Tackling differential distributions for Higgs production at higher orders

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In collaboration with:
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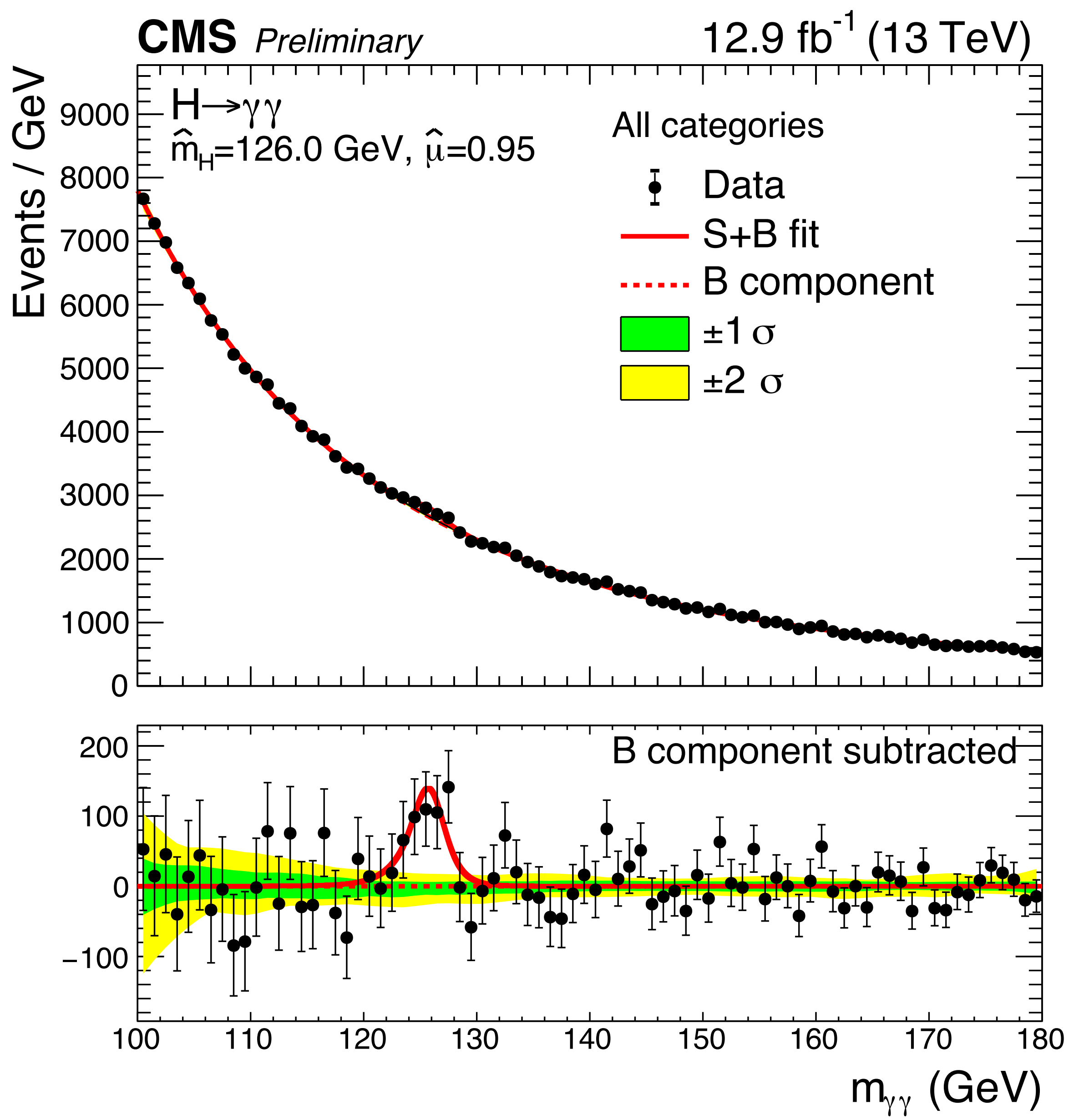
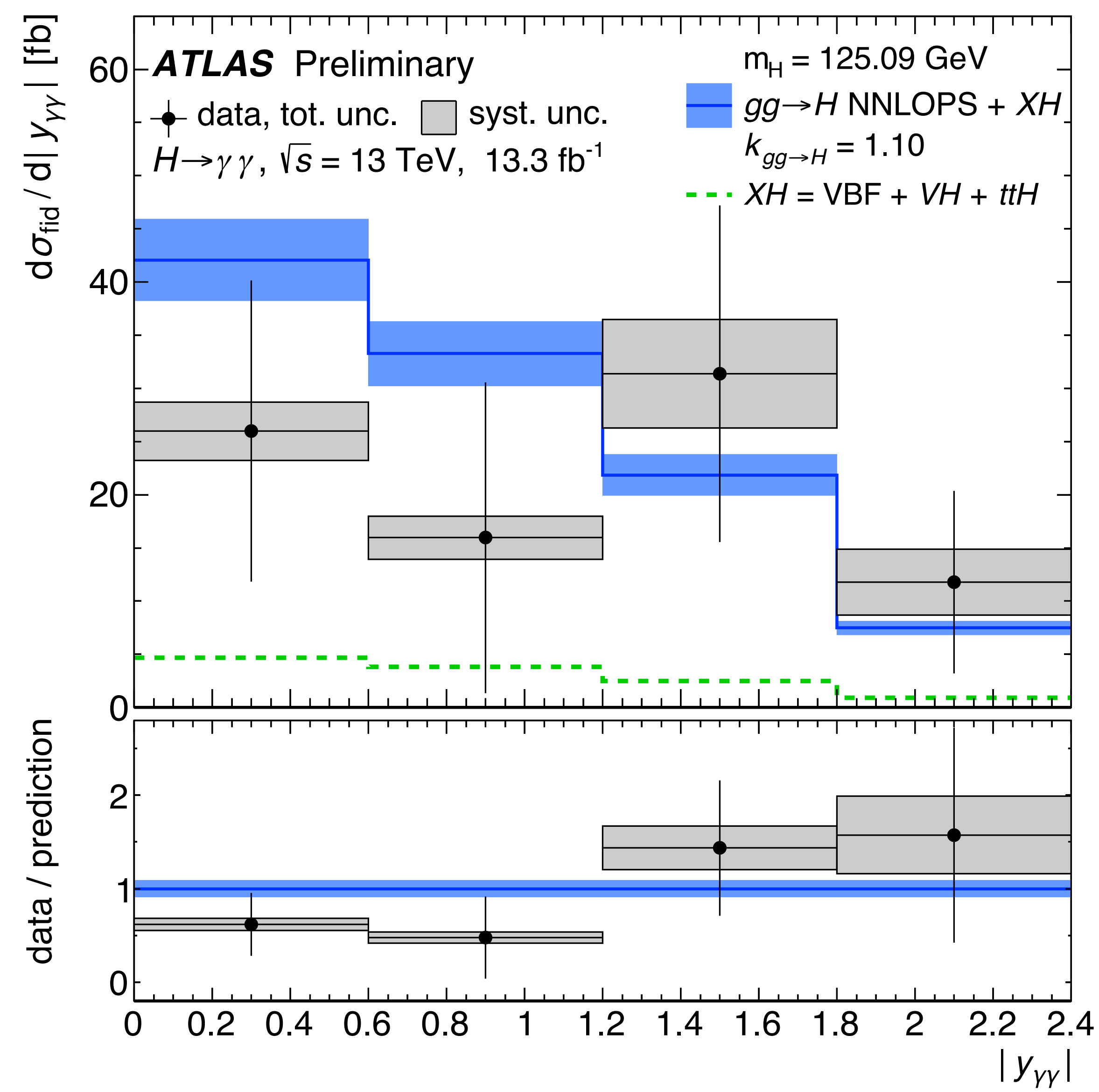
Motivation

After ICHEP 2016: Higgs looking great!





