Soft and Coulomb resummation for squark and gluino production at the LHC

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— RWTH Aachen —

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based on M.Beneke, J.Piclum, CS, C.Wever: arXiv:1607.07574 [hep-ph]

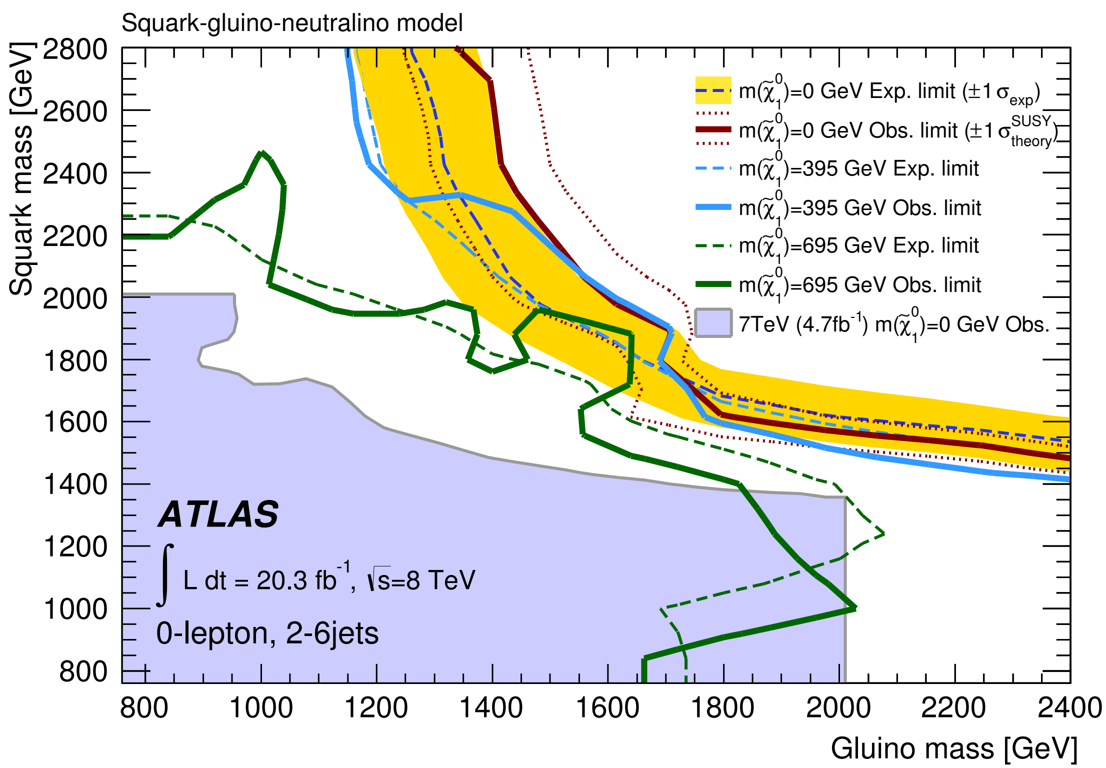
Squark and gluino production

main SUSY search channel at the LHC

Limits after first $13\,\mathrm{TeV}$ results

$$m_{\tilde{g}} \gtrsim 1.75\,\mathrm{TeV}\ , \qquad m_{\tilde{q}} \gtrsim 1.26\,\mathrm{TeV}$$

Ultimate LHC reach $\lesssim 4\,\mathrm{TeV}$



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Precise knowledge of cross sections:

- can help to distinguish models (if new particles observed)
- improve exclusion bounds (if no new particles observed)

Theoretical challenges at high masses

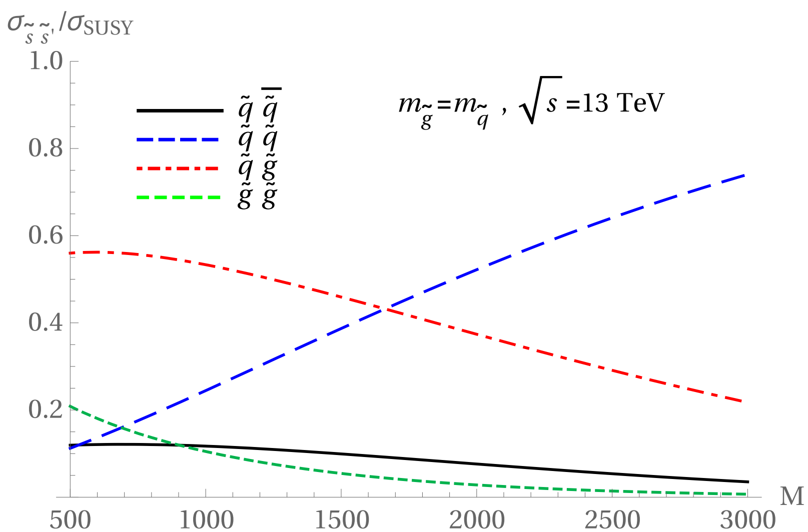
- up to 100% NLO SQCD corrections
- interplay of sizeable soft and Coulomb corrections

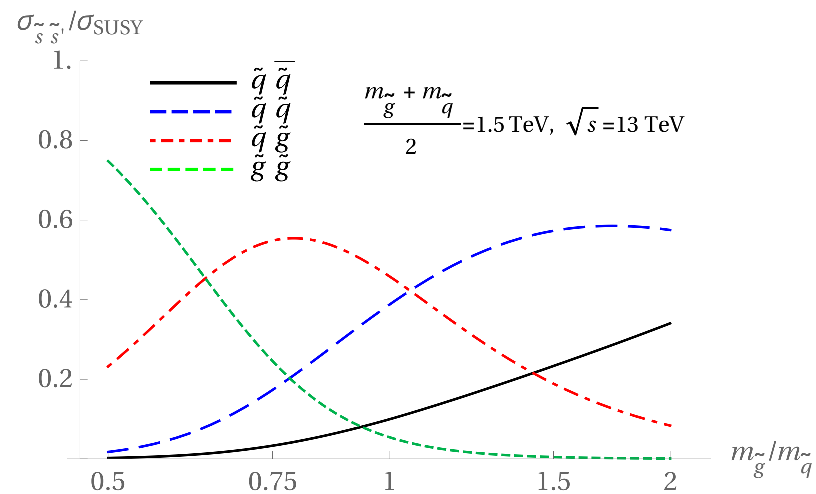
$\Rightarrow$ test case for QCD for very heavy particles

Production processes for squarks and gluinos at the LHC

$$gg,\, q\bar{q} \to \tilde{q}\bar{\tilde{q}}, \qquad qq \to \tilde{q}\tilde{q}, \qquad gq \to \tilde{q}\tilde{g}, \qquad gg,\, q\bar{q} \to \tilde{g}\tilde{g}$$

Relative contributions of production processes at the LHC





Production of stops treated separately, not discussed here

(Beenakker et al. 10/16; Falgari/CS/Wever 12; Broggio et al. 13)

NLO corrections

- NLO SUSY-QCD (Beenakker et al. 97, PROSPINO; Goncalves-Netto et al. 12, MADGOLEM)

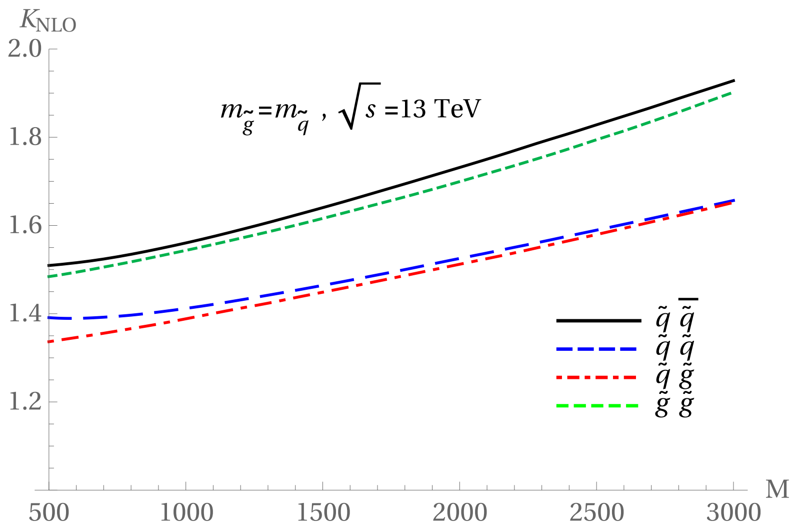
  +Parton-Shower matching (Gavin et al. 13, Degrande et al. 16)

