Crystalline Confined Phases in Lattice Gauge Theories

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based on
J. Stat. Phys. (2013); Phys. Rev. B (2014); Phys. Rev. B (2016)

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Rethinking Quantum Field Theory

Outline

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Introduction

- Lattice gauge theories $\rightarrow$ strongly correlated systems.
- Most non-perturbative computations done in Euclidean space with Wilson formulation.
- Ultra-cold atoms toolbox $\rightarrow$ quantum dynamics of gauge theories.
- Questions of real-time evolution and finite baryon density.
- Alternate formulation of gauge theories (Horn,1981; Orland, Rohrlich, 1990; Chandrasekharan, Wiese, 1997) and QCD with domain wall fermions (Brower, Chandrasekharan, Wiese, 1999) are particularly relevant.
- These realize **continuous** gauge symmetries using **discrete** quantum link variables, having finite dimensional Hilbert space $\rightarrow$ extension of Wilson formulation of gauge theories.
- Excellent candidate models to be implemented in cold-atom systems.
- Qualitatively **new** phases can be observed in these systems to be studied by Monte-Carlo + Quantum Simulators.

Outline

Hamiltonian $U(1)$ LGT: Quantum Links

- U(1) gauge invariant Hamiltonian:

$$H = \frac{g^2}{2} \sum_{x,i} E_{x,i}^2 - \frac{1}{2g^2} \sum_{\square} (U_{\square} + U_{\square}^{\dagger})$$

- $U = S^1 + iS^2 = S^+; \quad U^{\dagger} = S^1 - iS^2 = S^-$
  $\Rightarrow$ are operators in the Hamiltonian formulation, operating in a **finite** dimensional Hilbert space on a single link
- Electric field operator describes the kinematics of U:

$$E = S^3, \quad [E, U] = U, \quad [E, U^{\dagger}] = -U^{\dagger}; \quad [U, U^{\dagger}] = 2E$$

- U(1) gauge transformations generated by Gauss Law:

$$\begin{aligned} G_x &= \sum_i (E_{x,i} - E_{x-\hat{i},i}); \quad [G_x, H] = 0 \\ V &= \prod_x \exp(i\alpha_x G_x); \quad U'_{xy} = V U_{xy} V^{\dagger} = \exp(i\alpha_x) U_{xy} \exp(-i\alpha_y) \end{aligned}$$

The (2+1)-d U(1) Quantum Link model

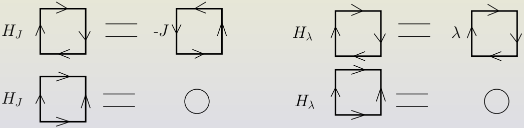
- Simplest Abelian pure gauge model: with spin $S = 1/2$

$$H = -J \sum_{\Box} \left(U_{\Box} + U_{\Box}^{\dagger} \right) + \lambda \sum_{\Box} \left(U_{\Box} + U_{\Box}^{\dagger} \right)^2$$

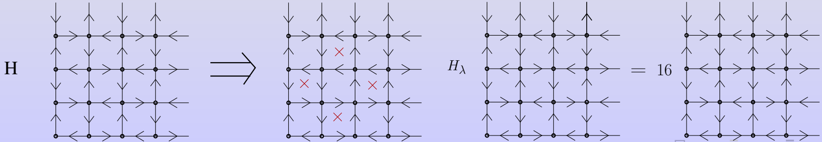
- Link states: 2-dim Hilbert space per link

$$E|\uparrow\rangle = \frac{1}{2}|\uparrow\rangle; \quad E|\downarrow\rangle = -\frac{1}{2}|\downarrow\rangle; \quad U|\uparrow\rangle = 0; \quad U|\downarrow\rangle = |\uparrow\rangle; \quad U^{\dagger}|\uparrow\rangle = |\downarrow\rangle; \quad U^{\dagger}|\downarrow\rangle = 0$$

E^2 contributes a constant for $S = 1/2$.



- Plaquettes are flipped only if they have flux in the right order; second term ($= H_\lambda$) counts the number of flippable plaquettes

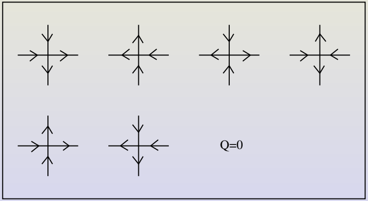


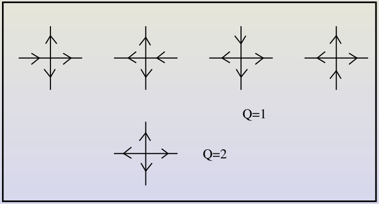
Gauss Law and Charge Sectors

To define the path integral $\mathcal{Z} = \mathrm{Tr}\left(\exp(-\beta H)\mathcal{P}_{\mathcal{G}}\right)$, the Gauss Law must be implemented :

$$\sum_i \left(E_{x,i} - E_{x-\hat{\imath},i} \right) = Q_x$$

There is zero charge everywhere (charge-0 sector) unless external static charges are placed at vertices.





