$\mathcal{N} = 3$ four dimensional field theories

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Hamburg, September 2016

Based on
[1512.06434] with I. García-Etxebarria

Goal and strategy

Goal:

- Understand F-theory at singularities with no supersymmetric smoothing. Only some examples!
- In particular, D3-branes probing codimension 4 terminal singularities.
- The simplest unknown cases lead to 4d $\mathcal{N}=3$ theories on the worldvolume of the D3s.

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Strategy:

- D3s probing an O3 from several perspectives:
  - Worldsheet
  - M/F-theory
  - 4d field theory
- Generalize the O3-plane.

O3s in perturbation theory

- In 2d CFT, O3s are defined as the quotient of 10d Type IIB by $\mathcal{I}(-1)^{F_L}\Omega$

$$\mathcal{I} : (z_1,\, z_2,\, z_3) \to (-z_1, -z_2, -z_3)$$

$(-1)^{F_L}$: left moving spacetime fermion number

Ω : orientation reversal on the worldsheet

$$\left.\begin{array}{l} (-1)^{F_L} \\ \Omega \end{array}\right\} \begin{pmatrix} B_2 \\ C_2 \end{pmatrix} \to \begin{pmatrix} -B_2 \\ -C_2 \end{pmatrix}$$

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- When including N parallel D3s, we need to specify an action on the Chan-Paton factors. [Gimon, Polchinski]

Before the quotient

After the quotient

$$4d\ \mathcal{N}=4\ \mathfrak{u}(N) \to 4d\ \mathcal{N}=4\ \mathfrak{so}(N)$$

$$4d\ \mathcal{N}=4\ \mathfrak{u}(N) \to 4d\ \mathcal{N}=4\ \mathfrak{usp}(N) \quad (N \in 2\mathbb{Z})$$

- There are different kinds of O3-planes.

O3s in M/F-theory (I)

- 10d Type IIB is given by the F-theory limit of M-theory on a torus.

M-th. on $\mathbb{R}^{1,2} \times \mathbb{C}^3 \times T^2$ $\xrightarrow{T^2 \to 0}$ IIB on $\mathbb{R}^{1,3} \times \mathbb{C}^3$

Complex structure of T^2 $\longrightarrow$ Axio-dilaton ($\tau \supset g_s$)

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$$\text{Complex structure of } T^2 \longrightarrow \text{Axio-dilaton } (\tau \supset g_s)$$

- The M-theory lift of the O3 is given by [Hanany, Kol]

$$\text{M-th. on } \mathbb{R}^{1,2} \times (\mathbb{C}^3 \times T^2)/\mathbb{Z}_2 \quad \text{with} \quad (z_1, z_2, z_3, u) \to (-z_1, -z_2, -z_3, -u)$$

$$(-1)^{F_L}\Omega \quad \text{lifts to:} \quad \mathcal{M}_{(-1)^{F_L}\Omega} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \in SL(2,\mathbb{Z})$$

This can be seen by looking at the action of the O3 on $\begin{pmatrix} B_2 \\ C_2 \end{pmatrix}$, which comes from reducing C_3 along the one-cycles in the torus.

O3s in M/F-theory (II)

- Four fixed points, which locally look like $\mathbb{C}^4/\mathbb{Z}_2$.
  This singularity has no susy smoothings: no low-energy dynamics associated to the O3. [Morrison, Stevens; Anno]

- D3-branes parallel to the O3-plane lift to M2-branes.

- In M-theory, this is precisely ABJM (at level $k=2$). [Aharony, Bergman Jafferis, Maldacena]

  The F-theory limit provides the 4d lift of ABJM.

  $k=1: \quad 4d\ \mathcal{N}=4\ \mathfrak{u}(N)$

  $k=2: \quad 4d\ \mathcal{N}=4\ \mathfrak{so}(N), \mathfrak{usp}(N)$

- Orientifold variants: discrete flux $\longrightarrow$ $O3^-, O3^+, \widetilde{O3}^-, \widetilde{O3}^+$.

[Hanany, Kol]

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- We have seen that $(-1)^{F_L}\Omega$ maps to $\mathbb{Z}_2^S \subset SL(2,\mathbb{Z})$.