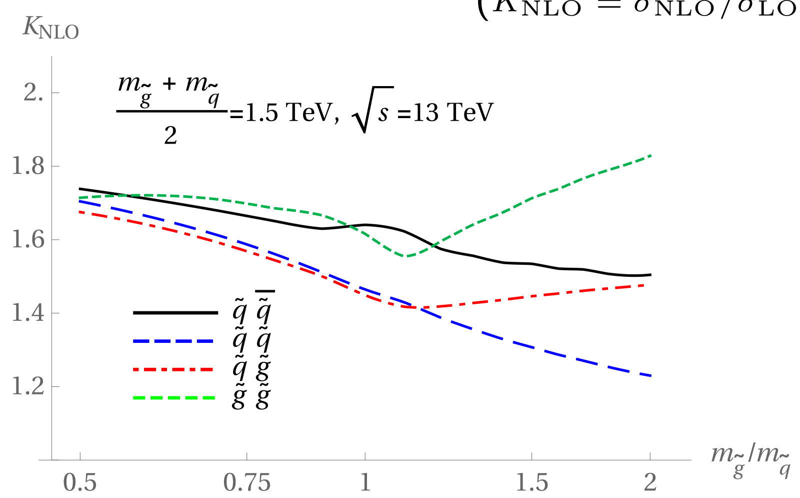
- NLO $\tilde{q}\tilde{q}$ production and decay (Hollik et al. 12; Gavin et al. 14)

- EW corrections (Bornhauser et al. 07; Germer/Hollik/Mirabella/Trenkel 08-11)

SQCD NLO corrections up to 100% , scale uncertainty ± 20–30%

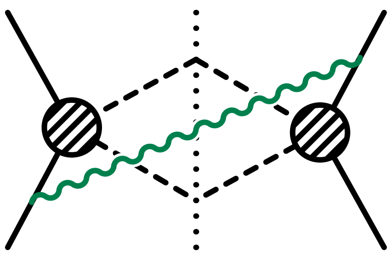
($K_{\rm NLO} = \sigma_{\rm NLO}/\sigma_{\rm LO}$)



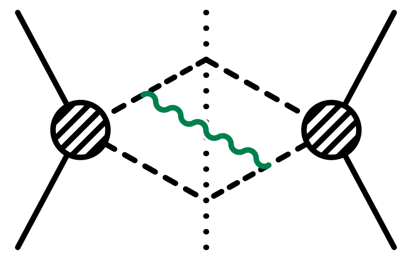


Threshold enhancement

Threshold logarithms $\sim \alpha_s \log^2 \beta, \ \beta = \sqrt{1 - \frac{4M^2}{\hat{s}}}$

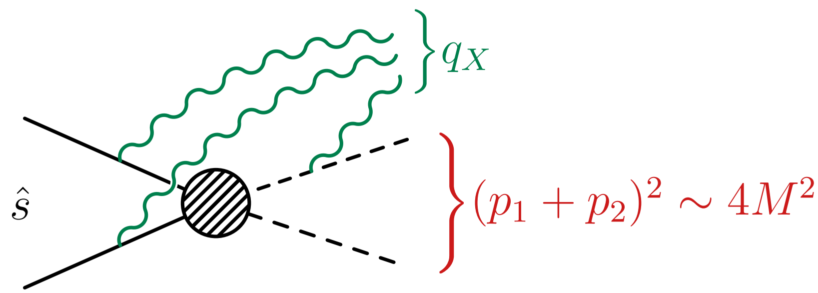


$\Rightarrow \alpha_s \ \log^2(8\beta^2)$



$\Rightarrow \alpha_s \ \log(8\beta^2)$

Remnants of cancellation of soft/collinear divergences between real and virtual corrections

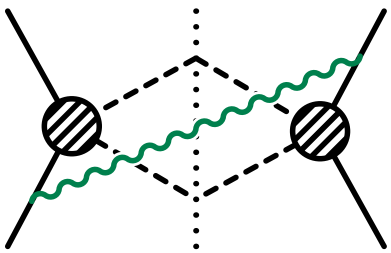


- real-gluon emission near threshold necessarily soft: $q_X \sim M\beta^2$
- structure of soft-gluon emission universal

$\Rightarrow$ can predict threshold logs at higher orders

(Sterman 87; Catani, Trentadue 89, Kidonakis,Sterman 97, Bonciani et.al. 98, ...)

Threshold logarithms $\sim \alpha_s \log^2 \beta$, $\beta = \sqrt{1 - \frac{4M^2}{\hat{s}}}$



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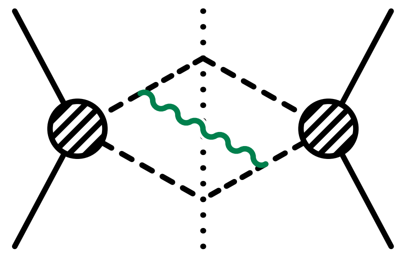
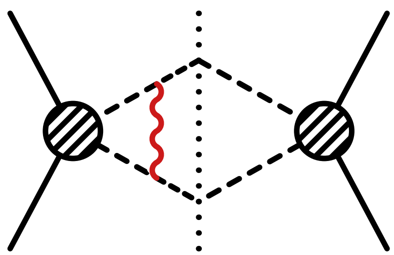


$\Rightarrow \alpha_s \ \log(8\beta^2)$

Coulomb gluon corrections (Fadin, Khoze 87; Peskin, Strassler 90,...)



$\Rightarrow \alpha_s \ \frac{1}{\beta}$

Resummation of $\frac{\alpha_s}{\beta}$ corrections: (Fadin, Khoze 87; Peskin, Strassler 90)
solve NR-Schrödinger equation for Green's function

Resummation of threshold-enhanced corrections

$$\hat{\sigma}_{pp'} \quad \propto \quad \sigma^{(0)} \exp\Big[\underbrace{\ln\beta\, g_0(\alpha_s \ln\beta)}_{\text{(LL)}} + \underbrace{g_1(\alpha_s \ln\beta)}_{\text{(NLL)}} + \underbrace{\alpha_s g_2(\alpha_s \ln\beta)}_{\text{(NNLL)}} + \dots\Big]$$
$$\times \sum_{k=0} \left(\frac{\alpha_s}{\beta}\right)^k \times \left\{1\,\text{(LL,NLL)}; \alpha_s, \beta\,\text{(NNLL)}; \dots\right\} :$$

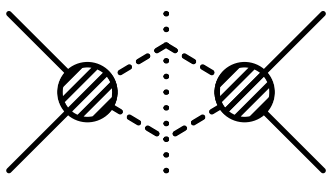
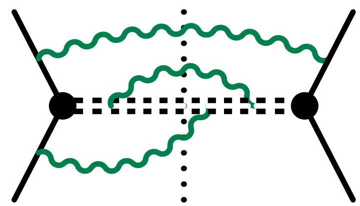
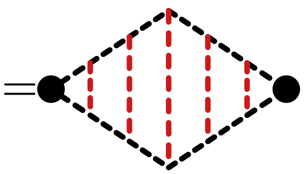
Theory status:

- (N)NLL soft resummation (Beenakker et al. 09-14, `NLLFast`; Broggio et al. 13)
  NNLO$_{\text{approx}}$ for $\tilde{q}\bar{\tilde{q}}$, $\tilde{g}\tilde{g}$, $\tilde{t}\bar{\tilde{t}}$ (Langenfeld et al. 09–12, Broggio et al. 13)
- Coulomb resummation (Hagiwara/Yokoya 09, Kauth et al. 11)
- Combined soft and Coulomb resummation (Beneke/Falgari/CS 09/10)
  - $t\bar{t}$ production at NNLL (Beneke/Falgari/Klein/CS 11)
  - squark and gluino production: NLL (Falgari/CS/Wever 12);
    NNLL (Beneke/Piclum/CS/Wever 13/16; Beenakker et al. 16)
  - Stoponium production (Kim/Idilbi/Mehen/Yoon 14)

