Large Charge Perturbation Theory

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based on

ongoing project

together with

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Domenico Orlando (Bern)
Susanne Reffert (Bern)

Large Charge Perturbation Theory

Overview

- Motivation for LCPT

- Fixing the charge
- Large-charge vacuum
- Quantum fluctuations

- Summary and Outlook

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- Analytic insight to a theory with no intrinsically small parameter
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  [Wilson,Fischer'72]

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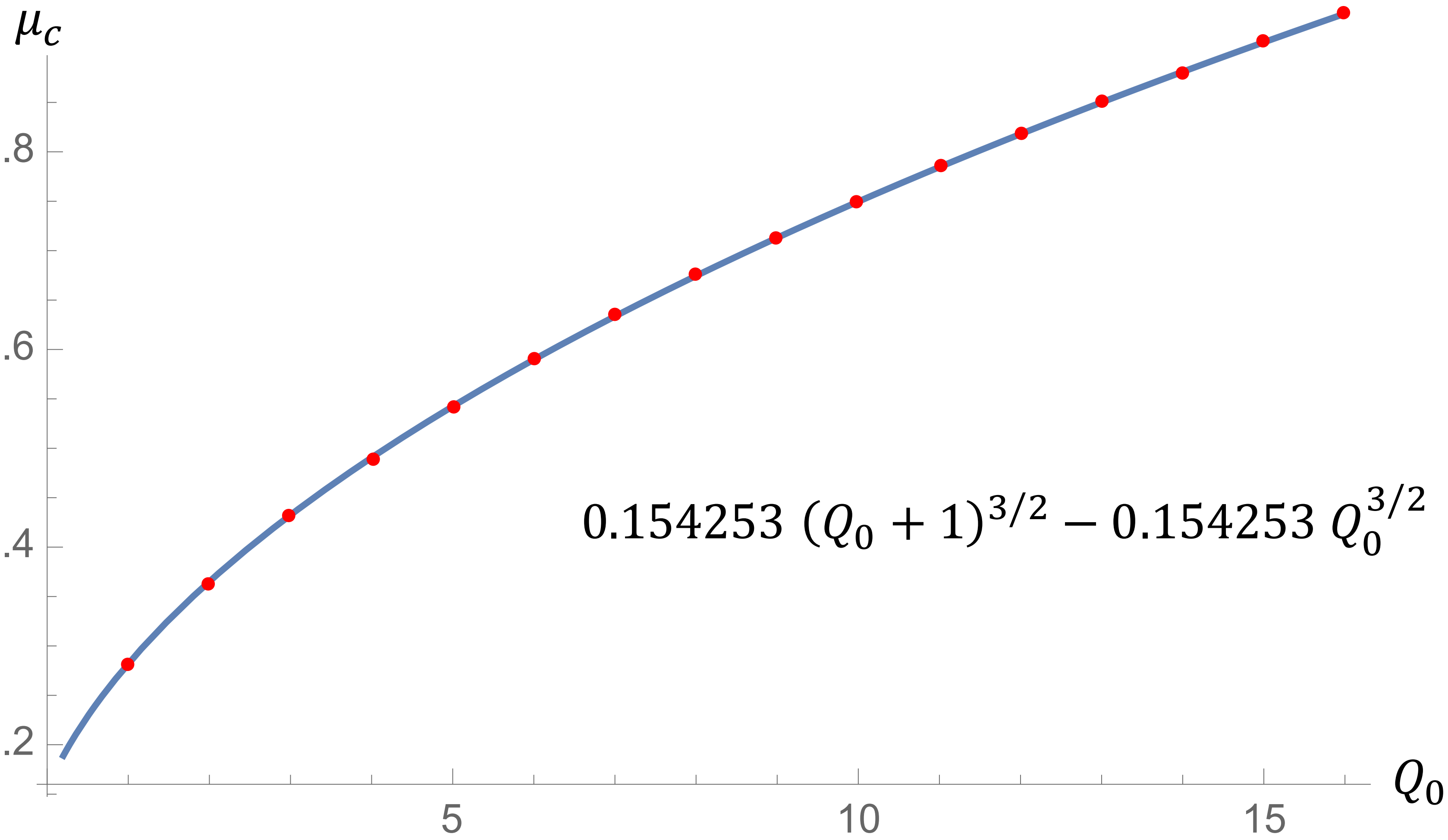
LCPT vs Lattice



╱ LCPT

● Lattice

[Banerjee,Chandrasekharan]

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  - take $Q_0 \gg 1$
  - setup perturbation series in "$1/Q_0$"

vacuum $|Q_0\rangle$ + Goldstone + Q_0 - suppressed corrections

Fixed charged theories

- Generically consider partition function or thermal sum at $T = \dfrac{1}{\beta}$

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The O(2) scalar theory

- Concrete class of models

$$\mathcal{L}\left[\phi,\phi^*\right] = \partial_\mu\phi^*\partial^\mu\phi - m^2|\phi|^2 - \frac{2^N}{2N}\,\lambda\,|\phi|^{2N}$$

  - complex scalar with self interaction in $D = d + 1$

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    - $\langle \chi \rangle = \langle \dot{\chi} \rangle\, t$ exhibits *non-constant* VEV

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- At $\beta \to \infty$ VEVs behave like their classical counterparts

isotropic oscillator in 2D	**VEVs of $O(2)$ at fixed Q_0**
radius $x^2 + y^2$	$v = \langle r \rangle$
polar angle $\tan^{-1} \frac{y}{x}$	$\langle \chi \rangle$
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    implies classical *Routhian*

$$\mathcal{R}_{Q_0}(\beta)\,[v, \rho_0] := \rho_0 \langle \dot{\chi} \rangle + \mathcal{V}_{Q_0}(\beta)\,[v, \langle \dot{\chi} \rangle] = \frac{\rho_0^2}{2v^2} + \frac{m^2}{2} v^2 + \frac{\lambda}{2N} v^{2N}$$