  $SL(2,\mathbb{Z})$ is a duality, not a symmetry: $\tau \to \dfrac{a\tau+b}{c\tau+d}$

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- Therefore, the orientifold corresponds to gauging $\mathbb{Z}_2^{O3} = \mathbb{Z}_2^R \cdot \mathbb{Z}_2^S$.
- Supercharges: $Q_{\alpha a}$ is charged under both $\mathbb{Z}_2^R$ and $\mathbb{Z}_2^S$. [Kapustin, Witten]

  $\mathbb{Z}_2^{O3} : Q_{\alpha a} \to Q_{\alpha a}$ (the O3 does not break SUSY further)

Beyond the O3

Different ways to look at the O3:

- Worldsheet: quotient by $\mathcal{I}(-1)^{F_L}\Omega$.
- M/F-theory: F-theory limit of $\mathbb{R}^{1,2} \times (\mathbb{C}^3 \times T^2)/\mathbb{Z}_2$.
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The last three admit a generalization: $\mathbb{Z}_2 \longrightarrow \mathbb{Z}_k$

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We call the associated objects $\mathrm{OF3}_k$-planes. $(\mathrm{OF3}_2 = \mathrm{O3})$

OF3s in M/F-theory (I)

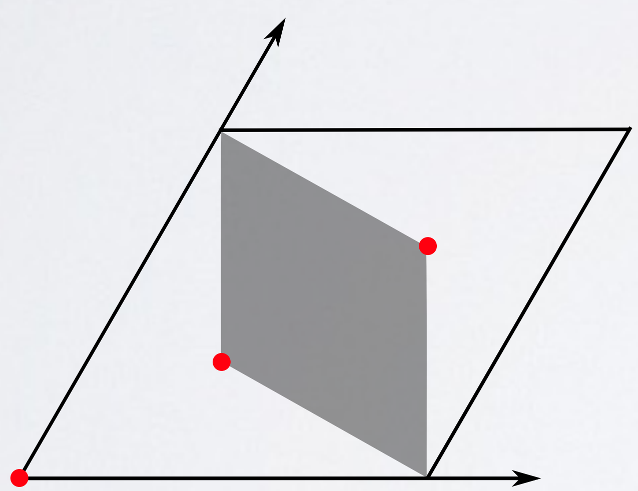
- We want to consider M/F-theory on $\mathbb{R}^{1,2} \times (\mathbb{C}^3 \times T^2)/\mathbb{Z}_k$

$$(z_1, z_2, z_3, u) \to (\zeta_k z_1, \bar{\zeta}_k z_2, \zeta_k z_3, \bar{\zeta}_k u) \quad \text{with} \quad \zeta_k = e^{2\pi i/k} \qquad (k = 2, 3, 4, 6)$$

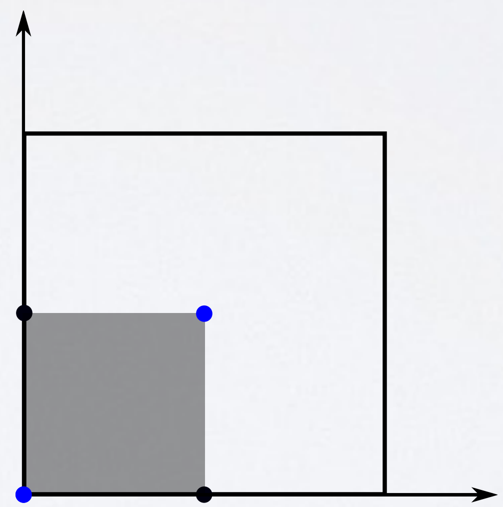
$\mathrm{OF3}_k$-planes exist only for some values of k.

Only well-defined for special values of the complex structure τ (g_s^{IIB}).

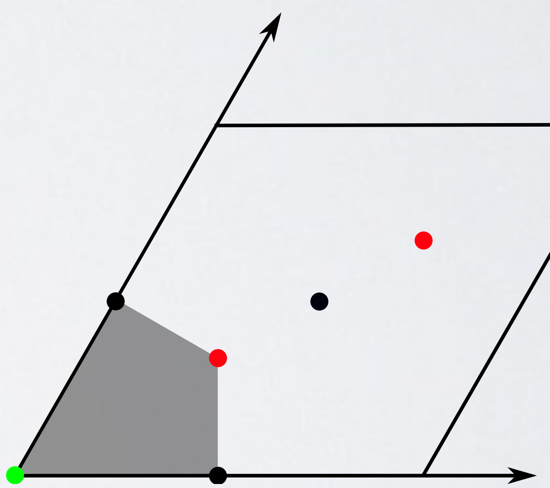
$T^2/\mathbb{Z}_k$:



$k = 3$, $\tau = e^{i\pi/3}$



$k = 4$, $\tau = i$



$k = 6$, $\tau = e^{i\pi/3}$

(Different kinds of singularities for a given k)

OF3s in M/F-theory (II)

- Similarly to $k = 2$, these do not have supersymmetric smoothings.