Factorization of cross section (Beneke, Falgari, CS 09/10)

$$\hat{\sigma}_{pp' \to HH'}|_{\hat{s} \to 4M^2} = \sum_R H_R W^R \otimes J^R$$

Hard, soft and Coulomb functions:

$H_R =$  , $W^R =$  , $J^R =$ 

Soft radiation "sees" only total colour state $R = 1, 8, \ldots$ of sparticles

Factorization scale dependence of H, W cancels against PDFs:

$$\frac{d\sigma}{d\mu} = \frac{d}{d\mu} \left(f_1 \otimes f_2 \otimes H \otimes W \otimes J \right) = 0$$

- $\frac{df_i}{d\mu} \Rightarrow$ Altarelli-Parisi equation (3-loop: Moch/Vermaseren/Vogt 04/05)
- $\frac{dH}{d\mu} \Rightarrow$ IR singularities (2-loop: Becher, Neubert; Ferroglia et.al. 09)

$\Rightarrow$ RGE for soft function (NNLL: Beneke/Falgari/CS; Czakon/Mitov/Sterman 09)

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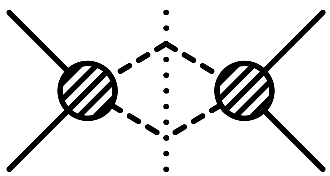
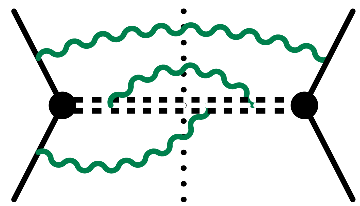
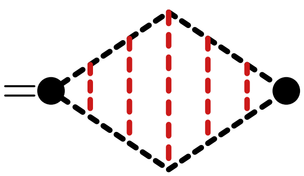
$H_R =$  , $W^R =$  , $J^R =$ 

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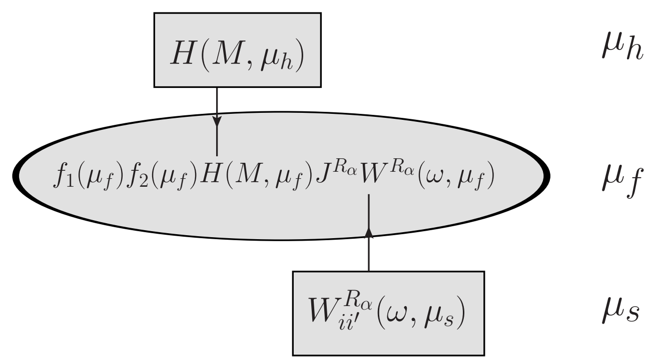
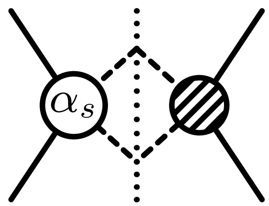
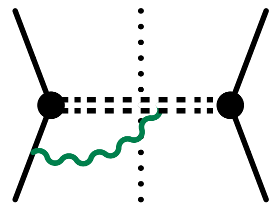
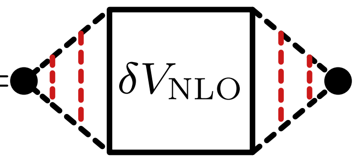
Momentum-space solution to RGE (Becher/Neubert/Pecjak 07)

- evolve hard function from
  $\mu_h \sim 2M$ to μ_f
- evolve soft function from
  $\mu_s \sim M\beta^2$ to μ_f
  choice of μ_s from relation to Mellin resummation
  (Sterman/Zeng 13; Bonvini et al. 14)



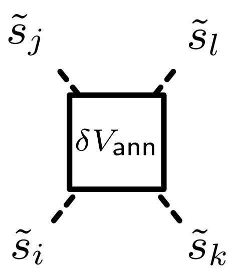
Input to resummation formula at NNLL

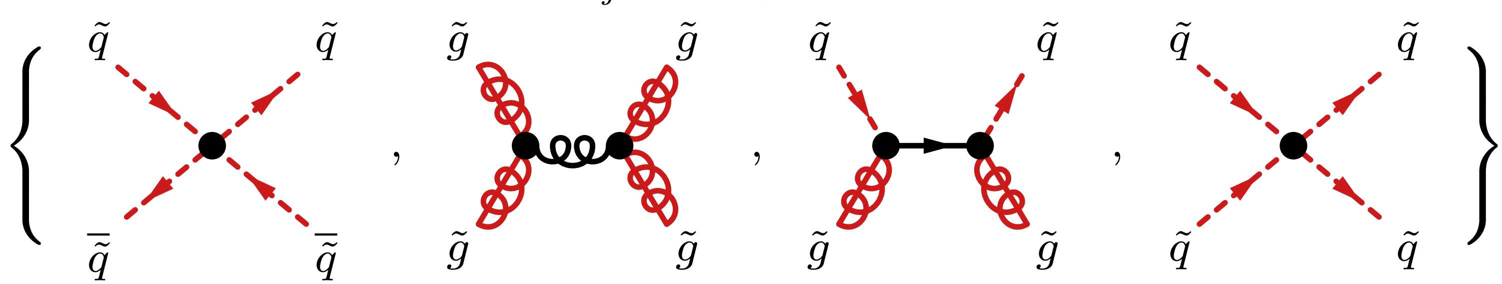
$H_i^{(1)} =$  , $W_i^{(1)R} =$  , $J^R_{\rm NLO} =$ 

- NLO hard functions for $\tilde{q}/\tilde{g}$ production (Beenakker et al. 11/13),
- one-loop soft function for arbitrary colour (Beneke/Falgari/CS 09)
- NLO Coulomb Green function (Beneke/Piclum/CS/Wever 16)

NLO potentials

- one-loop correction to Coulomb potential
- “non-Coulomb” potentials suppressed by $\alpha_s \frac{|\boldsymbol{q}|}{M}$, $\frac{\boldsymbol{q}^2}{M^2}$
- annihilation corrections from $\tilde{s}_i \tilde{s}'_j \to \tilde{s}_k \tilde{s}'_l$ scattering:



$=$


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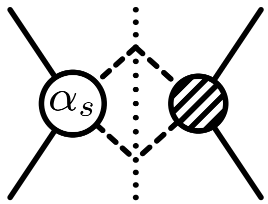
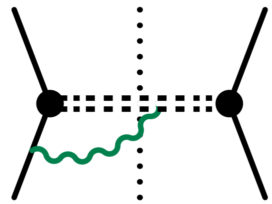
$H_i^{(1)} =$ 

, $W_i^{(1)R} =$ 
, $J^R_{\mathrm{NLO}} =$ 

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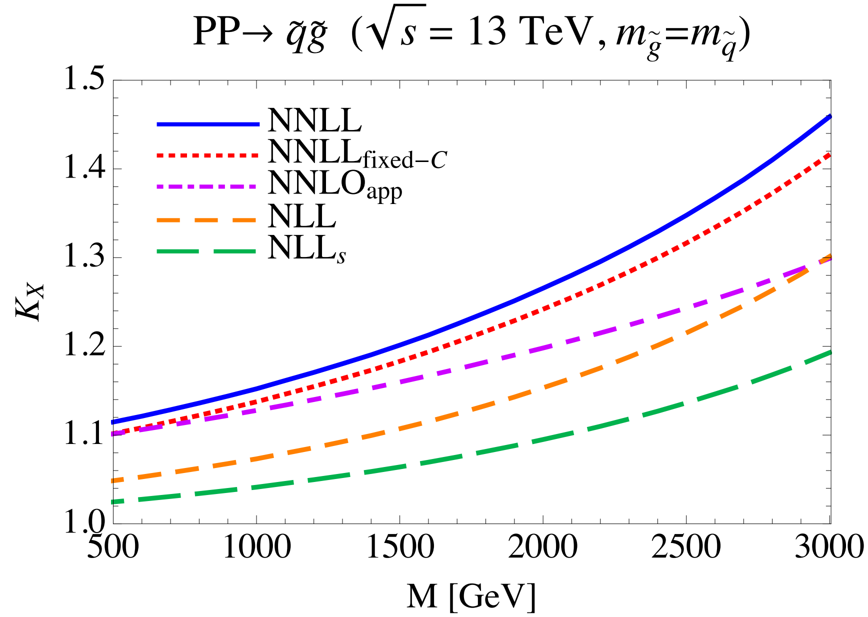
Matching to NLO+NNLO$_{\mathrm{approx}}$

$$\hat{\sigma}^{\mathrm{NNLL}}_{pp'\,\mathrm{matched}}(\hat{s}) = \left[\hat{\sigma}^{\mathrm{NNLL}}_{pp'}(\hat{s}) - \hat{\sigma}^{\mathrm{NNLL}(2)}_{pp'}(\hat{s})\right] + \hat{\sigma}^{\mathrm{NLO}}_{pp'}(\hat{s}) + \hat{\sigma}^{\mathrm{NNLO}}_{\mathrm{app},pp'}(\hat{s})\,.$$

