The Higgs boson as a five-dimensional gauge field

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Motivation

Gauge-Higgs Unification

- Origin of the potential responsible for the Brout-Englert-Higgs mechanism is unknown.
- In Gauge-Higgs Unification (GHU) models the Higgs field is associated with some extra-dimensional components of a gauge field [Manton, 1979].
- In the case of one extra dimension the Higgs potential is zero at tree level and is generated only through quantum effects.
- At 1-loop, Higgs mass is finite. Fermions are necessary to obtain a symmetry breaking minimum through the Hosotani mechanism [Hosotani, 1983].
- What happens non-perturbatively? $\longrightarrow$ lattice study.

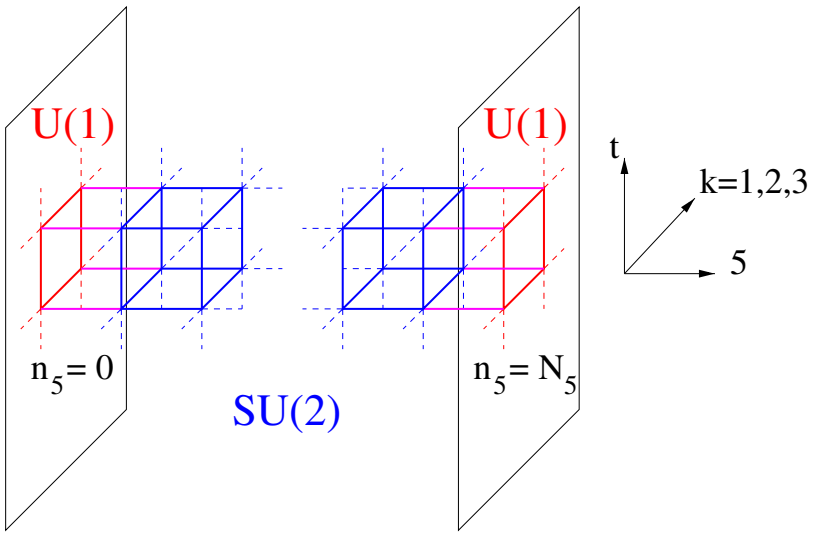
Lattice Formulation

Wilson Gauge action for bulk gauge group $\mathrm{SU}(2)$:

$$S_W^{orb} = \frac{\beta_4}{2} \sum_{P_4} w \cdot \mathrm{tr}\left\{1 - P_4\right\} + \frac{\beta_5}{2} \sum_{P_5} \mathrm{tr}\left\{1 - P_5\right\}$$

Boundary links satisfy $gUg^{-1} = U$ with $g = -i\sigma^3$

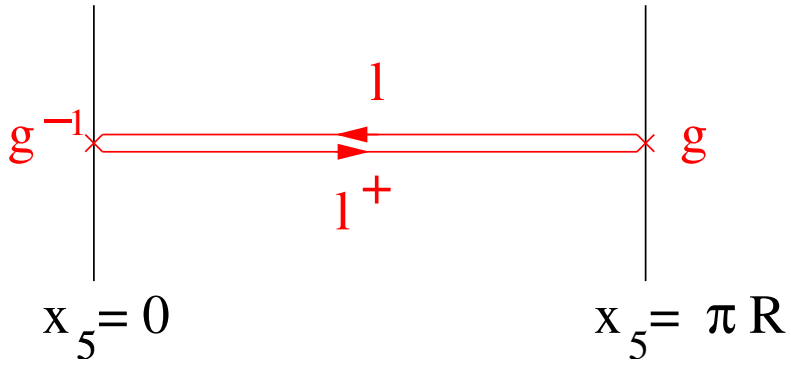
$$w = \begin{cases} \frac{1}{2} & \text{plaquette on boundary} \\ 1 & \text{otherwise} \end{cases}$$



5-d Orbifold

- bare anisotropy is $\gamma = \sqrt{\beta_5/\beta_4} \simeq a_4/a_5$
- N_5 is number of *links* in the fifth dimension
- $N_5 a_5 = \pi R$, R radius of extra dimension.