A staggered charge background $Q = \pm 1$ with the Hamiltonian H is called the Quantum Dimer model.

Symmetry breaking and phase transitions

- Discrete: Rotation by $\pi/2$, Reflection, **Charge Conjugation (C)**, **Translation($T = (T_x, T_y)$)**
- Symmetry breaking patterns can be deduced very well from exact diagonalizations.
- 2-component order parameter (M_A, M_B) to analyze the symmetry breaking patterns

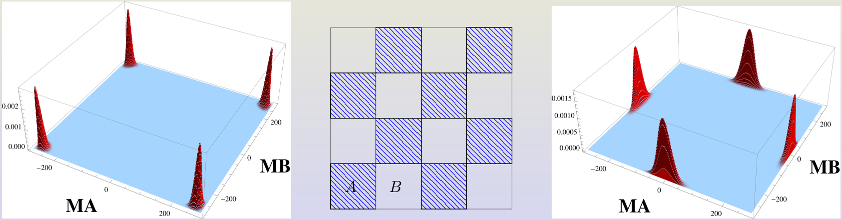


Figure: Order parameter(OP) distribution at $\lambda = -1$ (left) and at $\lambda = 0$ (right); Effect of symmetry operations on the OP (middle).

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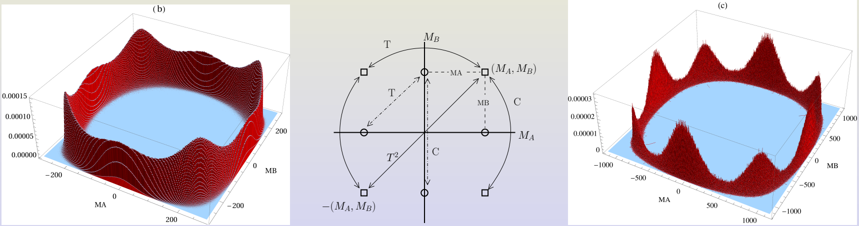
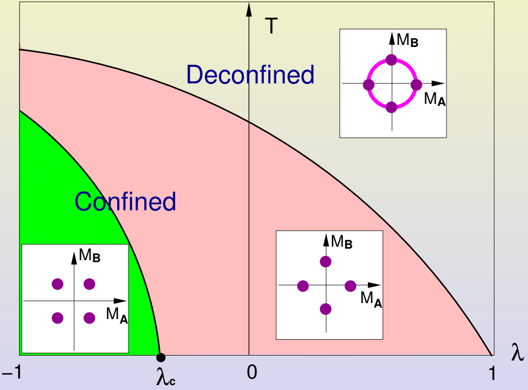
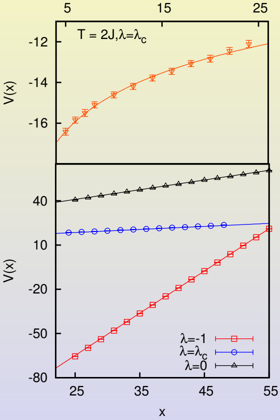


Figure: Order parameter(OP) distribution at $\lambda \sim \lambda_c$ for $L = 24$ (left) and at $L = 48$ (right); Effect of symmetry operations on the OP (middle).

Phase diagram

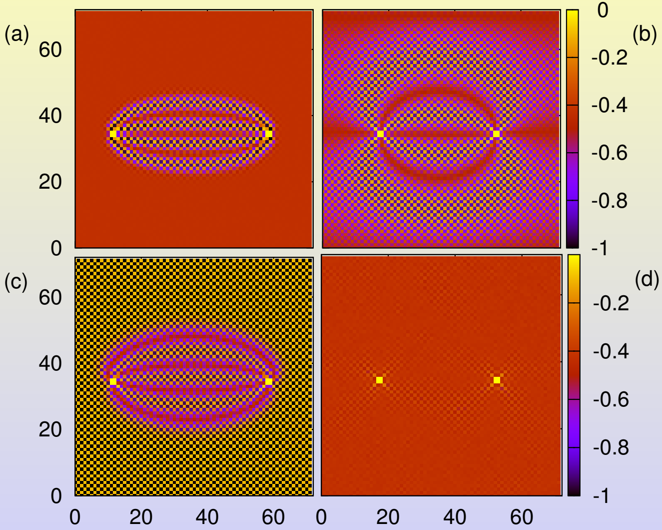
Explored with exact diagonalization and a newly developed cluster algorithm using dualization techniques.