- $\hat{\sigma}^{\mathrm{NNLL}(2)}$: NNLO expansion of NNLL
- $\hat{\sigma}^{\mathrm{NLO}}$: NLO cross section from Prospino
- $\hat{\sigma}^{\mathrm{NNLO}}_{\mathrm{app}}$: Threshold enhanced NNLO terms

  (Beneke/Czakon/Falgari/Mitov/CS 09; Beneke/Piclum/CS/Wever 16)

NNLL soft/Coulomb resummation (Beneke/Falgari/Piclum/CS/Wever 13/16)



$$K_X = \frac{\sigma_X}{\sigma_{\text{NLO}}}$$

PDF4LHC15_nnlo_30 PDFs

NNLL combined soft and Coulomb NNLL resummation

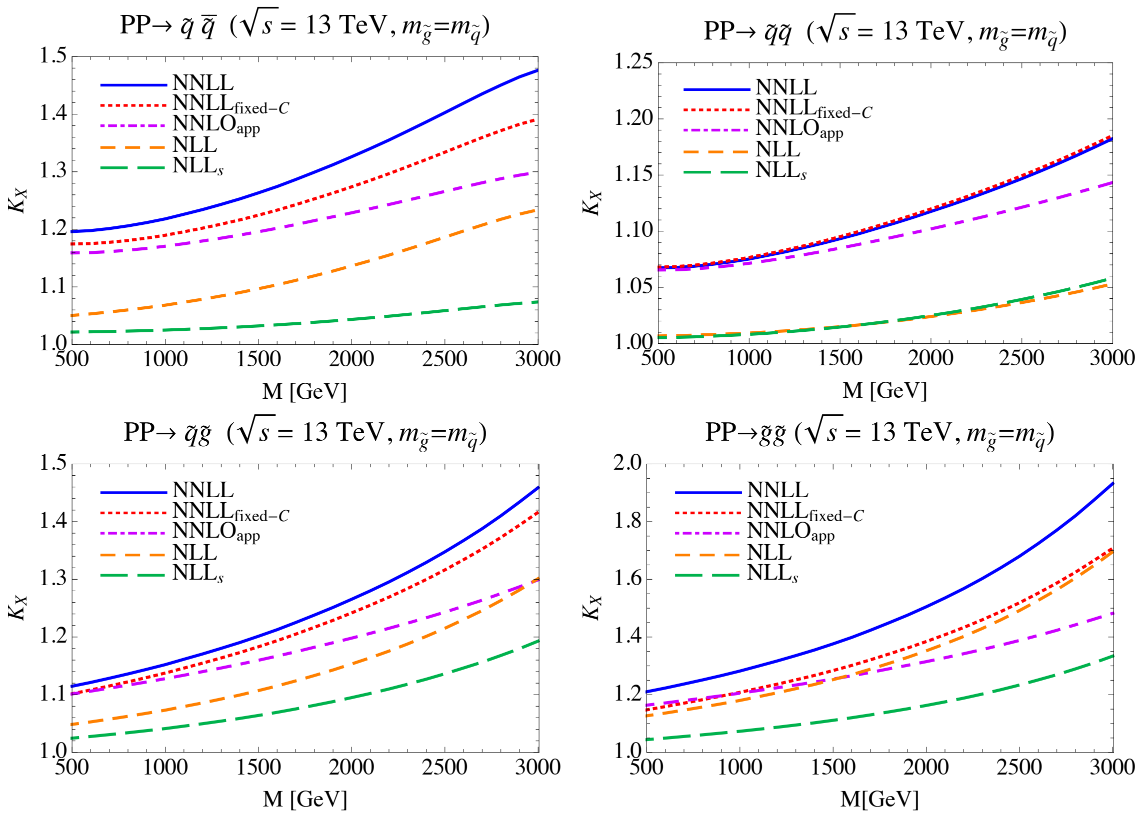
NNNL$_{\text{fixed-C}}$ soft resummation with fixed-order NNLO Coulomb corrections (corresponds to Beenakker et al. 14)

NNLO$_{\text{app}}$ NNLO threshold approximation

NLL combined soft/Coulomb NLL resummation (Falgari/CS/Wever 12)

NLL$_s$ soft NLL resummation (corresponds to NLLFast; Beenakker et al. 09)

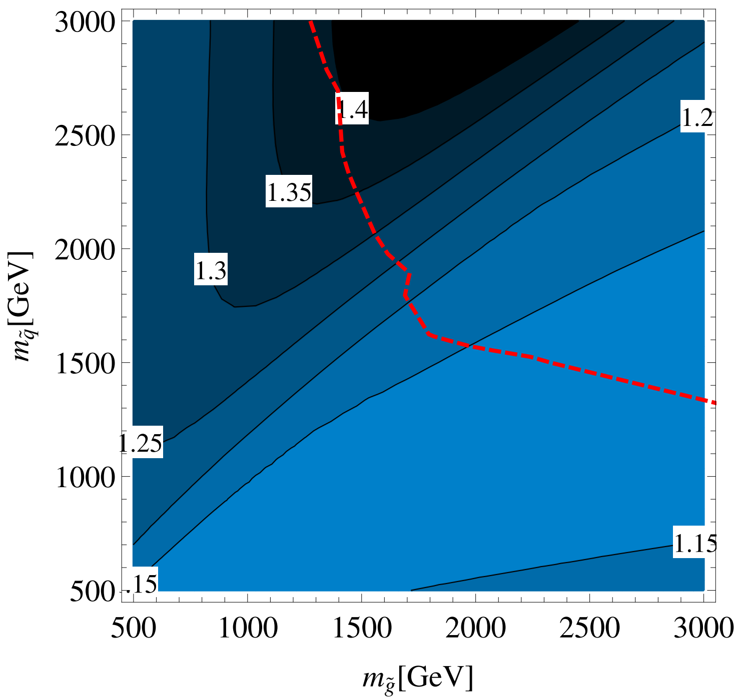
NNLL soft/Coulomb resummation (Beneke/Falgari/Piclum/CS/Wever 13/16)

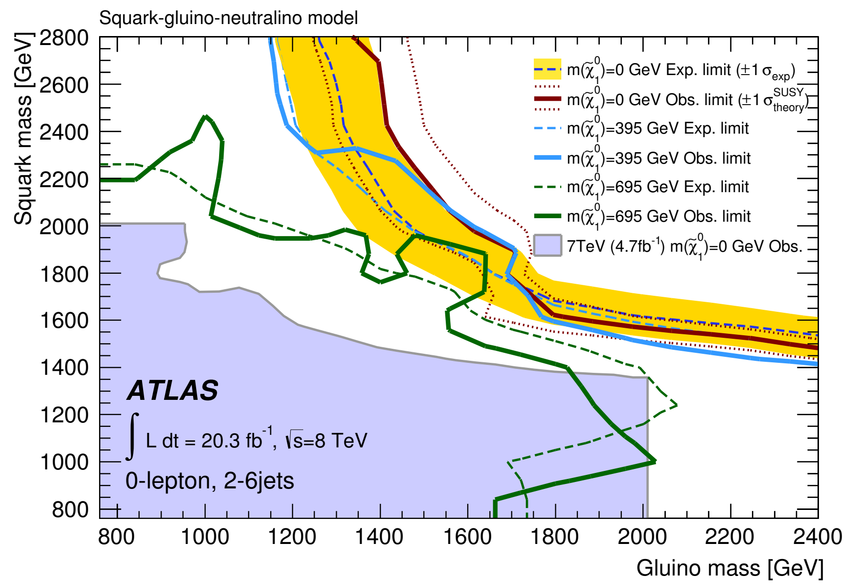


- NNLL corrections $10-30\%$ on top of NLL
- Coulomb resummation effects significant for $\tilde{q}\bar{\tilde{q}}$ and $\tilde{g}\tilde{g}$
- non-negligible corrections beyond NNLO for $M \gtrsim 1.5\,\mathrm{TeV}$