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centrifugal potential

Large-charge vacuum

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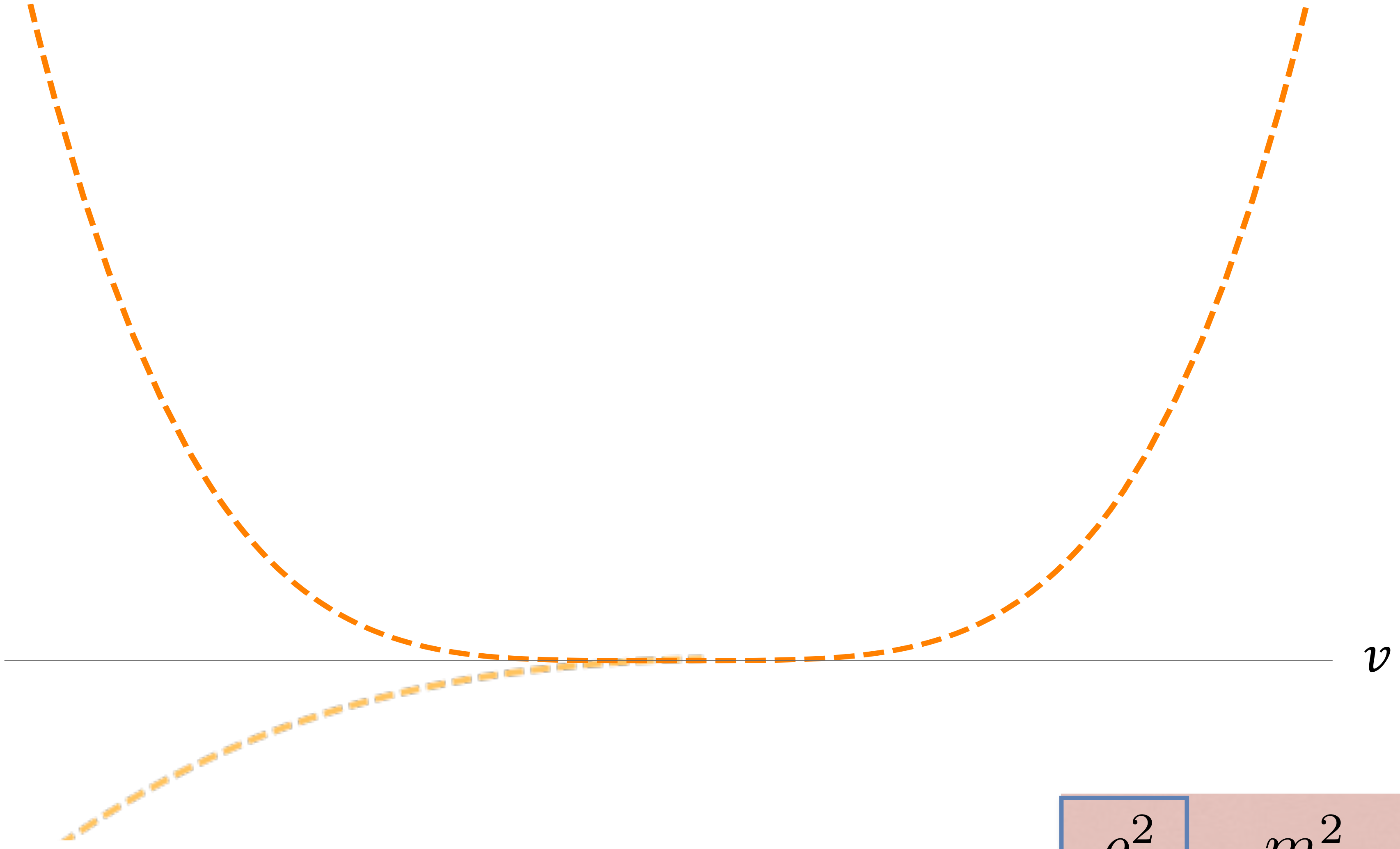
centrifugal potential

Large-charge vacuum

- For sufficiently large ρ_0 , there is minimum to expand, whatever the form of original potential