Motivation

Inclusive Higgs cross-section in gluon fusion: N^{3}LO QCD

[Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger]

$$
\begin{aligned}
48.58\,\mathrm{pb} = \;& 16.00\,\mathrm{pb} && (+32.9\%) && (\mathrm{LO,\ rEFT}) \\
& +20.84\,\mathrm{pb} && (+42.9\%) && (\mathrm{NLO,\ rEFT}) \\
& -\;\;2.05\,\mathrm{pb} && (-4.2\%) && ((t,b,c),\ \mathrm{exact\ NLO}) \\
& +\;\;9.56\,\mathrm{pb} && (+19.7\%) && (\mathrm{NNLO,\ rEFT}) \\
& +\;\;0.34\,\mathrm{pb} && (+0.2\%) && (\mathrm{NNLO},\ 1/m_t) \\
& +\;\;2.40\,\mathrm{pb} && (+4.9\%) && (\mathrm{EW,\ QCD\text{-}EW}) \\
& \boxed{+\;\;1.49\,\mathrm{pb} \quad (+3.1\%) \quad (\mathrm{N^3LO,\ rEFT})}
\end{aligned}
$$

"The fiducial cross section is measured to be $\hat{\sigma}_{\mathrm{fid}} = 69^{+18}_{-22}\,\mathrm{fb}\,[\ldots]$"

$$\hat{\sigma}_{\mathrm{fid}} = \sigma_H^{\mathrm{th}} \times \mathrm{BR}_{\gamma\gamma} \times f_{\mathrm{acc}}$$

N^3LO differential?

Success of N^3LO inclusive...

- **Reverse unitarity + IBP reduction**
- **Expansion around the soft limit**

... and challenges of NNLO differential!

- **Subtraction of infrared singularities**
- **Master integrals with multiple scales**

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Fully differential?

N^3LO differential?

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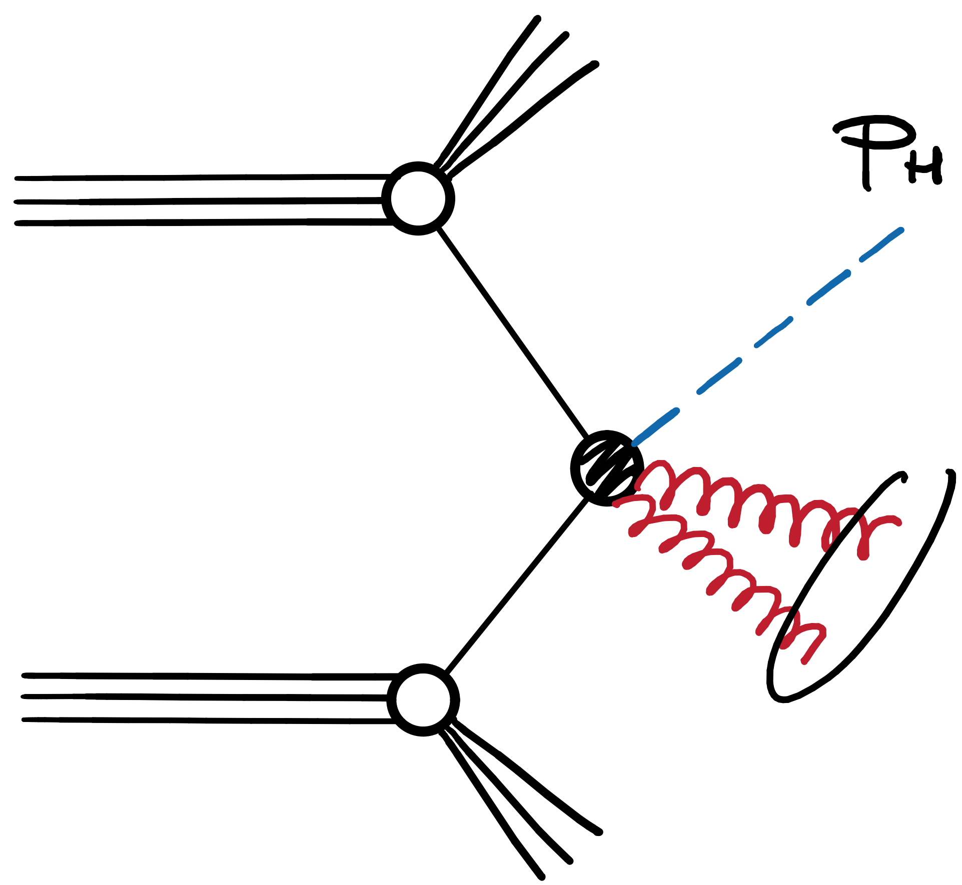
Single differential,
maybe rapidity?

Double differential

Integrate over extra QCD radiation analytically

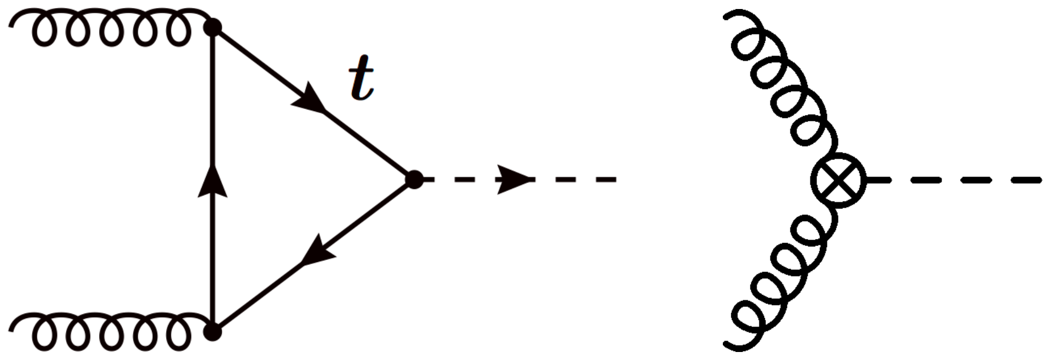
and

be completely differential in the Higgs.