An *approximate global SO(2) symmetry is emergent* at λ_c. A description in terms of a low-energy *effective theory suggests a weak 1st order transition*.

Crystalline confinement

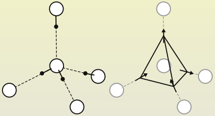


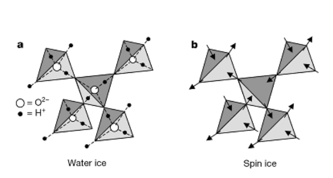
Energy density $\langle H_J \rangle$ of two charges $Q = \pm 2$ placed along the axis on $L = 72$ lattice

Outline

Ice and Spin-Ice

Gauge theories play a fundamental role in condensed matter systems as well $\longrightarrow$ Ice.





Ice rules can be encoded by the ground states of the Hamiltonian

$$H_1 = J \sum_{i,j} S_i^z S_j^z = J \sum_{\alpha} \left(\sum_{i_\alpha=1}^{4} S_{i_\alpha}^z \right)^2 \; ; \; J > 0$$

Constraints can select the sector of the theory by using an external magnetic field

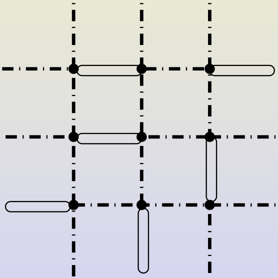
$$H_2 = J \sum_{\alpha} \left(\sum_{i_\alpha=1}^{4} S_{i_\alpha}^z \right)^2 - \sum_{i_\alpha} h S_{i_\alpha}^z = J \sum_{\alpha} \left(\sum_{i_\alpha=1}^{4} S_{i_\alpha}^z - h/2 \right)^2$$

QDM and the U(1) link model

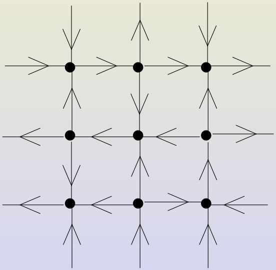
Rule: $E_{x,i} = (-1)^{x+y}[n_{x,i} - \frac{1}{2}]$

$E_{x,i}$: Electric flux at site x in dir i

$n_{x,i}$: Dimer number on the bond connecting sites x and x+$\hat{i}$



Dimer Model config



U(1) QLM config

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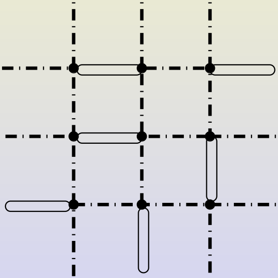
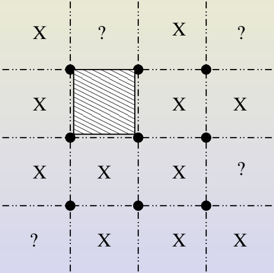


Dimer Model config



shaded : flippable plquette
crosses : non−flippable plaq

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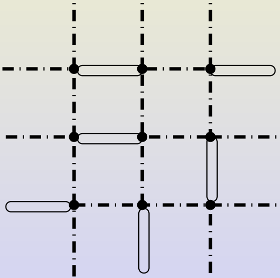

