$e^+ e^- \to$ 3 jets in ColorFulNNLO

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Introduction

Introduction

- α_s is a fundamental parameter of SM $\Longrightarrow$ most precise measurement is essential
- α_s is obtained through fitting $\Longrightarrow$ *quality* predictions are needed
- One way: from 3-jet *event shapes* in $e^+ e^-$ collisions
- In 3-jet production the Born is *proportional* to α_s
- All SMEs are known to carry out a calculation in NNLO QCD
- NNLO QCD computations are already performed[1] [2] (useful for cross-check and validation)
- Extensively measured by multiple collaborations in the past

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[1] Gehrmann-De Ridder et al. JHEP **0712** (2007) 094

[2] Weinzierl JHEP **0906** (2009) 041

The CoLoRFulNNLO method[3]

[3] Del Duca, Somogyi and Trócsányi

The CoLoRFulNNLO method

- Idea: do NNLO calculations as it was done with NLO: use *local* subtractions
- The NLO correction for a J jet function:

$$\sigma^{\mathrm{NLO}}[J] = \int_{m+1} \left[\mathrm{d}\sigma^{\mathrm{R}}_{m+1} J_{m+1} - \mathrm{d}\sigma^{\mathrm{R,A_1}}_{m+1} J_m \right]_{d=4} + \int_m \left[\mathrm{d}\sigma^{\mathrm{V}}_m J_m + \int_1 \mathrm{d}\sigma^{\mathrm{R,A_1}}_{m+1} J_m \right]_{d=4}$$

- The NNLO correction is composed of three contributions:

$$\sigma^{\mathrm{NNLO}}[J] = \int_{m+2} \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} + \int_{m+1} \mathrm{d}\sigma^{\mathrm{RV}}_{m+1} J_{m+1} + \int_m \mathrm{d}\sigma^{\mathrm{VV}}_m J_m$$

- Kinematic singularities in the $m+2$ and $m+1$ parton contribution
  - $m+2$: doubly and singly unresolved emission
  - $m+1$: singly unresolved radiation $\oplus$ ϵ poles from $m+1$ parton one-loop

- In the $m+2$ parton line subtractions are needed to regularize 1- and 2-parton emissions:

$$\sigma^{\mathrm{NNLO}}_{m+2} = \int_{m+2} \left\{ \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} J_m - \left[\mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} J_{m+1} - \mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2} J_m \right] \right\}_{d=4}$$

- A_1 and A_2 develop additional singularities $\implies$ A_{12} is needed to cancel
- In the $m+1$ line 1-parton emissions like in NLO but for one-loop-tree interference:

$$\sigma^{\mathrm{NNLO}}_{m+1} = \int_{m+1} \left\{ \left(\mathrm{d}\sigma^{\mathrm{RV}}_{m+1} + \int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right) J_{m+1} - \left[\mathrm{d}\sigma^{\mathrm{RV,A_1}}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right)^{\mathrm{A_1}} \right] J_m \right\}_{d=4}$$

- It contains the integrated A_1 from $m+2$ which is still singular $\implies$ subtraction is needed (last term)
- The m parton line contains the double virtual and integrated subtractions:

$$\sigma^{\mathrm{NNLO}}_{m} = \int_{m} \left\{ \mathrm{d}\sigma^{\mathrm{VV}}_{m} + \int_2 \left[\mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} - \sigma^{\mathrm{RR,A_{12}}}_{m+2} \right] + \int_1 \left[\mathrm{d}\sigma^{\mathrm{RV,A_1}}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right)^{\mathrm{A_1}} \right] \right\}_{d=4} J_m$$

- The scheme is completely general and worked out for processes with colorless inital state
- It is implemented in a `fortran90` program: `MCCSM` (Monte Carlo for the Colorful Subtraction Method) (AK)
  - Completely general
  - User friendly and flexible
  - Phase space is recursively constructed
  - Only needs the SMEs, color- and spin-correlated ones (having local subtractions)
- The implementation needed validation and cross-check $\Longrightarrow$ $e^+ e^- \to \mathbf{3\,jet}$ is a perfect candidate