Double differential at work

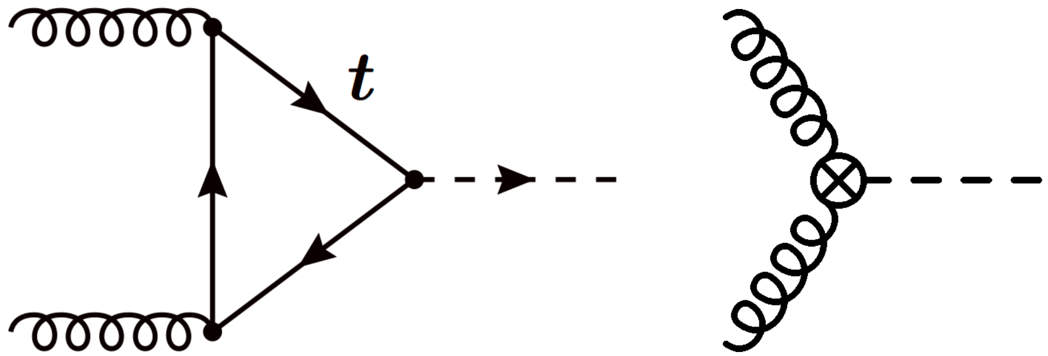
Gluon fusion in Higgs effective theory



NNLO QCD throughout.

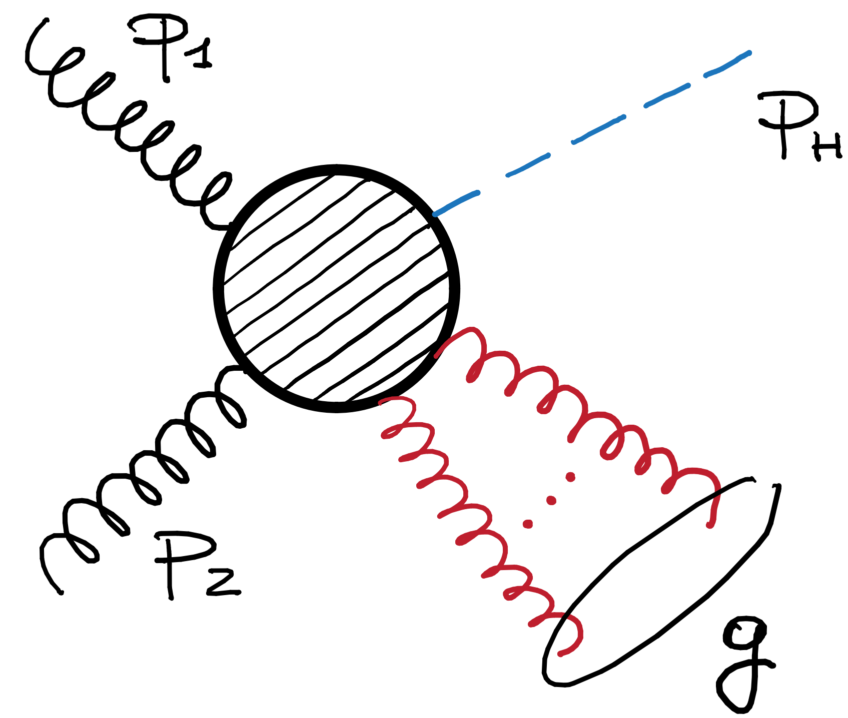
Double differential at work

Gluon fusion in Higgs effective theory



NNLO QCD throughout.

Partonic kinematics

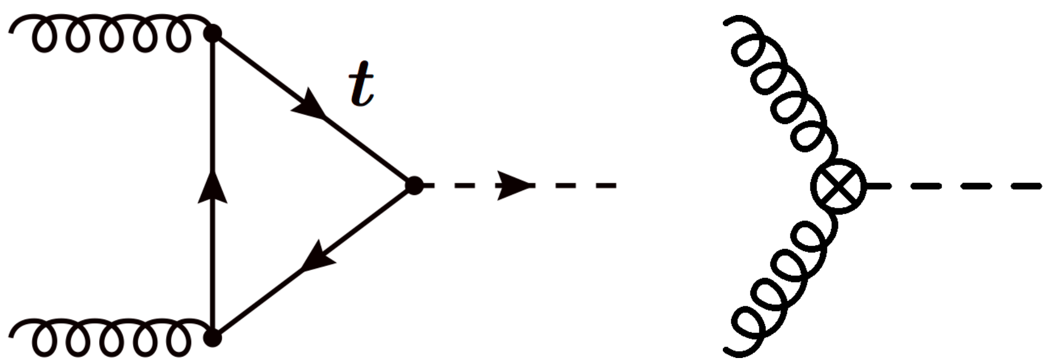


$$p_1 + p_2 = p_H + g$$

$$g = \sum_{i \text{ real}} g_i$$

Double differential at work

Gluon fusion in Higgs effective theory



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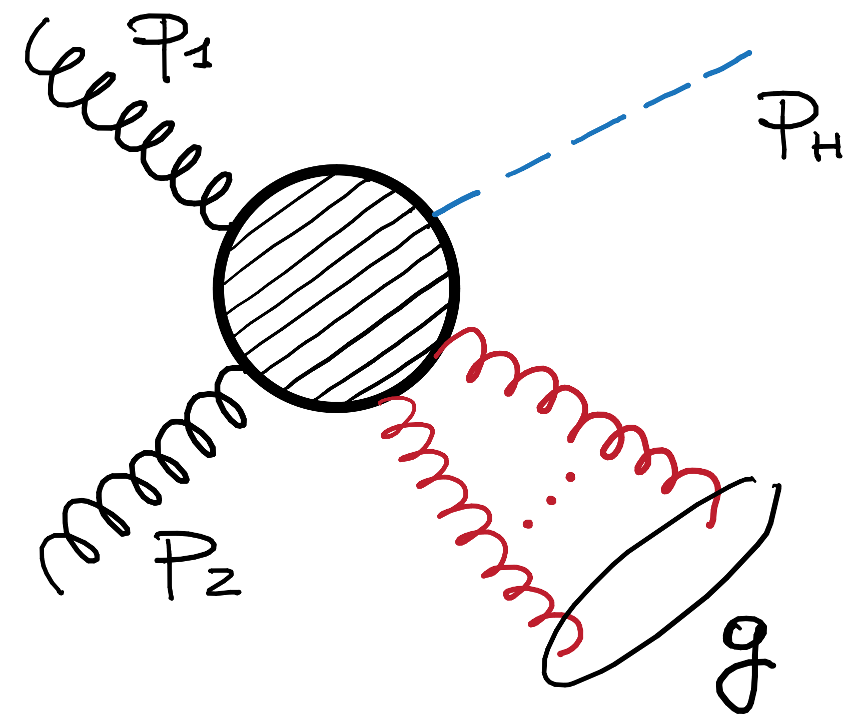

$$p_1 + p_2 = p_H + g$$

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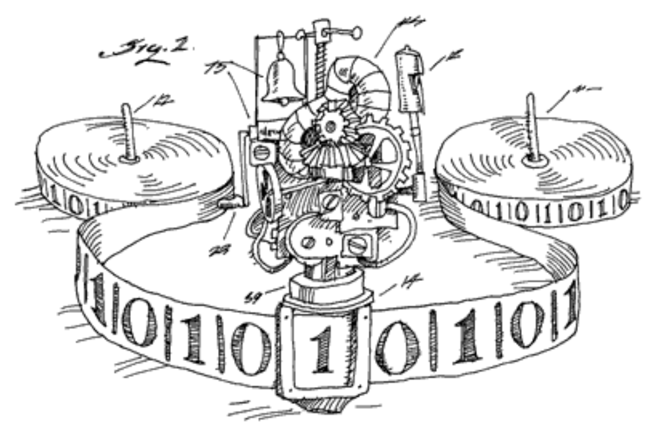
Differential in the Higgs momentum...

$$\frac{d^3\sigma}{d^3\vec{p}_H} \to \frac{d^2\sigma}{dp_H^T\, dY_H}$$

...double differential!

