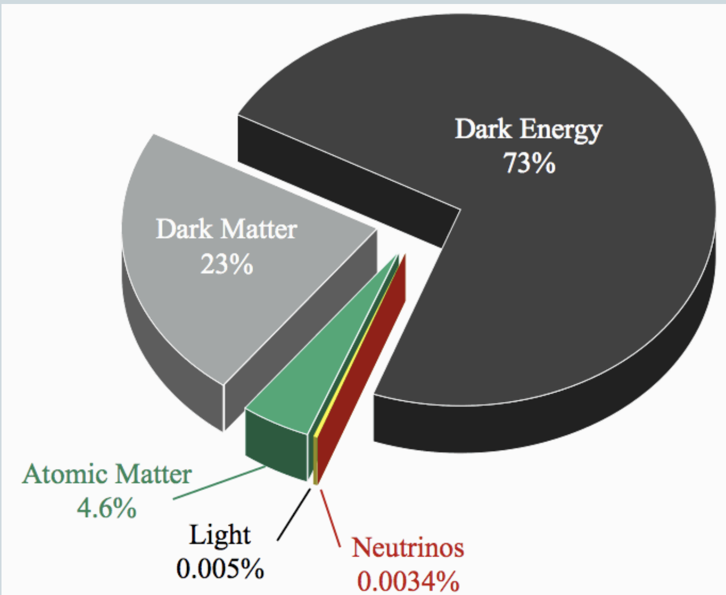
MA, STEEN HANNESTAD

DESY THEORY WORKSHOP
RETHINKING QUANTUM FIELD THEORY
27-30 SEPT 2016

# The role of neutrinos




$$\Delta m^2_{21} = 7.50^{+0.19}_{-0.17} \times 10^{-5} eV^2$$

$$\Delta m^2_{31} = 2.475^{+0.047}_{-0.047} \times 10^{-3} eV^2$$

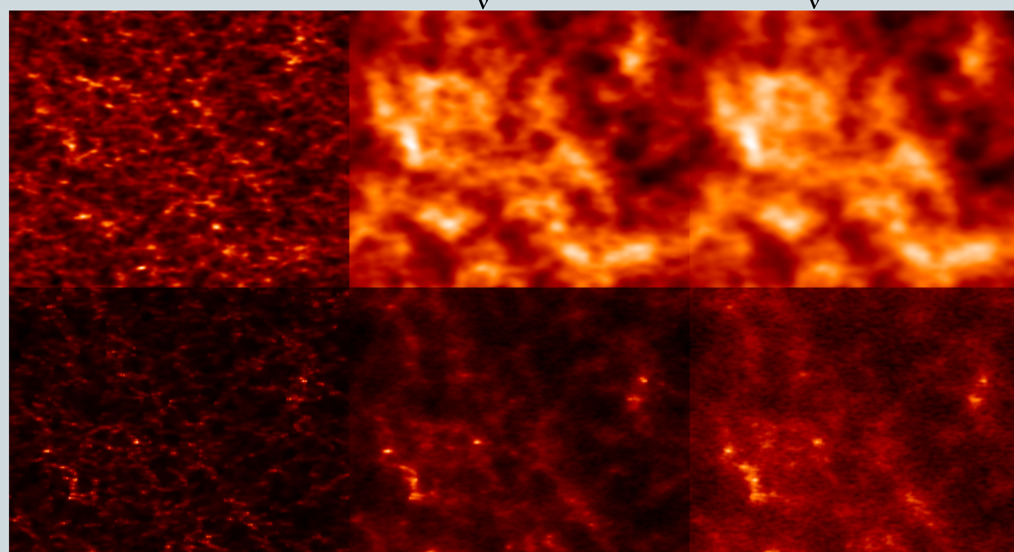
Gonzalez-Garcia et al. (2015)

HDM

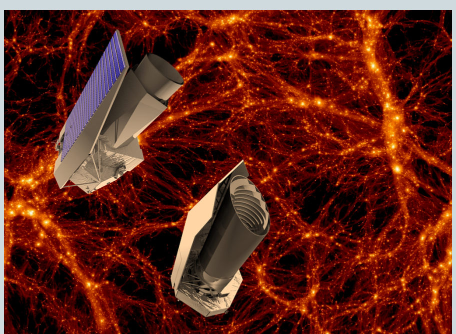
CDM | $\Sigma m_\nu = 0.6$ eV | $\Sigma m_\nu = 0.3$ eV

z = 4

z = 0



Brandbyge et al. (2008)