$e^+\,e^- \to 3\,\mathrm{jet}$ **production**

$e^+\,e^- \to 3\,\text{jet}$ **production**

- Computation was done at former LEP2 energy, $\sqrt{s} = Q = 91.2\,\text{GeV}$
- Six standard event shapes were calculated[4] [5], compared to Gehrmann et al. and Weinzierl
- Three event shapes are computed at NNLO QCD for the *first time*
- The used normalization for an event shape O:

$$\frac{O}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}O} = \frac{\alpha_\text{S}}{2\pi} O\,A(O) + \left(\frac{\alpha_\text{S}}{2\pi}\right)^2 O\,B(O) + \left(\frac{\alpha_\text{S}}{2\pi}\right)^3 O\,C(O) + \mathcal{O}(\alpha_\text{S}^4)$$

σ_0: LO cross section for $e^+\,e^- \to \text{hadrons}$

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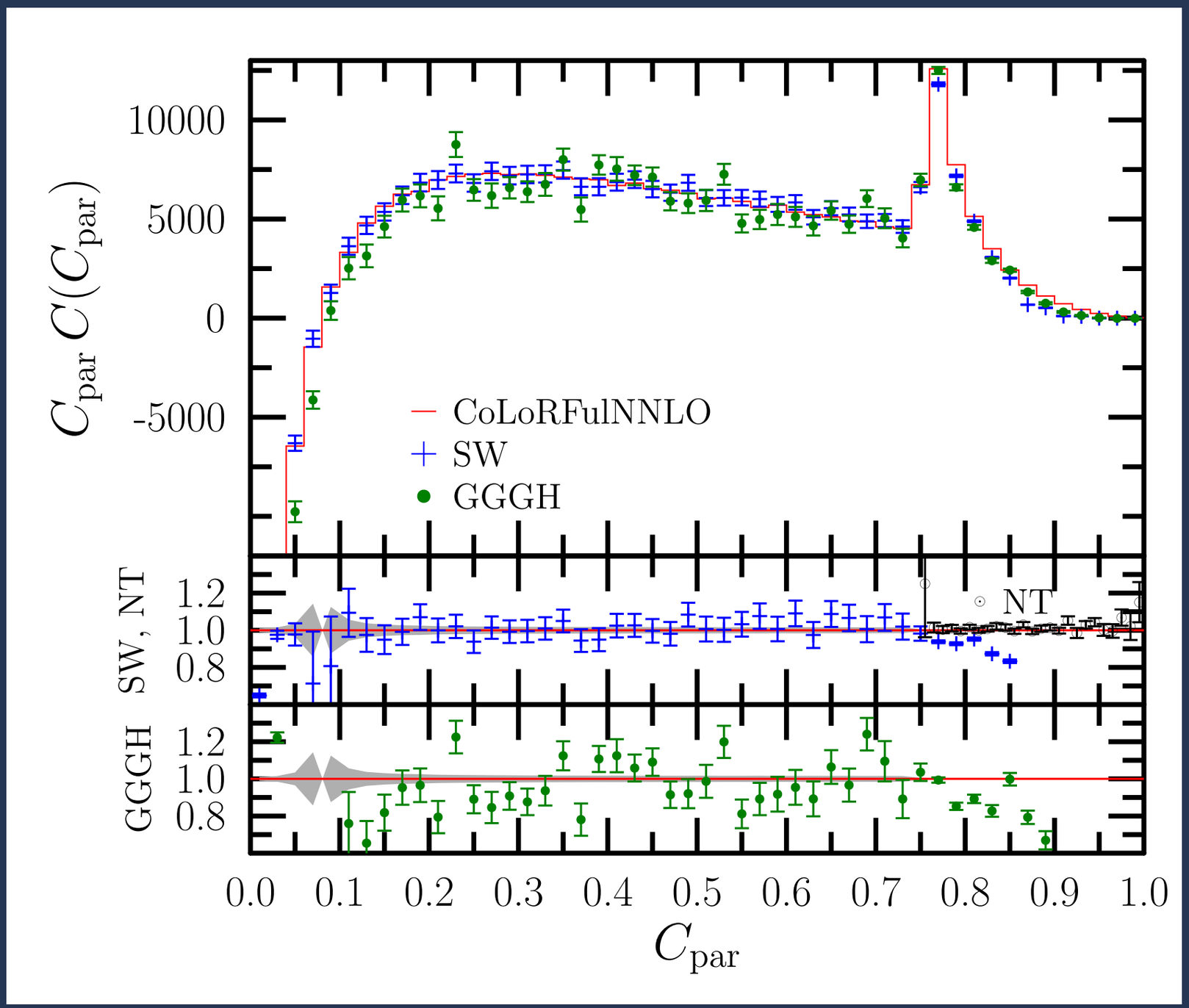
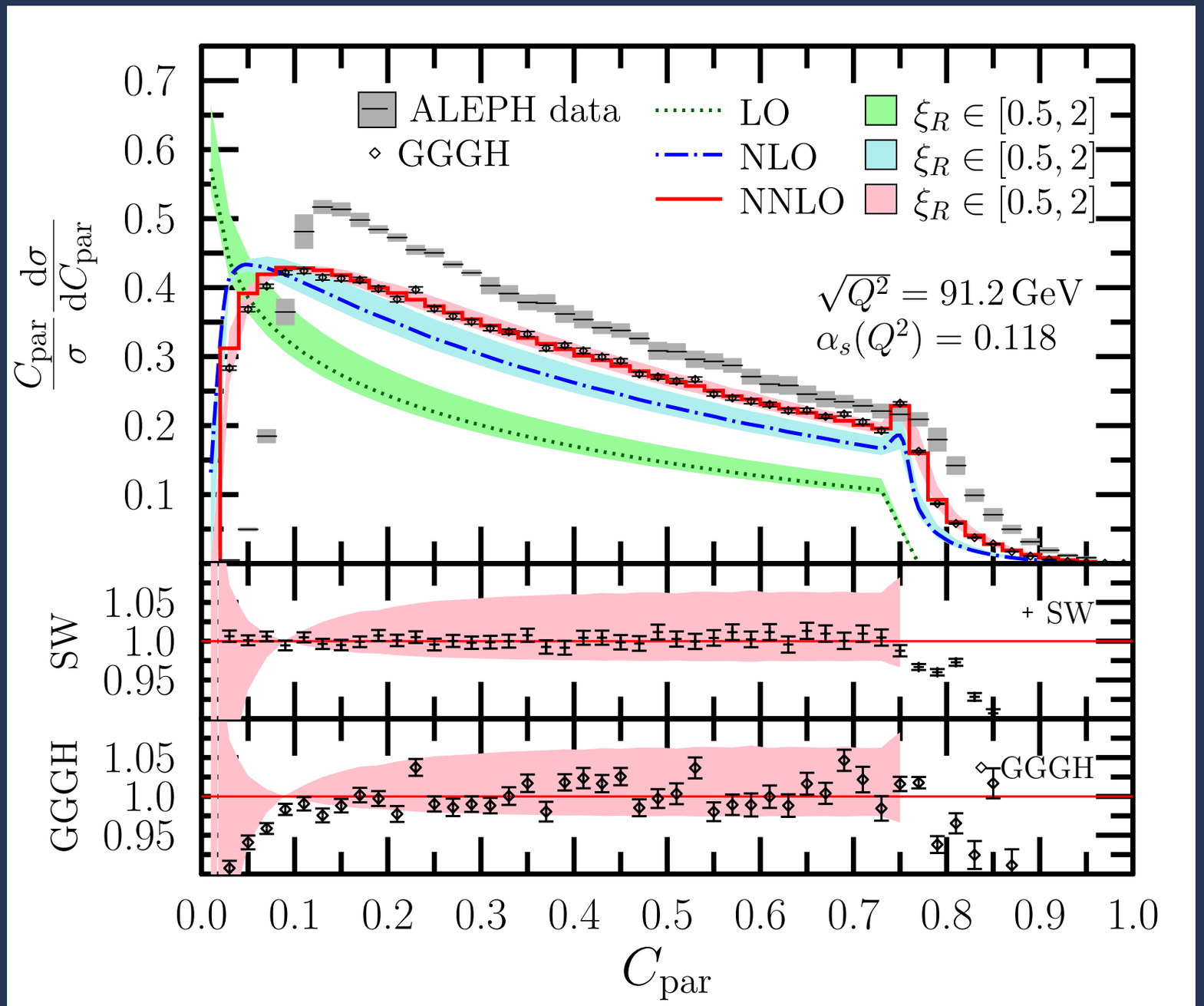
[4] Del Duca, Duhr, AK, Somogyi et al. arXiv:1603.08927, to appear in PRL