Double differential at work

Generate all 2→2 forward scattering diagrams



Apply integration-by-parts identities

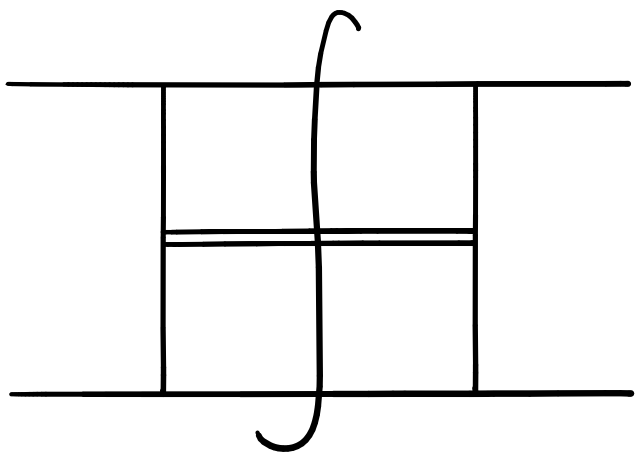
↓

Matrix elements:

products of rational coefficients and master integrals

Analytic integrals over radiation

$$g \equiv \sum_{i \text{ real}} g_i \qquad p_H = p_1 + p_2 - g$$



$$M = \int \prod_{i=1}^{\ell} [d^D g_i] \delta[p_H^T - p_H^T(g)]\, \delta[y_H - y_H(g)]$$

$$\times\, \delta[p_H^2(g) - m_H^2]\, f(g_i \cdot g_j, g_i \cdot p_1, g_j \cdot p_2)$$

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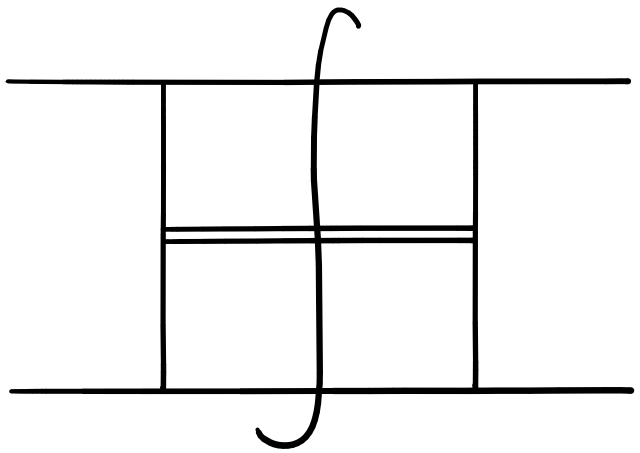

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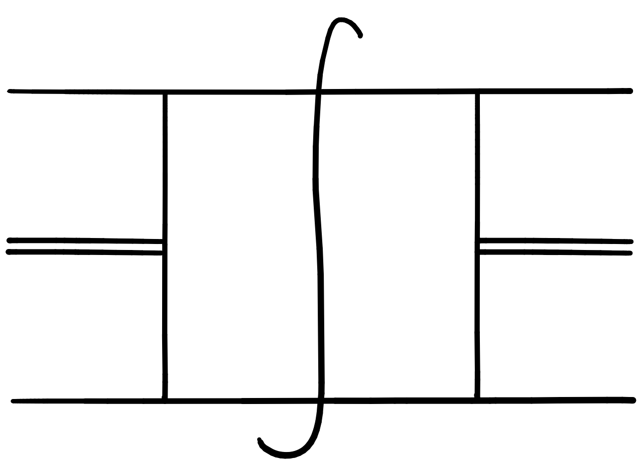
$$\times\, \delta[p_H^2(g) - m_H^2]\, f(g_i \cdot g_j, g_i \cdot p_1, g_j \cdot p_2)$$

Double differential

"Higgs loop" completely resolved!



$$M = \mathcal{F}(p_H^T, y_H) \times \int dg\, \delta[p_H^T - p_H^T(g)]$$
$$\times\, \delta[y_H - y_H(g)]\, \delta[p_H^2(g) - m_H^2]$$

Numeric integrals over Higgs variables

IR singularities: one variable

$$\int y^{-1+\alpha\epsilon} f(y)\, dy \to \int y^{-1+\alpha\epsilon}[f(y) - f(0)]\, dy + \frac{1}{\alpha\epsilon} f(0)$$