Total coloured sparticle production:

$\sigma_{\mathrm{NNLL}}/\sigma_{\mathrm{NLO}}$ for LHC13:

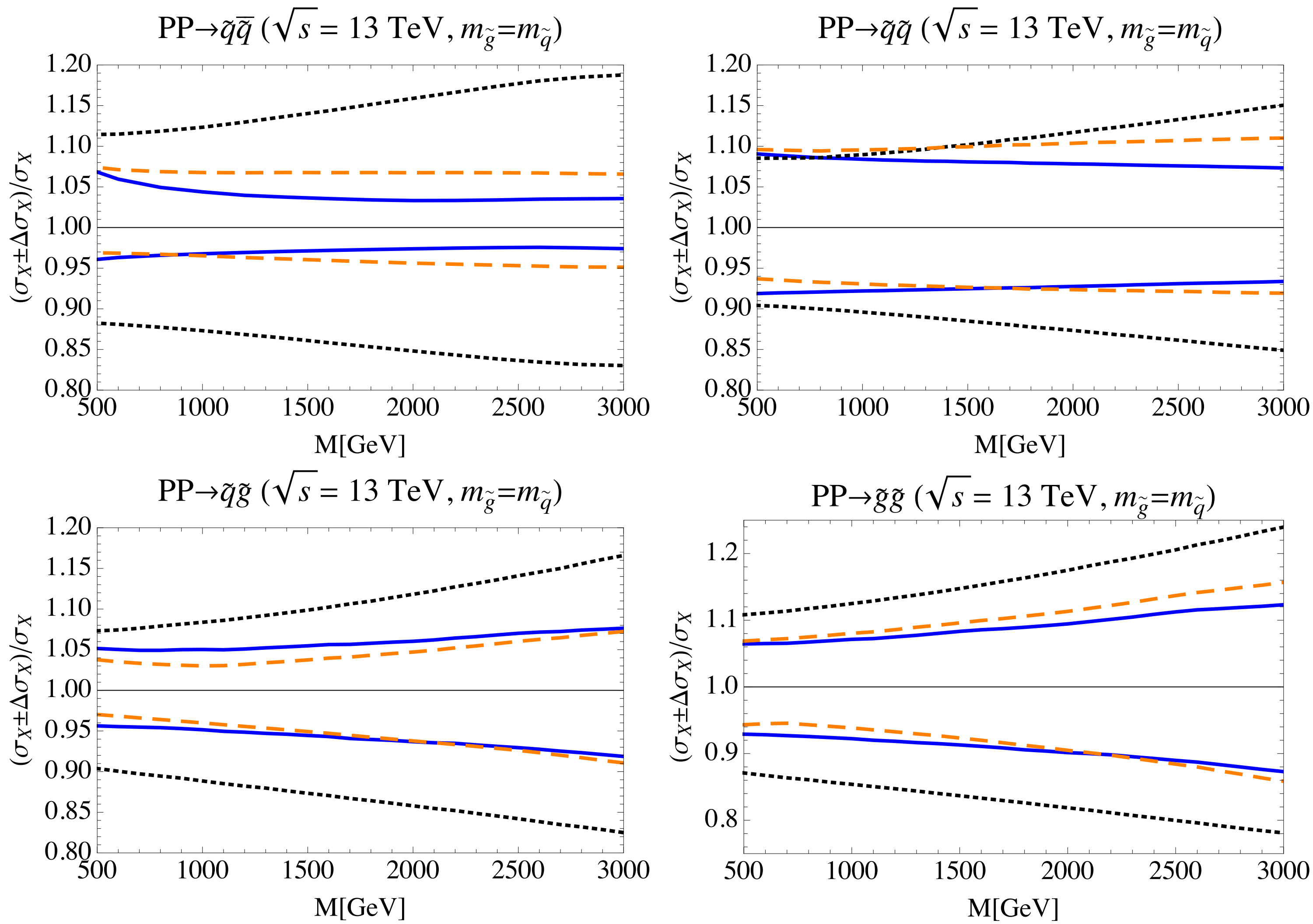




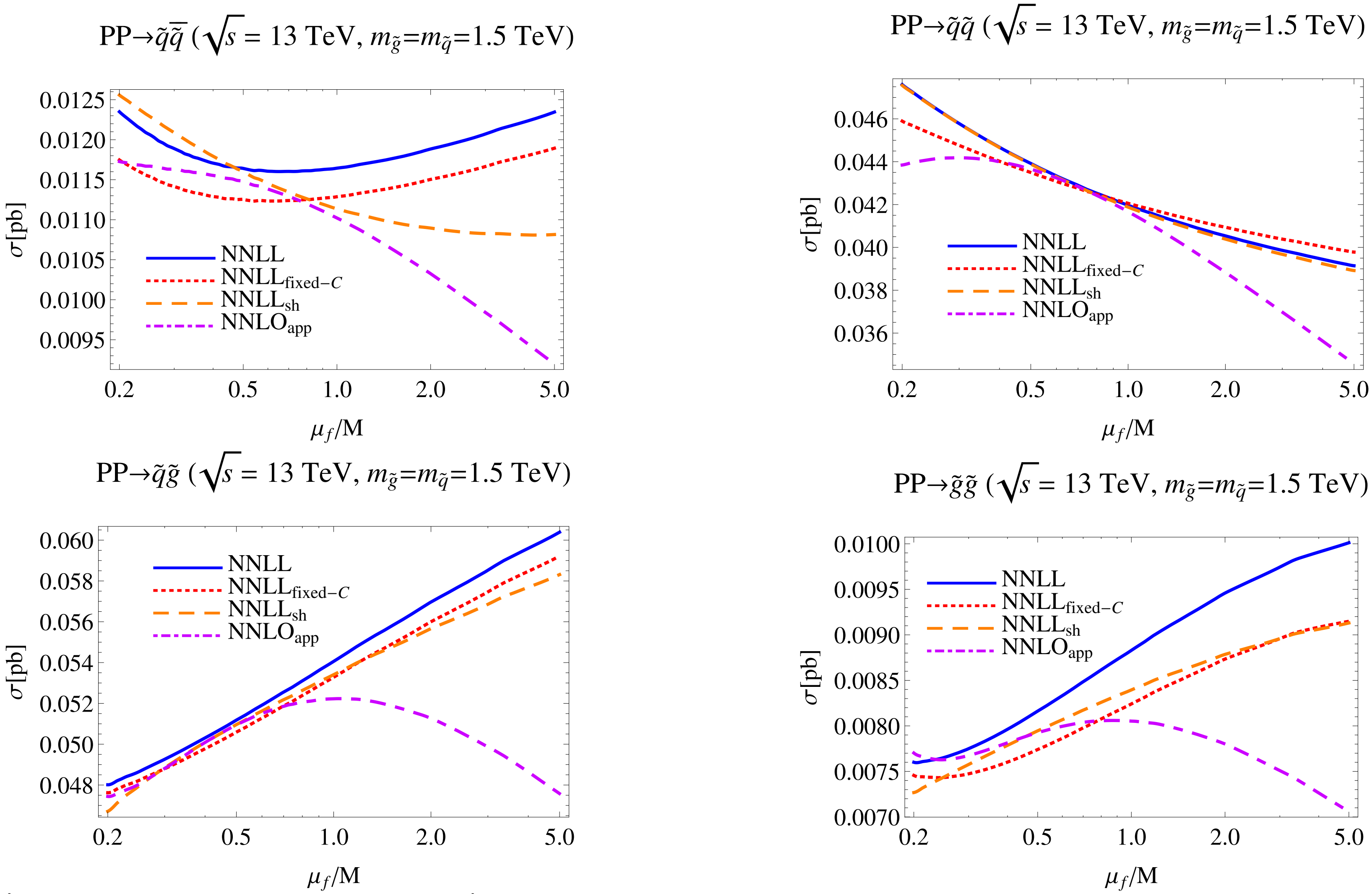
(ATLAS 20.3 fb^{-1} exclusion for simplified model with massless neutralino)

