Exceptional $F(4)$ Higher-Spin Theory at One-Loop and Other Tests of Duality

Rethinking Quantum Field Theory'2016, DESY

based on arXiv:1608.07582 with M.Gunaydin and T.Tran, see also S.Giombi, I.Klebanov and Z.Tan

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Main Messages

Higher-Spin Theories should be consistent quantum theories that incorporate gravity: want to quantize gravity — add higher spin fields (massive like in strings or massless like in HS theories)

We bridged the gap in SUSY HS: AdS_6 theory based on $F(4)$. The spectrum contains that of the Romans super-gravity.

The complete Lagrangian of HS theories is not known at present, including the $F(4)$. What can still be done? There are some global quantities — one-loop determinants on various backgrounds that require very few ingredients — spectrum! Many of such tests have been already done and we perform more. There is one puzzle left...

From Free CFT's to Higher-Spin Symmetry and Back

Let's take a free field $\Box\phi = 0$, $\partial\!\!\!/\psi = 0$ and others. They are free CFT's. There is an infinite-dimensional symmetry generated by Noether currents:

$$J_s = \phi\partial...\partial\phi \qquad\qquad \partial^m J_{ma...c} = 0$$

Currents, $J \rightarrow$ charges, $Q \rightarrow$ infinite-dimensional, higher-spin symmetry.

The opposite can be proved. Investigating $[Q, J] = J$ or the Jacobi identity $[Q, [Q, Q]] = 0$, (Fradkin, Vasiliev; Anselmi; Maldacena, Zhiboedov; Boulanger, Ponomarev, E.S, Taronna; Alba, Diab; Stanev) leads to

$$\boxed{\text{unbroken HS} = \text{free CFT}}$$

HS Symmetry is broken in $O(N)$ vector models!

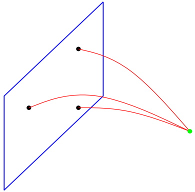
Higher-Spin Duals of Simple CFT's

By AdS/CFT free CFT's should be dual to some theories with massless fields $s = 0, 1, 2, 3, 4, ...$:

$$\partial^m J_{ma(s-1)} = 0 \qquad \Longleftrightarrow \qquad \delta\Phi_{\underline{a}(s)} = \nabla_{\underline{a}}\xi_{\underline{a}(s-1)}$$

The dual HS theory should be highly non-linear:

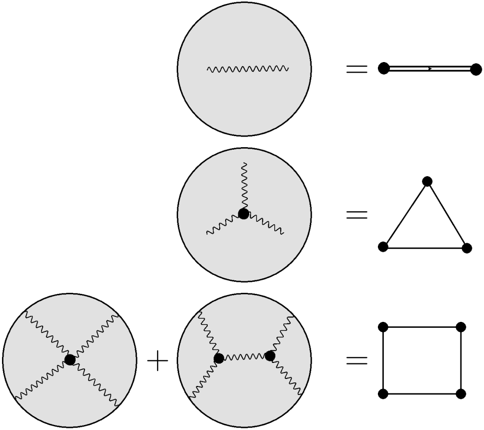
$$\langle J...J\rangle \neq 0 \quad \Longleftrightarrow \quad \int_{AdS} \Phi...\Phi \neq 0$$



While HS theories are generically duals of free CFT's (Sundborg, Sezgin-Sundell, Klebanov-Poyakov), not all of them are boring: quantization of gravity and interacting CFT's for a different choice of b.c. For example, $O(N)$ Wilson-Fisher CFT's ($3d$ Ising, λ-Helium, ...) are dual to HS theory in AdS_4 (Klebanov-Polyakov).

Higher-Spin Theories: Reconstruction

Given some CFT one can compute $\langle J ... J \rangle$ and try to engineer vertices in AdS_{d+1} as to reproduce the correlators:



$$(\Box + m_s^2)^{-1}$$

Bekaert, Erdmenger, Ponomarev, Sleight; Kessel, Lucena−Gomez, E.S., Taronna; E.S.; Sleight, Taronna Metsaev Giombi and Yin

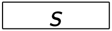
Bekaert, Erdmenger Ponomarev, Sleight

In principle, there are Vasiliev equations known in some of the cases, but the relation is indirect.

Higher-Spin Theories, to be constructed...