Lattice operators: Higgs



Scalar Polyakov loop (defined at $n = (n_\mu, 0)$)

$$p(n) \;=\; l(n)\; g\; l^\dagger(n)\; g^\dagger$$

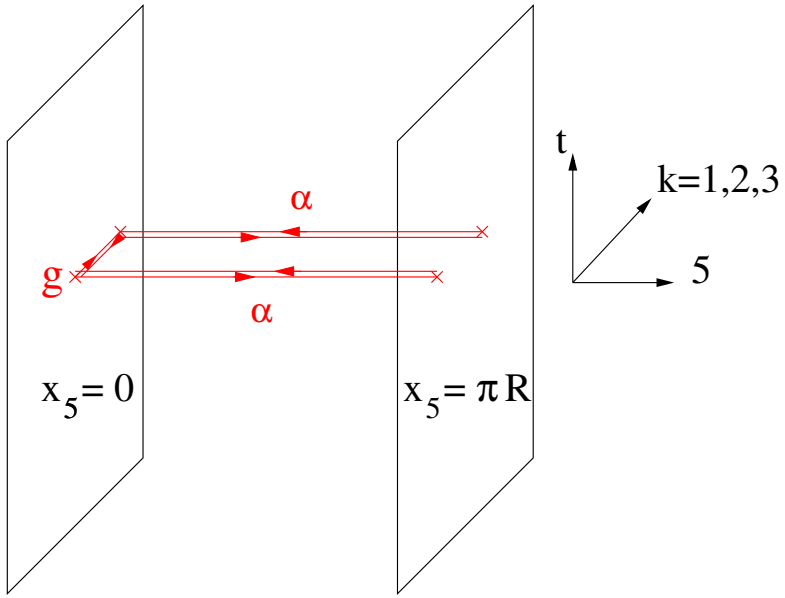
Higgs field

$$h(n) \;=\; [p(n) - p^\dagger(n), g]/(4N_5) \;\sim\; A_5^1\sigma^1 + A_5^2\sigma^2$$

Higgs operators

$$\mathcal{H}(n_0) = \sum_{n_1,n_2,n_3} \mathrm{tr}\,[hh^\dagger] \quad , \quad \mathcal{P}(n_0) = \sum_{n_1,n_2,n_3} \mathrm{tr}\,[p]$$

Lattice operators: Gauge boson



Gauge boson operators (defined at $n = (n_\mu, 0)$)

$$\mathcal{Z}(n_0, k) = \sum_{n_1,n_2,n_3} \mathrm{tr}\left[g\, U(n,k)\, \alpha(n + a_4\hat{k})\, U^\dagger(n,k)\, \alpha(n)\right]$$

$$\mathcal{Z}'(n_0, k) = \sum_{n_1,n_2,n_3} \mathrm{tr}\left[g\, U(n,k)\, l(n + a_4\hat{k})\, U^\dagger(n',k)\, l^\dagger(n)\right]$$

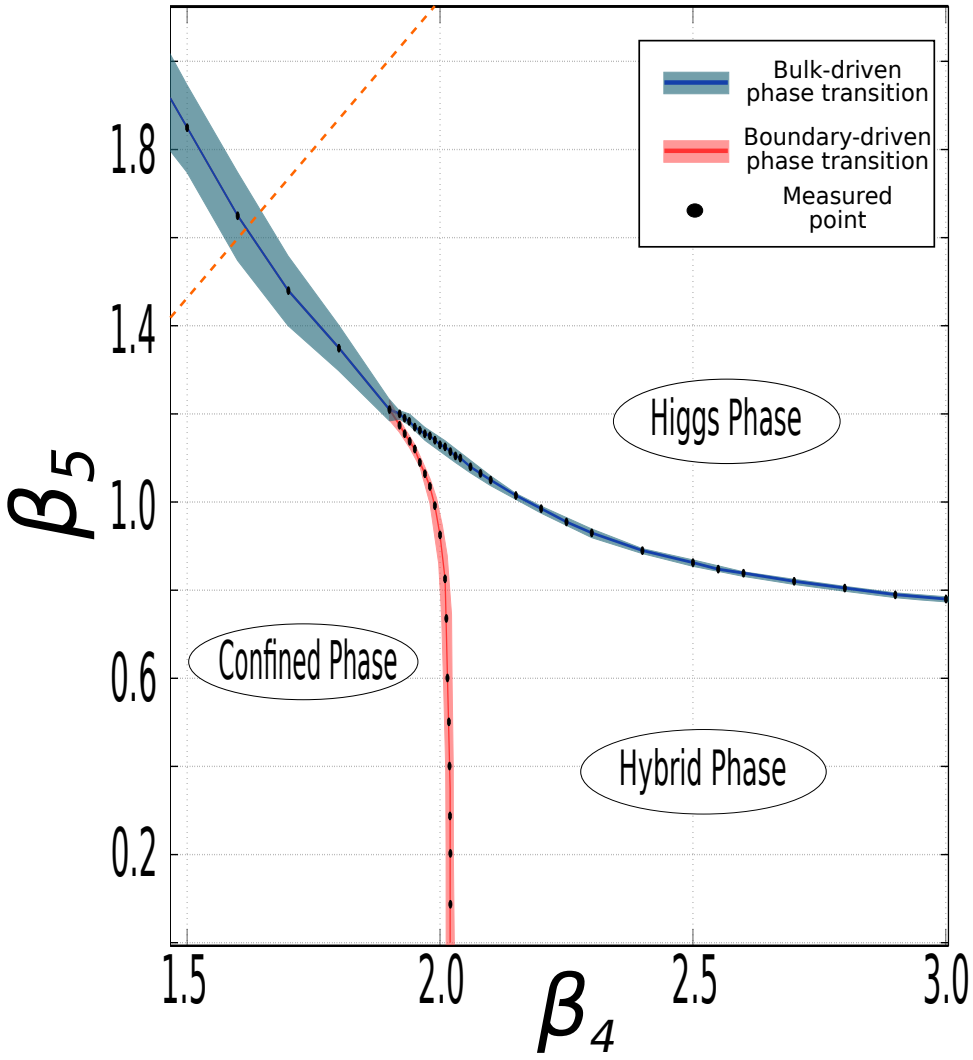
α is the $SU(2)$ projection of h; $n' = (n_\mu, N_5)$