[5] Del Duca, Duhr, AK, Somogyi et al. arXiv:1606.03453, to appear in PRD



NNLO QCD correction to C-parameter distribution



Physical prediction to C-parameter distribution

New predictions

- New predictions made for oblateness, energy-energy correlation and jet cone energy fraction
- Oblateness:

$$O = T_M - T_m$$

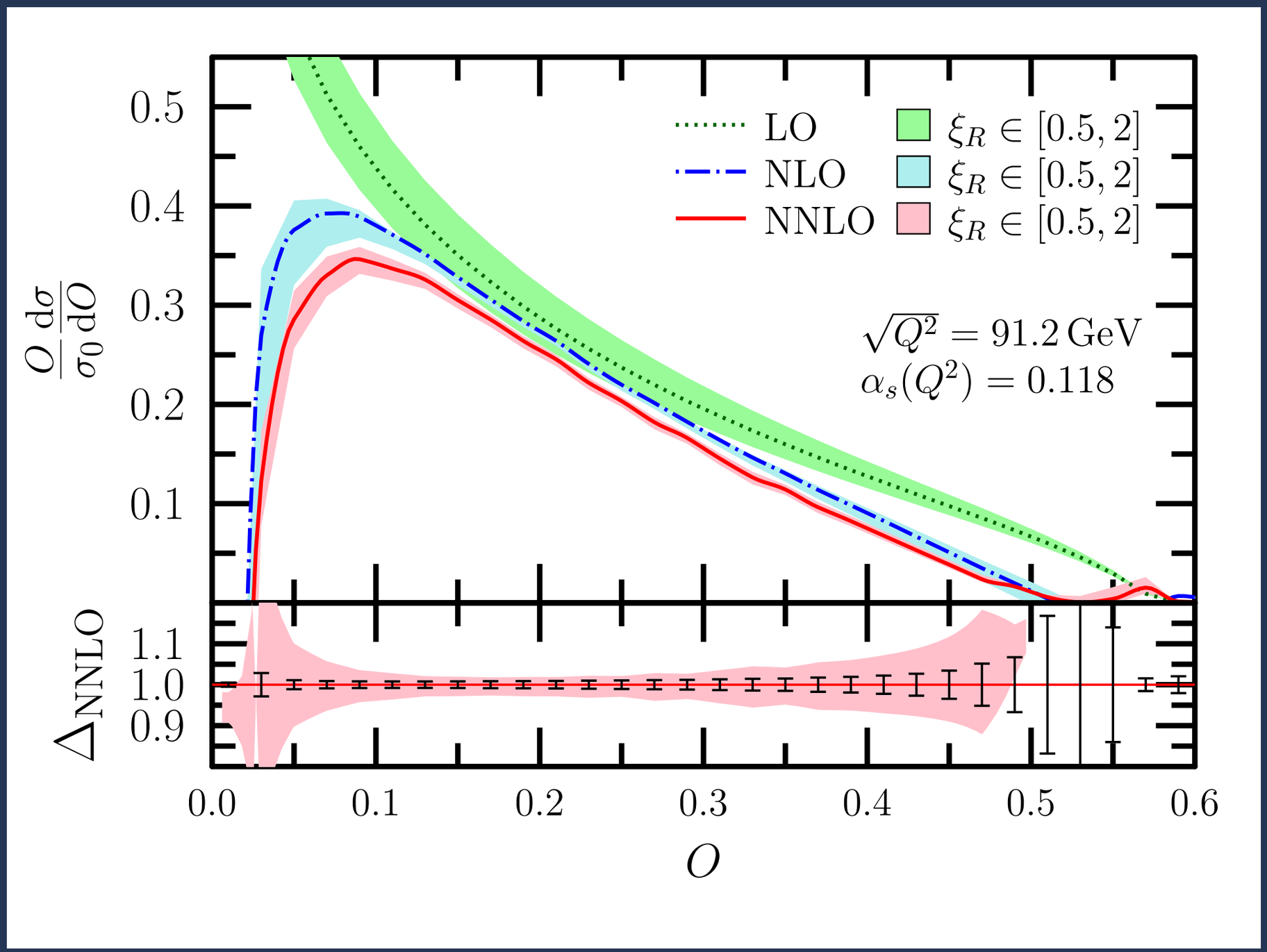
- Energy-energy correlation:

$$\frac{1}{\sigma_{\mathrm{had}}}\frac{\mathrm{d}\Sigma_{\mathrm{EEC}}}{\mathrm{d}\cos\chi} = \frac{1}{\sigma_{\mathrm{had}}}\sum_{i,j}\int \mathrm{d}\sigma_{e^+\,e^-\to i\,j+X}\frac{E_i E_j}{Q^2}\delta(\cos\chi + \cos\theta_{ij})$$