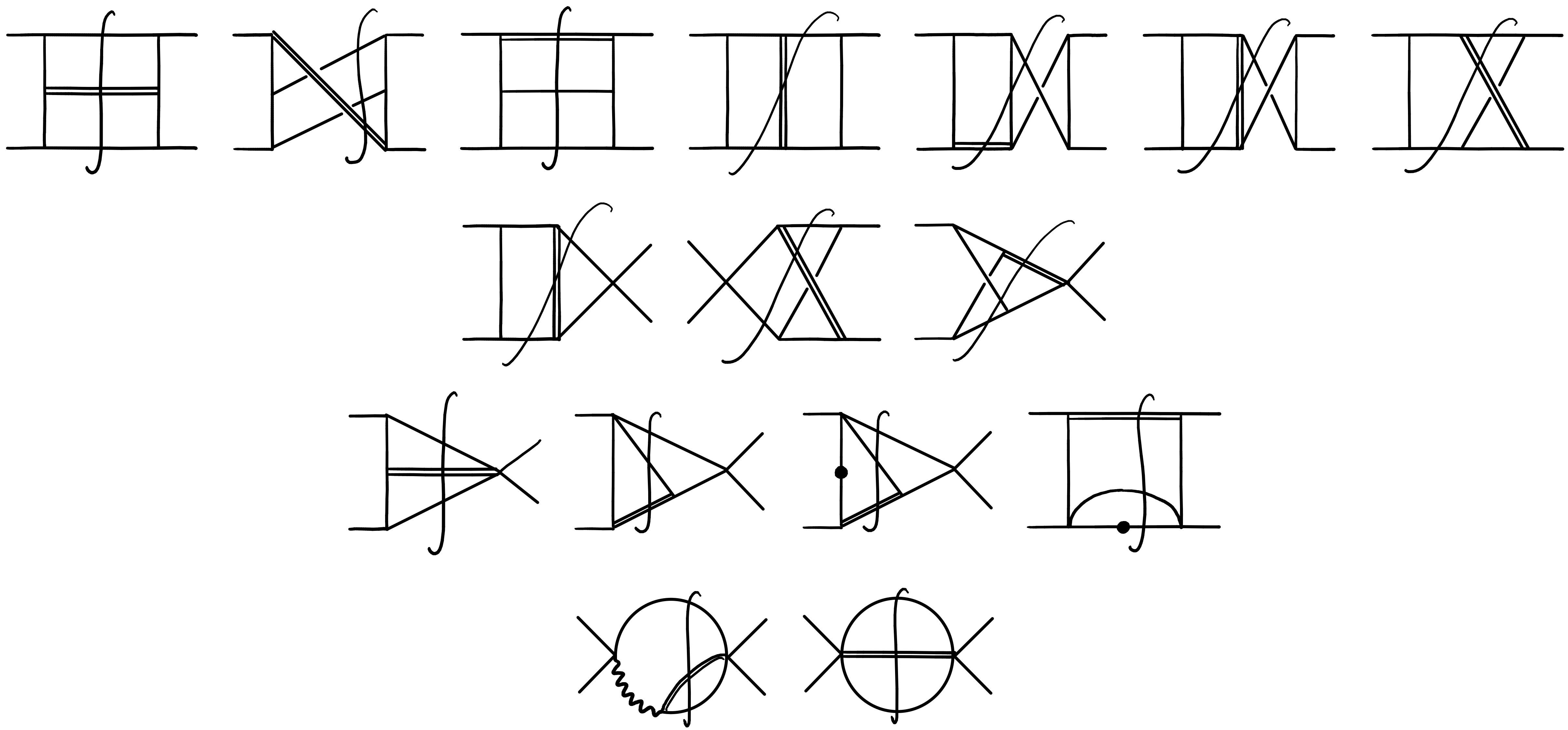
Phase space parametrisation

Soft	$g^\mu \sim \bar{z}$
Collinear	$2gp_2 \sim \bar{z}\lambda$
Radiation invariant mass	$g^2 \sim \bar{z}^2\lambda(1-\lambda)x$

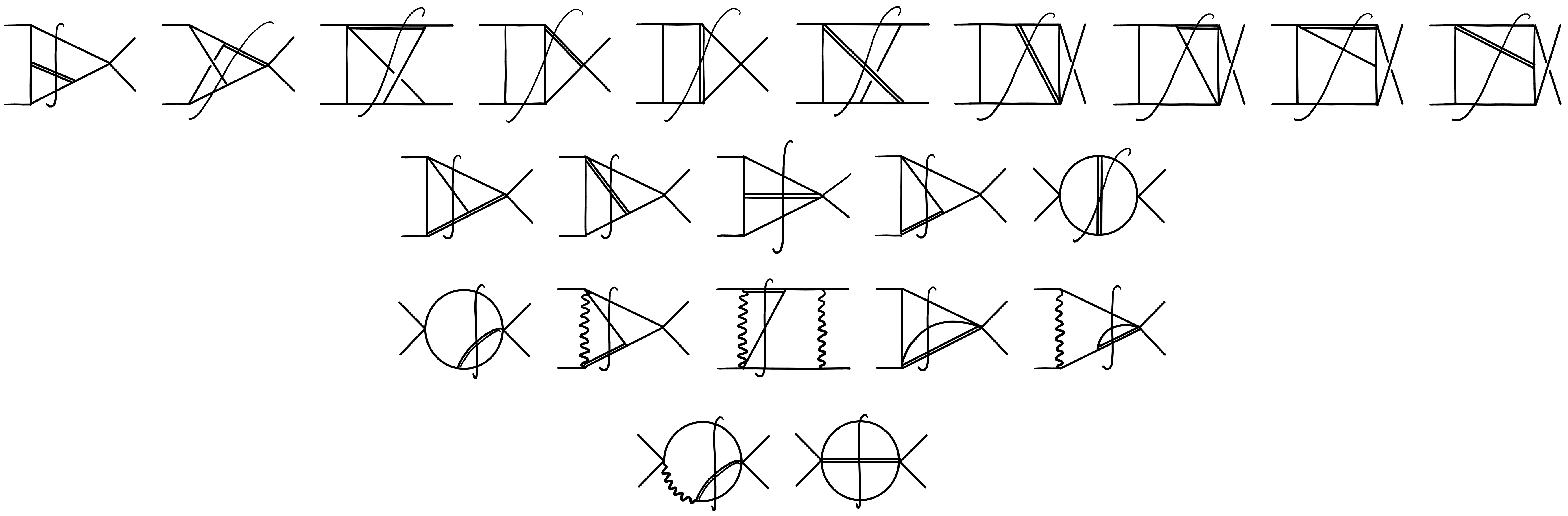
Just iterate!

- **Crucial for double-differential strategy to be viable**
- **Any color singlet, also more particles**
- **Higher orders ($\mathrm{N^nLO}$)**