Reminder

Requirements for a viable model

- ☐ it should reproduce the right physics.
- ☐ the fifth dimension should be “hidden”.
- ☐ the physics should be cut-off independent.

Phase Diagram



Notes

On the torus

- confined and Coulomb phase.

On the orbifold

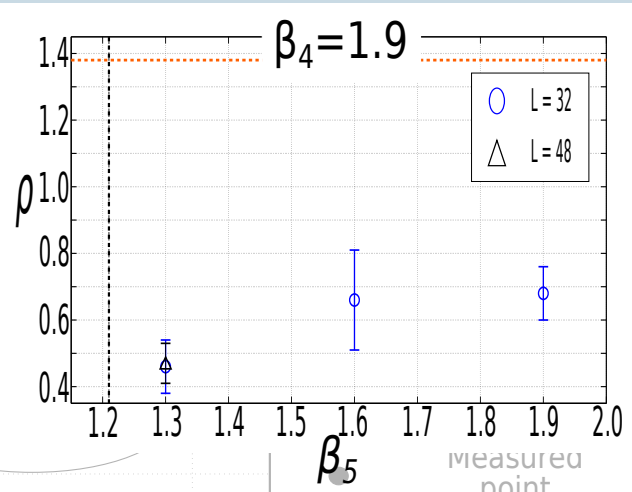
- One more phase: $U(1)$ gauge links deconfine separately.
- No compactification observed at $\gamma > 1$.
- Interesting physics is found at $\gamma < 1$.

First order phase transitions [Alberti, Irges, FK and Moir, 1506.06035]

Spectrum

Mass ratios

- In general, in the Higgs phase we measure $\rho = \frac{m_H}{m_Z} < 1$.



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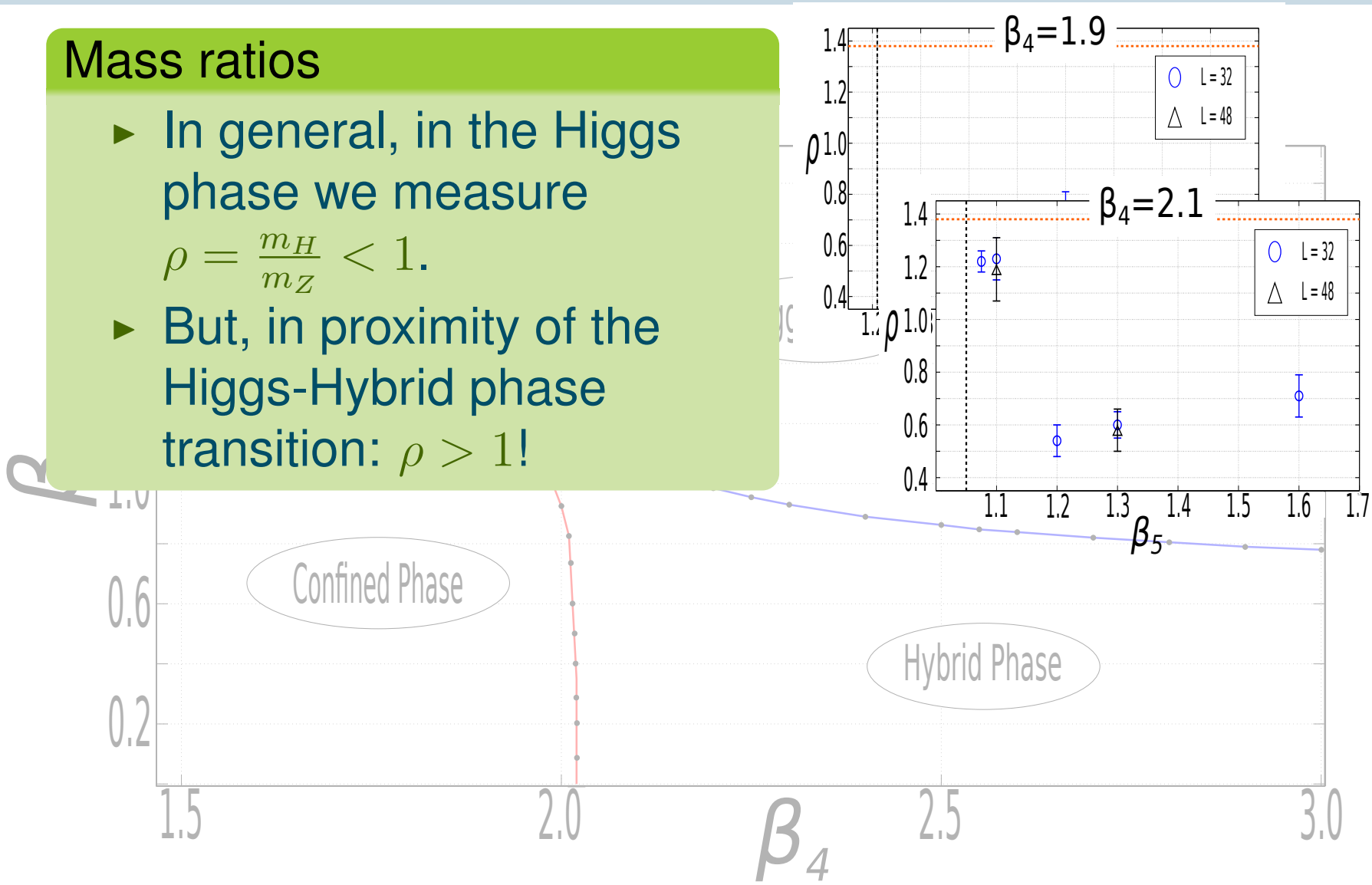