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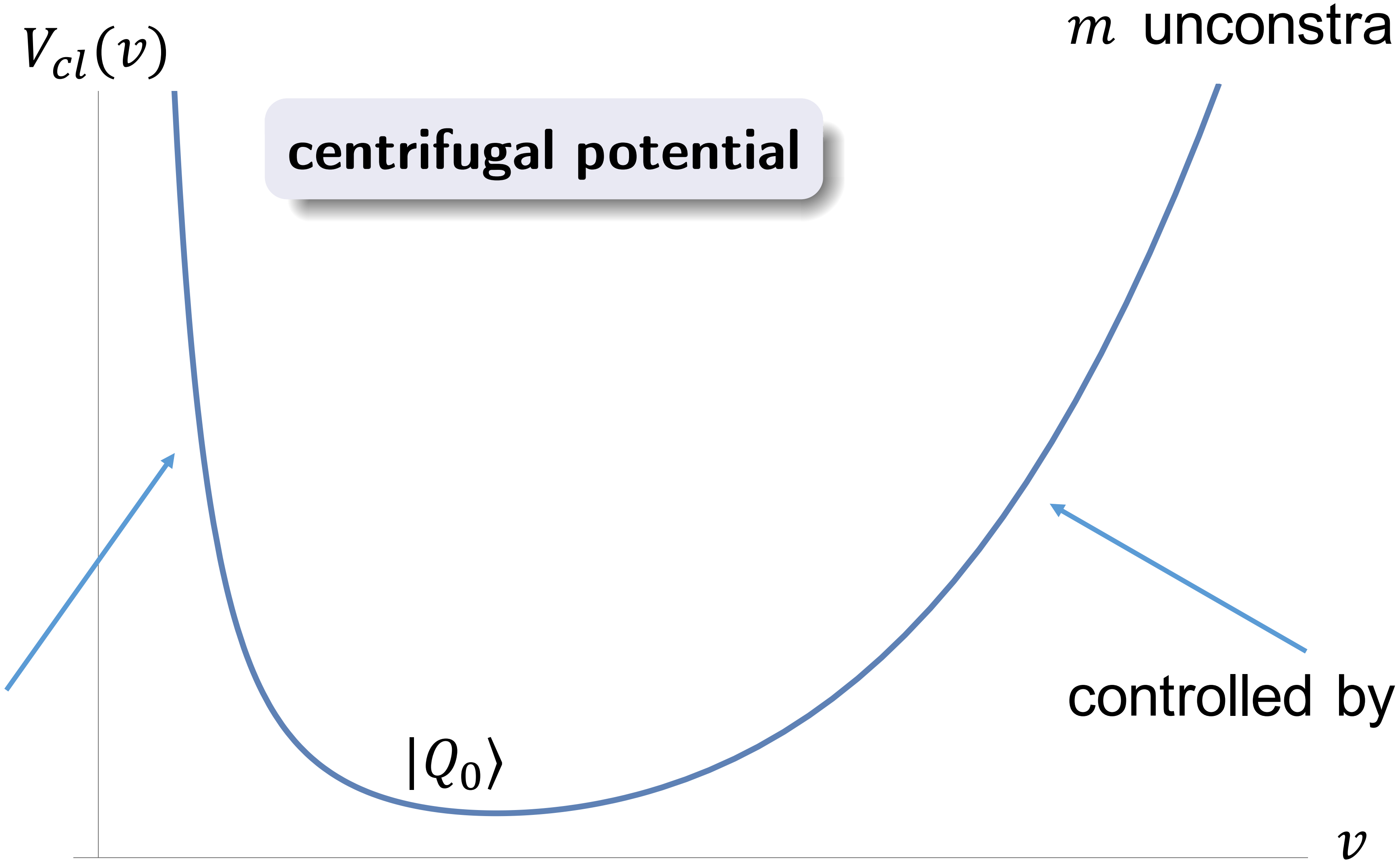

controlled by $\lambda > 0$

controlled
by $\rho_0 \gg 1$

ϕ^4 theory
at fixed charge

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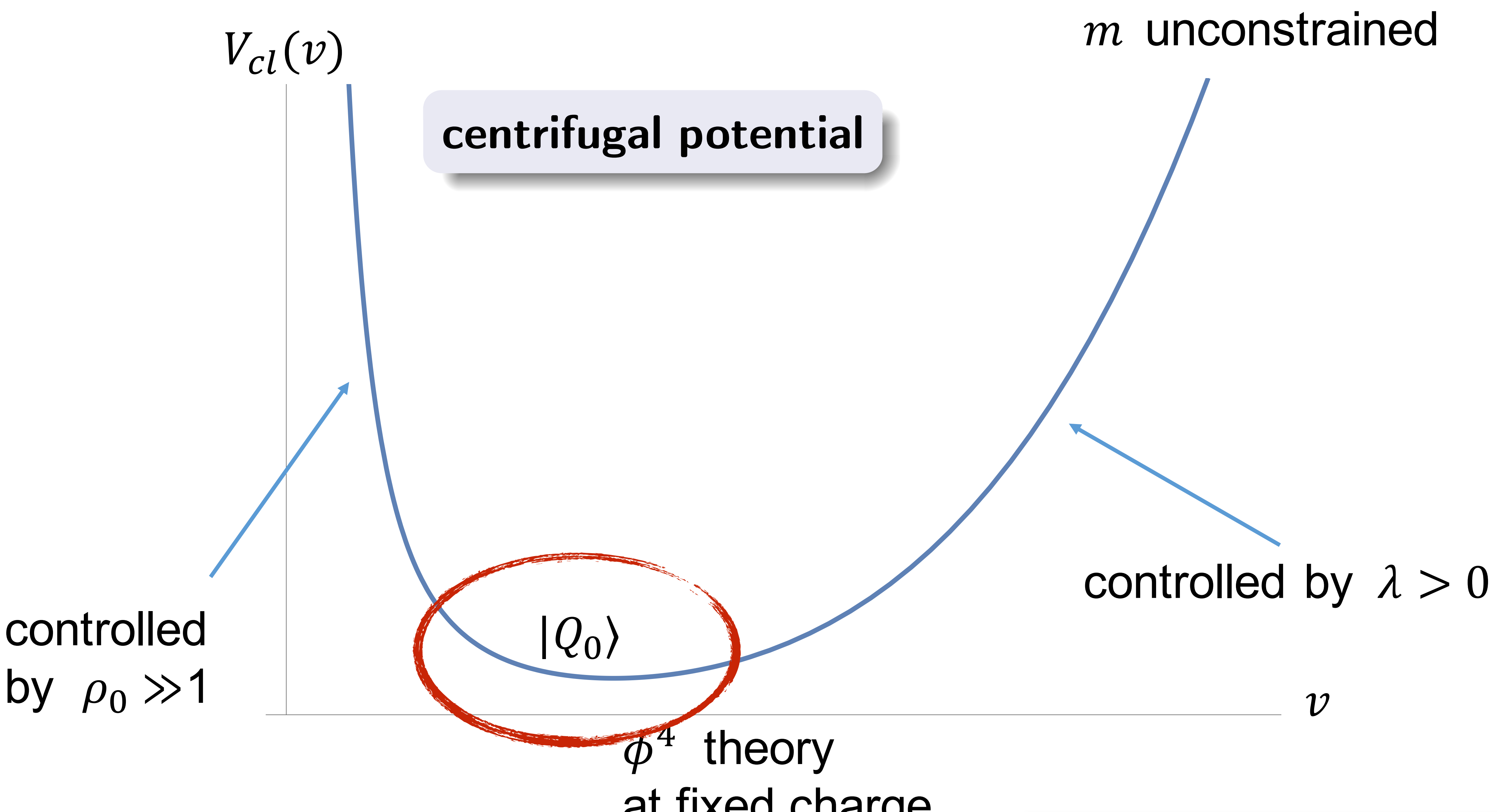

