

# Differential Higgs Boson Pair Production at Next-to-Next-to-Leading Order in QCD



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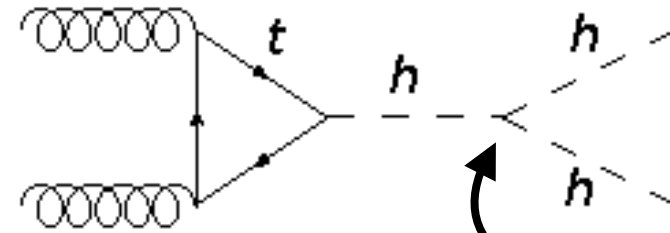
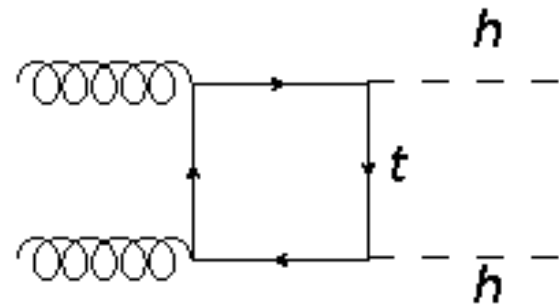
*work in collaboration with:*

Daniel de Florian, Massimiliano Grazzini, Catalin Hanga, Stefan Kallweit,  
Philipp Maierhöfer, Javier Mazzitelli, Dirk Rathlev  
*[arXiv:1606.09519, to appear in JHEP]*

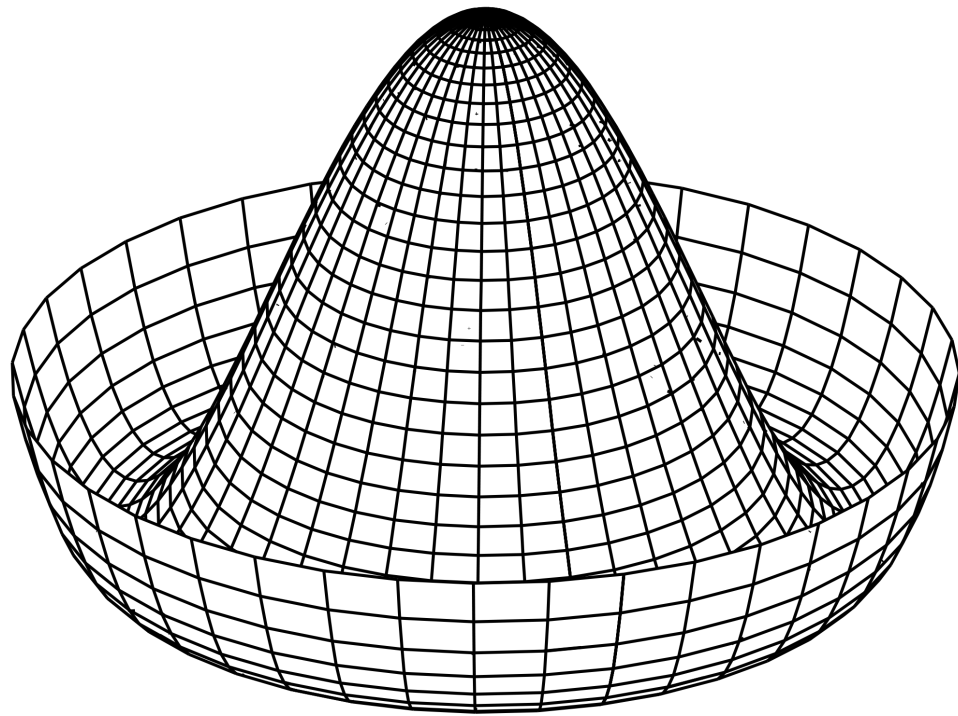
DESY Theory Workshop  
DESY, Hamburg, 28th September 2016

# Introduction

- $pp \rightarrow HH$  offers direct access to the trilinear Higgs coupling

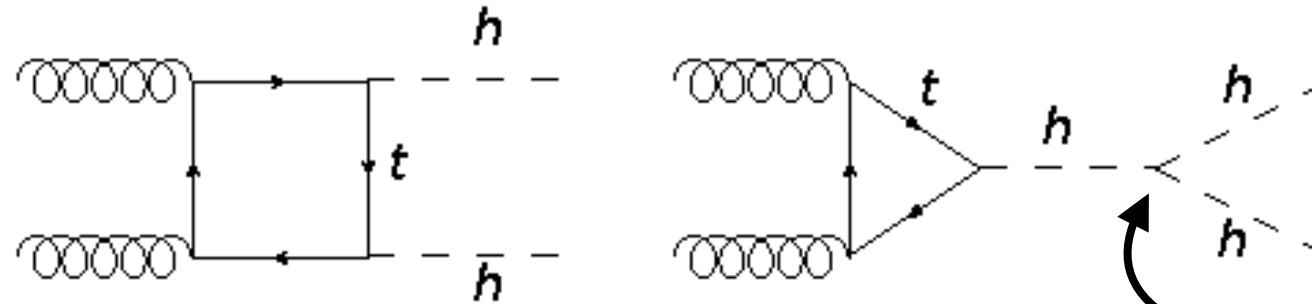


$$V(H) = \frac{1}{2}M_H^2 H^2 + \lambda v H^3 + \frac{1}{4}\lambda' H^4$$



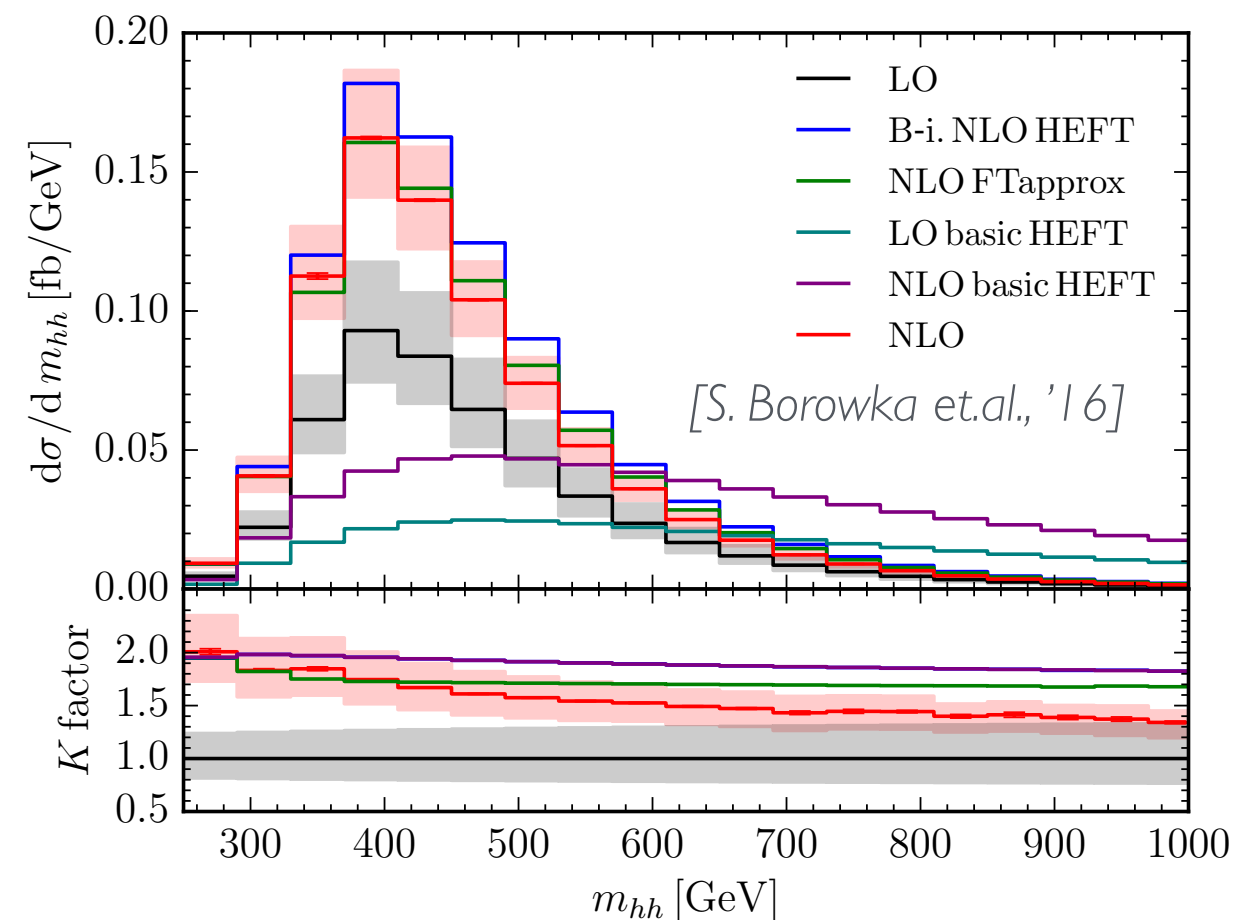
# Introduction

- $pp \rightarrow HH$  offers direct access to the trilinear Higgs coupling

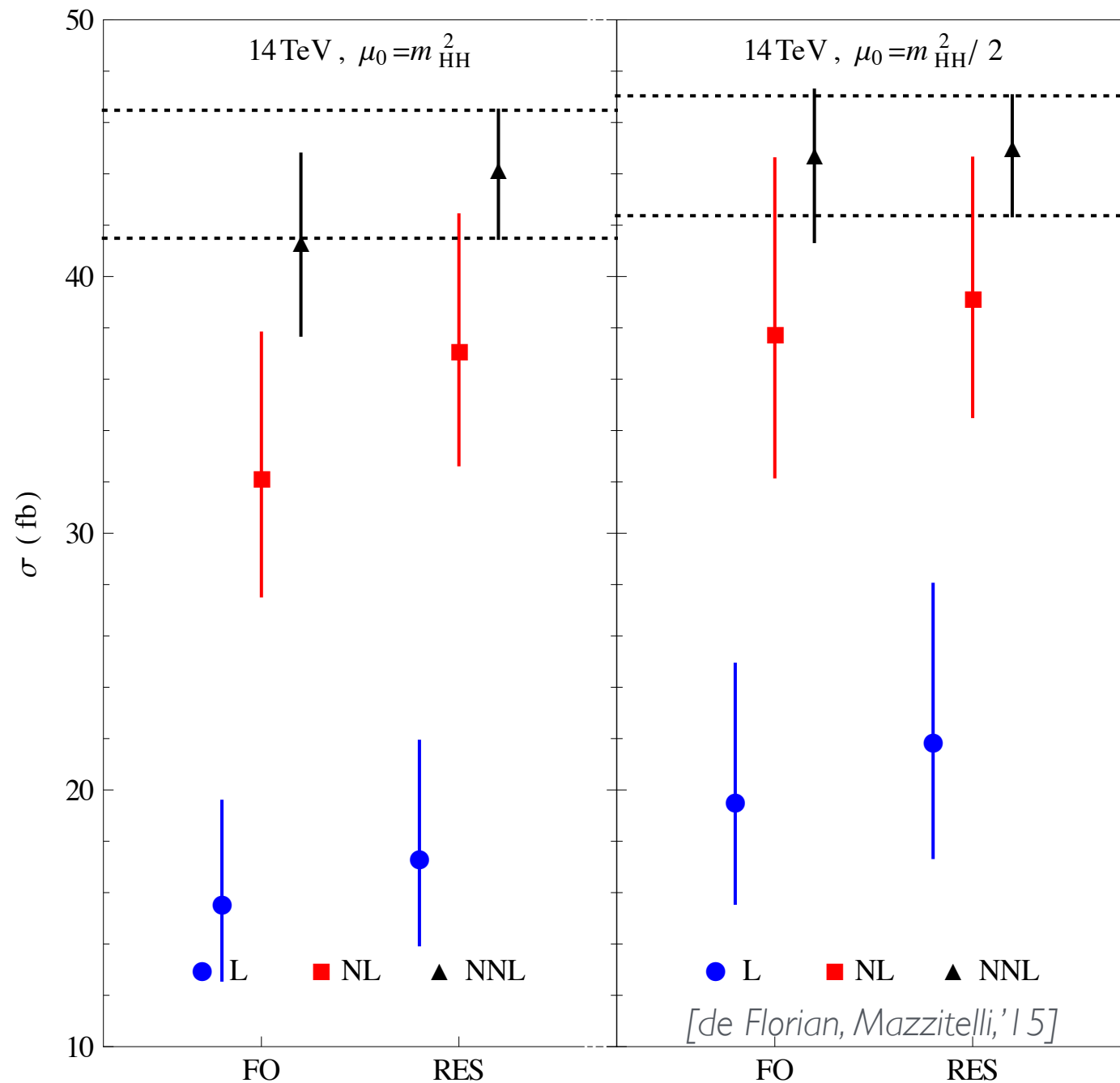


$$V(H) = \frac{1}{2} M_H^2 H^2 + \lambda v H^3 + \frac{1}{4} \lambda' H^4$$

- Loop-induced process with 4 mass scales  
→ higher orders extremely complicated!
- Full NLO result available since recently  
*[S. Borowka et.al., 2x '16]*
- based on purely numerical evaluation of the required two-loop amplitudes
- Indicates breakdown of HEFT  
(Higgs Effective Field Theory)
- NLO corrections are huge (~90%)  
& full results beyond NLO out of scope.  
→ how to go beyond NLO?