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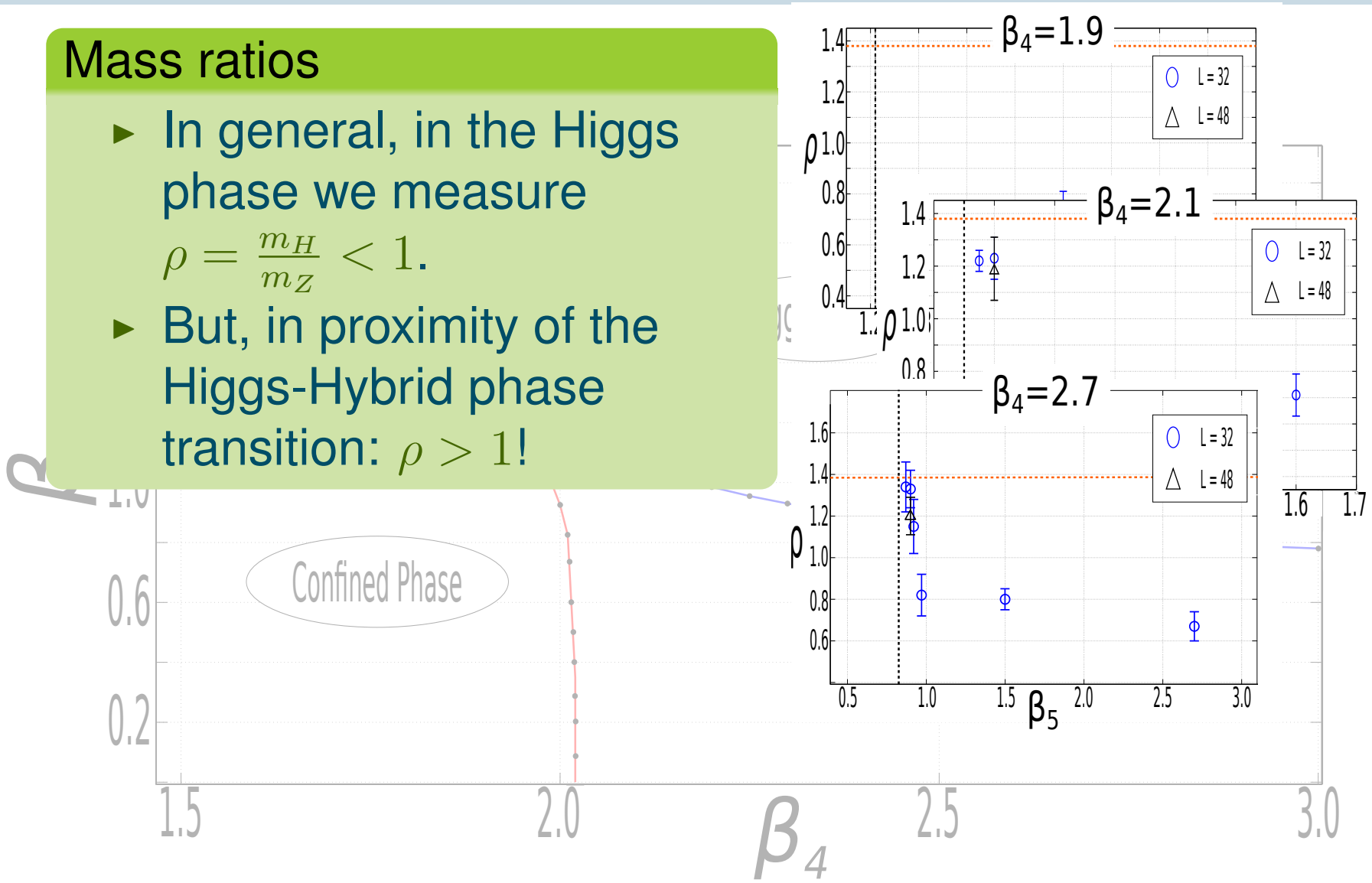