Theoretical uncertainty from scale choices and resummation ambiguities

reduced from ± 20–30% (NLO) $\Rightarrow \lesssim 5-10\%$ (NLL/NNLL)



Factorization scale dependence reduced by resummation;
nontrivial impact of soft/Coulomb interference



(NNLL$_{\text{sh}}$: only soft resummation)

Threshold corrections $\sim \log^n \beta$, $\frac{1}{\beta^n}$ to squark and gluino production

- combined Soft and Coulomb resummation possible

NNLL resummation of soft and Coulomb corrections

- results available at `http://users.ph.tum.de/t31software/SUSYNNLL/`
- Corrections from 20% $(\tilde{q}\tilde{q})$ to 90% $(\tilde{g}\tilde{g})$
- Coulomb corrections can be sizeable
- perturbative uncertainties reduced to ± 5–10%
- (not discussed: large PDF uncertainties for high masses)

Outlook:

- comparison to Mellin-space resummation; (Beenakker et al. 16) combination of predictions
- include stop production at NNLL, allow non-degenerate $m_{\tilde{q}}$

Bonus slides

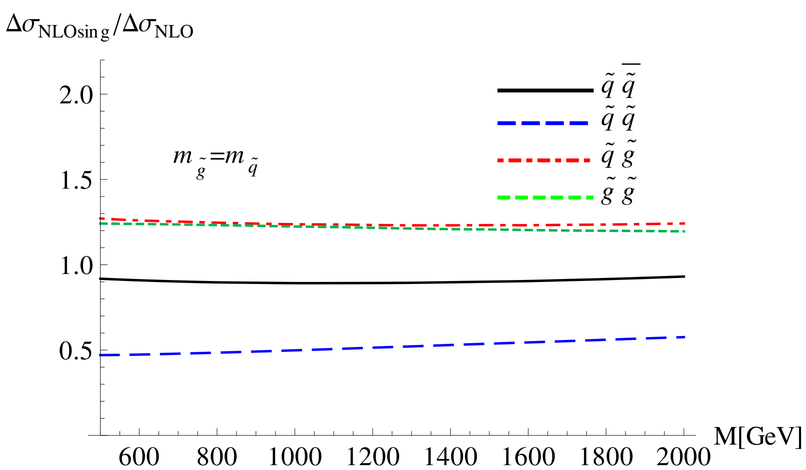
Universal limit $\beta = \sqrt{1 - \frac{4M^2}{\hat{s}}} \to 0$ (Beenakker et al. 97 , Beneke, Falgari, CS 09)

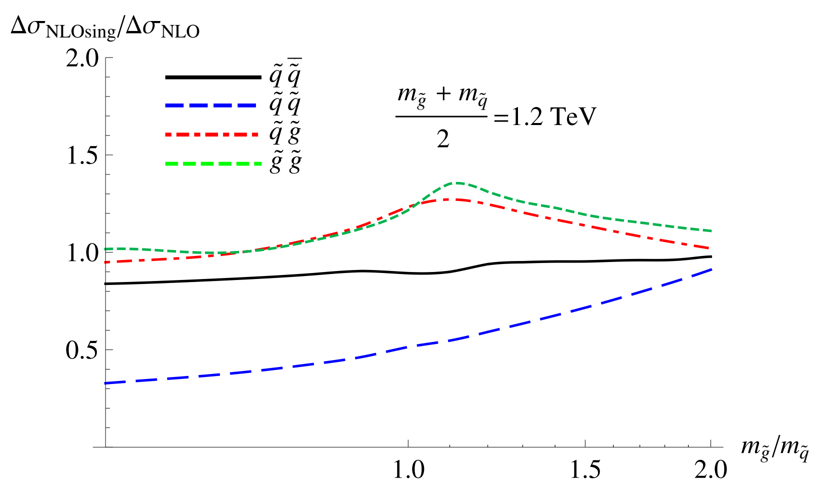
$$\sigma^{R_\alpha}_{\text{NLO,app}} = \sigma^{(0)} \frac{\alpha_s}{(4\pi)} \left\{ -\frac{2\pi^2 D_{R_\alpha}}{\beta} \sqrt{\frac{2m_{\text{red}}}{M}} + 4(C_r + C_{r'}) \ln^2 \left(8\beta^2\right) - 4(C_{R_\alpha} + 4(C_r + C_{r'})) \ln \left(8\beta^2\right) + \text{const} \right\} + \mathcal{O}(\beta^2)$$

(Average and reduced mass: $M = (m_s + m_{s'})/2$, $m_r = m_s m_{s'}/(m_s + m_{s'})$

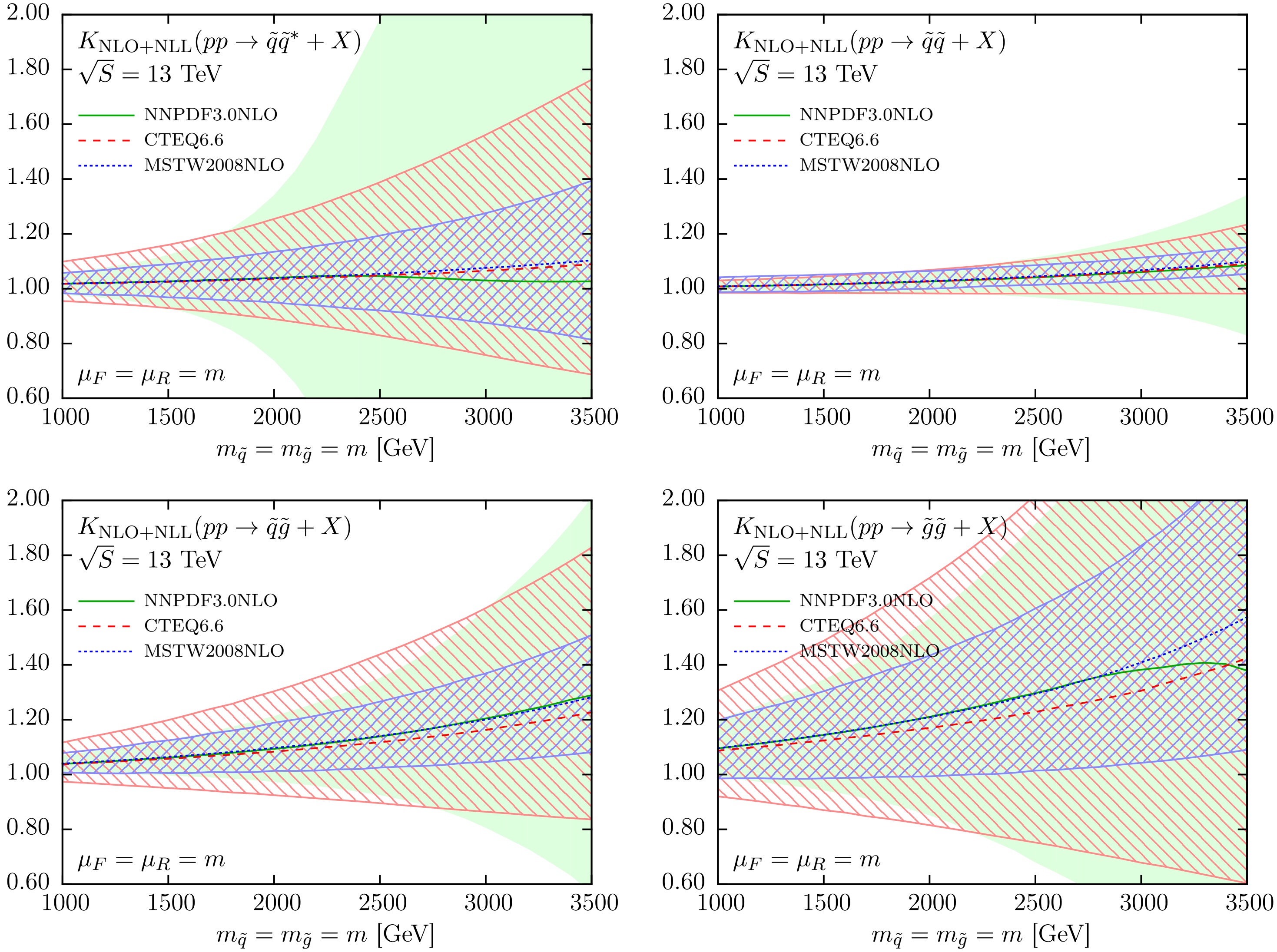
C_r: quadratic $SU(3)$ Casimir for rep. r Coulomb correction: $D_{R_\alpha} = \frac{1}{2}(C_{R_\alpha} - C_R - C_{R'})$)

Accuracy of threshold approximation: (NLO:Prospino, Plehn et al.)