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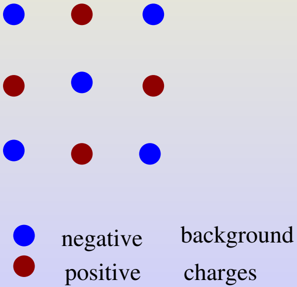
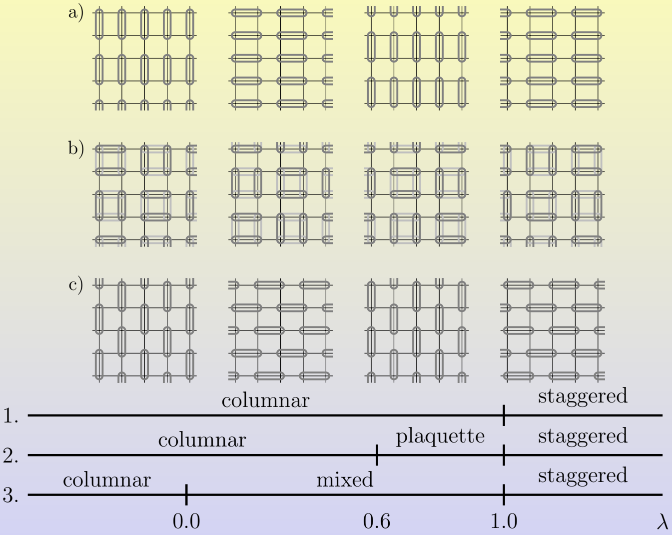
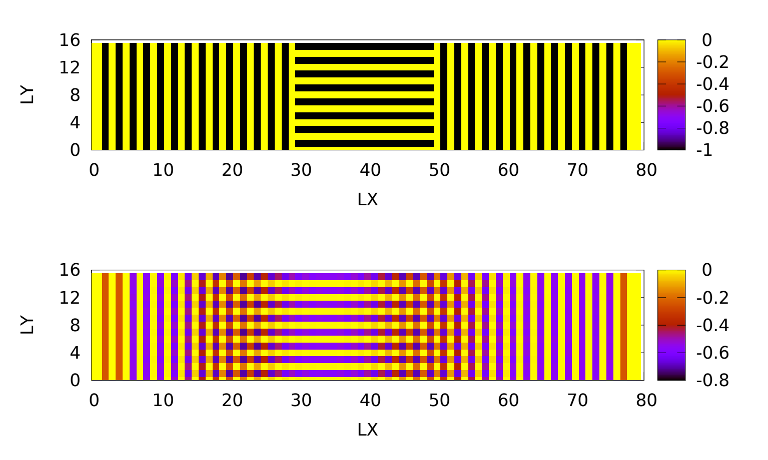


Candidate Phases and Phase Diagrams



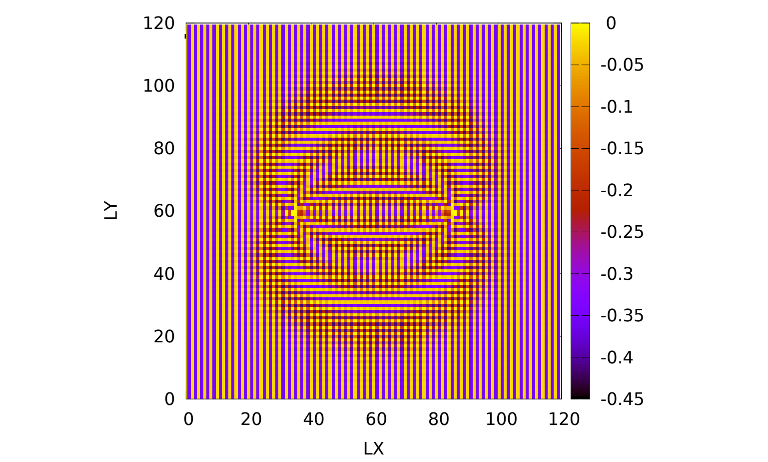
(a) Columnar (b) Plaquette (c) Staggered

Interfaces between columnar phases



Plaquette phases exist as interfaces between columnar phases $\lambda = -1.0$

Interfaces with charges



Energy density for a $2Q$ charge-anti-charge profile at $\lambda = -0.5$. Plaquette "phase" acts as interface carrying fractionalized flux $\frac{1}{4}$

Outline

Conclusion

- Interesting non-trivial physics, exotic phasess, interesting continuum limits.
- Extensively studied in condensed matter physics with reference to high-T_c phenomena.
- Interface physics: fractionalized flux.
- Broader applicability of the dualization techniques. Playground for ideas for extending the construction to other theories, particularly QCD.
- “Observing” this physics in engineered cold-atom systems.

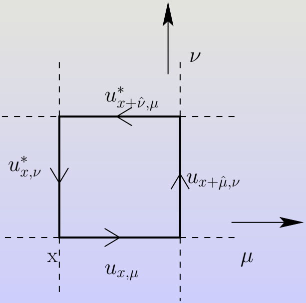
  Glätzle et. al., PRX (2014); D. Marcos et. al., Ann. Phys. (2014)
- Wilson-type gauge theories can be constructed by self-adjoint extensions which show similar ground states (under study).

Thank You for your attention!

