Conformal symmetry breaking and evolution equations in quantum chromodynamics

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Motivation & Introduction

RG-/Evolution equation

$$(M\partial_M + \beta(\alpha)\partial_\alpha + \hat{\gamma})[\mathcal{O}(x)] = 0$$

- calculation of mixing matrices $\hat{\gamma}$ can become a tedious task at higher orders
- straightforward calculation only feasible with modern computer algebra
- concept of evolution kernels $\mathbb{H}$: generalization to non-local operators
- conformal symmetry allows to obtain evolution kernel (and mixing matrices) with calculations one order beyond the desired accuracy (but with knowledge of anomalous dimensions to desired accuracy)

Conformal symmetry

Conformal transformations $\Delta = \frac{d-1}{2}$

Translation

$$x_\mu \mapsto x_\mu + a_\mu \qquad P_\mu = -i\partial_\mu$$

Rotation

$$x_\mu \mapsto \omega_{\mu\nu} x^\nu \qquad M_{\mu\nu} = i(x_\mu \partial_\nu - x_\nu \partial_\mu)$$

Dilatation

$$x_\mu \mapsto \lambda x_\mu \qquad D = -i(x \cdot \partial + \Delta)$$

Special conformal transformation

$$x_\mu \mapsto \frac{x_\mu - a_\mu x^2}{1 - 2a \cdot x + a^2 x^2} \qquad K_\mu = -i(2x_\mu x \cdot \partial - x^2 \partial_\mu + 2\Delta x_\mu)$$

QCD in $d = 4 - 2\epsilon$ at the critical point

Renormalized action

$$S_R = \int d^d x_E \left\{ \bar{q} \not{D} q + \frac{1}{4} F^a_{\mu\nu} F^{a,\mu,\nu} - \bar{c}^a \partial_\mu (D^\mu c)^a + \frac{1}{2\xi} (\partial_\mu A^{a,\mu})^2 \right\},$$

$$D_\mu = \partial_\mu - igM^\epsilon A^a_\mu t^a \,, \qquad F^a_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + gM^\epsilon f^{abc} A^b_\mu A^c_\mu \,.$$

QCD running coupling

$$\beta(a) = \frac{da}{d \ln(M)} = -2a \left(\epsilon + a\beta_0 + a^2 \beta_1 + \mathcal{O}(a^3) \right) \,, \quad a = \frac{\alpha_s}{4\pi} = \frac{g^2}{(4\pi)^2}$$

Conformal symmetry at critical point α_*:

$$\beta(a_*) = 0 \Leftrightarrow a_* = -\frac{1}{\beta_0}\epsilon - \frac{\beta_1}{\beta_0^3}\epsilon^2 + \mathcal{O}(\epsilon^3)$$

What did we consider

Non-local "light-ray" twist-two operator $n^2 = 0 \qquad D_+ := n_\mu D^\mu$

$$[\mathcal{O}^{(n)}(x; z_1, z_2)] = Z\bar{q}(x + nz_1)\not{n}q(x + nz_2)$$
$$= Z \sum_{l,m=0}^{\infty} \frac{z_1^l z_2^m}{l!m!} \bar{q}(x)(\overleftarrow{D}_+)^l \not{n}(\overrightarrow{D}_+)^m q(x)$$

We tacitely assume different flavors.

Renormalization factor in $\overline{MS}$ - scheme

$$Z = \mathbb{Z}Z_q^2 = 1 + \sum_{k=0}^{\infty} \frac{1}{\epsilon^k} \sum_{\ell=k}^{\infty} a^\ell Z_k^{(\ell)} \,,$$

Relation to evolution kernel

$$\mathbb{H} = -\frac{d}{d\ln M} \ln(\mathbb{Z})$$

General form

$$[\mathbb{H}\mathcal{O}](z_1, z_2) = \int_0^1 d\alpha \int_0^1 d\beta h(\alpha, \beta)\mathcal{O}(z_{12}^\alpha, z_{21}^\beta) \,, \quad z_{12}^\alpha = (1-\alpha)z_1 + \alpha z_2 \,.$$