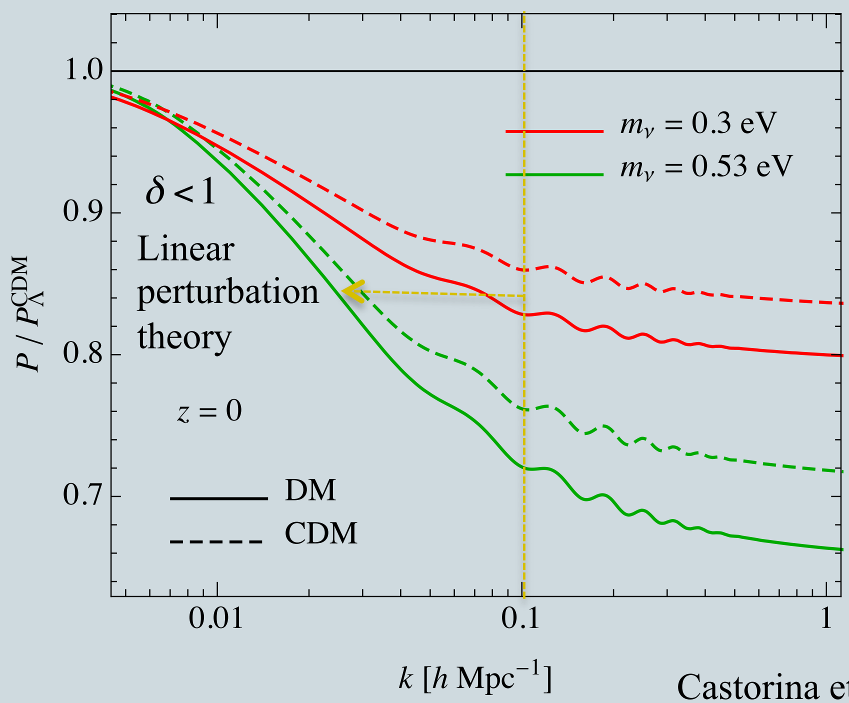
1% accuracy!

Euclid (2020)

# Matter power spectrum in the presence of $\nu$

$$v_{th} \equiv \frac{\langle p \rangle}{m} \approx \frac{3T_\nu}{m} \approx 150(1+z)\left(\frac{1eV}{m}\right) km\ s^{-1}$$

$$\lambda_{FS}(t) \equiv 2\pi\sqrt{\frac{2}{3}}\frac{v_{th}}{H(t)} \approx 7.7\frac{1+z}{\sqrt{\Omega_\Lambda + \Omega_m(1+z)^3}}\left(\frac{1eV}{m}\right) Mpc\ h^{-1}$$




Castorina et al. (2015)

# Neutrino perturbations in the linear regime

$$f(x^i, P_j, \tau) = f_0(q)\left[1 + \Psi(x^i, q, n_j, \tau)\right]$$

Phase-space distribution

$$\frac{\partial f}{\partial \tau} + \frac{\partial f}{\partial x^i}\frac{\partial x^i}{\partial \tau} + \frac{\partial f}{\partial q}\frac{\partial q}{\partial \tau} + \frac{\partial f}{\partial n^i}\frac{\partial n^i}{\partial \tau} = 0$$

Collisionless Boltzmann equation

$$\frac{\partial \Psi}{\partial \tau} + i\frac{q}{\varepsilon}(\vec{k}\cdot\hat{n})\Psi + \frac{d\ln f_0}{d\ln q}\left[\dot{\eta} - \frac{\dot{h} + 6\dot{\eta}}{2}(\vec{k}\cdot\hat{n})^2\right] = 0$$

Synchronous gauge, Fourier space

$$\Psi(\vec{k}, \hat{n}, q, \tau) = \sum_{l=0}^{\infty}(-i)^l(2l+1)\Psi_l(\vec{k}, q, \tau)P_l(\vec{k}\cdot\hat{n})$$

Legendre expansion

$$\dot{\Psi}_0 = -k\frac{q}{\varepsilon}\Psi_1 + \frac{1}{6}\dot{h}\frac{d\ln f_0}{d\ln q}$$

$$\dot{\Psi}_1 = k\frac{q}{3\varepsilon}(\Psi_0 - 2\Psi_2)$$

Energy-momentum conservation

$$\dot{\Psi}_2 = k\frac{q}{5\varepsilon}(2\Psi_1 - 3\Psi_3) - \left(\frac{1}{15}\dot{h} + \frac{2}{5}\dot{\eta}\right)\frac{d\ln f_0}{d\ln q}$$

$$\dot{\Psi}_l = k\frac{q}{(2l+1)\varepsilon}(l\Psi_{l-1} - (l+1)\Psi_{l+1}),\ \ l \geq 3$$