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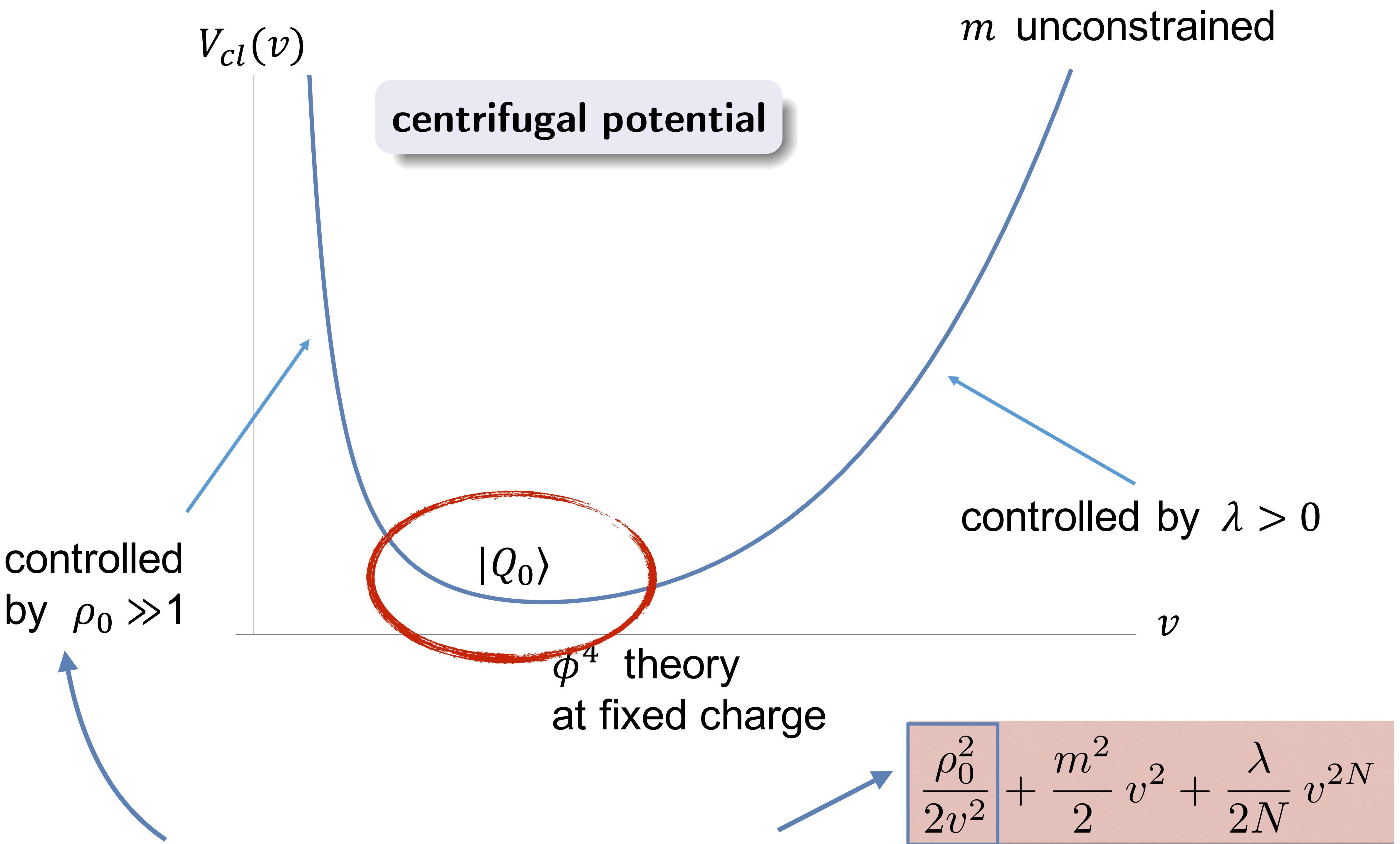