Type-A is the dual of the free scalar $\Box\phi = 0$:

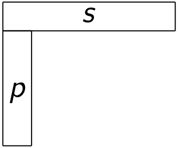
$$\delta\Phi_{\underline{m}(s)} = \nabla_{\underline{m}}\xi_{\underline{m}(s-1)}\,, \qquad s = 0, (1), 2, (3), \ldots$$



There are Vasiliev equations in any D. For another choice of b.c. should be dual to $O(N)$-models (Klebanov, Polyakov; Fei, Giombi, Klebanov).

Type-B is the dual of free fermion $\partial\!\!\!/\psi = 0$ (mixed-symmetry)

$$\Phi_{\underline{m}(s),\underline{a}[p]}\,, \qquad s = 0, (1), 2, (3), \ldots; p = 0, 1, \ldots$$



No Vasiliev equations are known in $D > 4$, but the theory should exists. In $D = 4$ the dual is Gross-Neveu (Leigh, Petkou; Sezgin, Sundell).

SUSY HS Theories

The duals of fermion+boson are SUSY HS theories:

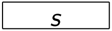
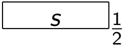
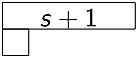
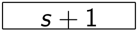
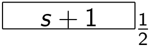
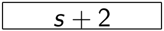
$$\begin{pmatrix} \text{Type-A} \sim \phi\phi & \phi \times \psi \\ \psi \times \phi & \text{Type-B} \sim \psi\psi \end{pmatrix} = \sum \begin{pmatrix} \Phi^{\underline{a}(s)} & \Psi^{\underline{a}(s-\frac{1}{2});\underline{\alpha}} \\ \Psi^{\underline{a}(s-\frac{1}{2});\underline{\alpha}} & \Phi^{\underline{a}(s),\underline{m}[p]} \end{pmatrix}$$

- $\mathcal{N} \leq 8$ restriction does not formally apply to HS theories (Fradkin, Vasiliev);
- HS theories can extend spectrum of SUGRAs with HS fields;
- $\mathcal{N} > 8$ HS theories may not have usual SUSY (higher dimensions);
- AdS/CFT rolls back SUSY HS theories to $\mathcal{N} \leq 8$ via boundary conditions for interacting CFT's.

Exceptional $F(4)$ Theory

The $AdS_{4,5,\not{6},7}$ gauged SUGRAs are covered by $su(n|m)$ and $osp(n|m)$. The gap of AdS_6 was bridged by Romans with the help of $F(4)$. The bosonic part is $so(5,2) \oplus su(2)_R$.

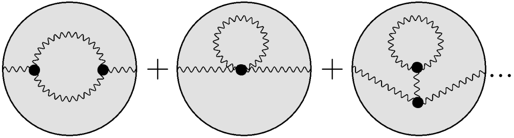
Gunaydin, Sudarshan constructed the $F(4)$ super-singleton, which consists of $2\times$ scalars $\oplus$ fermion. Scalars are in $su(2)_R$ doublet. Romans multiplet is the first in the tensor square:

Scalar tower:	$D(3+s;(s,0)_D)$	
Spinor tower:	$D^r(7/2+s;(s,1)_D)$	
Tensor field tower:	$D(4+s;(s,2)_D)$	
Vector field tower:	$D^a(4+s;(s+1,0)_D)$	
Gravitino tower:	$D^r(9/2+s;(s+1,1)_D)$	
Graviton tower:	$D(5+s;(s+2,0)_D)$	

plus a short $L(8|8)$ multiplet that is found in $5d$ conformal supergravities (Bergshoeff et al).

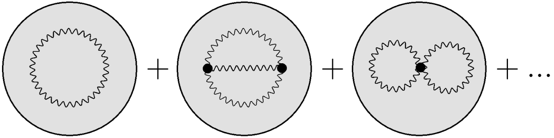
One-Loop

The part of the action known at present does not allow to compute legged diagrams.



Neither do we know the classical action on *AdS*

S_{cl} +



But the one-loop determinant can be computed! (Giombi, Klebanov, Safdi, Tseytlin, Beccaria, Joung, Lal, Bekaert, Boulager, ...)