# Inclusive NNLO in HEFT



- Inclusive NNLO:  $\sigma_{\text{HEFT}}^{\text{NNLO}} / \sigma_{\text{HEFT}}^{\text{NLO}} \approx 20\%$   
[de Florian, Mazzitelli, '13, '13]

- $\sigma_{\text{HEFT}}^{\text{NNLO}} \approx \sigma_{\text{HEFT}}^{\text{NNLL}}$  for  $\mu_0 = m_{HH}/2$   
[de Florian, Mazzitelli, '15]

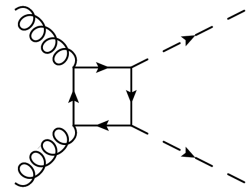
→ fully differential NNLO



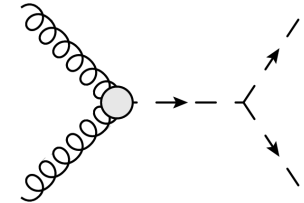
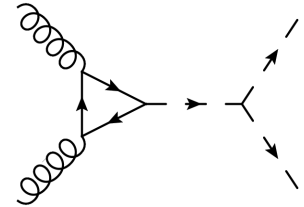
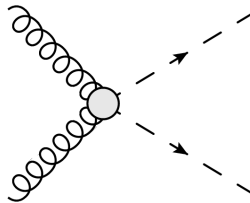
# HH up to NNLO in HEFT

[Djouadi, Spira, Zerwas, '91; Dawson, Dittmaier, Spira, '98]

LO



LO HEFT



Integrate out tops

$$\mathcal{L}_{\text{HEFT}} = -\frac{1}{4}G_{\mu\nu}G^{\mu\nu} \left( C_H \frac{H}{v} - C_{HH} \frac{H^2}{v^2} \right)$$

$$C_X = -\frac{\alpha_S}{3\pi} \sum_{n \geq 0} C_X^{(n)} \left( \frac{\alpha_S}{\pi} \right)^n, \text{ with } X = H, HH$$

Matching coefficients  
known up to NNLO

[Grigo, Melnikov, Steinhauser, '14]

# HH up to NNLO in HEFT

[Djouadi, Spira, Zerwas, '91; Dawson, Dittmaier, Spira, '98]

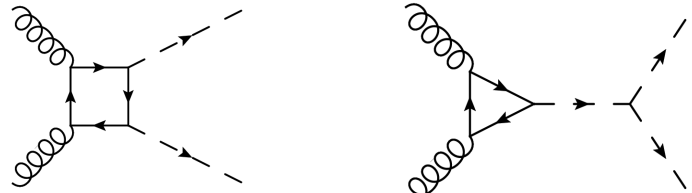
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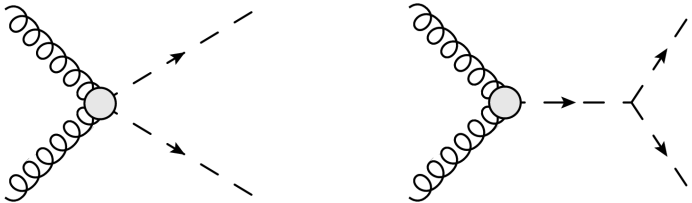
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LO

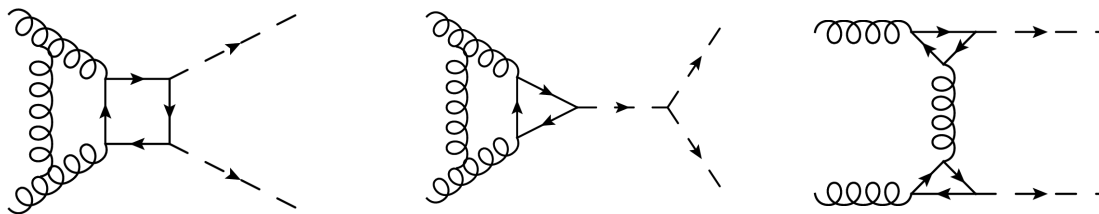


LO HEFT

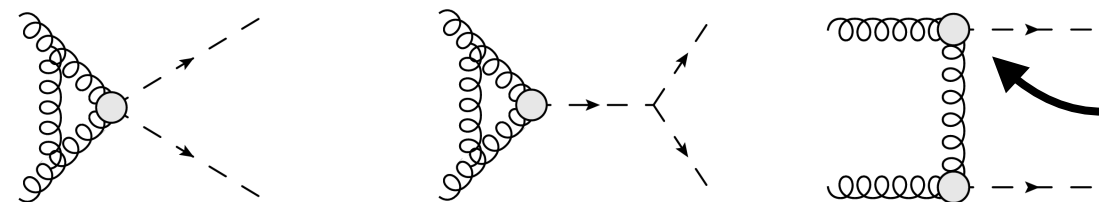


Integrate out tops

NLO



NLO HEFT



tree-level double operator insertion

# HH up to NNLO in HEFT

[Djouadi, Spira, Zerwas, '91; Dawson, Dittmaier, Spira, '98]

$$\mathcal{L}_{\text{HEFT}} = -\frac{1}{4}G_{\mu\nu}G^{\mu\nu} \left( C_H \frac{H}{v} - C_{HH} \frac{H^2}{v^2} \right)$$

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Matching coefficients  
known up to NNLO

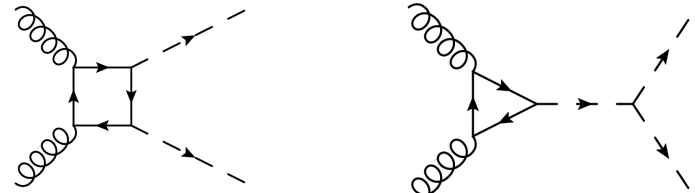
[Grigo, Melnikov, Steinhauser, '14]

Integrate out tops

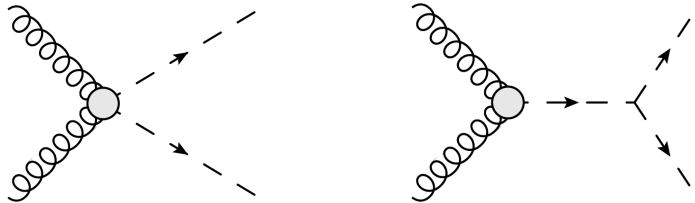
tree-level double operator insertion

one-loop double operator insertion

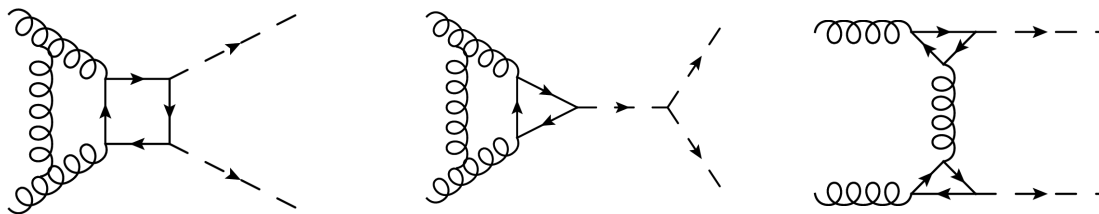
LO



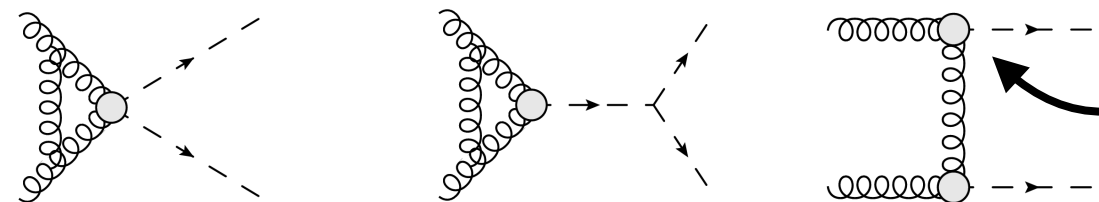
LO HEFT



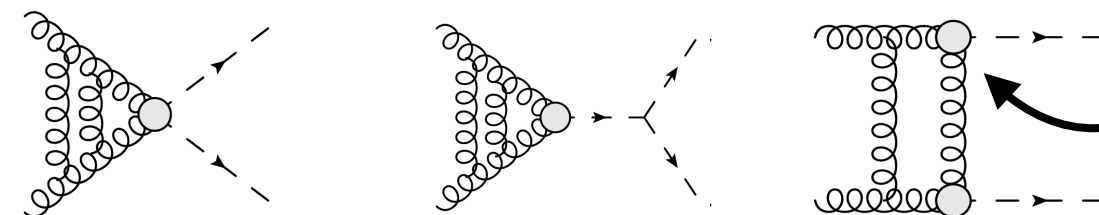
NLO



NLO HEFT



NNLO HEFT



# HH two-loop virtual

- easily fits on one slide (finite part):

$$\frac{d\sigma_{\text{fin}}^{(2)}}{dt} = F_{\text{LO}} \{ |C_{\text{LO}}|^2 \mathcal{F}^{(2)} + \text{Re}(C_{\text{LO}}) \mathcal{R}^{(2)} + \text{Im}(C_{\text{LO}}) \mathcal{I}^{(2)} + \mathcal{V}^{(2)} + \mathcal{O}(\epsilon) \}$$

$$C_{\text{LO}} = \frac{6 \lambda v^2}{s - M_H^2 + i M_H \Gamma_H} - 1$$

$$\mathcal{V}^{(2)} = \frac{1}{(3stu)^2} [M_H^8(t+u)^2 - 2M_H^4 tu(t+u)^2 + t^2 u^2 (4s^2 + (t+u)^2)] ,$$

$$\mathcal{I}^{(2)} = 4\pi \left( 1 + \frac{2M_H^4}{s^2} \right) \log \left( \frac{(M_H^2 - t)(M_H^2 - u)}{t u} \right) ,$$

$$\begin{aligned} \mathcal{F}^{(2)} = & \left( \frac{8N_f}{3} + \frac{19}{2} \right) \log \left( \frac{s}{M_t^2} \right) + N_f \left( \frac{217\zeta_2}{12} - \frac{17\zeta_3}{6} - \frac{3239}{108} \right) \\ & - \frac{11\zeta_2 N_f^2}{18} - \frac{249\zeta_2}{2} - \frac{253\zeta_3}{4} + \frac{45\zeta_4}{8} + \frac{8971}{36} , \end{aligned}$$