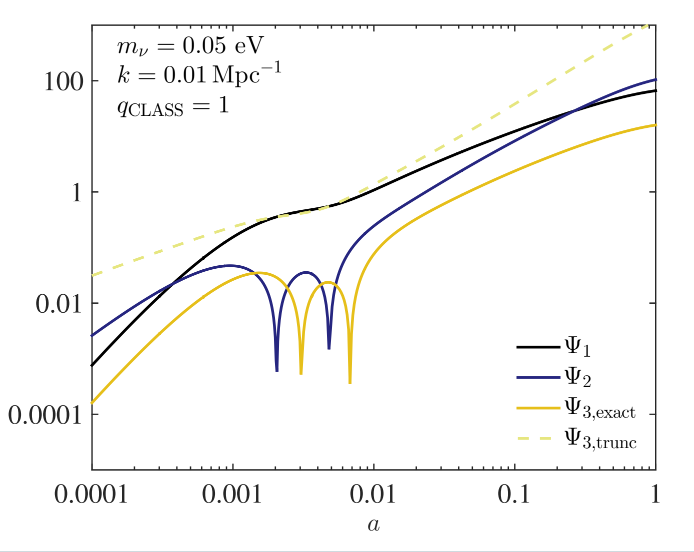
Neutrino free-streaming

# The moment hierarchy truncation

$$\dot{\Psi}_l = k\frac{q}{(2l+1)\varepsilon}\left(l\Psi_{l-1} - (l+1)\Psi_{l+1}\right), \;\; l \geq 3$$

In the absence of any gravitational source term, or for very high $l$ $\Psi_l(k\tau) \propto j_l(k\tau)$

$$\Psi_{l+1} = \frac{2l+1}{k\tau}\Psi_l - \Psi_{l-1}$$




$l = 2$

$$\dot{\Psi}_2 = k\frac{q}{5\varepsilon}(2\Psi_1 - 3\Psi_3) - \left(\frac{1}{15}\dot{h} + \frac{2}{5}\dot{\eta}\right)\frac{d\ln f_0}{d\ln q}$$

$$\Psi_3 \approx \frac{5\varepsilon}{k\tau}\Psi_2 - \Psi_1$$

Archidiacono & Hannestad, arXiv:1510.02907

# A new moment hierarchy truncation

In a non-expanding Universe and in the absence of gravity, the Boltzmann hierarchy:

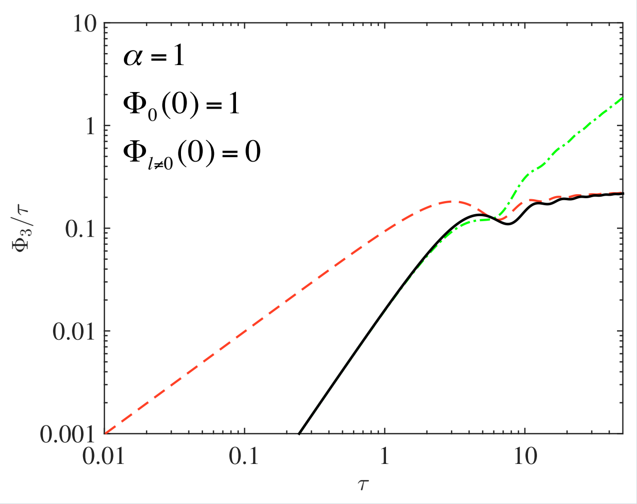
$$\dot{\Psi}_l = \frac{\alpha}{(2l+1)}\left(l\Psi_{l-1} - (l+1)\Psi_{l+1}\right)$$

with solutions $\Psi_l \propto j_l(\alpha\tau)$

$$\dot{\Phi}_l = \frac{\alpha}{(2l+1)}\left(l\Phi_{l-1} - (l+1)\Phi_{l+1}\right) + f(\tau)(\delta_{l0} + \delta_{l2})$$

$$\Phi_l = \frac{g(\tau)}{\sqrt{2l+1}} \qquad \alpha\tau >> 1 \rightarrow \Phi_3 \approx \sqrt{\frac{5}{7}}\Phi_2$$

$$\Phi_l = \frac{\alpha\tau}{2l+1}\Phi_{l-1} \qquad \alpha\tau \le 1 \rightarrow \Phi_3 \approx \frac{\alpha\tau}{7}\Phi_2$$