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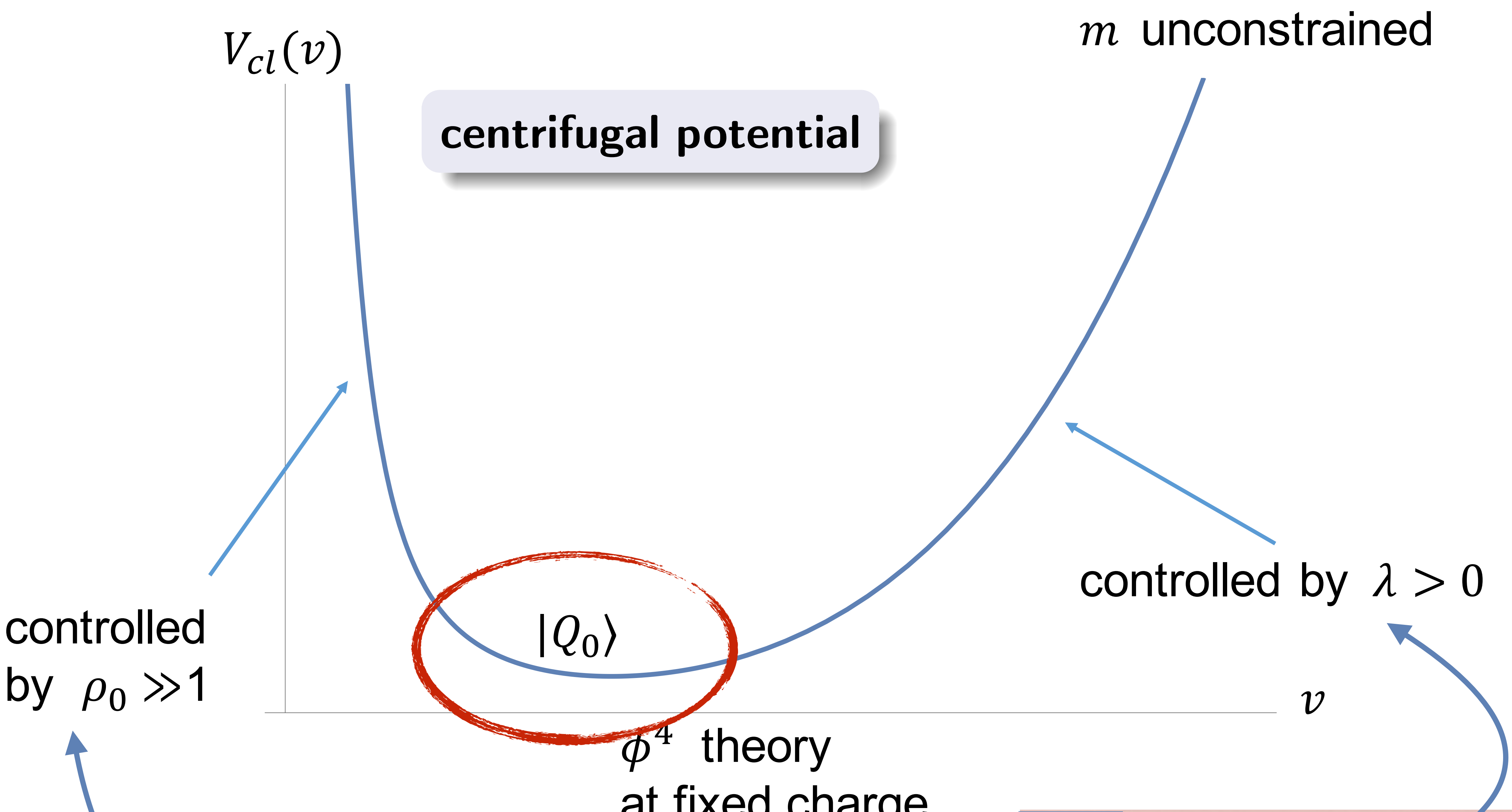
Large-charge vacuum



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Large-charge vacuum

- large condensates $(m = 0)$

$$v^2 = \left(\frac{\rho_0^2}{\lambda}\right)^{\frac{1}{N+1}} \quad , \quad \mathcal{R}_{Q_0} = \frac{N+1}{2N}\left(\lambda\,\rho_0^{2N}\right)^{\frac{1}{N+1}}$$

$$\langle\dot{\chi}\rangle = \left(\lambda\,\rho_0^{N-1}\right)^{\frac{1}{N+1}}$$

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Quantum fluctuations on top of $|Q_0\rangle$

- $\phi(x) = \frac{1}{\sqrt{2}}\left(v + \alpha(x)\right) \mathrm{e}^{i\langle\dot{\chi}\rangle t + i\psi(x)/v}$

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massless field $\psi(x)$	*Goldstone* with higher derivative terms
massive field α	$m_\alpha^2 \sim (N-1)\left(\lambda\,\rho_0^{N-1}\right)^{\frac{2}{N+1}} + \dots$

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- Integrate-out massive mode to obtain infrared physics around $|Q_0\rangle$