Mimicking QLM physics in Wilson-type theories

$$\mathcal{H} = \mathcal{H}_E + \mathcal{H}_B = \frac{e^2}{2}\sum_{x,i}\left(-i\partial_{\phi_{x,i}}\right)^2 - \frac{1}{4e^2}\sum_{x,i>j}\left[u_{x,i}u_{x+i,j}u^*_{x+j,i}u^*_{x,j} + h.c.\right]$$

$$\left[e_{x,i}, u_{y,j}\right] = \delta_{x,y}\delta_{i,j}u_{x,i} \quad [e_{x,i}, u^\dagger_{y,j}] = -\delta_{x,y}\delta_{i,j}u^\dagger_{x,i}$$



- $u_{x,i} = \exp(i\phi_{x,i})$
- flux-basis: $m_{x,i} \in \mathbb{Z}$
- "ground state" when all $m_{x,i} = 0$
- very different from the QLM
- However, one parameter of self-adjoint extension possible
- $e_{x,i} = -i\partial_{\phi_{x,i}} + \frac{\theta_{x,i}}{2\pi}$
- Motivation: quantum rotor

Quantum Rotor: Particle on a ring

- $\mathbb{H} = \frac{\mathbb{L}^2}{2I}$, $\mathbb{L} = -i\partial_\phi$
- $\psi_m = A\exp(im\phi)$, $E_m = \frac{m^2}{2I}$, $m_{x,i} \in \mathbb{Z}$
- ψ need not be single-valued: twisted boundary condition $\psi(\phi + 2\pi) = e^{i\theta}\psi_\phi$
- Equivalently, demand $\psi(\phi + 2\pi) = \psi(\phi)$, but introduce θ in the Hamiltonian
- $\widetilde{\mathbb{H}} = \frac{\widetilde{\mathbb{L}}^2}{2I}$, $\widetilde{\mathbb{L}} = -i\partial_\phi + \frac{\theta}{2\pi}$
- Energies shifted: $E_m = \frac{1}{2I}(m - \frac{\theta}{2\pi})^2$
- θ : Vector potential of a magnetic field penetrating the “ring”
- Key idea: Use a similar construction to change the spectrum of the Wilson type theory.
- Flux can be made half-integral. In particular, the ground state has the same amount of flux as the QLM

Height Model

- DOF: h_x where $x = (x_1, x_2, x_3)$ (dual lattice)
- odd and even lattice sites: $(-1)^{(x+y)}$
- Partition function of a classical spin model:

$$\mathcal{Z} = \prod_{x \text{ even}} \sum_{h_x \in \mathbb{Z}} \prod_{x \text{ odd}} \sum_{h_x + \frac{1}{2} \in \mathbb{Z}} \exp(-S[h]); \quad S[h] = \frac{e^2}{2} \sum_{x,\mu} (h_{(x+\hat{\mu})} - h_x)^2$$

- Integer shift symmetry: $h'_x = h_x + l; l \in \mathbb{Z}$
  Dual to the gauge symmetry
- Charge conjugation: ${}^C h_x = -h_x$
- Spatial Translation: ${}^T h_x = h_{x+\hat{i}} + \frac{1}{2}$

Height Model: Order Parameters

- Any OP must be invariant under shift symmetries
- $O_{C,T}[h] = \sum_{x \text{ even}} h_x - \sum_{x \text{ odd}} h_x$
- Sensitive to both C and T symmetry breaking

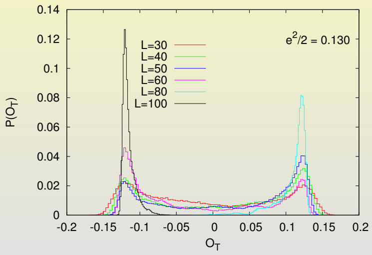
$$O_{C,T}[h'] = O_{C,T}[h], \quad O_{C,T}[{}^{C}h] = -O_{C,T}[h], \quad O_{C,T}[{}^{T}h] = -O_{C,T}[h]$$

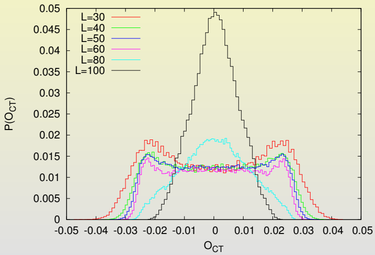
- $O_T[h] = \sum_{x \text{ even}}(h_x - \overline{h})^2 - \sum_{x \text{ odd}}(h_x - \overline{h})^2; \quad \overline{h} = \frac{1}{V}\sum_x h_x$
- Sensitive to T only

$$O_T[h'] = O_T[h], \quad O_T[{}^{C}h] = O_T[h], \quad O_T[{}^{T}h] = -O_T[h]$$

- Sign problem associated with θ gone
- Can be simulated with a variety of algorithms: Metropolis, Over-relaxation, Cluster

Histograms: Distribution of O_T, O_{CT}





Broken translational symmetry, but charge conjugation is unbroken. $\lambda = 0$ phase in QLM.