[Morrison, Stevens ; Anno]

- Preserve twelve supercharges, $\mathcal{N}_{3d} = 6$ or $\mathcal{N}_{4d} = 3$. $(k > 2)$

- ABJM at level $k > 2$ preserves $\mathcal{N}_{3d} = 6$. The lift only works for some values of k, because there has to be a torus in M-theory.

- M-theory geometry admits discrete flux $\longrightarrow$ Different $\mathrm{OF3}_k$

[Aharony, Tachikawa]

OF3s in field theory

- The theory on N D3s probing an $\mathrm{OF3}_k$ should arise as a $\mathbb{Z}_k$ quotient of $4d$ $\mathcal{N}=4$ $\mathfrak{u}(N)$ SYM.

- Just like before, $\mathbb{Z}_k^{\mathrm{OF}} = \mathbb{Z}_k^R \cdot \mathbb{Z}_k^S$ with

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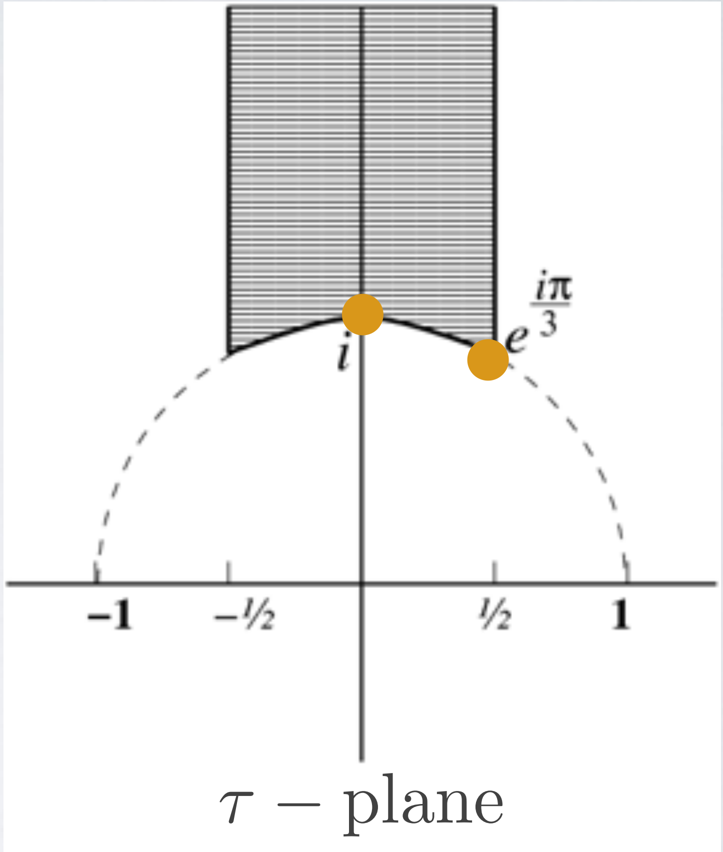
  Restriction in both k and τ. The theory is stuck at strong coupling.



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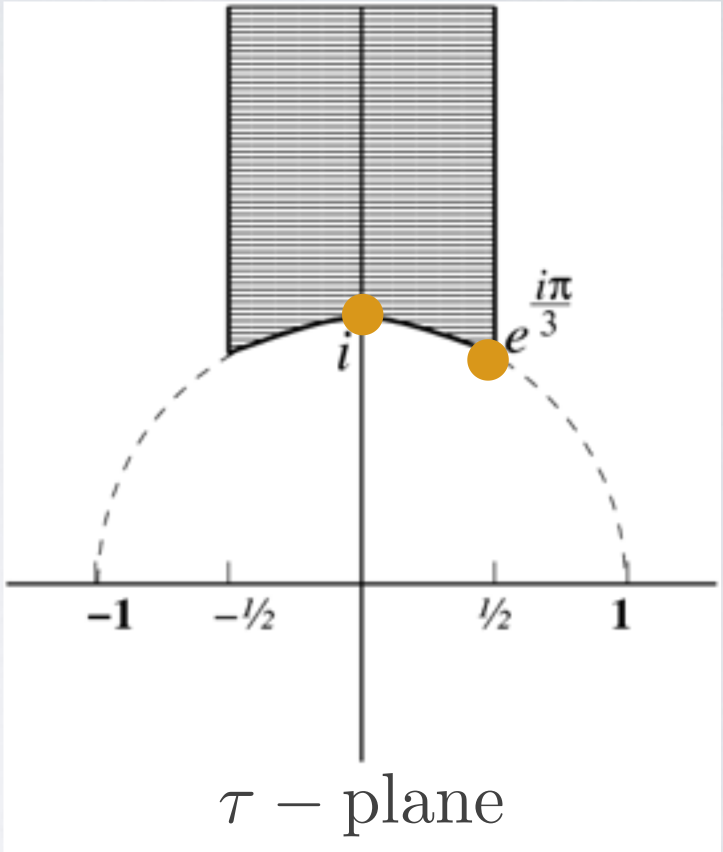
- The action on the supercharges shows that $\mathcal{N}=3$.