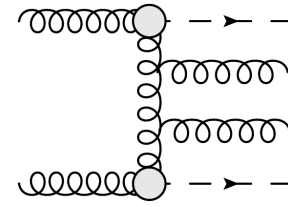
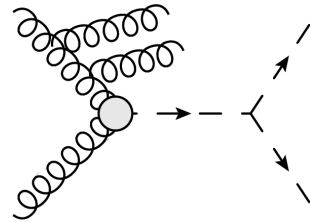
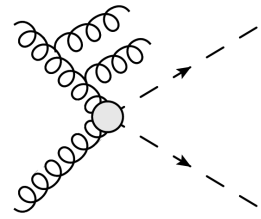
$$\begin{aligned} \mathcal{R}^{(2)} = & - \left( 1 + \frac{2M_H^4}{s^2} \right) \left\{ -\frac{24}{3}\zeta_2 + 2\text{Li}_2 \left( 1 - \frac{M_H^4}{t u} \right) + 4\text{Li}_2 \left( \frac{M_H^2}{t} \right) + 4\text{Li}_2 \left( \frac{M_H^2}{u} \right) \right. \\ & + 4 \log \left( 1 - \frac{M_H^2}{t} \right) \log \left( -\frac{M_H^2}{t} \right) + 4 \log \left( 1 - \frac{M_H^2}{u} \right) \log \left( -\frac{M_H^2}{u} \right) - \log^2 \left( \frac{t}{u} \right) \Big\} \\ & + \frac{4M_H^2}{s} + \frac{314}{9} - \frac{20}{27}N_f - \frac{33 - 2N_f}{9} \log \left( \frac{t u}{s^2} \right) + 8(C_H^{(2)} - C_{HH}^{(2)}) . \end{aligned}$$

[de Florian, Mazzitelli, '13]

[Grigo, Melnikov, Steinhauser, '14]

# HH real-virtual and real-real

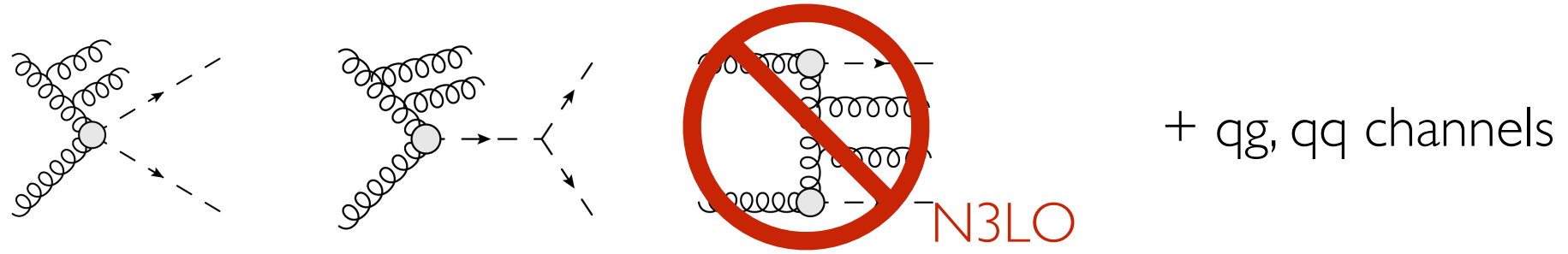
real-real



+ qg, qq channels

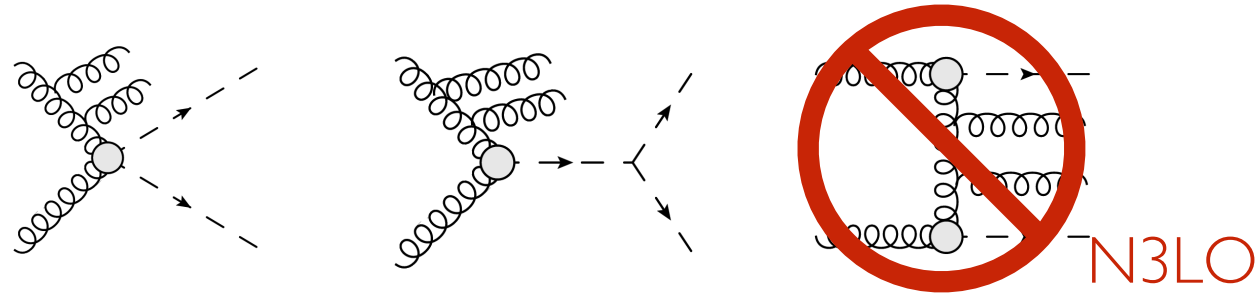
# HH real-virtual and real-real

real-real



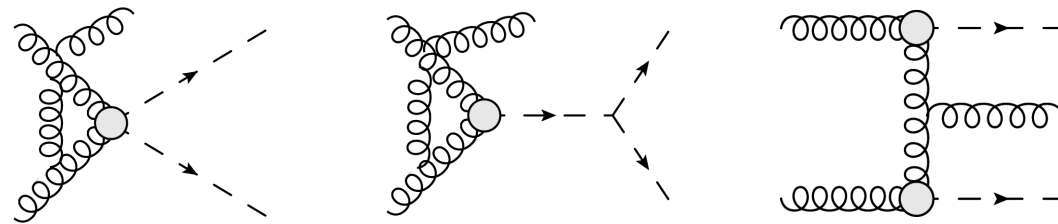
# HH real-virtual and real-real

real-real



+ qg, qq channels

real-virtual



+ qg, qq channels

► obtained from **OpenLoops** [JML, Maierhöfer, Pozzorini]

► provides very fast and stable tree & one-loop amplitudes for

$$\begin{aligned} &pp \rightarrow H, \quad pp \rightarrow H+j, \quad pp \rightarrow H+jj, \quad pp \rightarrow H+jjj \\ &pp \rightarrow HH, \quad pp \rightarrow HH+j, \quad pp \rightarrow HH+jj \end{aligned}$$



# qT Subtraction

[Catani, Grazzini, '12]

[Catani, Cieri, de Florian, Ferrera, Grazzini, '14]

[Gehrmann, Lübbert, Yang, '14]



$$d\sigma_{\text{NNLO}}^{HH} = \mathcal{H}_{\text{NNLO}}^{HH} \otimes d\sigma_{\text{LO}}^{HH} + \left[ d\sigma_{\text{NLO}}^{HH+\text{jet}} - d\sigma_{\text{NNLO}}^{\text{CT}} \right]$$

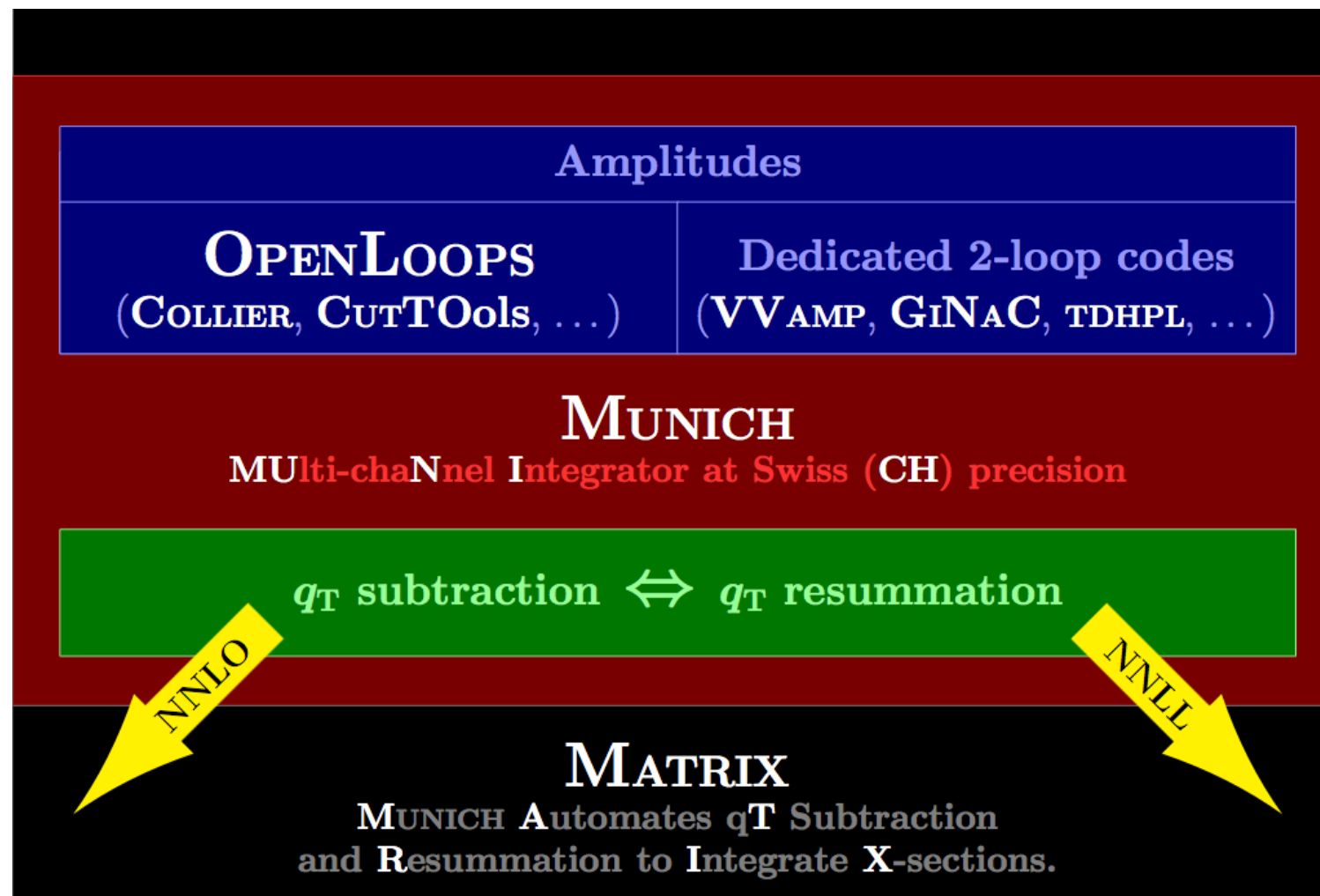
hard-collinear coefficient  
includes two-loop virtual

real-virtual + real-real

process-independent counterterm

IR singular in the limit  $q_{T,HH} \rightarrow 0$ , but sum is finite!

# MATRIX Framework



Available processes in MATRIX @ NNLO(+NNLL) QCD:

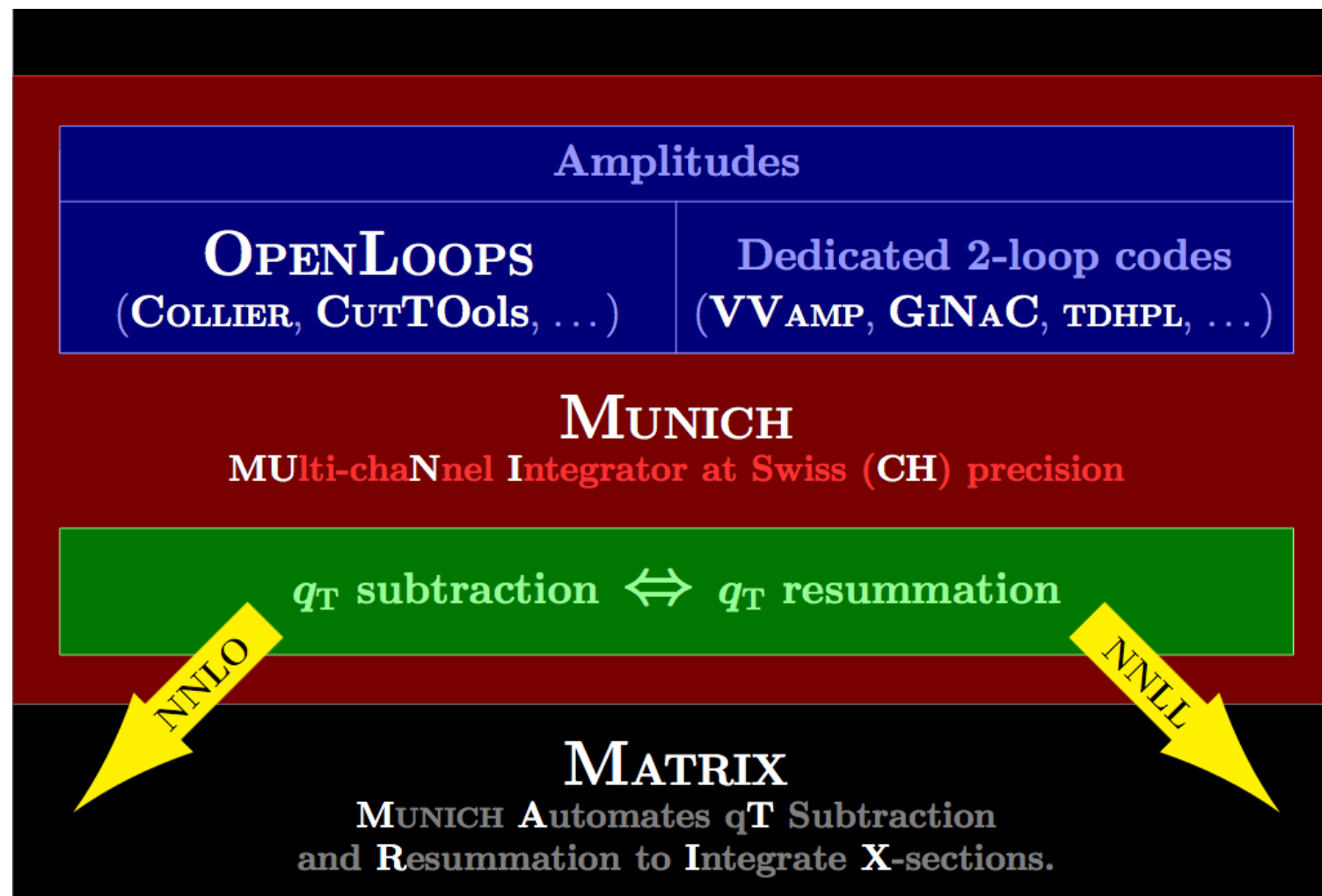
**$pp \rightarrow V$** ,  $V=W,Z$  and  $pp \rightarrow ll$ ,  $pp \rightarrow lv$