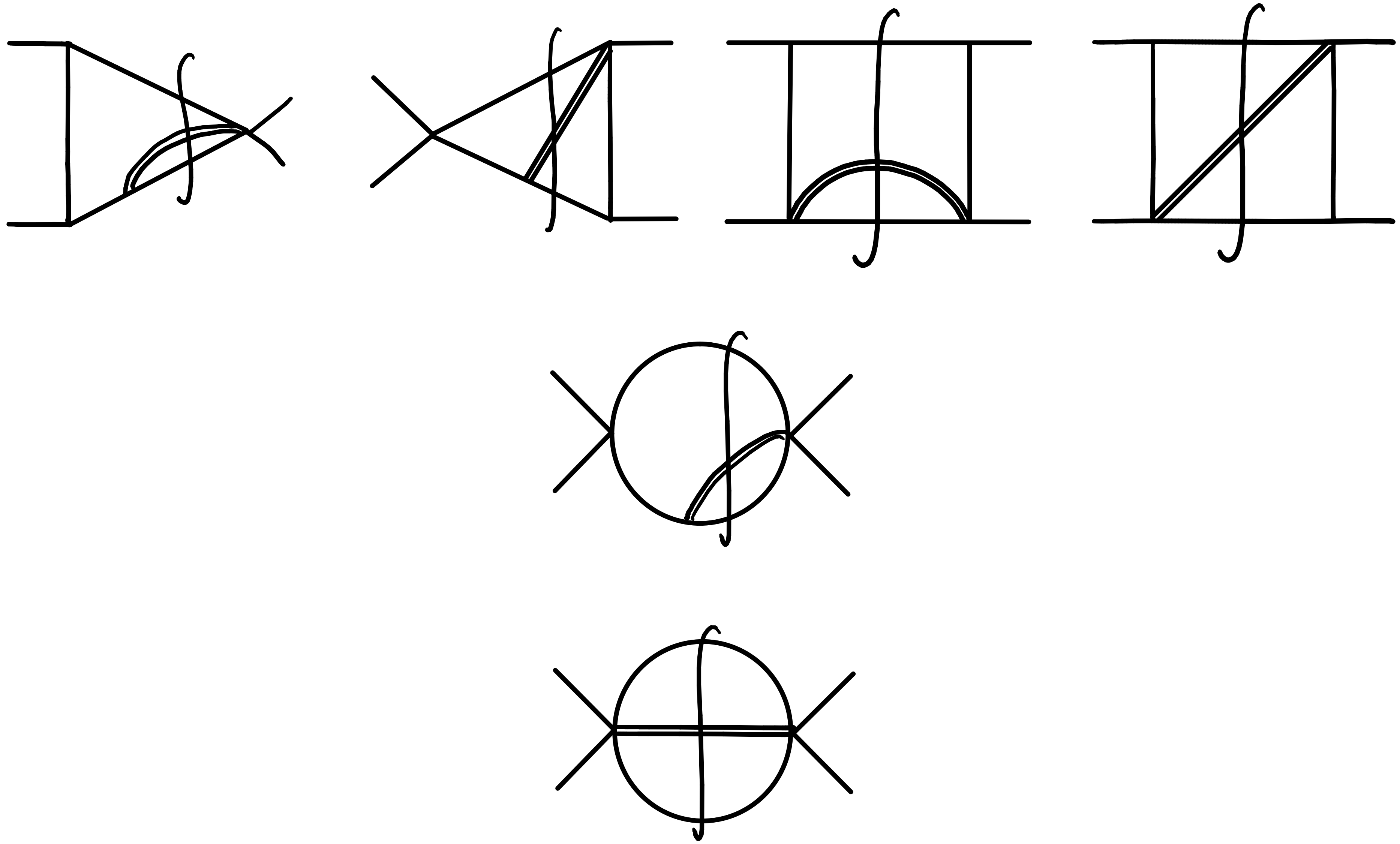
RR master integrals: inclusive



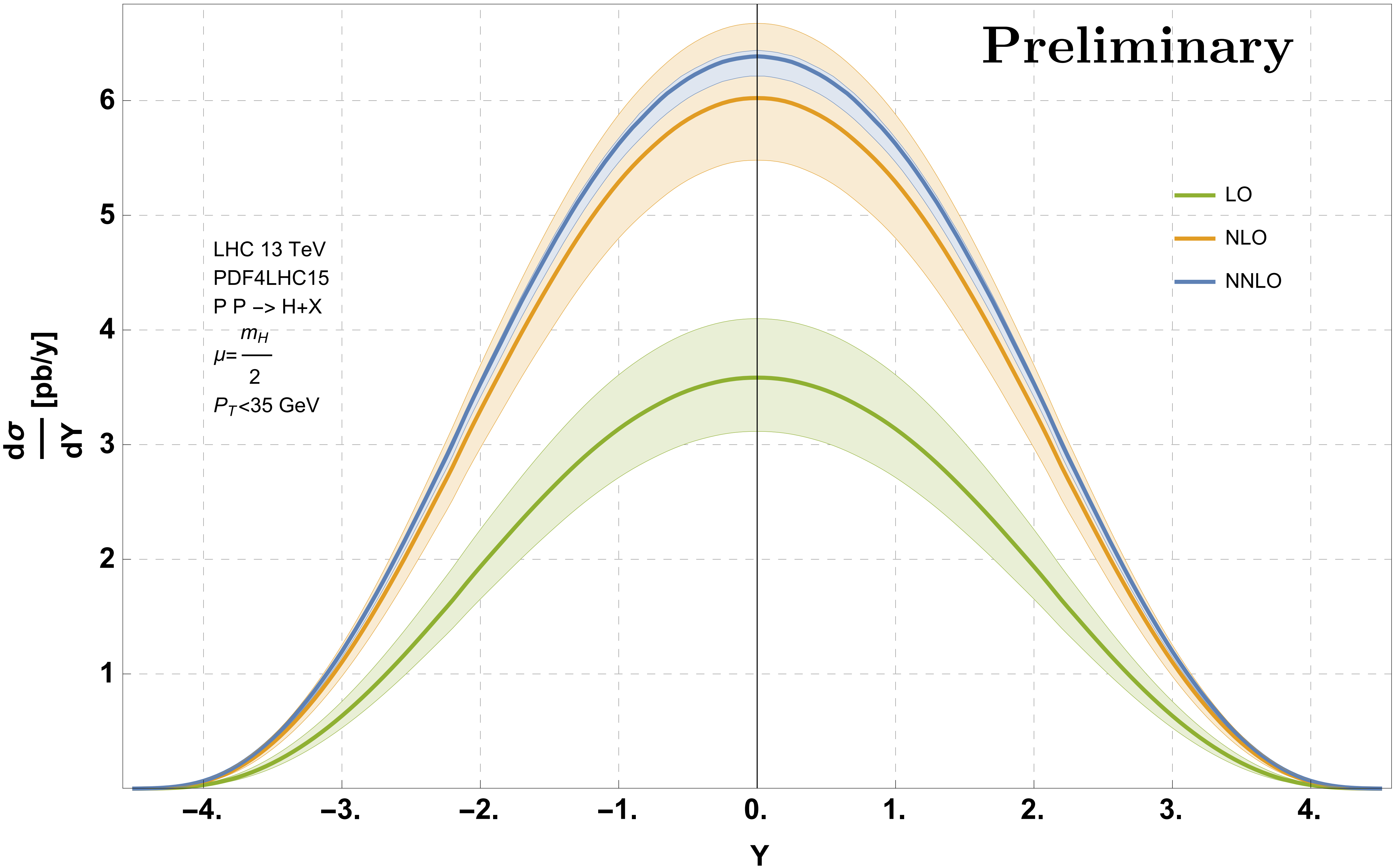
RR master integrals: single differential



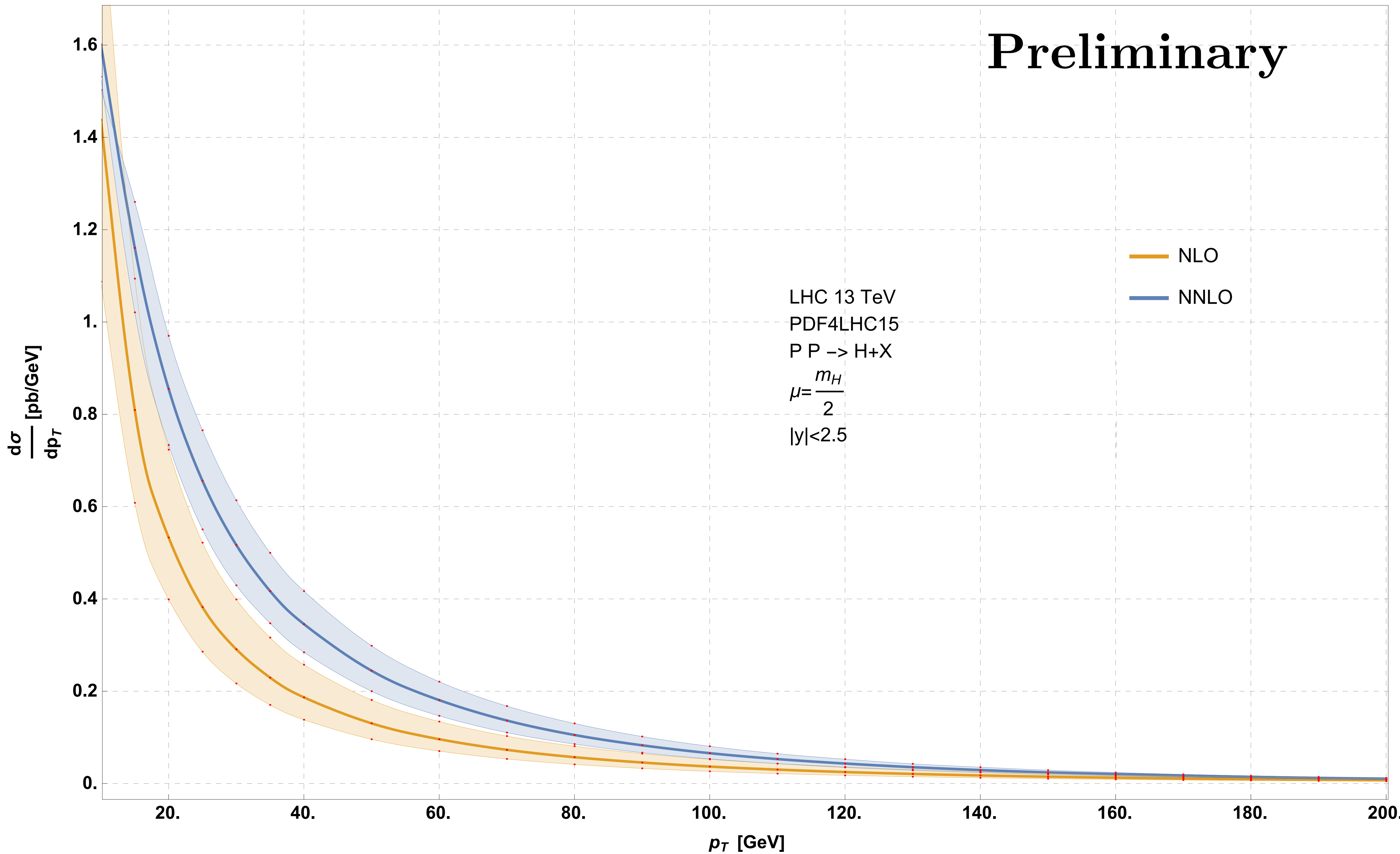
RR master integrals: double differential