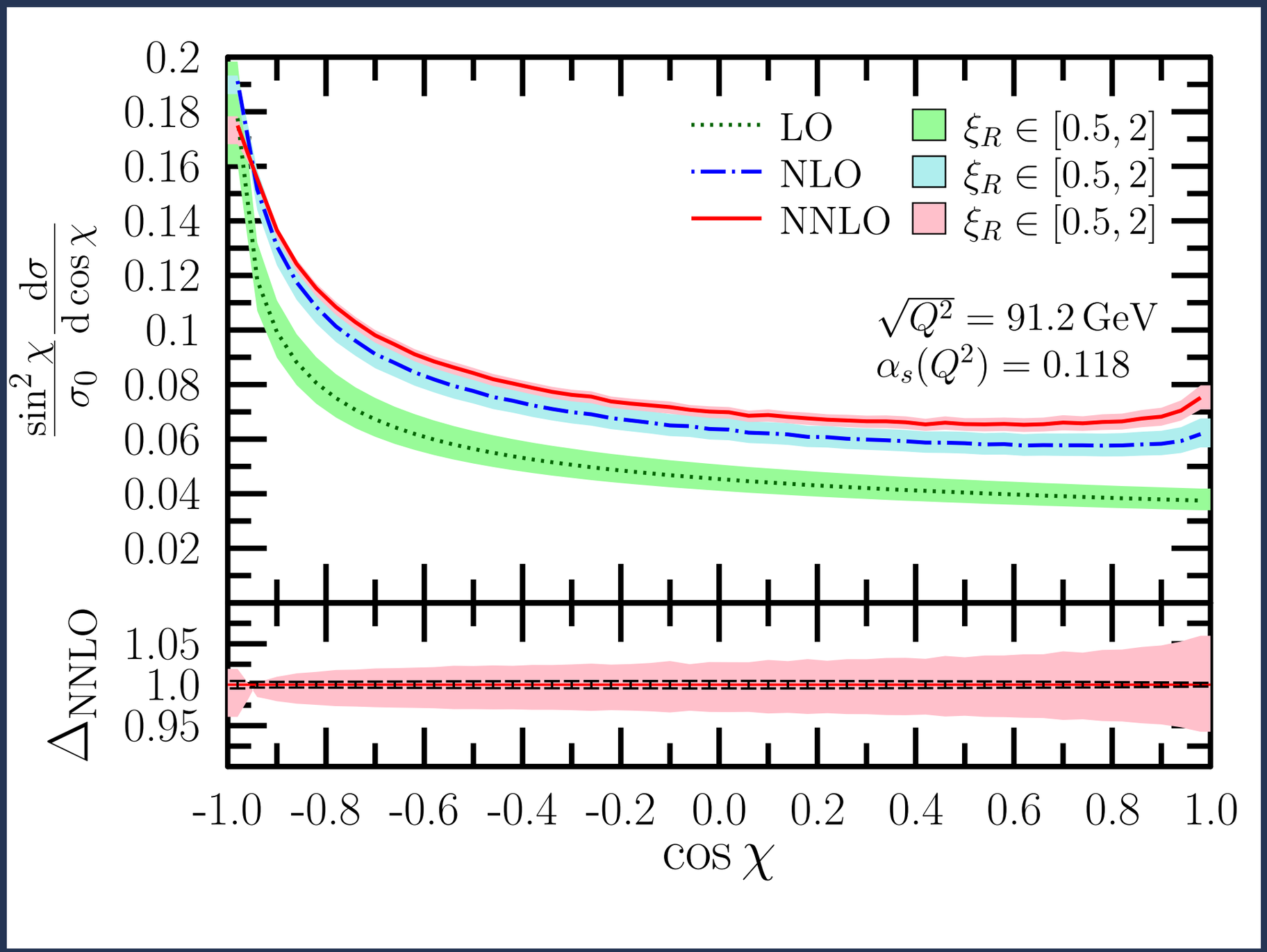
- Jet cone energy fraction:

$$\frac{\mathrm{d}\Sigma_{\mathrm{JCEF}}}{\mathrm{d}\cos\chi} = \sum_i \int \mathrm{d}\sigma_{e^+\,e^-\to i+X}\frac{E_i}{Q}\delta\left(\cos\chi - \frac{\vec{p}_i\cdot\vec{n}_T}{|\vec{p}_i|}\right)$$