**$pp \rightarrow VV'$** ,  $V=W,Z,\gamma$  and  $pp \rightarrow 4l$ ,  $pp \rightarrow 2l2v$ ,  $pp \rightarrow 2l\gamma$ ,  $pp \rightarrow lv\gamma$

**$pp \rightarrow H$**

**$pp \rightarrow HH$**

# MATRIX Framework



Available processes in MATRIX @ NNLO QCD

$pp \rightarrow V$ ,  $V=W, Z$  and  $pp \rightarrow VV$

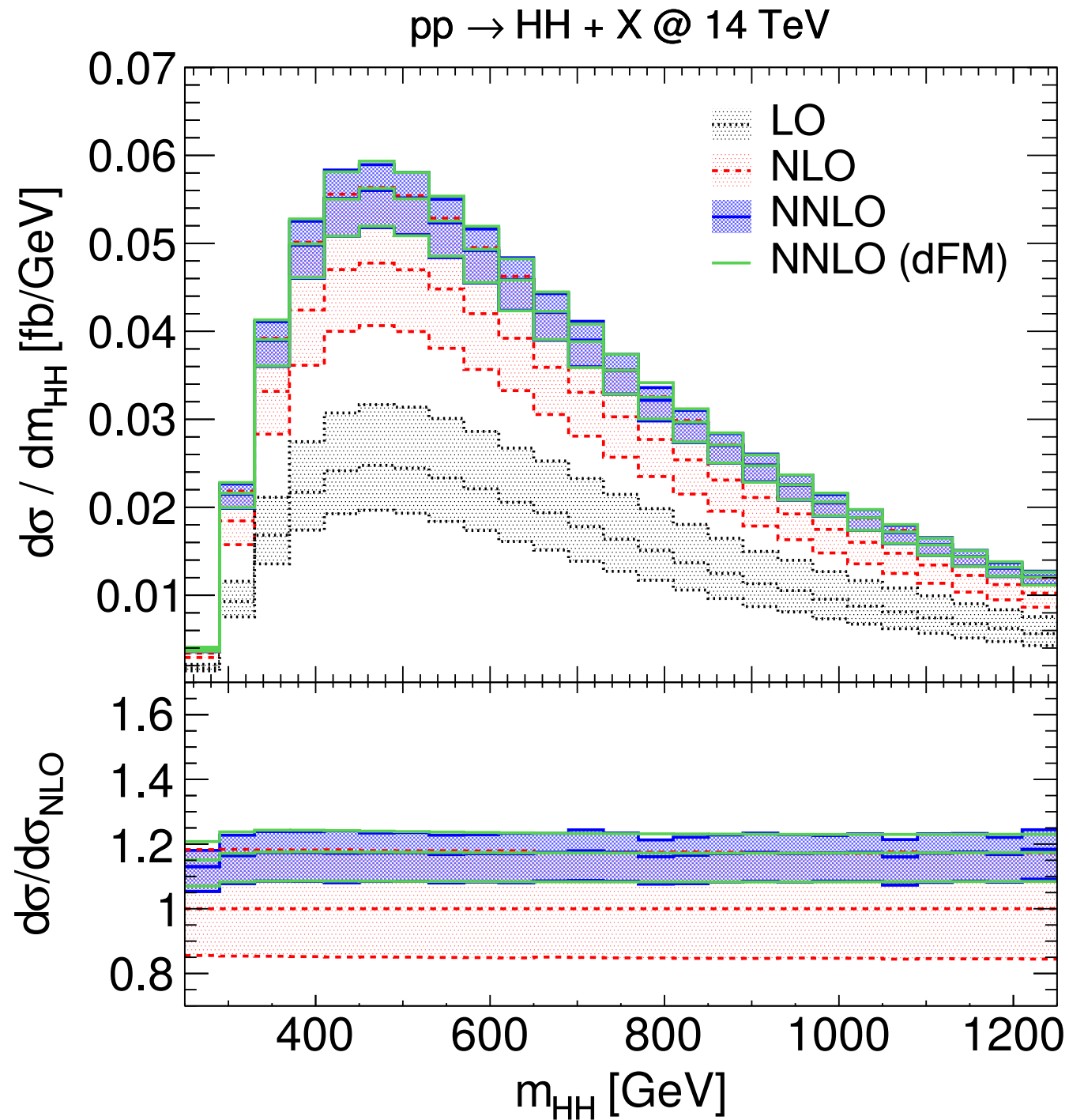
$pp \rightarrow VV$

$pp \rightarrow 2l2\nu$ ,  $pp \rightarrow 2l\gamma$ ,  $pp \rightarrow lv\gamma$

all single- and di-boson processes of the SM (✓)

$pp \rightarrow HH$

# Results: LHC 14 TeV



$\sqrt{s}$ [TeV]	$\sigma_{LO}$ [fb]	$\sigma_{NLO}$ [fb]	$\sigma_{NNLO}$ [fb]
13	13.8059(13) $^{+31.5\%}_{-22.5\%}$	25.829(3) $^{+17.8\%}_{-15.4\%}$	30.38(3) $^{+5.2\%}_{-7.7\%}$
14	17.0778(16) $^{+30.7\%}_{-22.1\%}$	31.934(3) $^{+17.5\%}_{-15.1\%}$	37.52(4) $^{+5.2\%}_{-7.6\%}$

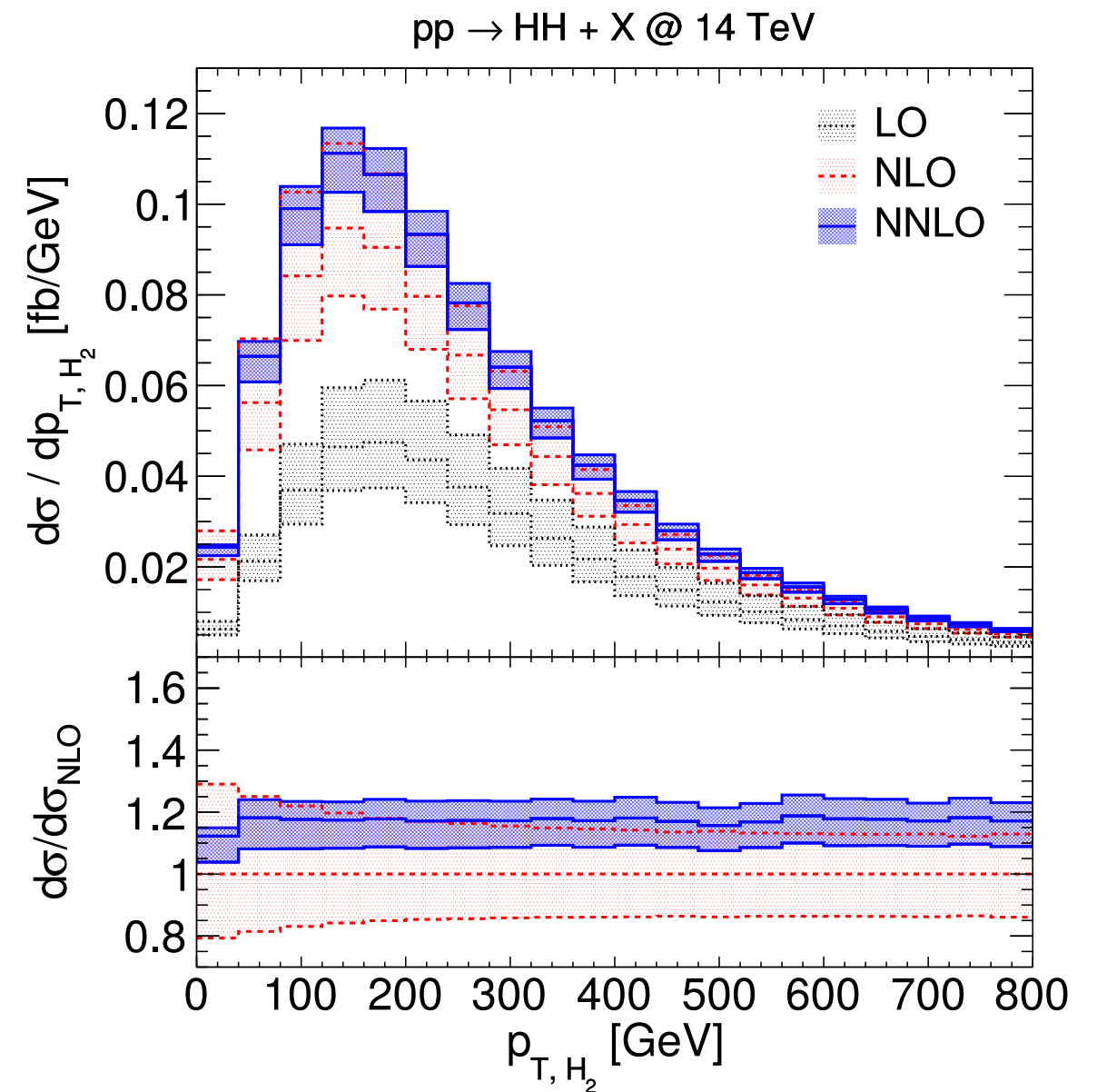
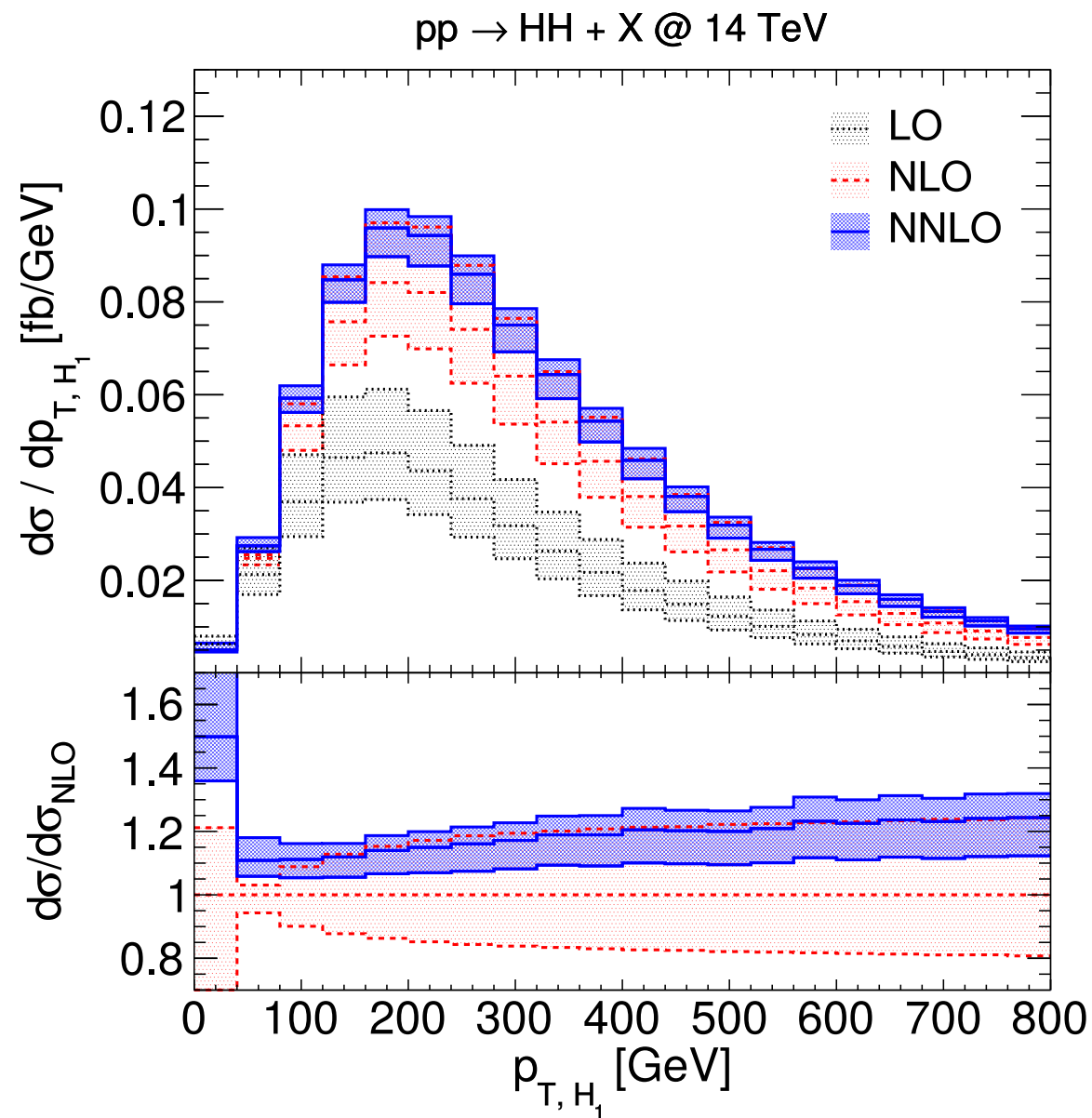
87 %

