Classification of Maximally Supersymmetric Backgrounds in Supergravity Theories

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Introduction

- Supersymmetric solutions in supergravity are well studied, many are classified.
- They are believed to uplift to solutions of the full string theory and are related to string solitons, i.e. non-perturbative string theory.
- Here: Maximally supersymmetric solutions for all gauged and deformed supergravity theories in $D \geq 3$ space-time dimensions.

Outline / Results

A) Set the stage to discuss solutions in a generic framework.

B) Solutions *without* fluxes:

- Only two possible cases:

$$Mink_D \qquad \text{and} \qquad AdS_D$$

(only gauged / deformed SUGRAs) → AdS_D

C) Solutions *with* (non-trivial) fluxes:

- Exist only for a small class of theories.
- Solutions coincide with those of the corresponding ungauged theories.
- For these theories all solutions are known and classified.

→ Exhaustive list of solutions.

Maximally supersymmetric backgrounds

- Classical solutions / backgrounds for which

$$\langle \delta_\epsilon B \rangle \;=\; \langle \delta_\epsilon F \rangle \;= 0\,.$$

SUSY variations of bosonic / fermionic fields

- Here: bosonic solutions, i.e. $\langle F \rangle = 0$

$$\Rightarrow \quad \langle \delta_\epsilon B \rangle \sim \langle F \rangle = 0\,.$$

$\Rightarrow$ only remaining condition: $\langle \delta_\epsilon F \rangle = 0$

- indep. supersymmetry paramters ϵ $\quad\leftrightarrow\quad$ preserved supercharges

Supersymmetry variations

- gravitini:

$$\delta\psi^i_\mu \;=\; D_\mu\epsilon^i \quad + \quad (\mathcal{F}_{0\mu})^i_{\;j}\,\epsilon^j \quad + \quad A^i_{0\,j}\,\gamma_\mu\epsilon^j$$

fluxes from the gravity multiplet → $(\mathcal{F}_{0\mu})^i_{\;j}$

gaugings / deformations → $A^i_{0\,j}$

covariant derivative: $D_\mu\epsilon^i = \nabla_\mu\epsilon^i + (\mathcal{Q}_\mu)^i_{\;j}\,\epsilon^j$

- spin-1/2 fermions from the gravity multiplet ("dilatini"):

$$\delta\chi^a \;=\; (\mathcal{F}_1)^a_{\;i}\,\epsilon^i \quad + \quad A^a_{1\,i}\,\epsilon^i$$

- spin-1/2 fermions from other multiplets ("gaugini", "hyperini", ...):

$$\delta\lambda^s \;=\; (\mathcal{F}_2)^s_{\;i}\,\epsilon^i \quad + \quad A^s_{2\,i}\,\epsilon^i$$

fluxes from other multiplets → $(\mathcal{F}_2)^s_{\;i}$

Solutions without fluxes (1)

- Easiest case: all fluxes vanish:

$$\mathcal{F}_{0\mu} = \mathcal{F}_1 = \mathcal{F}_2 = 0$$

- Supersymmetric solutions of gauged supergravity without background fluxes need to satisfy the Killing spinor equation:

$$\delta\psi^i_\mu = \nabla_\mu \epsilon^i + A^i_{0\,j}\gamma_\mu \epsilon^j = 0$$

- Integrability condition:

$$\left(\frac{1}{4} R_{\mu\nu}{}^{\alpha\beta}\delta^i_k + 2 A^i_{0\,j} A^j_{0\,k}\delta^\alpha_\mu \delta^\beta_\nu\right)\gamma_{\alpha\beta}\epsilon^k = 0$$

⇒ Unbroken supersymmetry (without fluxes):
$\mathbf{Mink_D}$ and **$\mathbf{AdS_D}$** are the only possible solutions.

Solutions without fluxes (2)

- Spin-1/2 variations:

$$\delta\chi^a = A_{1\,i}^{a}\,\epsilon^i = 0\,, \qquad \delta\lambda^s = A_{2\,i}^{s}\,\epsilon^i = 0$$

Algebraic conditions

$${A_0}^2 = -\tfrac{\Lambda}{2(D-1)(D-2)}\,\mathbb{1}\ , \qquad A_1 = A_2 = 0$$

- Compare with the potential:

$$V = -c_0\mathrm{tr}(A_0^\dagger A_0) + c_1\mathrm{tr}(A_1^\dagger A_1) + c_2\mathrm{tr}(A_2^\dagger A_2)\,,$$

- **Mink$_\mathbf{D}$** ($\Lambda = 0$) solutions exist for all theories.
- For $D \leq 7$ *most* gauged theories admit **AdS$_\mathbf{D}$** ($\Lambda < 0$) solutions.