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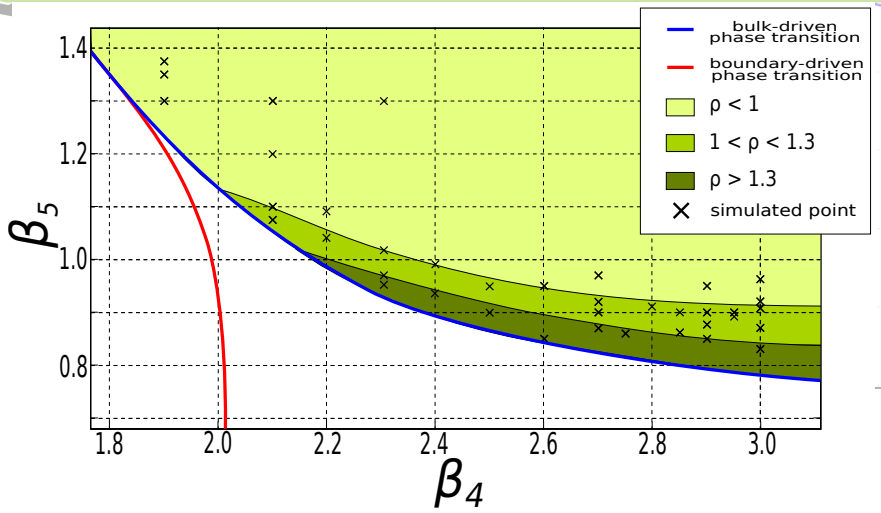