Rapidity distribution



p_T distribution



Summary

- **3rd order radiative QCD corrections relevant for Higgs phenomenology**

- **Rethought how to compute differential cross-sections at higher orders**

- **Proof of concept: NNLO double differential Higgs production via gluon fusion**

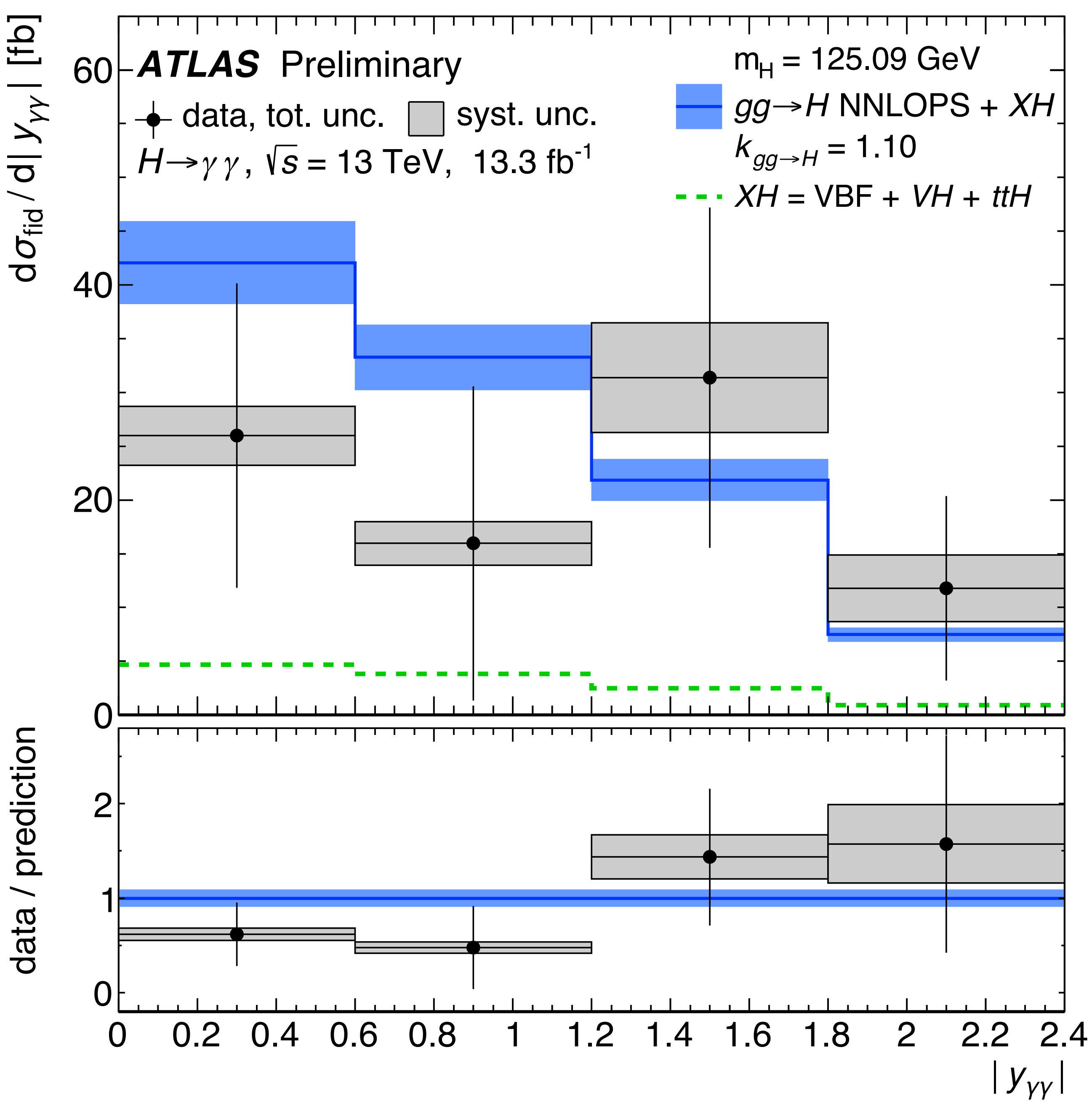
Outlook

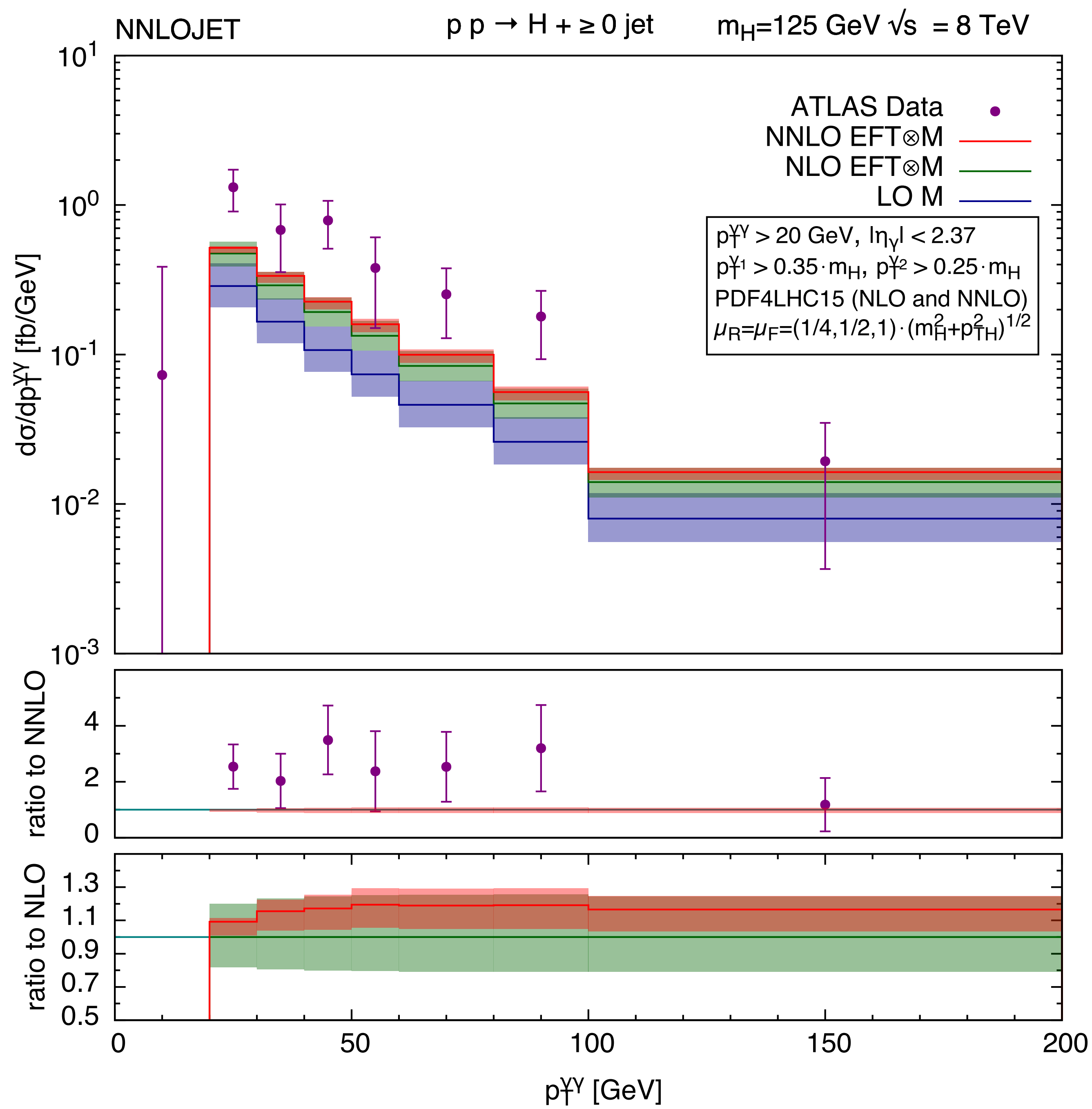
- **Combine with other results to obtain fully differential**

- **Stay tuned for N^3LO updates!**

Backup

Differential distributions





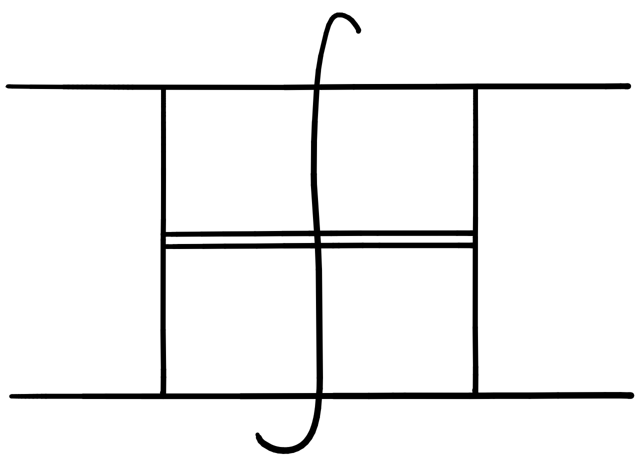
[Chen, Cruz-Martinez, Gehrmann, Glover, Jaquier]

Extra radiation: analytic integrals

$$(p_H^T, y_H) \;\rightarrow\; (Q^2, W)$$

$$g \equiv \sum_{i \text{ real}} g_i \qquad Q^2 \equiv g^2 \qquad W \equiv 2g(p_1 - p_2)$$

$$M = \int d^D p_H \prod_i [d^D g_i] \delta^{(D)}(p_1 + p_2 - g - p_H)$$



$$\times\, \delta[g^2 - Q^2]\, \delta[W - 2g(p_1 - p_2)]\, \delta[p_H^2 - m_H^2]$$

$$\times\, f(g_i \cdot g_j, g_i \cdot p_1, g_j \cdot p_2)$$