[de Alwis, Louis, McAllister, Triendl, Westphal '13] [Louis, Triendl '14] [Louis, SL '15] [Louis, Triendl, Zagermann '15] [Louis, Muranaka '16]

Solutions with non-trivial flux (1)

- For more “interesting” solutions: Allow for non-vanishing flux.
- Firstly: spin-1/2 variations:

$$\delta\chi^a = (\mathcal{F}_1)^a_i\,\epsilon^i + A^a_{1i}\,\epsilon^i = 0\,, \quad \delta\lambda^s = (\mathcal{F}_2)^s_i\,\epsilon^i + A^s_{2i}\,\epsilon^i = 0$$

- Unbroken supersymmetry:

$$\mathcal{F}_1 = \mathcal{F}_2 = A_1 = A_2 = 0 \qquad \Rightarrow \qquad \mathcal{F}_{0\mu} = 0$$

- Generically no non-trivial fluxes possible!

Two exceptions

1. theories without χ's in the gravitational multiplet.
2. chiral theories with selfdual fluxes.

Solutions with non-trivial flux (2)

- Secondly: gravitino variations:

$$\delta\psi^i_\mu = \nabla_\mu \epsilon^i + (\mathcal{Q}_\mu)^i_j + (\mathcal{F}_{0\mu})^i_j\,\epsilon^j + A^i_{0\,j}\gamma_\mu\epsilon^j = 0$$

- Integrability condition:

$$\left(\tfrac{1}{4}R_{\mu\nu\rho\sigma}\gamma^{\rho\sigma}\delta^i_j - (\mathcal{H}_{\mu\nu})^i_j + \ldots\right)\epsilon^j = 0\,,$$

where $\mathcal{H}_{\mu\nu}$ is the field strength corresponding to $\mathcal{Q}_\mu$.

- Unbroken supersymmetry:

$$\mathcal{H}_{\mu\nu} = 0 \qquad \Rightarrow \qquad \mathcal{Q}_\mu = A_0 = 0$$

(See also [Hristov,Looyestijn,Vandoren] [Gauntlett,Gutowski] [Akyol,Papadopoulos])

- The supersymmetry variations take the same form as in the ungauged / undeformed case.

Solutions with non-trivial flux (3)

SUGRAs with non supersymmetry breaking flux:

dimension	supersymmetry	q	possible flux	classification
$D = 11$	$N = 1$	32	$F^{(4)}$	[Figueroa-O'Farrill,Papadopoulos]
$D = 10$	IIB	32	$F_+^{(5)}$	[Figueroa-O'Farrill,Papadopoulos]
$D = 6$	$N = (2,0)$	16	$5 \times F_+^{(3)}$	[Chamseddine,Figueroa-O'Farrill,Sabra]
$D = 6$	$N = (1,0)$	8	$F_+^{(3)}$	[Gutowski,Martelli,Reall]
$D = 5$	$N = 2$	8	$F^{(2)}$	[Gauntlett,Gutowski,Hull,Pakis,Reall]
$D = 4$	$N = 2$	8	$F^{(2)}$	[Tod]

The maximally supersymmetric solutions are classified:

- $AdS_p \times S^{(D-p)}$ and $AdS_{(D-p)} \times S^p$, for p-form flux.
- Hpp-wave as Penrose-limit of $AdS \times S$ solutions.

  [Penrose;Gueven;Blau,Figueroa-O'Farrill,Hull,Papadopoulos]
- Exceptional solutions in $D = 5$. [Gauntlett,Gutowski,Hull,Pakis,Reall]

All maximally supersymmetric solutions

(with non-trival flux)

dim.	SUSY	q	$AdS \times S$	H*pp*-wave	others
$D = 11$	$\mathcal{N} = 1$	32	$\mathrm{AdS}_4 \times S^7$ $\mathrm{AdS}_7 \times S^4$	KG_{11}	-
$D = 10$	IIB	32	$\mathrm{AdS}_5 \times S^5$	KG_{10}	-
$D = 6$	$\mathcal{N} = (2, 0)$ $\mathcal{N} = (1, 0)$	16 8	$\mathrm{AdS}_3 \times S^3$	KG_6	-
$D = 5$	$\mathcal{N} = 2$	8	$\mathrm{AdS}_2 \times S^3$ $\mathrm{AdS}_3 \times S^2$	KG_5	Gödel-like, NH-BMPV*
$D = 4$	$\mathcal{N} = 2$	8	$\mathrm{AdS}_2 \times S^2$	KG_4	-

* = near-horizon limit of the BMPV [Breckenridge,Myers,Peet,Vafa] black hole

Summary

- Systematic classification of all maximally supersymmetric supergravity backgrounds:
- no fluxes: only flat space-time and anti-de Sitter.
- fluxes: - generically not possible
  - in the gauged case: same solutions as for ungauged theories
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Thank You!