18 %

- NNLO corrections:
  - almost flat in  $m_{HH}$
  - at the level of 18%
  - scale uncertainties at the level of 10%
  - overlap with NLO uncertainty band
- perfect agreement with analytical result of [de Florian-Mazzitelli, '13]

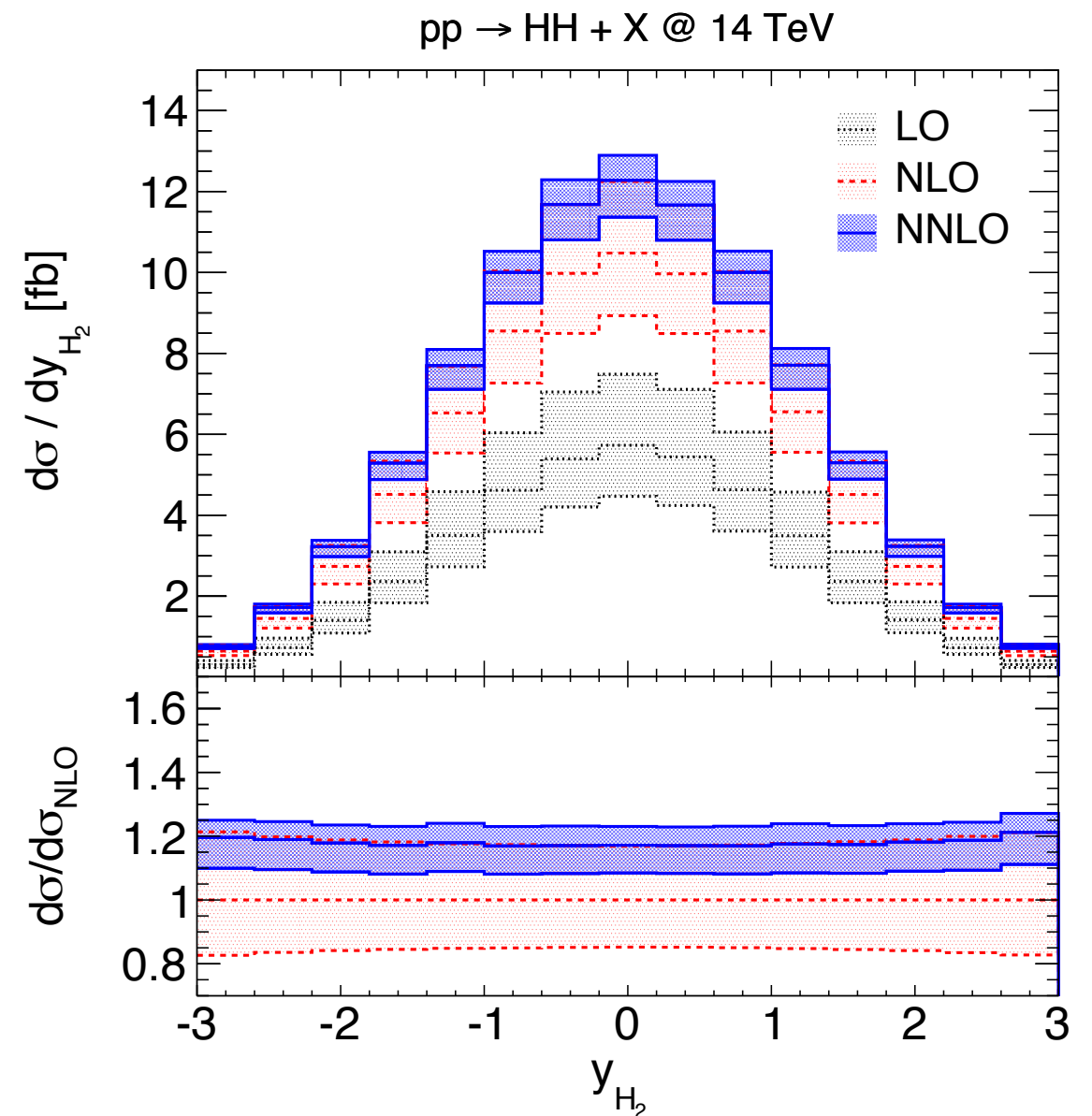
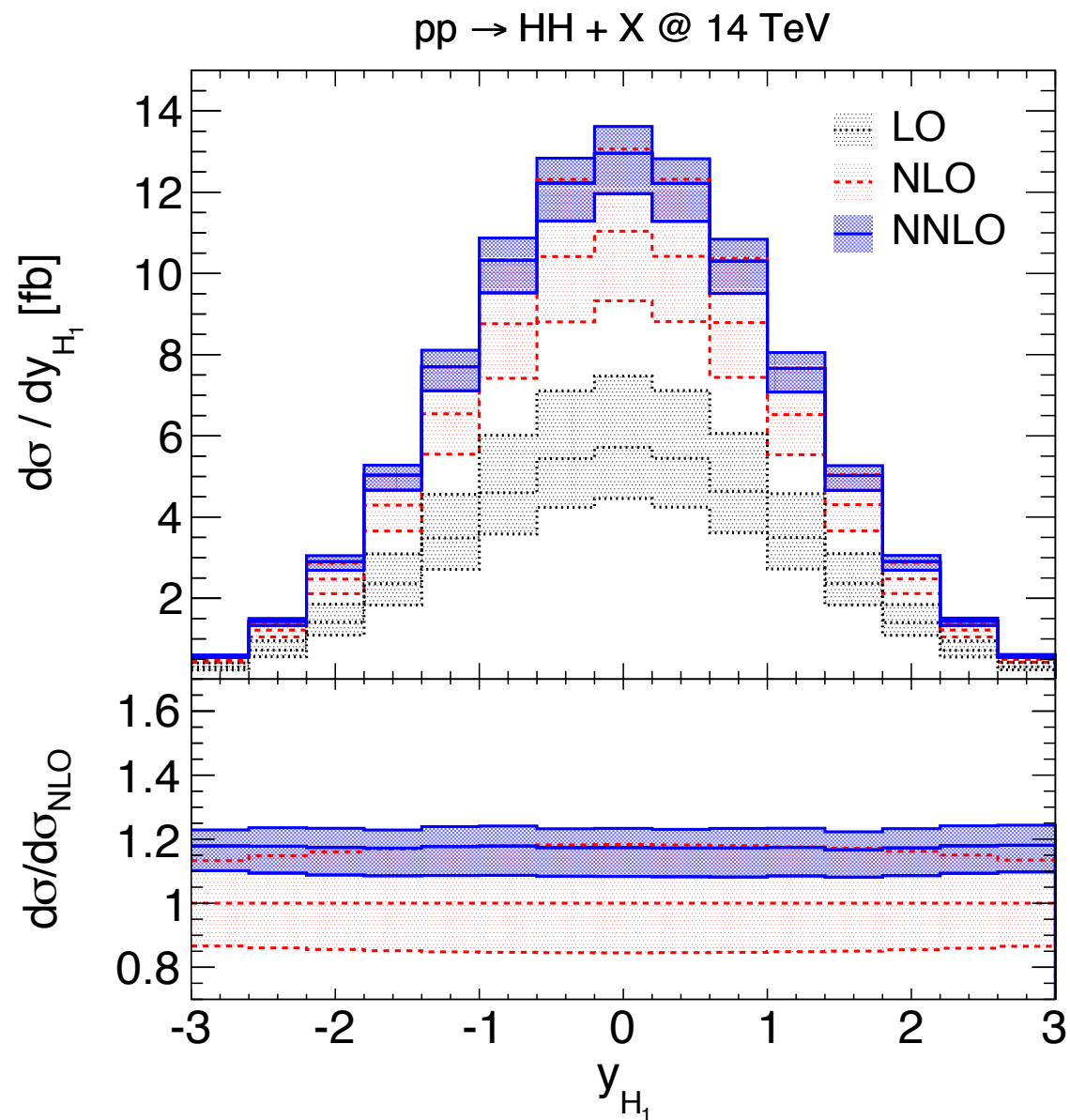
Setup:  $\mu_0 = m_{HH}/2 + 7$  pt. variation, PDF4LHC\_nlo for NLO & LO / PDF4LHC\_nnlo for NNLO,  $m_H = 125$  GeV