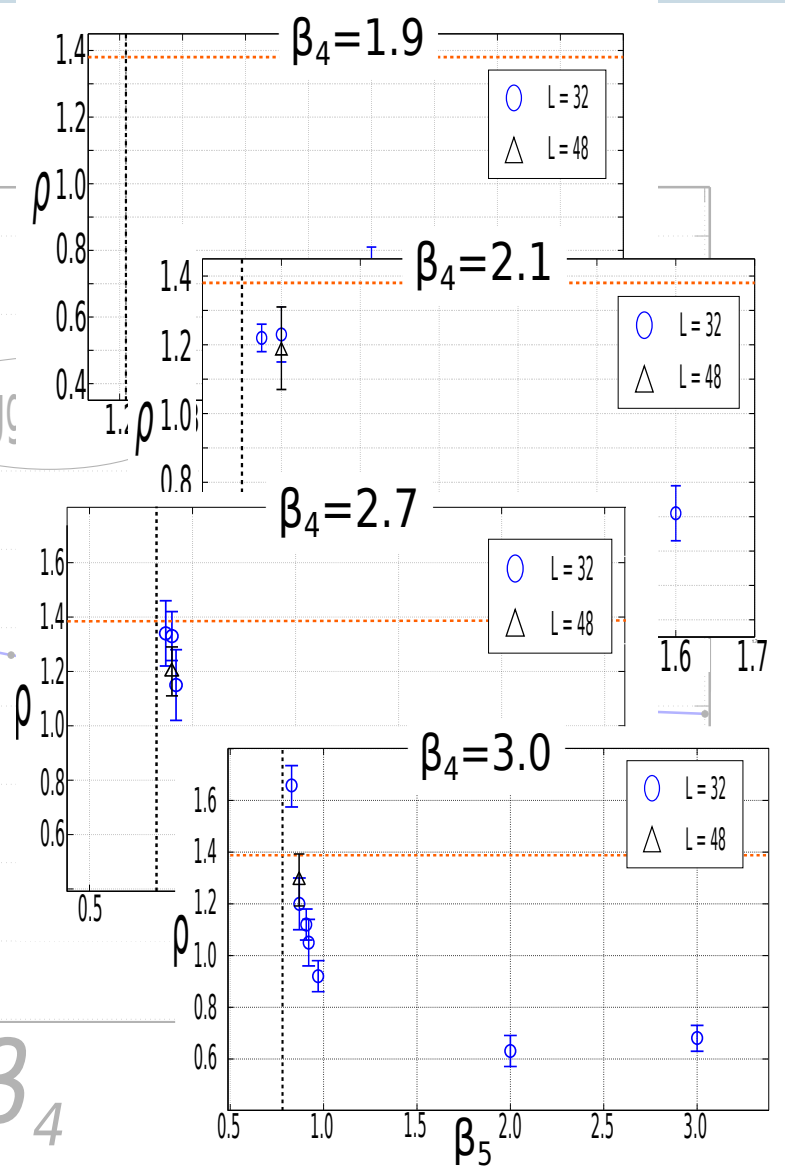

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Spontaneous symmetry breaking

Elitzur's theorem [Elitzur, 1975]

In the gauge-invariant lattice theory, spontaneous symmetry breaking (SSB) must originate from the breaking of a *global* symmetry.

Stick symmetry

Global transformation [Ishiyama, Murata, So and Takenaga, 0911.4555]

$$\begin{cases} U(n_5 = 0, 5) \to g_s^{-1}\, U(n_5 = 0, 5) \\ U(n_5 = 0, \mu) \to g_s^{-1}\, U(n_5 = 0, \mu)\, g_s \end{cases} \quad \text{with } g_s = -i\sigma^1$$

Gauge boson operators $\mathcal{Z}$, $\mathcal{Z}'$ are odd under the stick symmetry, they are the order parameters of SSB [Irges and FK, arXiv:1312.3142]

Gauge-Higgs Unification

Gauge-Higgs Unification on the Orbifold

Does it work?

Requirements of a viable model

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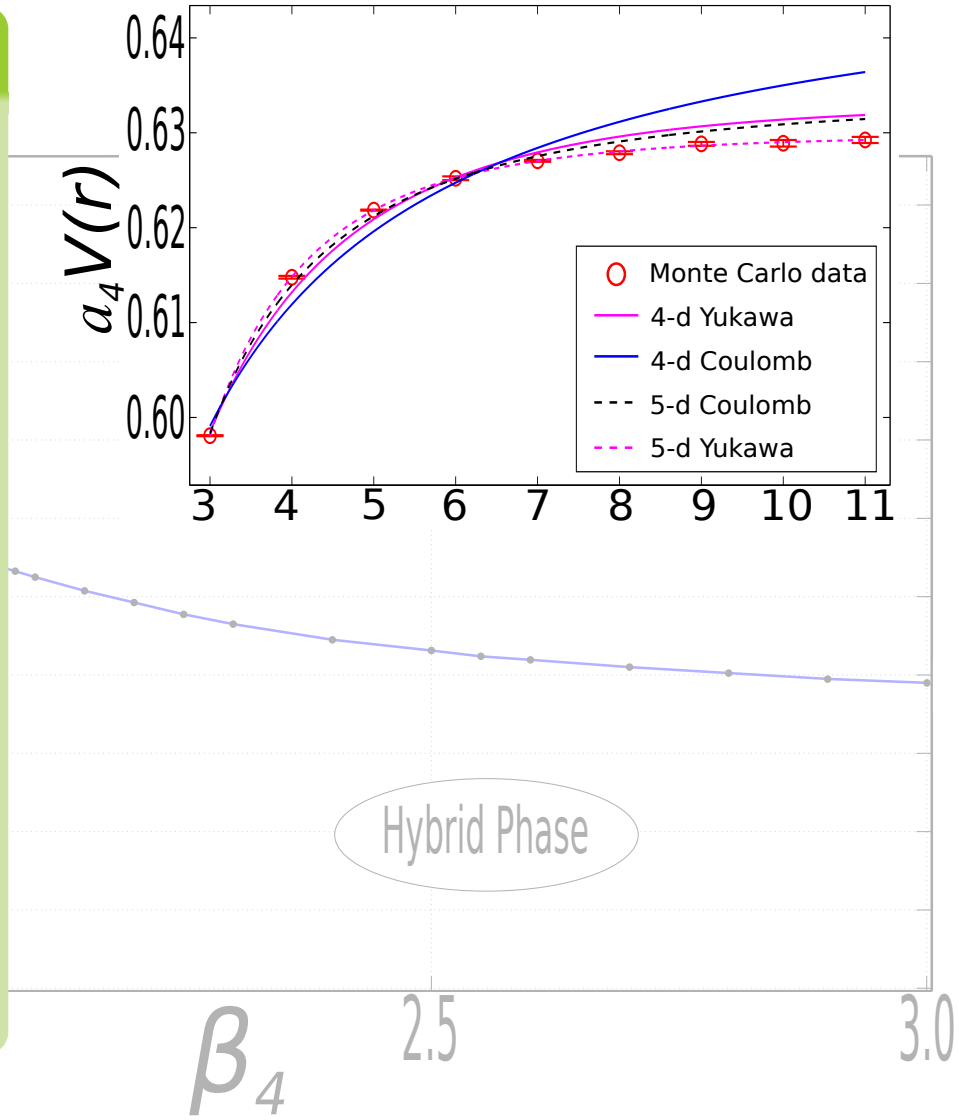
Dimensional Reduction

Higgs phase

- The orbifold's $SU(2)$ bulk feels the fifth dimension.