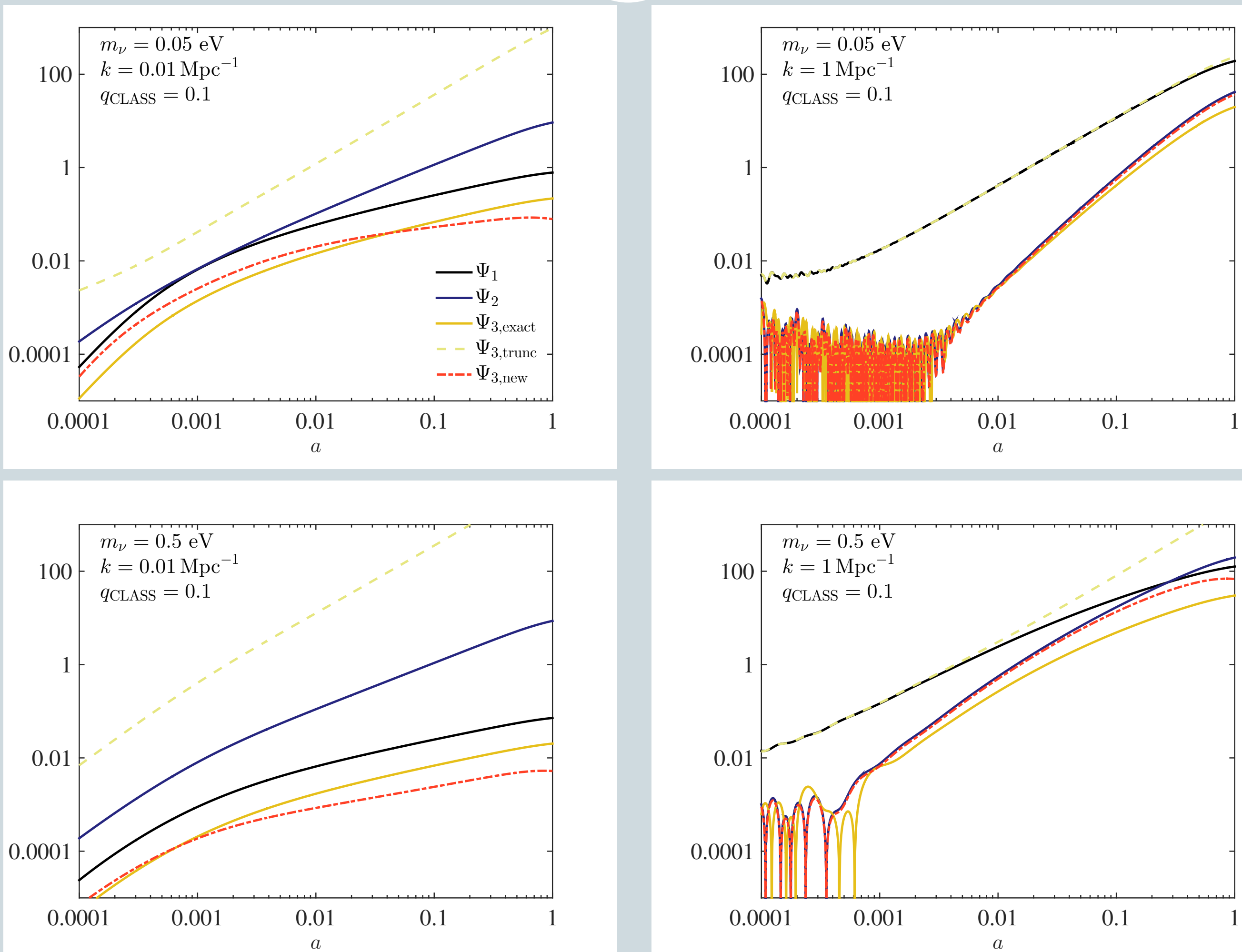



Ansatz:

$$\Psi_3 \approx \left(\frac{\frac{x}{7}\frac{\beta}{x} + \sqrt{\frac{5}{7}}\frac{x}{\beta}}{\frac{\beta}{x} + \frac{x}{\beta}}\right)\Psi_2, \qquad x = \frac{qk\tau}{\varepsilon}$$