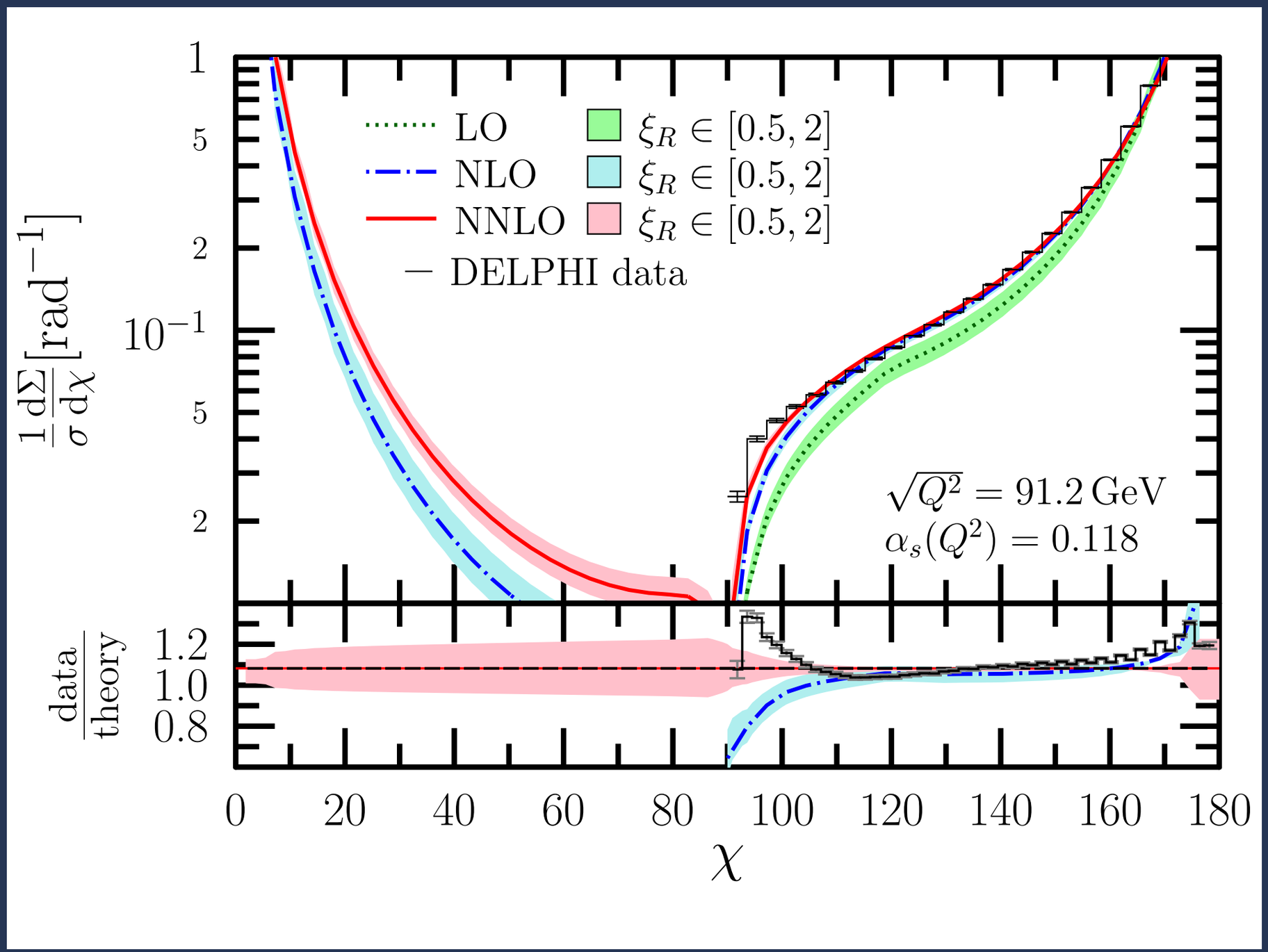
T_M: thrust major, T_m: thrust minor, σ_{had}: total hadronic XS, $\vec{n}_T$: thrust axis pointing from heavy jet mass hemisphere