Collinear subgroup $SL(2,\mathbb{R})$ and spin generators

Generators of infinitesimal transformations

$$-in\cdot P \equiv S_- = -\partial_z$$

$$\frac{i}{2}\left(D - M_{\mu\nu}n^\mu\bar{n}^\nu\right) \equiv S_0 = z\partial_z + 2j \qquad [\bar{n}^2 = 0, \bar{n}\cdot n = 1]$$

$$\frac{i}{2}\bar{n}\cdot K \equiv S_+ = z^2\partial_z + 2jz$$

obey the $s\ell(2)$ - algebra

$$[S_+, S_-] = 2S_0 \qquad [S_0, S_\pm] = \pm S_\pm$$

QCD at the classical level

Evolution equation commutes with all three spin generators

$$[S_{\pm,0}, \mathbb{H}] = 0\,.$$

Shape of the evolution kernel fixed

$$[\mathbb{H}\mathcal{O}](z_1, z_2) = \int_0^1 d\alpha \int_0^1 d\beta h\,(\tau)\, O(z_{12}^{\alpha}, z_{21}^{\beta})\,, \qquad \tau = \frac{\alpha\beta}{\bar{\alpha}\bar{\beta}}\,.$$

Anomalous dimension γ_N of local operators with spin N are eigenvalues

$$\mathbb{H}(z_1 - z_2)^N = (z_1 - z_2)^N \int_0^1 d\alpha \int_0^1 d\beta h\,(\tau)\,(1 - \alpha - \beta)^N = (z_1 - z_2)^N \gamma_N$$

This can be inverted for the kernel $h(\tau)$!

Breakdown and restoration of conformal symmetry

- canonical conformal symmetry broken, even for $\beta(a_*) = 0$
- exact symmetry can be reconstructed order by order
- done by conformal Ward identities $\delta_C \langle [O(x; z_1, z_2)][O(y; w_1, w_2)] \rangle = 0$

Corrections added to canonical generators

$$
\begin{aligned}
S_- &= S_-^{(0)} \\
S_0 &= S_0^{(0)} + \Delta S_0(a_*) \\
S_+ &= S_+^{(0)} + \Delta S_+(a_*)
\end{aligned}
$$

$\mathcal{O}(a_*)$: V.M. Braun, A.N. Manashov, Eur.Phys.J. C73 (2013) 2544
$\mathcal{O}(a_*^2)$: V.M. Braun, A.N. Manashov, S. Moch, M. S. , JHEP 03 (2016) 142

Ward Identities

Consider Green function of two light-ray operators

$$G(x,z,w) = \langle [\mathcal{O}^{(n)}](0,z)[\mathcal{O}^{(\bar{n})}](x,w)\rangle \,.$$

Conformal Ward Identity $\delta_C G(x,z,w) = 0$

$$\langle \delta_C [\mathcal{O}^{(n)}](0,z)[\mathcal{O}^{\bar{n}}](x,w)\rangle + \langle [\mathcal{O}^{(n)}](0,z)\delta_C[\mathcal{O}^{\bar{n}}](x,w)\rangle = \langle \delta_C S_R [\mathcal{O}^{(n)}](0,z)[\mathcal{O}^{\bar{n}}](x,w)\rangle \,.$$

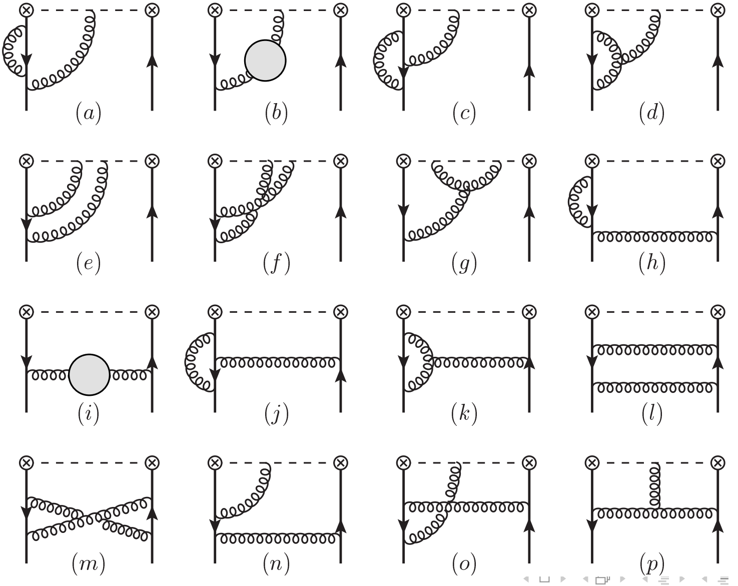
Variation of the renormalized action

$$\delta_D S_R = \epsilon \int d^d x \mathcal{N}(x)\,, \qquad \delta_K S_R = \epsilon \int d^d x 2(nx)\mathcal{N}(x) + \ldots\,,$$