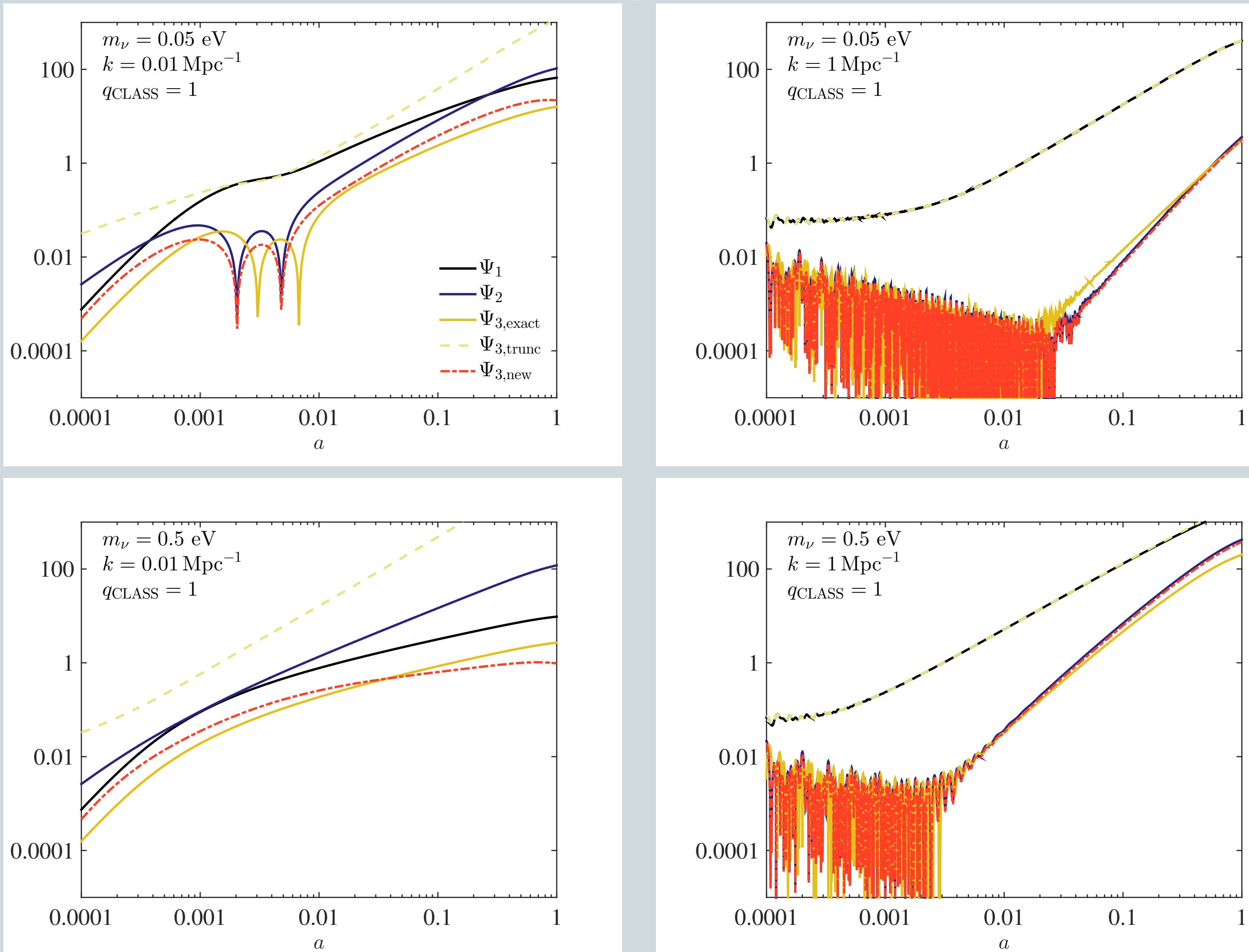
$$\Psi_3 \approx \left(\frac{\frac{1}{7} + \sqrt{\frac{5}{7}}x}{\frac{1}{x} + x}\right)\left[\frac{k}{1h/Mpc}\right]^{0.12}\Psi_2$$