  i.e. an effective action for the Goldstone

$$
\begin{aligned}
S\,[\psi]_{Q_0} = {} & \beta V \,\frac{N-1}{2N}\,\left(\lambda\,\rho_0^{\,2N}\right)^{\frac{1}{N+1}} \\
& + \int \mathrm{d}\tau \int \mathrm{d}^d\mathbf{x}\, \left\{ \frac{1}{2}\left[\left(\frac{N+1}{N-1}\right)(\partial_0\psi)^2 - (\nabla\psi)^2\right]\right. \\
& + \left(\frac{1}{\rho_0}\right)^{\frac{N}{N+1}} \left(\frac{1}{\lambda}\right)^{\frac{1}{2N+2}} \left[\frac{N+1}{3(N-1)^2}\,(\partial_0\psi)^3 - \frac{1}{N-1}\,(\partial_0\psi)\,(\nabla\psi)^2\right] \\
& - \frac{g(d)}{2\sqrt{2(N-1)}}\,\frac{|\mathbf{\Lambda}|^d}{\rho_0}\left[\frac{4N^2-9N+4}{N-1}\,(\partial_0\psi)^2 + (\nabla\psi)^2\right] + \int_{X,Y} j_{\partial\psi}(x)\, D_0(x-y)_a\, j_{\partial\psi}(y) \\
& + f(d)\,\frac{N}{(N-1)^2}\left(\frac{\sqrt{\lambda}}{\rho_0^{N+2}}\right)^{\frac{1}{N+1}} |\mathbf{\Lambda}|^{\frac{3D-4}{2}} \quad \times \\
& \times \int \mathrm{d}^D y\, \left(\partial_0\psi(y)\right) \int \mathrm{d}^D z\, \frac{1}{|\mathbf{y}+\mathbf{z}|^{\frac{D}{2}}}\left[\frac{N-2}{N-1}\,\left(\partial_0\psi(z)\right)^2 + \left(\nabla\psi(z)\right)^2\right. \\
& \qquad\qquad\qquad\qquad \left. + \frac{1}{N-1}\left(\frac{1}{\lambda^{\frac{3}{2}}\,\rho_0^{N-2}}\right)^{\frac{1}{N+1}} \partial_\mu\partial^\mu\partial_0\psi(z)\right] + \;\ldots\; + \\
& + \frac{1}{2(N-1)^2}\left(\frac{1}{\lambda^2\rho_0^{2N-2}}\right)^{\frac{1}{N+1}} \partial_\mu(\partial_0\psi)\partial^\mu(\partial_0\psi) \quad + \mathcal{O}\left(\frac{|\mathbf{\Lambda}|^{2d}}{\rho_0^{\frac{N+2}{N+1}}}\right) \left.\right\}
\end{aligned}
$$

condensate

$$S\left[\psi\right]_{Q_0} = \beta V \, \frac{N-1}{2N} \left(\lambda \, \rho_0^{2N}\right)^{\frac{1}{N+1}}$$

$$+ \int \mathsf{d}\tau \int \mathsf{d}^d\mathbf{x} \left\{ \, \frac{1}{2} \left[\left(\frac{N+1}{N-1} \right) (\partial_0 \psi)^2 - (\nabla \psi)^2 \right] \right.$$

$$+ \left(\frac{1}{\rho_0} \right)^{\frac{N}{N+1}} \left(\frac{1}{\lambda} \right)^{\frac{1}{2N+2}} \left[\frac{N+1}{3(N-1)^2} (\partial_0 \psi)^3 - \frac{1}{N-1} (\partial_0 \psi) (\nabla \psi)^2 \right]$$

$$- \frac{g(d)}{2\sqrt{2(N-1)}} \, \frac{|\mathbf{\Lambda}|^d}{\rho_0} \left[\frac{4N^2 - 9N + 4}{N-1} (\partial_0 \psi)^2 + (\nabla \psi)^2 \right] + \int_{X,Y} j_{\partial\psi}(x) \, D_0(x-y)_a \, j_{\partial\psi}(y)$$

$$+ f(d) \, \frac{N}{(N-1)^2} \left(\frac{\sqrt{\lambda}}{\rho_0^{N+2}} \right)^{\frac{1}{N+1}} |\mathbf{\Lambda}|^{\frac{3D-4}{2}} \quad \times$$

$$\times \int \mathsf{d}^D y \, (\partial_0 \psi(y)) \int \mathsf{d}^D z \, \frac{1}{|\mathbf{y}+\mathbf{z}|^{\frac{D}{2}}} \left[\frac{N-2}{N-1} (\partial_0 \psi(z))^2 + (\nabla \psi(z))^2 \right.$$

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condensate

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Goldstone

$$+\left(\frac{1}{\rho_0}\right)^{\frac{N}{N+1}}\left(\frac{1}{\lambda}\right)^{\frac{1}{2N+2}}\left[\frac{N+1}{3(N-1)^2}\,(\partial_0\psi)^3-\frac{1}{N-1}\,(\partial_0\psi)\,(\nabla\psi)^2\right]$$

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Goldstone

quantum corrections

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fixing the charge

$$\times\int \mathrm{d}^D y\;(\partial_0\psi(y))\int \mathrm{d}^D z\,\frac{1}{|\mathbf{y}+\mathbf{z}|^{\frac{D}{2}}}\left[\frac{N-2}{N-1}\,(\partial_0\psi(z))^2+(\nabla\psi(z))^2\right.$$

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cut-off

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Quantum fluctuations on top of $|Q_0\rangle$

- Already leading Goldstone dispersion relation manifestly non-Lorentz invariant

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- This breakdown of Lorentz-covariance is fundamental due to (rapidly) rotating angular VEV $\chi = \langle\dot{\chi}\rangle t + \ldots$ assigned in temporal direction

- Formally, $\langle\chi\rangle$ is *not* constant
  - generalised Goldstone's theorem is needed to prove existence of massless mode [Nicolis,Piazza'12]