$\mathcal{N}=3$ is not always $\mathcal{N}=4$

- Having $\mathcal{N}=3$ in 4d is surprising. There is an argument saying that, in the absence of gravity,

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- Loophole: notion of elementary field (vector). This only applies to Lagrangian theories. [García-Etxebarria, DR; Aharony, Evtikhiev]

- In our case, we have massless "electrons" and "monopoles", so it's reasonable not to have a Lagrangian.

- Actually, since we have $\mathcal{N}=3$, we conclude that it cannot have a Lagrangian description.

Conclusions and outlook

Conclusions:

- We have built the first examples of $\mathcal{N}=3$ field theories in 4d as quotients of $\mathcal{N}=4$ SYM by particular R-symmetry and $SL(2,\mathbb{Z})$ symmetries.
- Only works for specific values of the coupling. Isolated field theories.
- The worldvolume theory of D3s probing OF3s (generalized orientifolds).
- Can be thought of as the 4d version of ABJM (only for some k).
- (Large N limit as a quotient of $AdS_5 \times S^5$ acting on the IIB coupling.)

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- Other $\mathcal{N}=3$ theories? Classification?
- Connection to class S theories [To appear]
- Better understanding in M-theory (BPS states).
- Other (less supersymmetric) isolated singularities.

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Thank you!

Holographic dual (O3)

[Witten]

- The holographic dual can be obtained from the IIB construction.
- Before introducing the O3, we have $N \gg 1$ D3-branes, whose near horizon limit is $AdS_5 \times S^5$. [Maldacena]
- The geometric action $\mathbb{Z}_2$ gives IIB on $AdS_5 \times S^5/\mathbb{Z}_2$ (which is not supersymmetric by itself).
- There is also an $SL(2,\mathbb{Z})$ bundle on $S^5/\mathbb{Z}_2$ (now supersymmetric).

  ↑ $\mathbb{Z}_2$
- The near horizon geometry in F-theory is $AdS_5 \times (S^5 \times T^2)/\mathbb{Z}_2$.
- The different O3 variants are given by turning on discrete fluxes $[H_3, F_3] \in H^3(S^5/\mathbb{Z}_2, \tilde{\mathbb{Z}}) = \mathbb{Z}_2$

Holographic dual (OF3)

- Just like for the usual O3, we can derive it from the IIB construction.
- Before introducing the OF3, we have N D3-branes, whose near horizon limit is $AdS_5 \times S^5$.
- In the presence of an OF3, we have Type IIB on $AdS_5 \times S^5/\mathbb{Z}_k$ with an $SL(2,\mathbb{Z})$ bundle. Or F-theory on $AdS_5 \times (S^5 \times T^2)/\mathbb{Z}_k$.

  ↑ $\mathbb{Z}_k$

  [Aharony, Tachikawa]

- We see that:
  - Smooth, weakly curved geometry.
  - Stuck at strong string coupling. No marginal deformation in the CFT.

Other results in the literature

- [Aharony, Evtikiev] General properties of 4d N=3 SCFTs, assuming they exist. Many results: no (N=3 preserving) relevant or marginal deformations, a=c, etc.

- [García-Etxebarria, DR] First examples of 4d N=3.

- [Nishinaka, Tachikawa] Rank-one 4d N=3 theories. Moduli space is C^3/Z_k for k=3,4,6. Compute the central charge. 2d chiral algebra.

- [Argyres, Lolito, Lü, Martone] Classification of 4d N=2 SCFTs. The N=3 theories seem to fit in their classification.

- [Aharony, Tachikawa] Classification of the different OF3 variants. Large N limit, discrete gaugings.

- [Imamura, Yokoyama] Superconformal index (large N).

- [Imamura et al. ; Agarwal et al.] N=3 to N=4 enhancement.