# Results: LHC 14 TeV



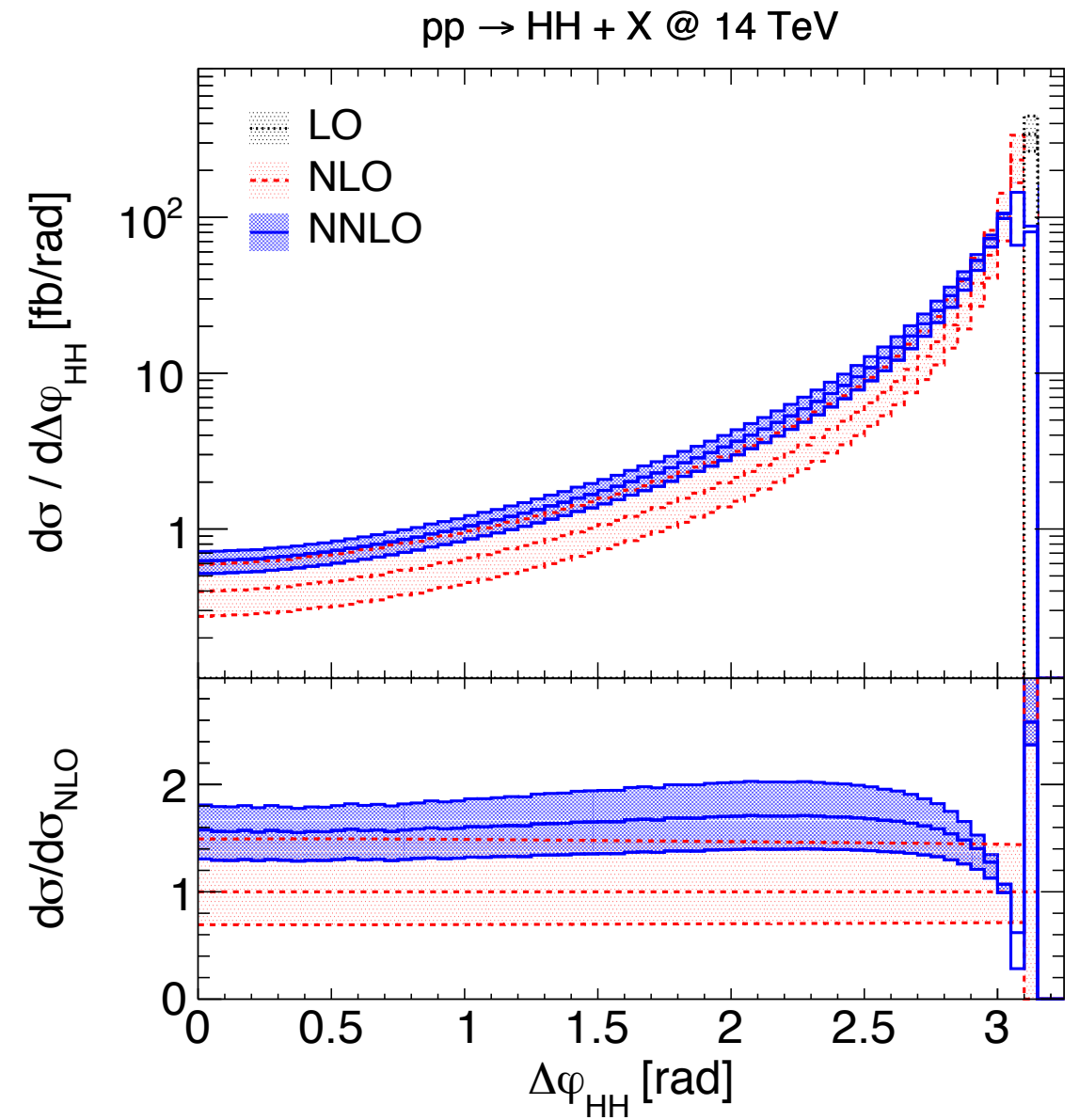
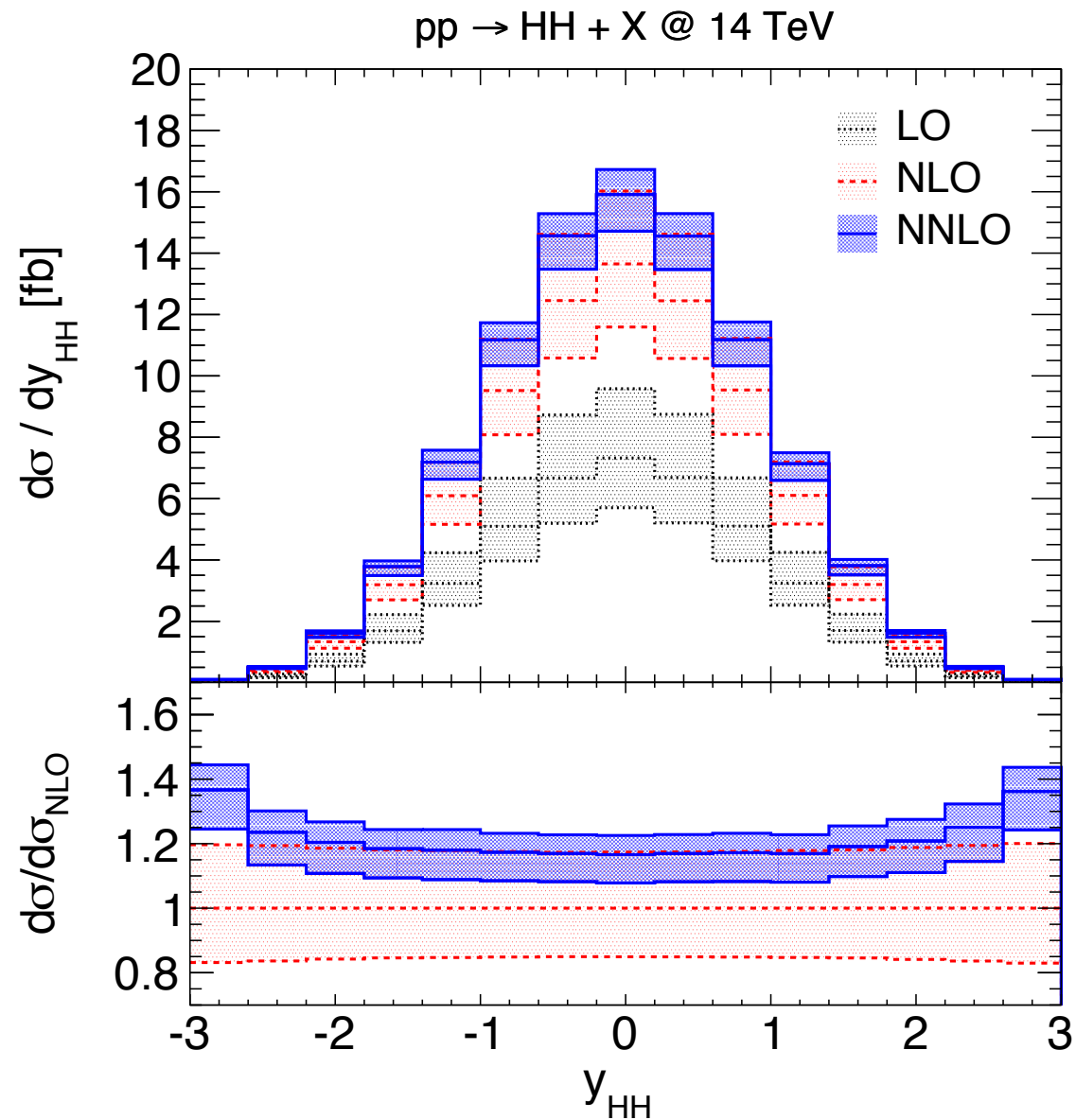
- mild increase of NNLO/NLO for large  $p_{T,H1}$  : 10% – 25%
- flat NNLO/NLO for large  $p_{T,H2}$
- scale uncertainties 5-10%

# Results: LHC | 4 TeV



► almost flat NNLO/NLO

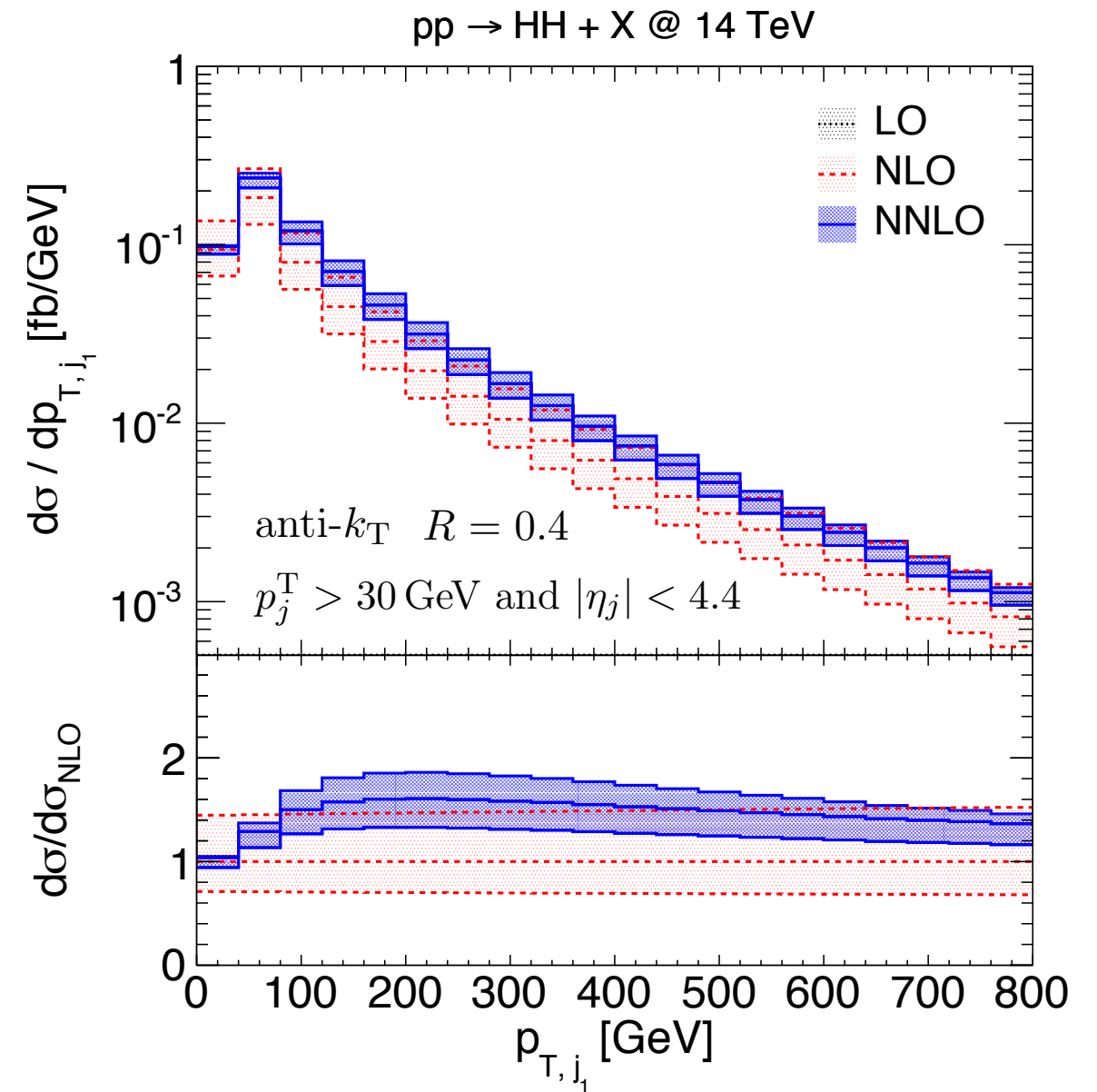
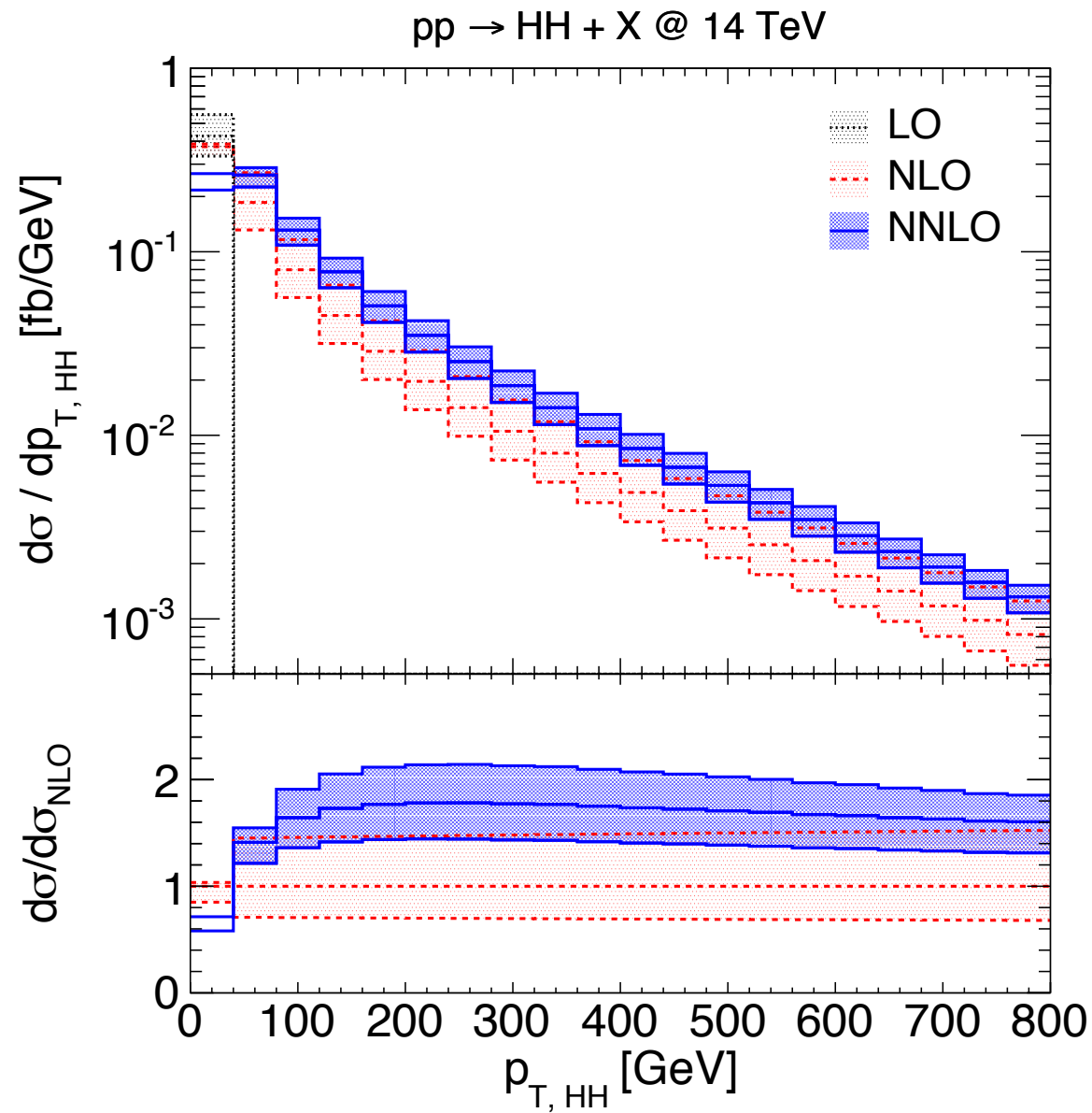
# Results: LHC 14 TeV