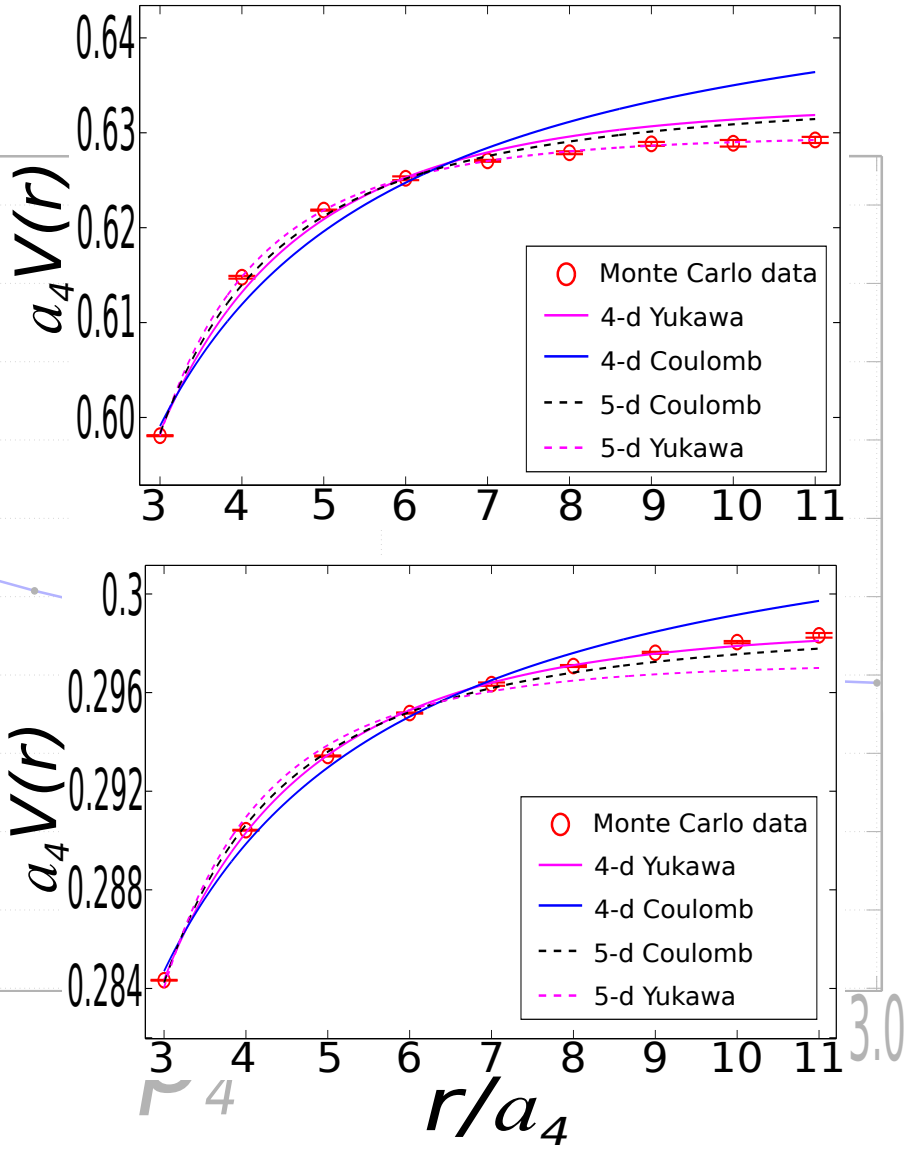


Dimensional Reduction

Higgs phase

- The orbifold's $SU(2)$ bulk feels the fifth dimension.
- But near the PT the boundary remains four-dimensional.
- The fitted Yukawa masses agree with the measured one.