Free Energy AdS/CFT Expansion

On both sides of the duality we expect

$$F_{AdS} = \frac{1}{G} F^0_{AdS} + F^1_{AdS} + \ldots$$
$$F_{CFT} = N F^0_{CFT} + F^1_{CFT} + \ldots$$

Free Energy AdS/CFT Expansion

On both sides of the duality we expect

$$F_{AdS} = \frac{1}{G} F^0_{AdS} + F^1_{AdS} + \dots$$
$$F_{CFT} = N F^0_{CFT}$$

- On general grounds $G^{-1} \sim N$;
- The first term is not available in AdS;
- In free CFT's the second and others vanish identically;
- Generally we would expect to find boring $0 = 0$

$$F^1_{AdS} \sim \sum_s \log\det[\Box + m_s^2] = 0$$

- On contrary, in many cases $F^1_{AdS} \neq 0$ and is an integer multiplet of F^0_{CFT}, which naturally leads to (Giombi, Klebanov)

$$G^{-1} = a(N + \text{integer})$$

What Determinant Can Tell

One-loop determinant can be computed as

$$F = -\zeta(0)\log \Lambda l - \frac{1}{2}\zeta'(0)$$

Whenever the first term is non-zero, the finite part is ill-defined. Depending on the background, various information can be extracted.

Let's restrict to Euclidian AdS vs. free energy F on a sphere:

$$F^{even} = a \log R \qquad\qquad F^{odd} = \text{number}$$

In AdS there is also a volume divergence:

$$\mathrm{vol}\, AdS_{2n+1} \sim \log R \qquad\qquad \mathrm{vol}\, AdS_{2n+2} \sim \text{const}$$

Sum Rules

Instead of something like supermultiplet sum rules:

$$\sum_s (-)^{2s} d(s) s^p = 0$$

we have infinite sums that may run over bosonic fields only, so no usual SUSY cancellation is possible.

The recipe that was shown to work in many examples is to use (Hurwitz) zeta-function. For example, from Giombi, Klebanov:

$$\frac{1}{360} + \sum_s \left(\frac{1}{180} - \frac{s^2}{24} + \frac{5s^4}{24} \right) = 0$$

Zeta for Romans $F(4)$ Multiplets

SUSY does help in the higher-spin case too. For example, in AdS_6 for bosonic spin-s field we find

$$\zeta(0) = -\frac{(s+1)^2(7s(s+2)(s(s+2)(9s(s+2)+13)+2)-20)}{30240}$$

If we sum over the Romans spin-s $F(4)$ multiplet we find

$$\zeta_{\text{Romans},s}(0) = -\frac{3}{8}s^4$$

The regulated sum is $3/8$, which is then cancelled by the $L(8|8)$ multiplet.

We also showed that fermionic HS fields $s = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, ...$, give exactly zero in all dimensions, which is necessary to make SUSY HS consistent.

a-anomaly, AdS_{2k+1}/CFT^{2k}

a-anomaly can be extracted as $\log R$ term in

$$F_{AdS} = -\zeta(0)\log \Lambda l - \frac{1}{2}\zeta'(0)$$

where $\log R$ comes from volume of AdS_{2k+1}. Giombi, Klebanov, Safdi, Tseytlin, Beccaria showed on many examples that for Type-A (symmetric HS fields)

$$\zeta'_{HS,\text{non-min.}}(0) = 0$$
$$\zeta'_{HS,\text{min.}}(0) = -2a_\phi \log R$$

therefore $G^{-1} = N - 1$. Here a^d_ϕ is the Weyl-anomaly coefficient of the free scalar field in CFT^d:

$$a^4_\phi = \frac{1}{90}\,, \qquad a^6_\phi = -\frac{1}{756}\,, \qquad a^8_\phi = \frac{23}{113400}$$

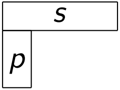
a-anomaly, AdS_{2k+1}/CFT^{2k}

a-anomaly can be extracted as $\log R$ term in

$$F_{AdS} = -\zeta(0)\log \Lambda l - \frac{1}{2}\zeta'(0)$$

where $\log R$ comes from volume of AdS_{2k+1}. We looked at the Type-B that contains mixed-symmetry fields

$$\zeta'_{HS,\text{non-min.}}(0) = 0$$
$$\zeta'_{HS,\text{min.}}(0) = -2a_\psi \log R$$



where a^d_ψ is the Weyl-anomaly coefficient of the free fermion in CFT^d:

$$a^4_\psi = \frac{11}{180}\,, \qquad a^6_\psi = -\frac{191}{7560} \qquad a^8_\psi = \frac{2497}{226800}$$

AdS zeta-function can be used (Giombi, Klebanov, Pufu, Safdi, Tarnopolsky; Beccaria, Tseytlin) to compute Weyl a-anomaly directly from the *AdS* side without having to deal with any infinities: the ratio of partition functions with Dirichlet and Neumann boundary conditions = det boundary operator.