- ▶ NNLO/NLO increases to 40% for large  $y_{HH}$
- ▶ at LO HH back-to-back → NNLO  $\Delta\phi_{HH}$  effectively NLO: 60-80% corrections

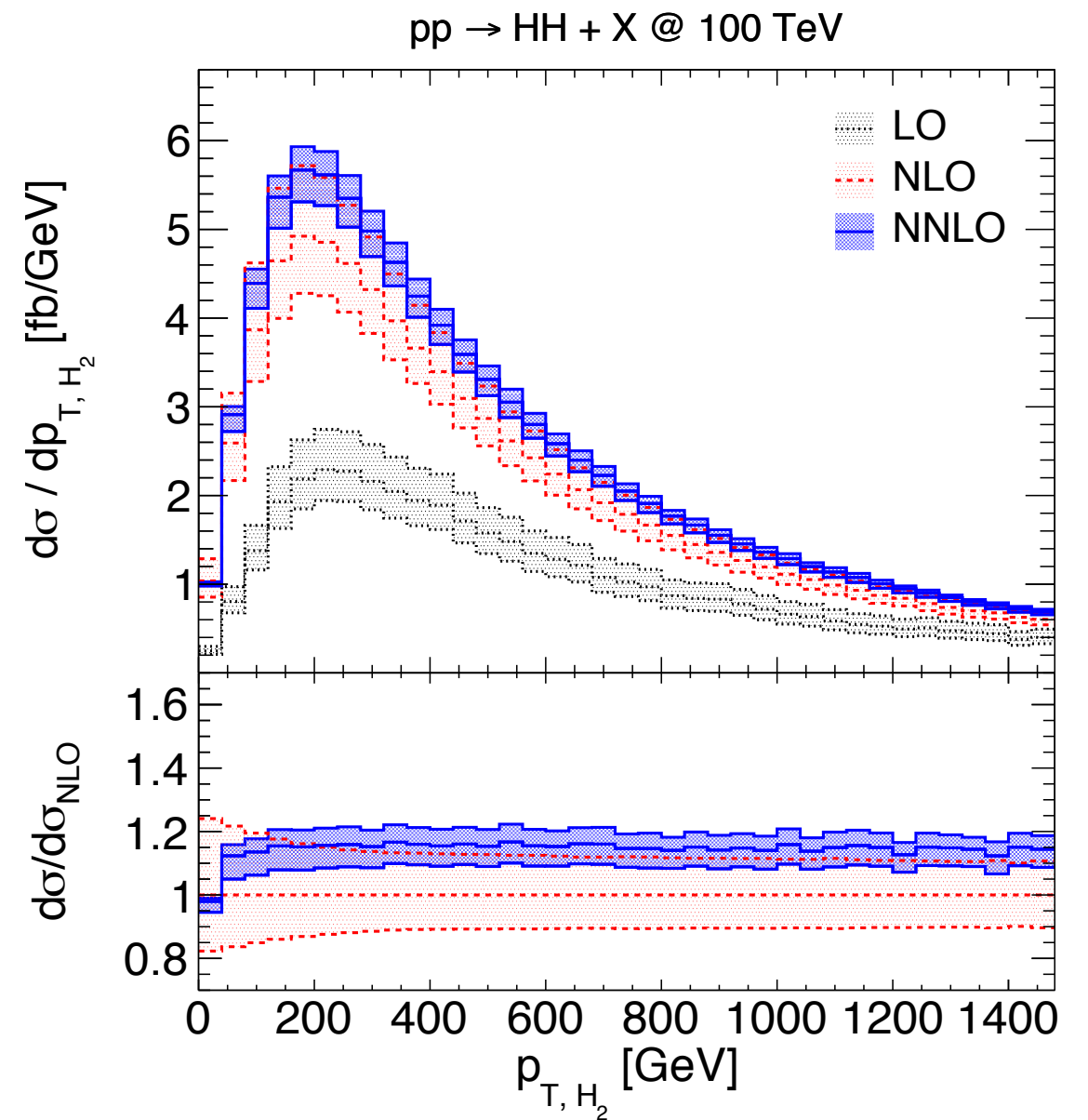
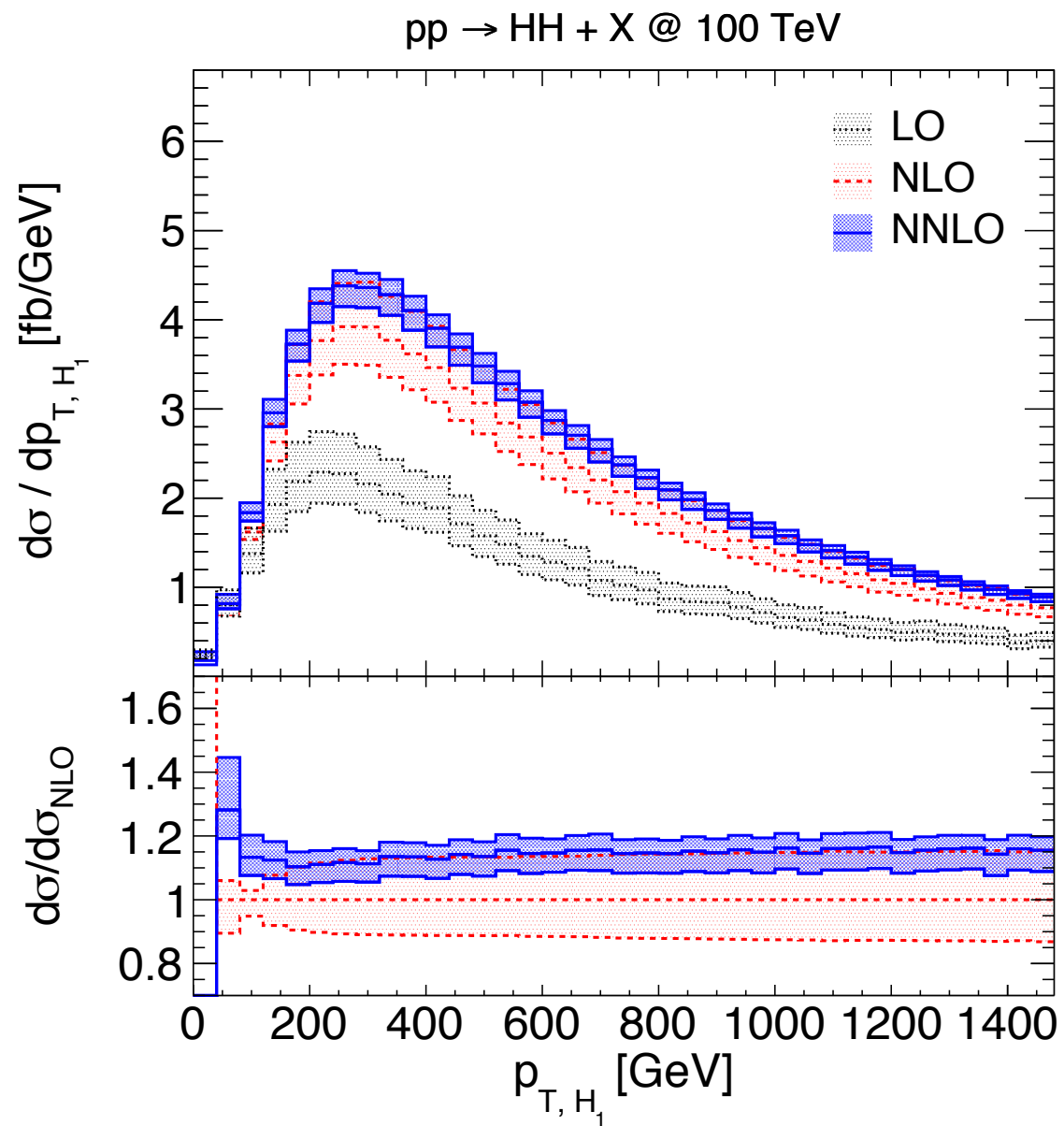


# Results: LHC 14 TeV



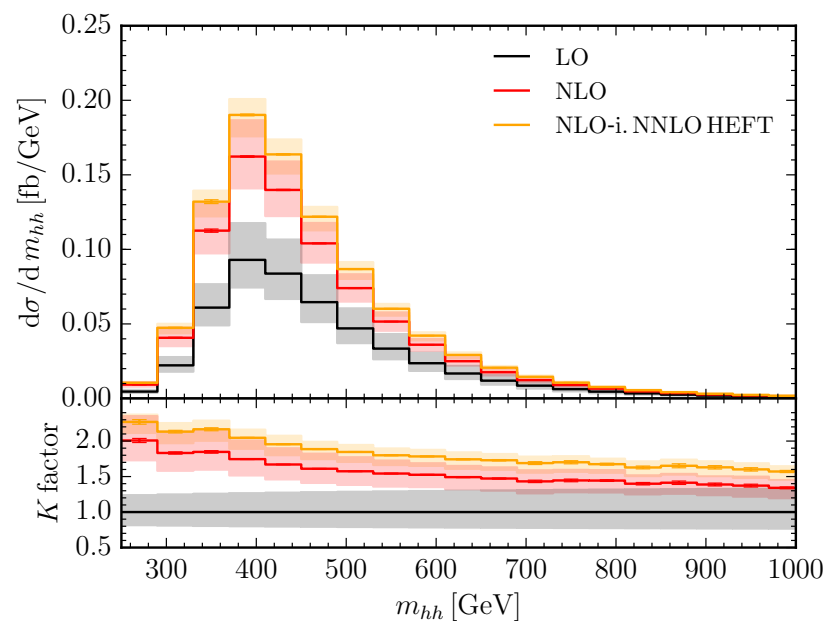
- ▶ effectively NLO: 60-80% NNLO/NLO corrections
- ▶ in the limit  $p_{T,HH} \rightarrow 0$  large log terms  $\log^n(p_{T,HH}/m_{HH}) \rightarrow$  resum