PDF uncertainties very large for high M (Beenakker et al. 15)



Resummation of $\frac{\alpha_s}{\beta}$ corrections: Zero-distance Green function

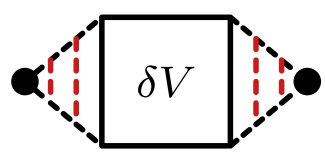
$$G_{\mathrm{C}}^{R_\alpha(0)}(0,0;E) \quad = \quad -\frac{(2m_{\text{red}})^2}{4\pi}\left\{\sqrt{-\frac{E}{2m_{\text{red}}}} + (-D_{R_\alpha})\alpha_s\left[\frac{1}{2}\ln\left(-\frac{8\,m_{\text{red}}E}{\mu^2}\right)\right.\right.$$
$$\left.\left. -\frac{1}{2} + \gamma_E + \psi\left(1 - \frac{(-D_{R_\alpha})\alpha_s}{2\sqrt{-E/(2m_{\text{red}})}}\right)\right]\right\}.$$

LO Potential function:

$$J_{R_\alpha}^{(0)}(E) = 2\mathrm{Im}\,G_{R_\alpha}^{(0)}(0,0;E) = = \begin{cases} \frac{M^2\pi D_R\alpha_s}{2\pi}\left(e^{\pi D_R\alpha_s\sqrt{\frac{M}{E}}} - 1\right)^{-1} & E > 0 \\ \sum_{n=1}^{\infty}\delta(E-E_n)2R_n & E < 0 \end{cases}$$

Bound-state poles at $E_n = -\dfrac{\alpha_s^2 D_R^2 M}{4n^2}$

NLO potential function from perturbation theory

$$\delta G_{R_\alpha}^{(1)}(0,0,E) =$$ 

$$= \int d^3z\, G_{R_\alpha}^{(0)}(0,\vec{z},E)\,(i\delta V^{R_\alpha}(\vec{z}))\,iG_{R_\alpha}^{(0)}(\vec{z},0,E)$$

NLO Potentials:

- one-loop correction to Coulomb potential,

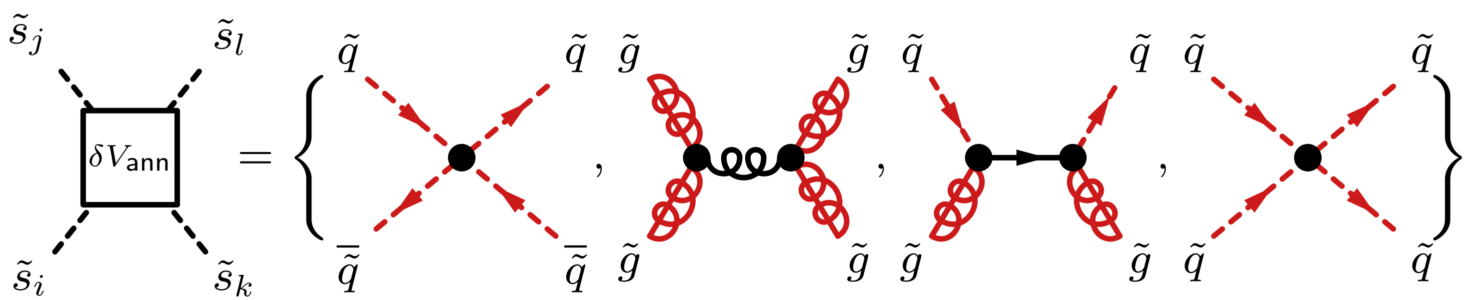
$$\delta\tilde{V}_{\mathrm{C}}^{R_\alpha}(\boldsymbol{q}) = \frac{D_{R_\alpha}\alpha_s^2(\mu)}{\boldsymbol{q}^2}\left(a_1 - \beta_0 \ln\frac{\boldsymbol{q}^2}{\mu^2}\right), \qquad a_1 = \frac{31}{9}C_A - \frac{20}{9}n_l T_f,$$

- “non-Coulomb” potentials suppressed by $\alpha_s \frac{|\boldsymbol{q}|}{M}$, $\frac{\boldsymbol{q}^2}{M^2}$

$$\delta\tilde{V}_{\mathrm{nC}}^{R_\alpha}(\boldsymbol{p},\boldsymbol{q}) = \frac{4\pi D_R \alpha_s}{\boldsymbol{q}^2}\left[\frac{\pi\alpha_s|\boldsymbol{q}|}{8m_{\mathrm{red}}}\left(\frac{D_R m_{\mathrm{red}}}{M} + C_A\right) + \frac{\boldsymbol{p}^2}{m_1 m_2} + \frac{\boldsymbol{q}^2}{4m_{\mathrm{red}}^2}\nu_{\mathrm{spin}}^S\right]$$

(generalisation of results for $e^-e^+ \to t\bar{t}$ to other colour/spin quantum numbers)

- Annihilation correction from $\tilde{s}_i\tilde{s}'_j \to \tilde{s}_k\tilde{s}'_l$ scattering:



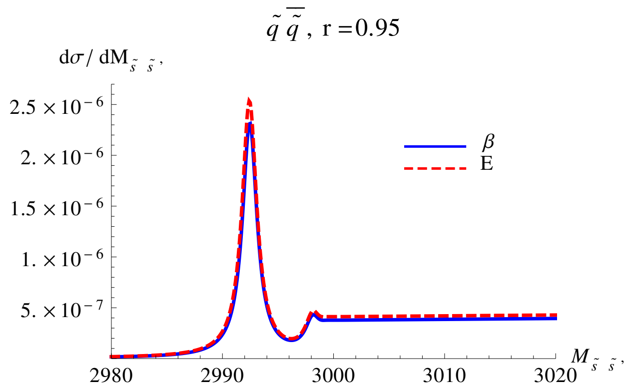
Finite decay width $E \to E + \mathrm{i}\Gamma$ smear out bound-state poles

Different signatures depending on $\Gamma_{\tilde{s}}$:

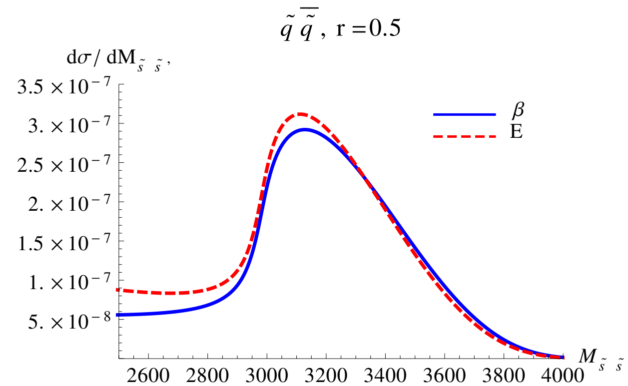
- $\Gamma_{\tilde{s}} < \Gamma_{\rm Bound}$: Boundstate formation ("stoponium", "gluinonium"); decay of bound state to $\gamma\gamma$, gg. (Resummation: Kim et al. 14)
- $\Gamma_{\rm Bound} < \Gamma_{\tilde{s}} \ll |E_1|$: $\tilde{q}, \tilde{g}$ decay signatures, peaks in $M_{\tilde{s}\tilde{s}'}$-spectrum
- $\Gamma_{\rm Bound} \ll \Gamma_{\tilde{s}} \simeq |E_1|$: peaks washed out

Default: include bound-states with $\Gamma = 0$

Finite-width effects on $\sigma_{\rm tot}$ negligible for $\Gamma/M \lesssim 5\%$ (Falgari, CS, Wever 12)