# Comparison: perturbations




Archidiacono & Hannestad, arXiv:1510.02907

# Comparison: perturbations




Archidiacono & Hannestad, arXiv:1510.02907

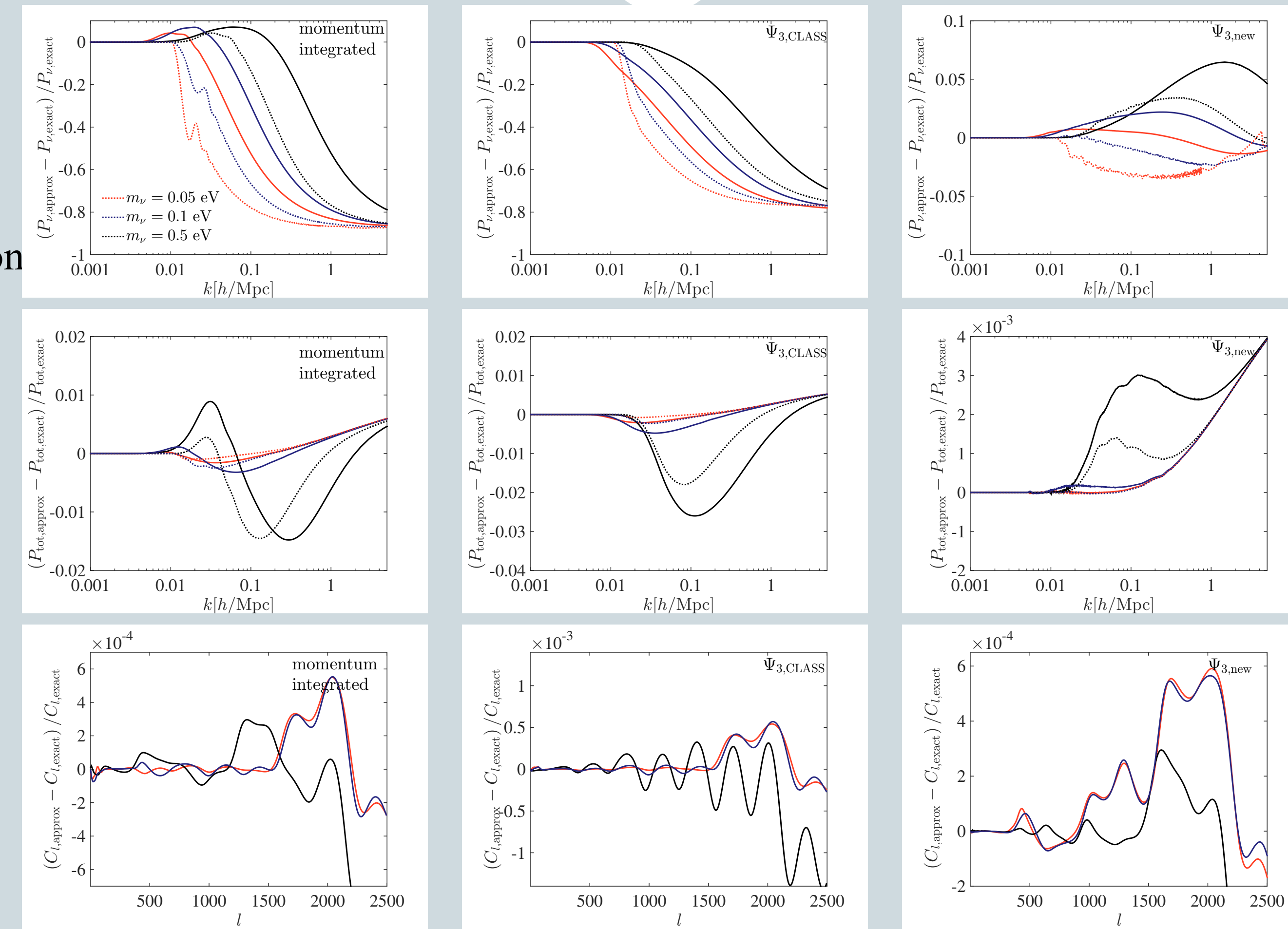
# Comparison: observables

$k\tau \leq 31$

$l = 17$

$k\tau > 31$

approximation
or
truncation




z=0

z=10

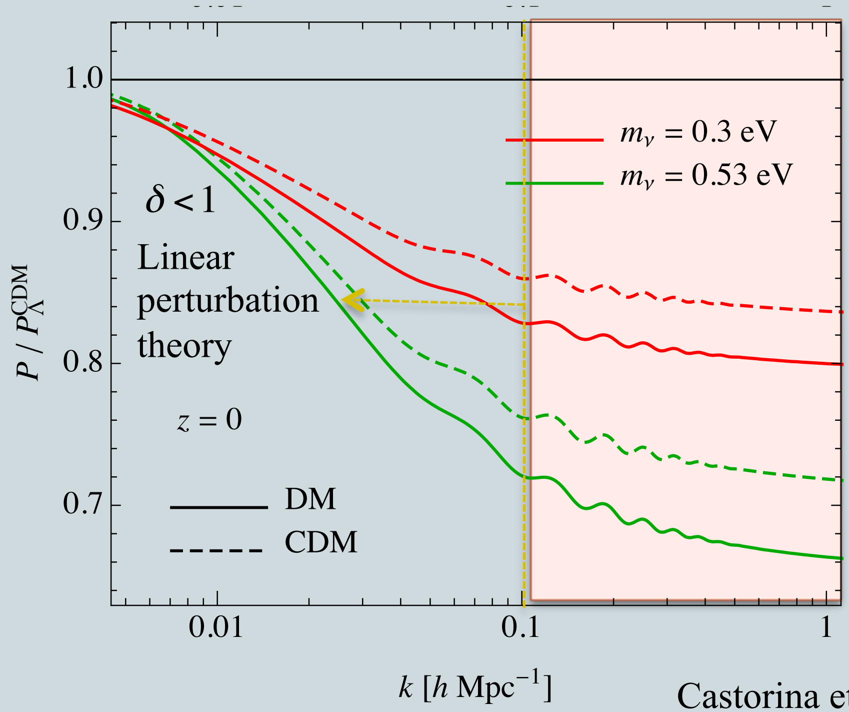
(< 1% Euclid)