# Results: FCC 100 TeV

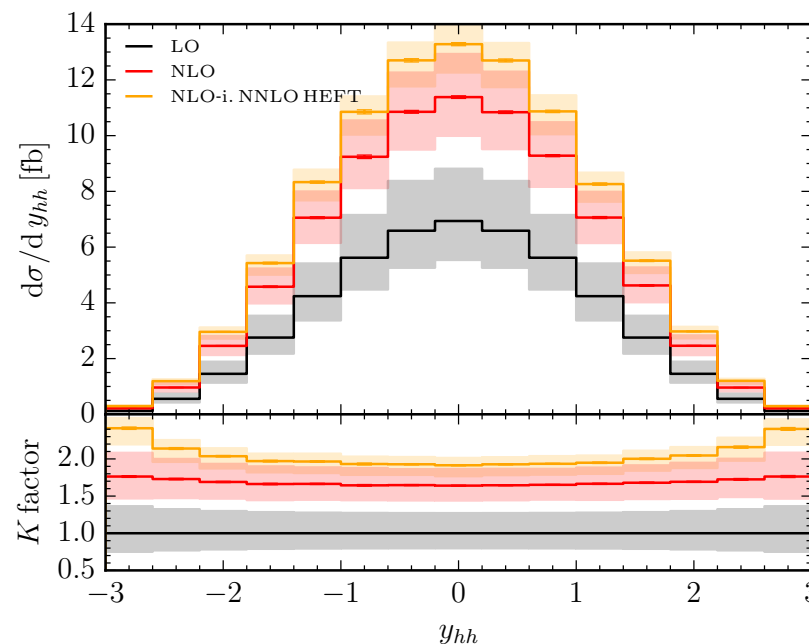


► similar behaviour of NNLO/NLO for  $p_{T,H_1}$  and  $p_{T,H_2}$  as at 14 TeV

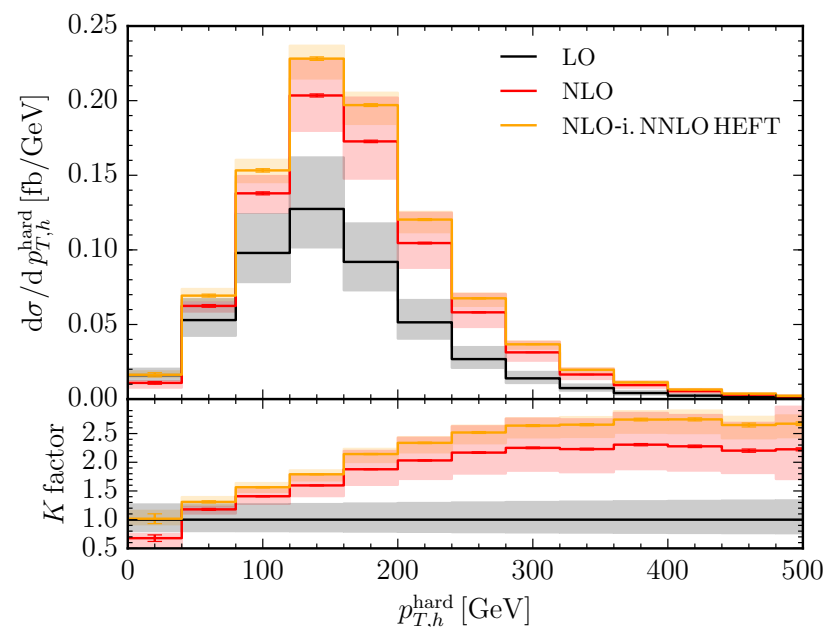
# NLO-improved NNLO HEFT



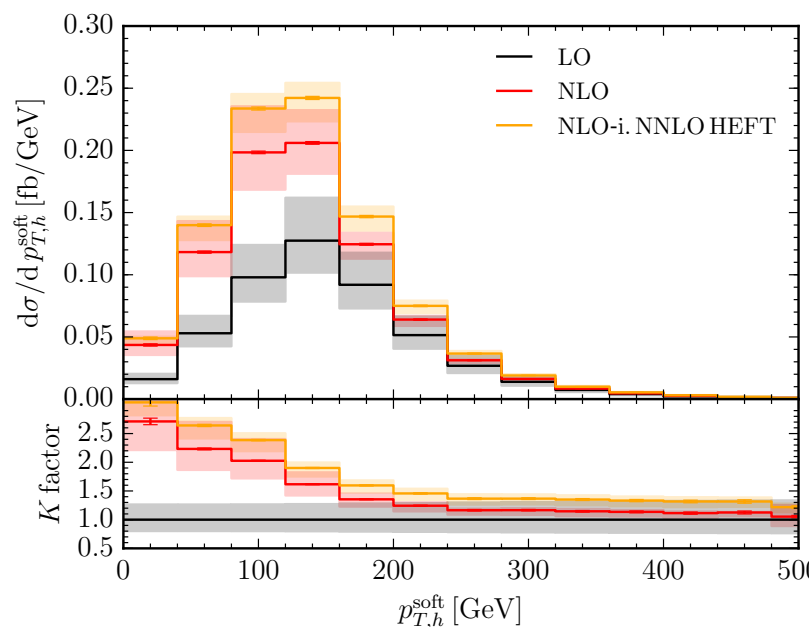
(a) 14 TeV,  $m_{hh}$



(b) 14 TeV,  $y_{hh}$



(c) 14 TeV, leading  $p_T$



(d) 14 TeV, subleading  $p_T$

[S. Borowka et.al., arXiv:1608.04798]

First steps combining full NLO and NNLO HEFT have been presented recently in

[S. Borowka et.al., arXiv:1608.04798]

Idea: apply differential NNLO/NLO K-factor to full NLO calculation

$$d\sigma^{\text{NLO-i. NNLO HEFT}} = d\sigma^{\text{NLO}} \frac{d\sigma^{\text{NNLO basic HEFT}}}{d\sigma^{\text{NLO basic HEFT}}}$$

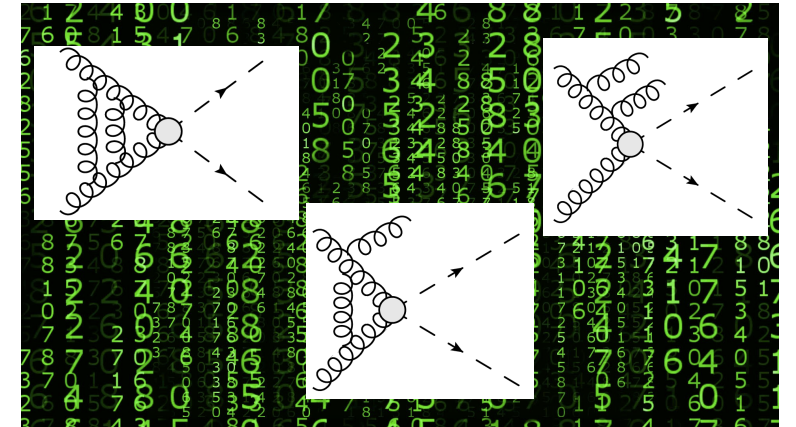
$$\sigma^{\text{NLO-i. NNLO HEFT}} = 38.67^{+5.2\%}_{-7.6\%} \text{ fb @ 14 TeV}$$

$$\sigma^{\text{B-i. NLO HEFT}} = 38.32^{+18.1\%}_{-14.9\%} \text{ fb @ 14 TeV}$$

Further refinements possible:

- fully-differential: projection to Born phase-space,...
- ....

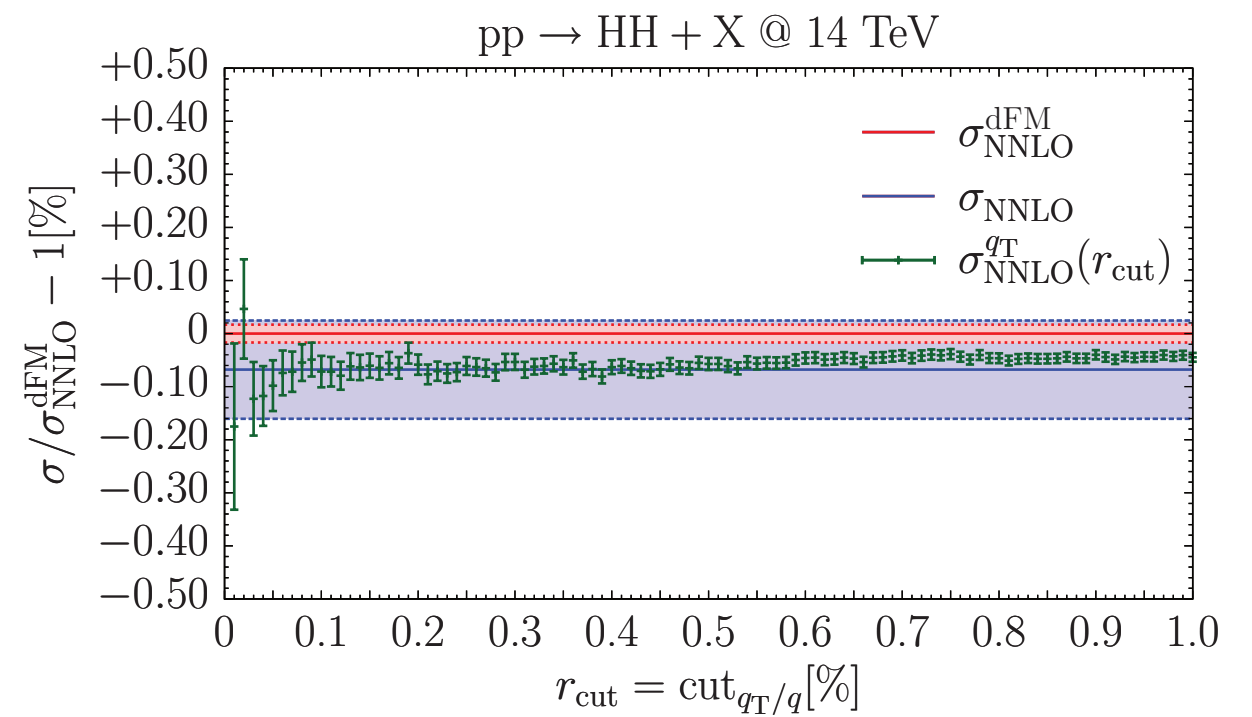
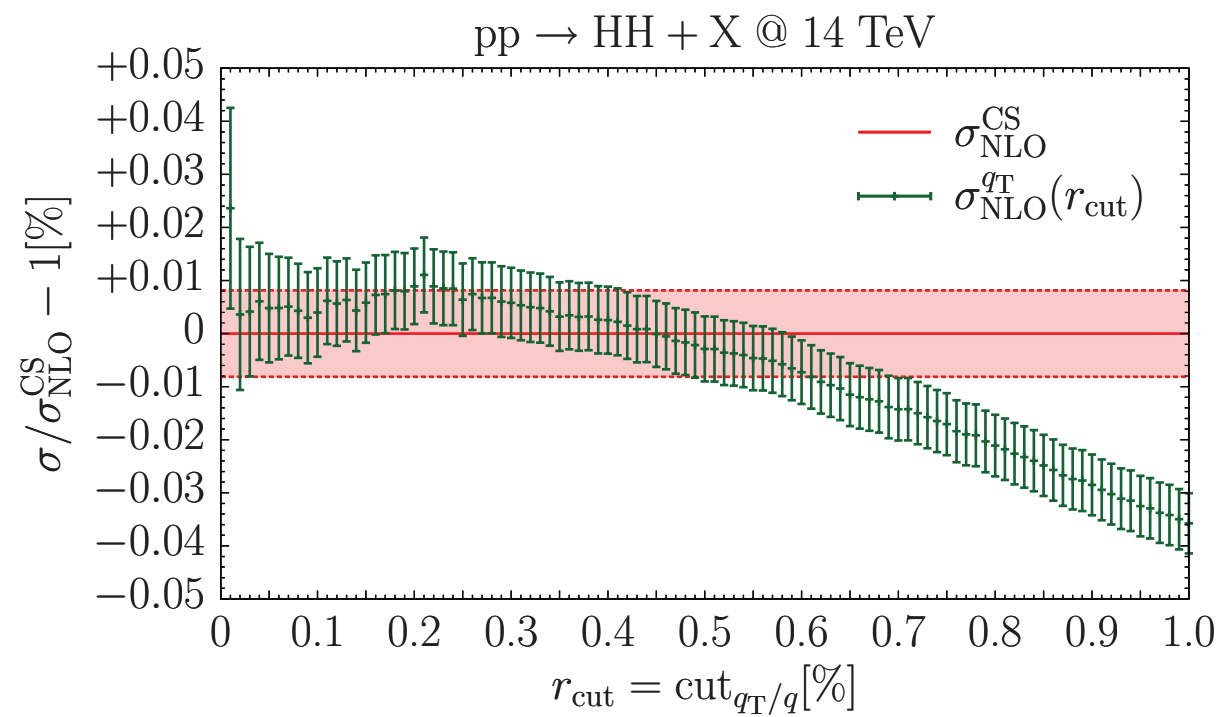
# Conclusions



- ▶ HH @ NNLO in HEFT available in the MATRIX
- ▶ mild phase space dependence of NNLO corrections
- ▶ NNLO/NLO at the level of 10-25%
- ▶ scale uncertainties at NNLO at the level of 5-15%
- ▶ mostly overlapping uncertainty bands between NNLO and NLO
- ▶ Outlook:
  - Higgs decays
  - NNLO+NNLL
  - Refine combination with full NLO

# Backup slides

# Validation & Stability



# Results: LHC 14 TeV