Summary

vacuum $|Q_0\rangle$ + Goldstone + Q_0 - suppressed corrections

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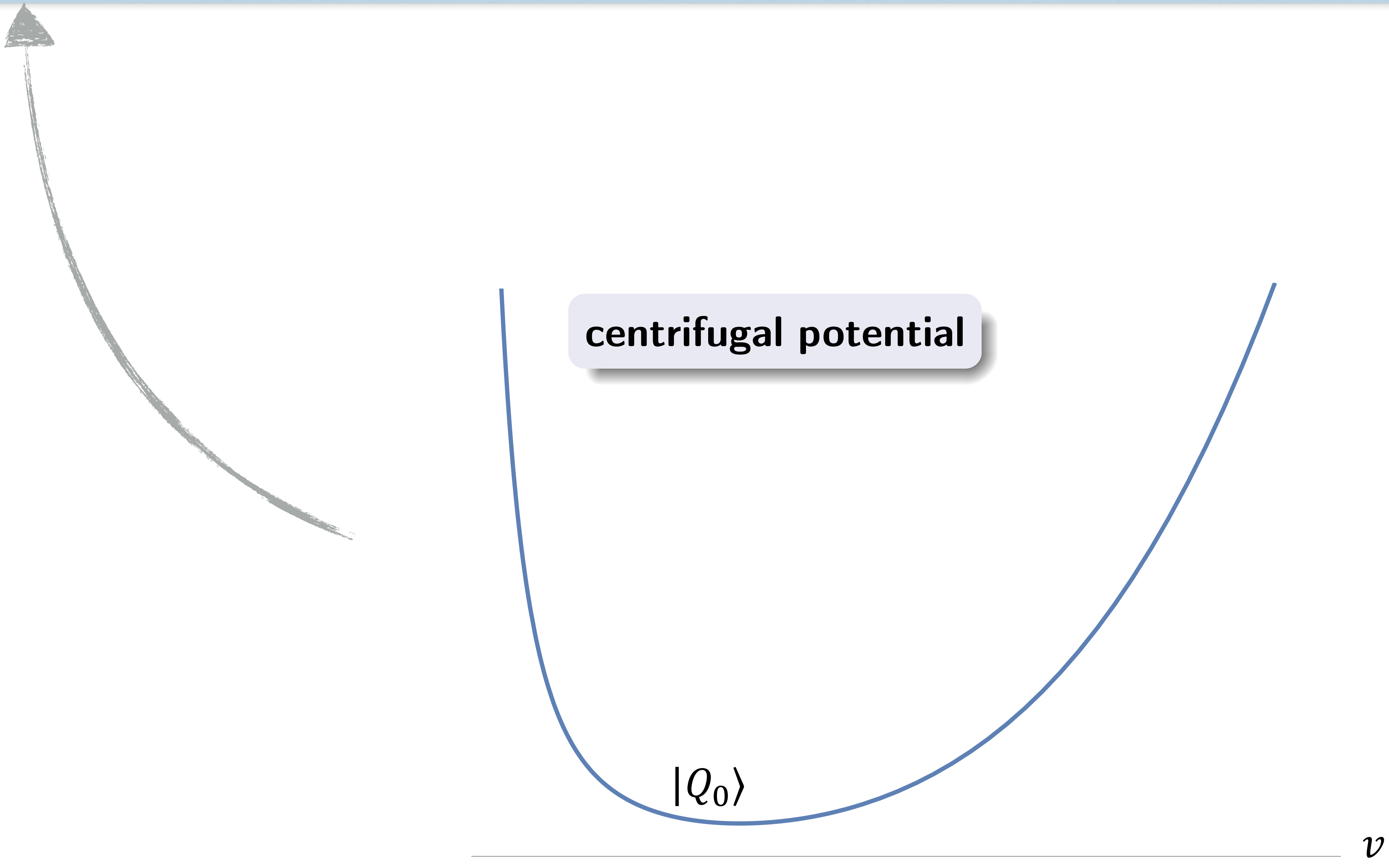

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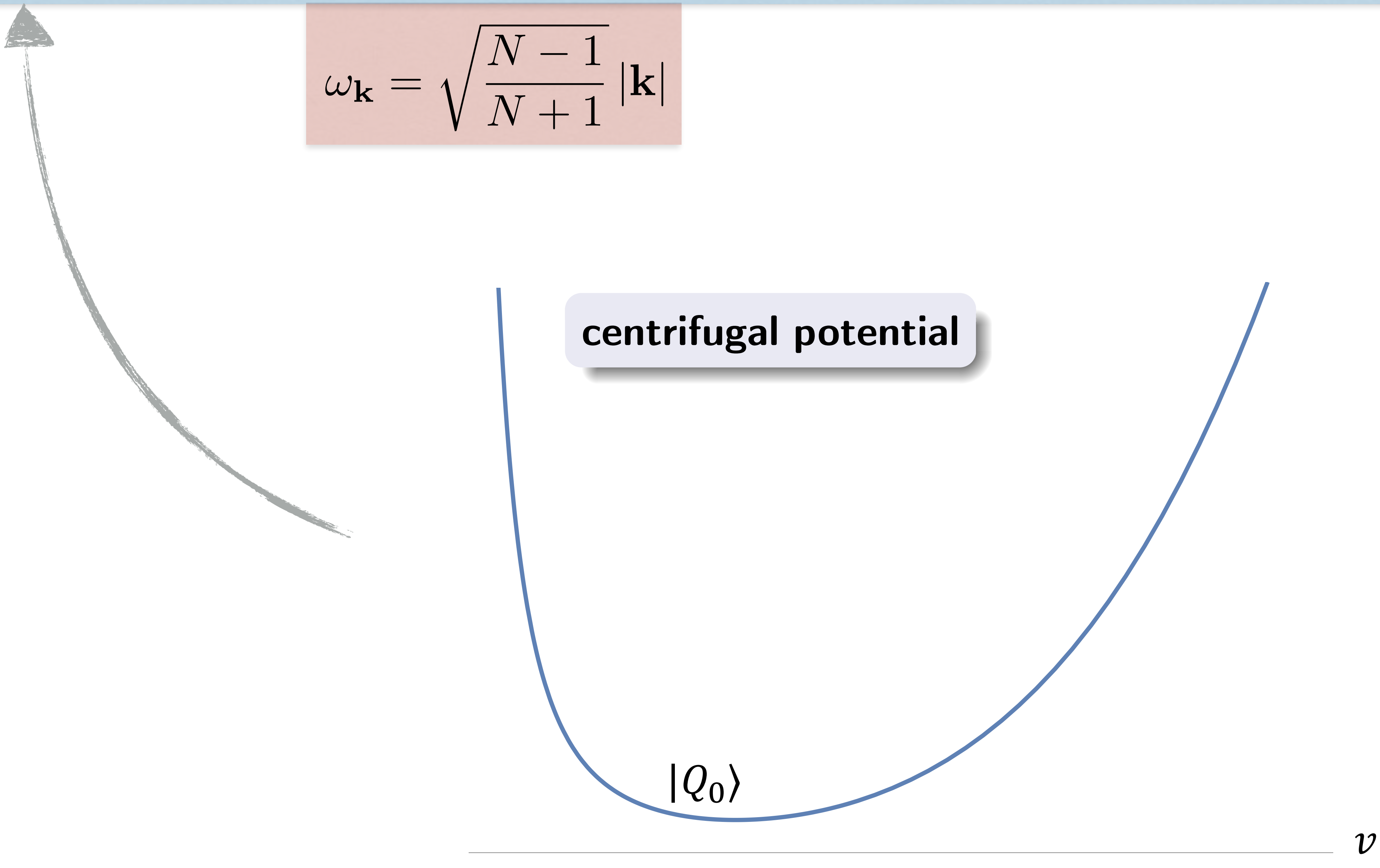