Physical prediction to oblateness



Physical prediction to energy-energy correlation



Physical prediction to jet cone energy fraction

Conclusions

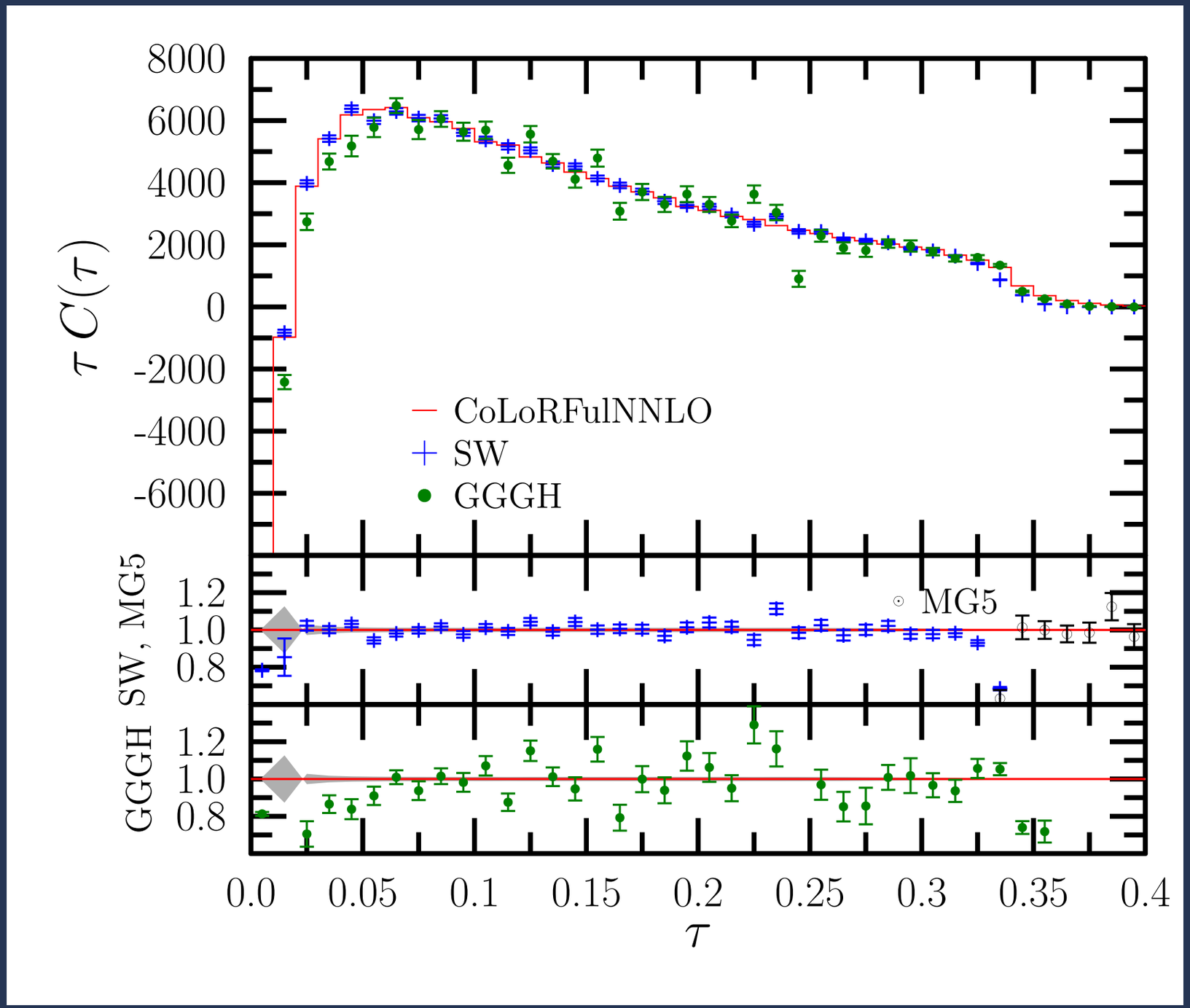
Conclusions

- The CoLorFulNNLO subtraction method is briefly introduced
- The scheme is fully worked out for colorless initial states
- Application to $e^+ e^- \to \mathbf{3\,jet}$ is shown
- Validated through standard event shape variables
- New predictions at NNLO QCD for oblateness, energy-energy correlation and jet cone energy fraction
- The extension to hadron colliders is in progress