With the help of ζ one can obtain a closed form for

$$a'(\Delta) = \frac{1}{\log R}\frac{1}{2\Delta - d}\frac{\partial}{\partial\Delta}\zeta'_\Delta(0)\,,$$

which is then integrated to a-coefficient

$$a(\Delta) = \frac{1}{\log R}\zeta'_\Delta(0) = \int_{d/2}^{\Delta} dx\,(2x-d)a'(x)\,.$$

$\zeta(z)$ is more complicated. In addition $\zeta(0)$ does not vanish for each field, only for specific spectra. Giombi, Klebanov, Safdi showed on many examples

$$\zeta_{HS,non-min}(0) = 0 \qquad \zeta_{HS,min.}(0) = 0$$
$$\zeta'_{HS,non-min}(0) = 0 \qquad \zeta'_{HS,min.}(0) = F^{\phi}_{d}$$

where F is sphere free energy of one scalar field

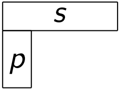
$$F^3_\phi = \frac{1}{16}(2\log 2 - \frac{3\zeta(3)}{\pi^2})$$
$$F^5_\phi = \frac{-1}{2^8}(2\log 2 + \frac{2\zeta(3)}{\pi^2} - \frac{15\zeta(5)}{\pi^4})$$

F-energy, Type-B, AdS_{2k+2}/CFT^{2k+1}

The Type-B theory that is dual to the singlet sector of $\partial\!\!\!/\psi = 0$ leads to some puzzles. For example,

$$AdS_4: \quad -\frac{1}{2}\zeta'(0) = -\frac{\zeta(3)}{8\pi^2}$$

$$AdS_6: \quad -\frac{1}{2}\zeta'(0) = -\frac{\pi^2\zeta(3)+3\zeta(5)}{96\pi^4}$$



These numbers look a bit different from F-energy of free fermion field

$$F^3_\psi = \frac{1}{16}\left(2\log 2 + \frac{3\zeta(3)}{\pi^2}\right)$$

$$F^5_\psi = \frac{-1}{2^8}\left(6\log 2 + \frac{10\zeta(3)}{\pi^2} + \frac{15\zeta(5)}{\pi^4}\right)$$

but they are not random — many strange numbers like Gleisher-Kinkelin constant are gone.

The Type-B theory that is dual to the singlet sector of $\partial\!\!\!/\psi=0$ leads to some puzzles. The change in F due to double-trace deformation (Giombi, Klebanov):

$$\delta F^{\psi}_{\Delta}=\frac{2}{\Gamma(d+1)}\int_0^{\Delta-d/2}\cos(\pi u)\Gamma\left(\frac{d+1}{2}+u\right)\Gamma\left(\frac{d+1}{2}-u\right)du\,.$$

The free energy can computed as

$$F^{\psi}_{d}=(-)^{\frac{d-1}{2}}\delta F^{\psi}_{\Delta=\frac{d-1}{2}}\,.$$

The numbers that resulted from the tedious computations in AdS_{2n+2} arrange themselves into:

$$-\frac{1}{2}\zeta'_{HS}(0)=-\frac{1}{4}\delta F^{\psi}_{\Delta=\frac{d-2}{2}}$$

The same as in Giombi, Klebanov, Tan.

Conclusions

The square of the $F(4)$ supersingleton gives the spectrum of a SUSY higher-spin theory in AdS_6 with the lowest multiplet corresponding to the Romans one. This bridges the gap of AdS_6 in SUSY HS.

Despite the lack of information about the action of higher-spin theories some quantum tests of the HS AdS/CFT duality can be done. Many HS spectra pass the tests. SUSY HS is safe.

Mixed-symmetry fields of Type-B that is dual to free fermion do pass the tests in AdS_{2n+1}/CFT^{2n}, but the F-energy is naively different for AdS_{2n+2}/CFT^{2n+1}, but the numbers are not random.

Using AdS/CFT one can compute a-anomaly for any CFT field.