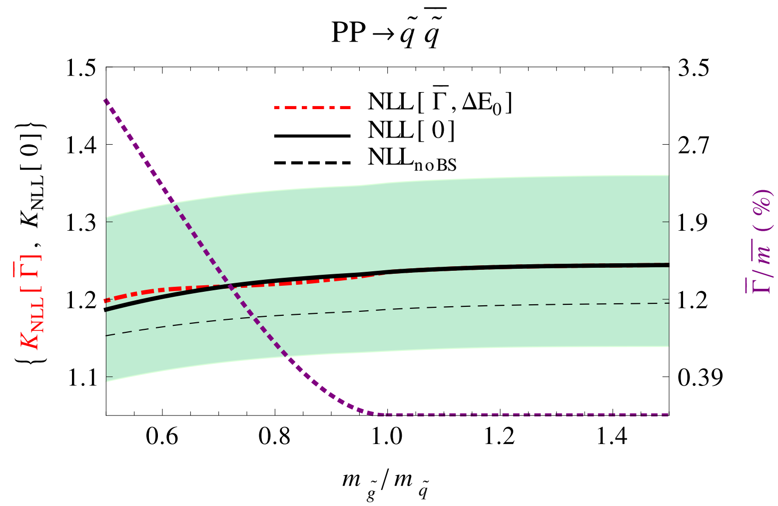
$\left(\frac{\Gamma_{\tilde{q}}}{m_{\tilde{q}}} = 0.05\%\right)$

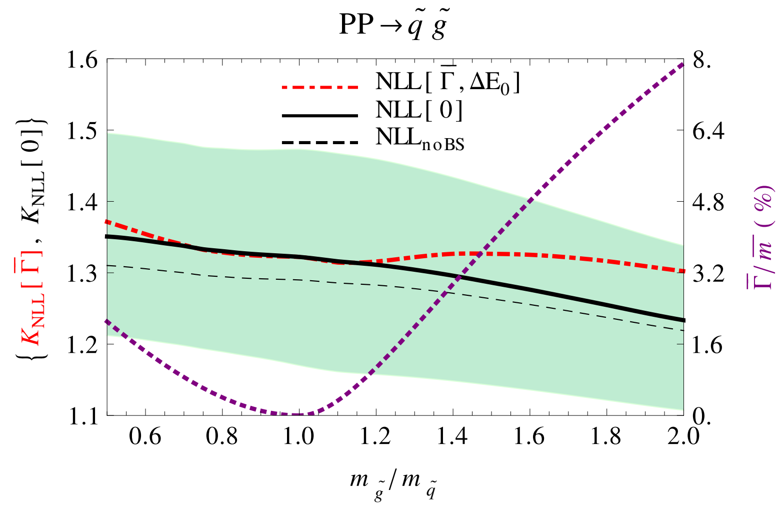


$\left(\frac{\Gamma_{\tilde{q}}}{m_{\tilde{q}}} = 3.5\%\right)$

Total cross sections, LO-SQCD decays as example:

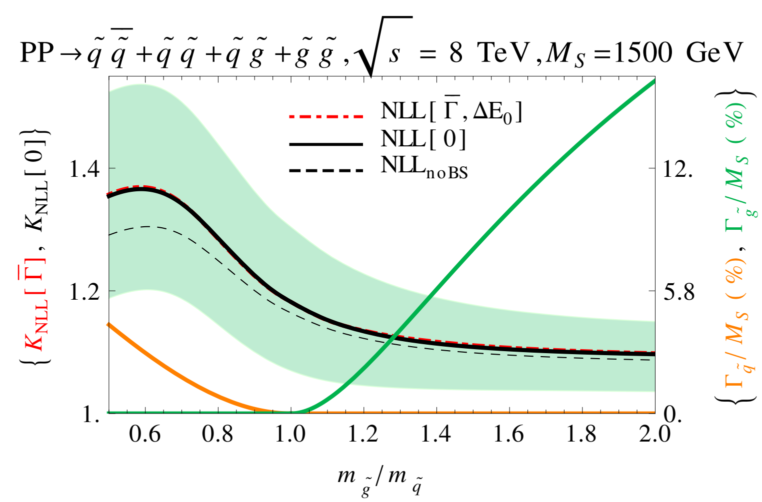
$$\Gamma_{\tilde{q}\to q\tilde{g}} = \frac{\alpha_s C_F m_{\tilde{q}}}{2}\left(1-\left(\frac{m_{\tilde{g}}}{m_{\tilde{q}}}\right)^2\right)^2 , \qquad m_{\tilde{q}}>m_{\tilde{g}}, \qquad \Gamma_{\tilde{g}\to q\bar{\tilde{q}}} = \frac{\alpha_s n_f m_{\tilde{g}}}{2}\left(1-\left(\frac{m_{\tilde{g}}}{m_{\tilde{q}}}\right)^{-2}\right)^2 , \qquad m_{\tilde{q}}<m_{\tilde{g}}.$$





Negligible effect
on total SUSY production rate

but relevant bound-state corrections



Factorization scale dependence of H, W cancels against PDFs:

$$\frac{d\sigma}{d\mu} = \frac{d}{d\mu}\left(f_1 \otimes f_2 \otimes H \otimes W \otimes J\right) = 0$$

- $\frac{df_i}{d\mu} \Rightarrow$ Altarelli-Parisi equation (3-loop: Moch/Vermaseren/Vogt 04/05)
- $\frac{dH_i}{d\mu} \Rightarrow$ related to IR singularities (2-loop: Becher, Neubert; Ferroglia et.al. 09)

$$\frac{d}{d\log\mu} H_i(M,\mu) = \gamma_{\text{cusp}}(C_r + C_{r'}) \ln\left(\frac{4M^2}{\mu^2}\right) + 2(\gamma_{H.s}^{R_\alpha} + \underbrace{\gamma^r + \gamma^{r'}}_{\text{incoming partons}})$$

$\Rightarrow$ RGE for soft function (NNLL: Beneke/Falgari/CS; Czakon/Mitov/Sterman 09)

$$\frac{d}{d\log\mu} W_i^{R_\alpha}(z^0,\mu) = \Big(2\gamma_{\text{cusp}}(C_r + C_{r'}) \log\left(\frac{iz_0\bar{\mu}}{2}\right) - 2(\gamma_{H.s}^{R_\alpha} + \gamma_s^r + \gamma_s^{r'}\Big) W_i^{R_\alpha}(z^0,\mu)$$

Soft anomalous dimension (Beneke, Falgari, CS 09; Czakon, Mitov, Sterman 09)

$$\gamma_{H,s}^{R_\alpha} = \frac{\alpha_s}{4\pi}(-2C_{R_\alpha}) + \left(\frac{\alpha_s}{4\pi}\right)^2 C_{R_\alpha}\left[-C_A\left(\frac{98}{9} - \frac{2\pi^2}{3} + 4\zeta_3\right) + \frac{40}{18}n_f\right] + \mathcal{O}(\alpha_s^3).$$

(extracted from Becher/Neubert 09, Korchemsky/Radyushkin 92, Kidonakis 09)

Relation to Mellin-space resummation (Sterman/Zeng 13; Bonvini et al. 14)

Single-power approximation for parton luminosity

$$L_{pp'}(\tau,\mu) \approx L_{pp'}(\tau_0,\mu)\left(\frac{\tau_0}{\tau}\right)^{s_{1,pp'}(\tau,\mu)} \qquad \tau_0 = 4M^2/s$$

with $s^{(1)}_{pp'}(\tau_0,\mu) = -\frac{d\ln L_{pp'}(\tau,\mu)}{d\ln\tau}\Big|_{\tau=\tau_0}.$

hadronic cross section ⇔ Mellin transform

$$\sigma(s) \approx L_{pp'}(\tau_0)\int_{\tau_0}^{1} d\tau\,(\tau/\tau_0)^{-s^{(1)}}\hat{\sigma}(\tau s) \approx L_{pp'}(\tau_0)\hat{\sigma}^{N=s^{(1)}}$$

Equivalence to Mellin-space resummation for soft scale

$$\mu_s = \frac{2Me^{-\gamma_E}}{s^{(1)}_{pp'}}$$

⇒ default choice