$$\mathcal{N}(x) = 2\mathcal{L}_R^{YM+gf}\,.$$

… denotes terms which vanish for gauge-invariant correlators

Diagramatics at $\mathcal{O}(a_*^2)$



Short excerpt from our results for $S_{+,0}(a_*)$ at NNLO

$$
\begin{aligned}
\Delta S_0^{(2)} =& \mathbb{H}^{(2)}\,, \\
\Delta S_+^{(2)} =& (z_1+z_2)(\beta_1+\mathbb{H}^{(2)}) + \frac{1}{4}[\mathbb{H}^{(2)}, z_1+z_2] + (z_1-z_2)\Delta_+^{(2)}\,, \\
\Delta_+^{(2)} f(z_1,z_2) =& \int_0^1 d\alpha \int_0^{\bar\alpha} d\beta \big[\omega(\alpha,\beta) + \omega_{\mathbb{P}}(\alpha,\beta)\mathbb{P}\big]\big[f(z_{12}^\alpha, z_{21}^\beta) - f(z_{12}^\beta, z_{21}^\alpha)\big] \\
&+ \int_0^1 du \int_0^1 dt \varkappa(t)\big[f(z_{12}^{(ut)}, z_2) - f(z_1, z_{21}^{(ut)})\big]\,. \\
\omega(\alpha,\beta) =& \beta_0 C_F \omega_{\beta F}(\alpha,\beta) + C_F^2 \omega_{FF}(\alpha,\beta) + C_F C_A \omega_{FA}(\alpha,\beta)\,, \\
\omega_{FA}(\alpha,\beta) =& 2(\frac{1}{\alpha}-\alpha)\left(\mathrm{Li}_2(\frac{\beta}{\bar\alpha}) - 2\mathrm{Li}_2(\alpha) - \mathrm{Li}_2(\beta) - \ln(\alpha)\ln(\bar\alpha)\right) \\
&- \bar\beta\ln(\alpha) - \frac{\bar\alpha}{\alpha}\ln(\bar\alpha) + \frac{\alpha}{\tau}(\tau\ln(\tau) + \bar\tau\ln(\bar\tau))
\end{aligned}
$$

Evaluation of Ward Identities: special conformal transformation

Special conformal Ward Identity

- at critical point: $\boxed{[S_+(a_*), \mathbb{H}(a_*)] = 0}$ $\quad\leftrightarrow\quad [S_0(a_*), S_+(a_*)] = S_+(a_*)$.
- compare in perturbative expansion:

$$[S_+^{(0)}, \mathbb{H}^{(\ell)}] = \sum_{k=1}^{\ell-1} [S_+^{(k)}, \mathbb{H}^{(\ell-k)}]$$

- on r.h.s. appear only operators at order $\leq \ell - 1$.
- we gain one order in perturbation theory
- fixes $\mathbb{H}^{(\ell)}$ only up to "invariant" part $[S_+^{(0)}, \mathbb{H}^{(\ell)}] = 0$: can be reconstructed from anomalous dimensions.

What is this good for?

Applications

- construct full evolution kernel to NNLO accuracy $\mathbb{H}^{(3)}$ (anomalous dimensions known tho three-loop accuracy S. Moch , J.A.M. Vermaseren , A. Vogt: Nucl. Phys. B 688 (2004) 101–134)
- construct nondiagonal elements of anomalous dimension matrix $\hat{\gamma}^{(3)}$ for local operators
- construct transformation to so-called conformal renormalization scheme → determines the NNLO distribution amplitude for mesons.

Thank you for
your attention

Any questions?