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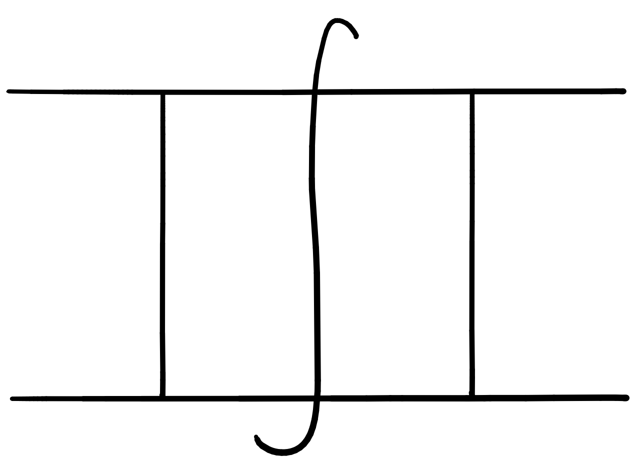

$$\mathcal{F}(Q^2, W) = \int \prod_i [d^D g_i] \delta^{(D)}(p_1 + p_2 - g - p_H)$$
$$\times\; f(g_i \cdot g_j, g_i \cdot p_1, g_j \cdot p_2)$$

✓ **Direct integration**

✓ **Differential equations**

Extra radiation: analytic integrals

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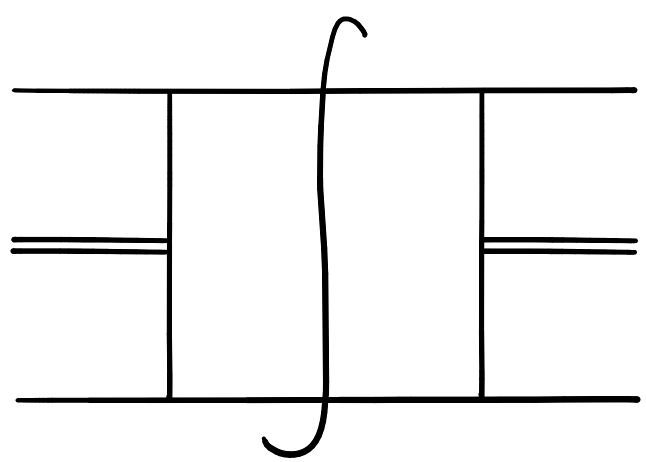

Double differential:

"Higgs loop" completely resolved

Numeric integrals over Higgs variables

Parametrization of phase space

$$\bar{z} = 1 - \frac{m_H^2}{2p_1p_2} \in [0, 1-\tau] \qquad \tau = \frac{m_H^2}{S}$$

$$\lambda = \frac{2p_2g}{2p_1p_2 - m_H^2} \in [0,1]$$

$$x = \frac{g^2\ 2p_1p_2}{2p_1g\,2p_2g} \in [0,1]$$

Soft
$g^\mu \sim \bar{z}$

Collinear
$2p_2g \sim \bar{z}\lambda$

Radiation invariant mass
$g^2 \sim \bar{z}^2\lambda(1-\lambda)x$

- **Singularities:** $\bar{z} = 0; \quad \lambda = 0, 1; \quad x = 0, 1.$
- **Soft expansion: powers of** $\bar{z}$**, nontrivial rapidity distribution**
- **(Asymmetric but rational)**