Summary

$$\frac{1}{\sqrt[d]{V}} \ll \Lambda \ll \frac{Q_0}{\sqrt[d]{V}}$$

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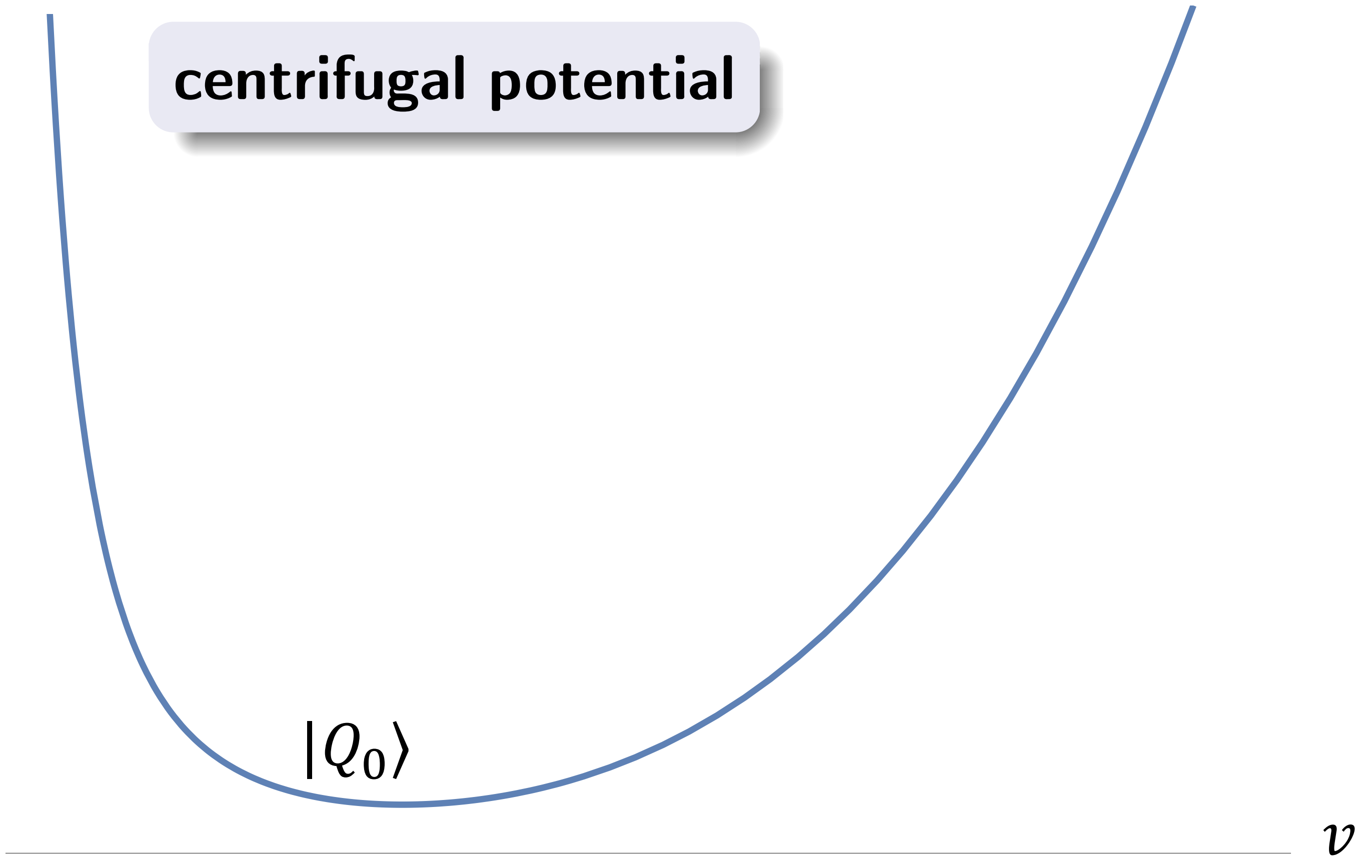

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    - e.g. non-Abelian $\mathrm{O}(2n)$ [work in progress]
- LCPT for fermions
    - fermions do not fundamentally condensate
    - composite particle (?)

Thank you

for your attention

Overview

Large Charge Perturbation Theory (LCPT)

- Motivation

- Fixing the charge
- Large-charge vacuum
- Quantum fluctuations

- Summary and Outlook