Archidiacono & Hannestad, arXiv:1510.02907

# The non-linear regime

$$v_{th} \equiv \frac{\langle p \rangle}{m} \approx \frac{3T_\nu}{m} \approx 150(1+z)\left(\frac{1eV}{m}\right) km\ s^{-1}$$

$$\lambda_{FS}(t) \equiv 2\pi \sqrt{\frac{2}{3}} \frac{v_{th}}{H(t)} \approx 7.7 \frac{1+z}{\sqrt{\Omega_\Lambda + \Omega_m (1+z)^3}} \left(\frac{1eV}{m}\right) Mpc\ h^{-1}$$




Data: Ly-α
Euclid
21cm

Constraints: ν
WDM
SIDM
DE
…

Castorina et al. (2015)

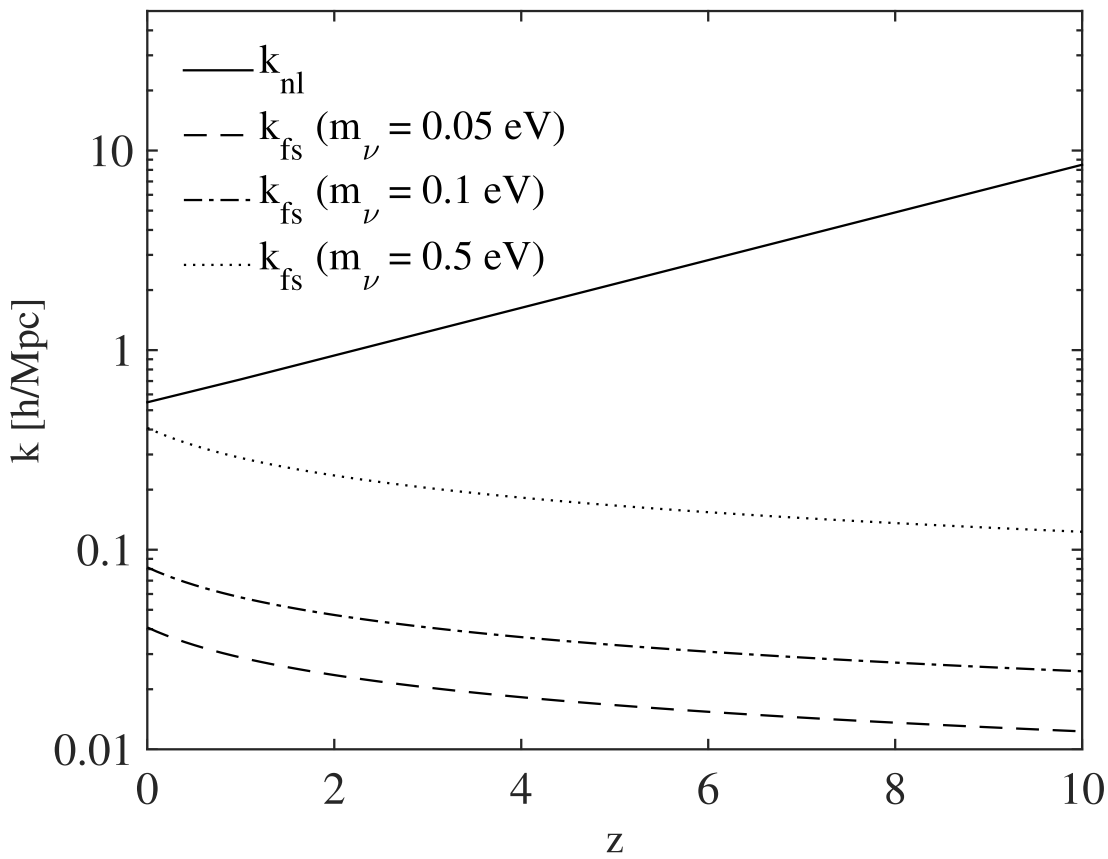
# Non-linear methods

- Beyond linear order perturbation theory
  - Fuhrer & Wong (2014)
  - Blas, Garny, Konstandin, Lesgourgues (2014)
  - Dupuy & Bernardeau (2014)