$\Rightarrow$ dimensional reduction via localization



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  $\rightarrow$ Search for Lines of Constant Physics

Effective theory

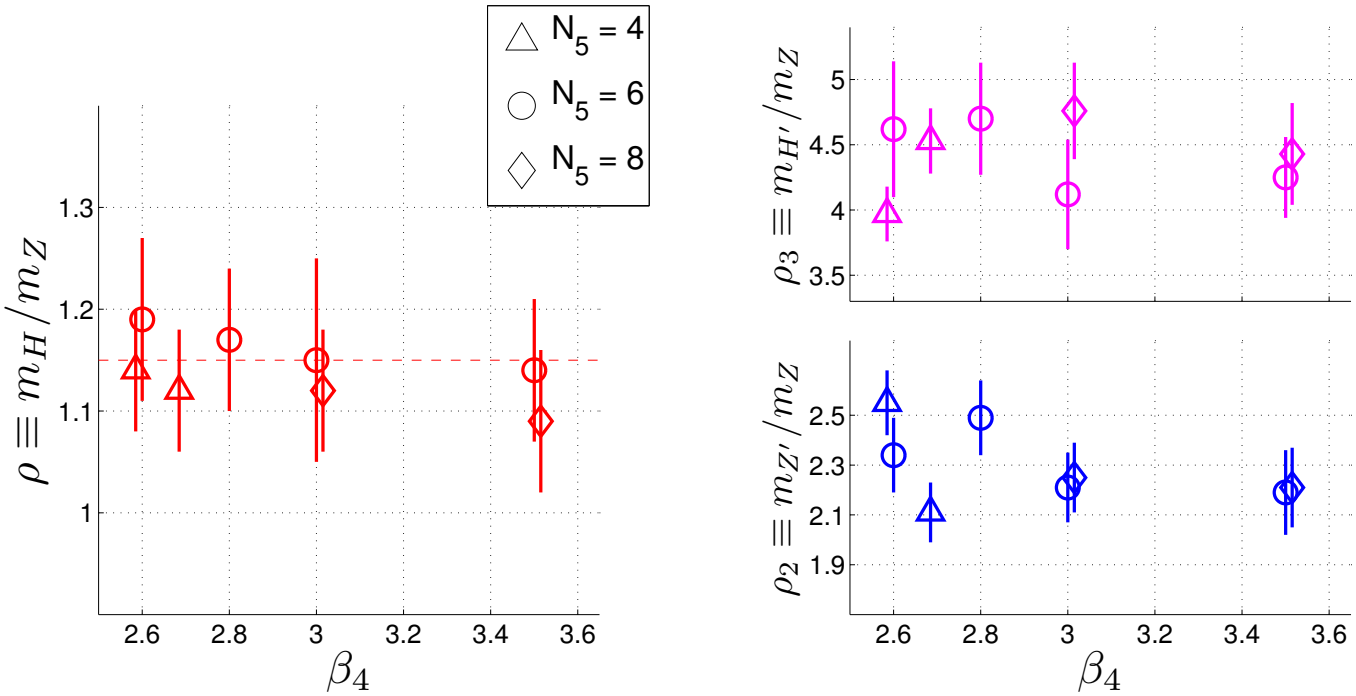
Non-renormalizability

- Gauge theories in five dimensions are perturbatively non-renormalizable.
- On the lattice so far the only known continuum limit $a_4 \to 0$ is perturbative and thus trivial.
- **Effective theory for energies $E \ll a_4^{-1}$: is there a range where cut-off effects are small?**

LPCP

Theory has *three* parameters: β_4, β_5 and N_5
Lines of Constant Physics (LCP): keep *two dimensionless* quantities fixed, e.g. $\rho = m_H/m_Z$ and $F_1 = m_H R$ and *vary* $a_4 m_Z$.
To begin: for several $N_5 = 4, 6, 8$ construct Lines of Partially Constant Physics (LPCP) defined by $\rho = 1.15$.

Lines of Partially Constant Physics



- On LPCPs $a_4 m_Z$ mainly depends on N_5. It diminishes by 20%, going from 0.155 at $N_5 = 4$ to 0.125 at $N_5 = 8$.
- Energies of excited Higgs (H') and excited gauge (Z') bosons are approximately constant, $\rho_3 = m_{H'}/m_Z \approx 4.5$ and $\rho_2 = m_{Z'}/m_Z \approx 2.3$.

[Alberti, Irges, FK and Moir, arXiv:1609.07004]

Conclusions

Non-perturbative Gauge-Higgs Unification

- $\mathrm{SU}(2)$ pure gauge theory on a 5d orbifold has a Higgs phase with the Higgs mechanism realized as a quantum and bosonic effect.
- A Standard-Model like spectrum can be reproduced.
- Localization on the 4d boundaries is observed.
- Cut-off effects appears to be small. Excited state energies are 2–5 times larger than the ground states.

Outlook

- Pursue the construction of LCPs.
- Study connection to 4d Abelian Higgs model.
- Larger gauge group, warping, fermions, . . .

Review on lattice works on extra dimensions [FK and Rinaldi, `arXiv:1605.04341`].