- N-body simulations
  - Hybrid methods: Brandbyge & Hannestad (2009 & 2010)
  - Semi-linear methods: Ali-Haimoud & Bird (2012)

- Our approach: using HALOFIT, we account for the non-linear growth of cold dark matter overdensities and gravitational potential, then we evolve linear neutrino perturbations in the "non-linear" gravitational potential. The entire computation is in Fourier k space.

# Free-streaming scale vs non-linear scale

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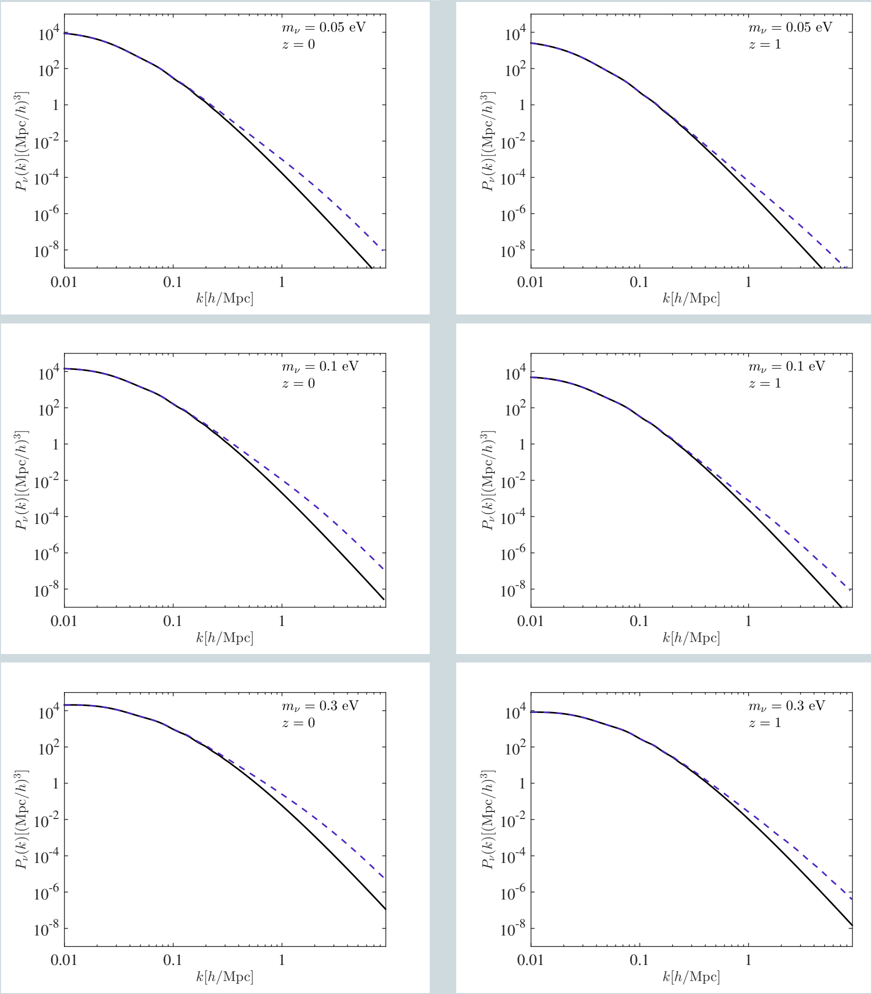
# Neutrino power spectrum




Consistent with results from N-body simulations

Archidiacono & Hannestad, arXiv:1510.02907

# Conclusions

- We have demonstrated that the neutrino evolution hierarchy can be solved very accurately even if truncated at $l = 2$. Our approximation for the $l = 3$ term allowed us to reliably calculate the neutrino power spectrum to better than ~5% precision for masses up to 1.5 eV. The matter power spectrum has a precision of better than 0.5% because of the relatively small direct contribution of neutrinos to this quantity. The new approximation to $\Psi_3$ is significantly more precise than previously used once.

- We showed how the neutrino power spectrum can be calculated using the full non-linear gravitational potential, but keeping the entire computation in k-space. The results obtained using this technique are completely consistent with those from N-body simulations implementing neutrinos in Fourier-space. However, in our case the neutrino power spectrum can be obtained in a few seconds whereas the N-body technique requires far bigger computational resources.