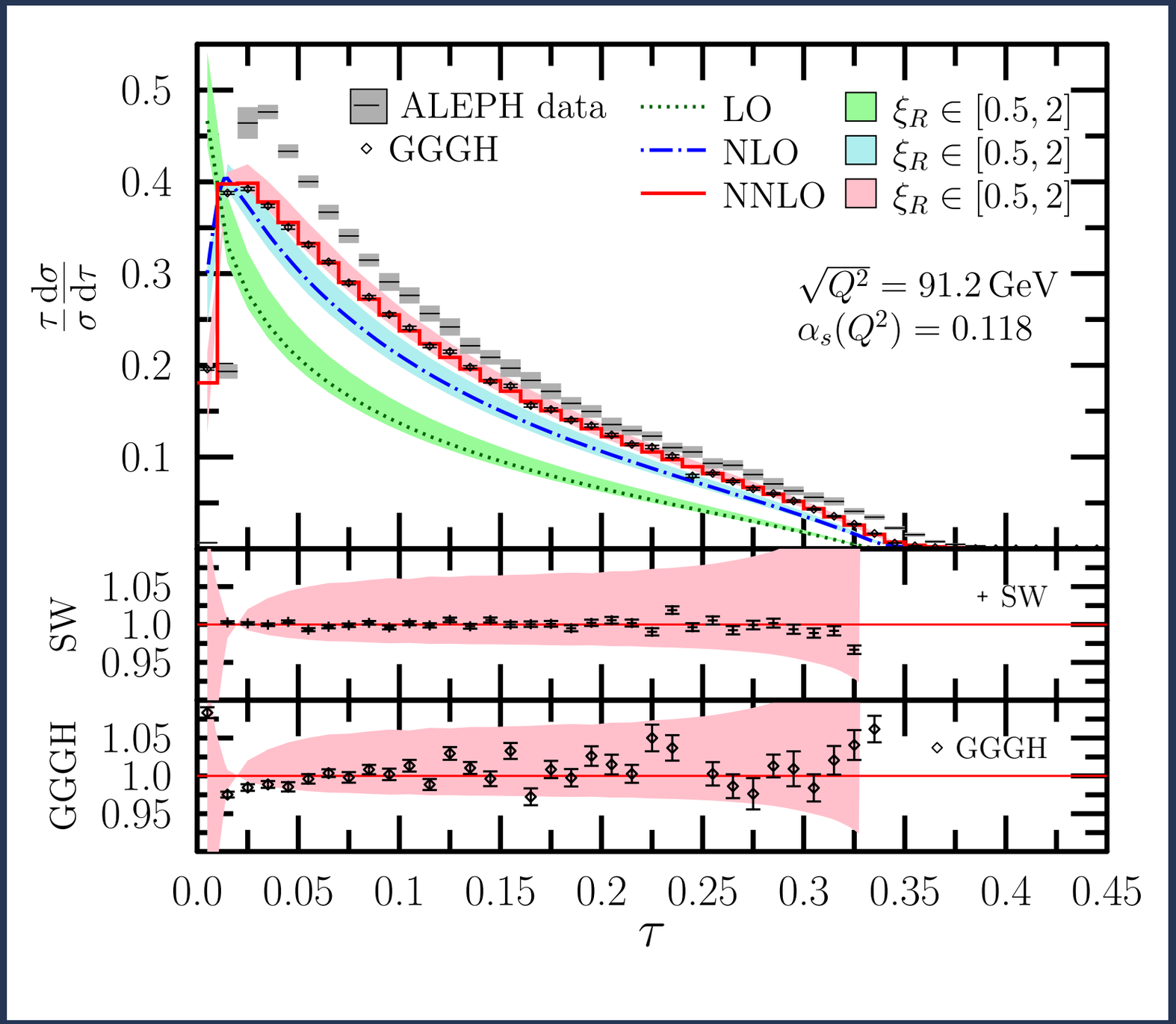
Thank you for your attention!

Extra slides

The $1 - T$ distribution

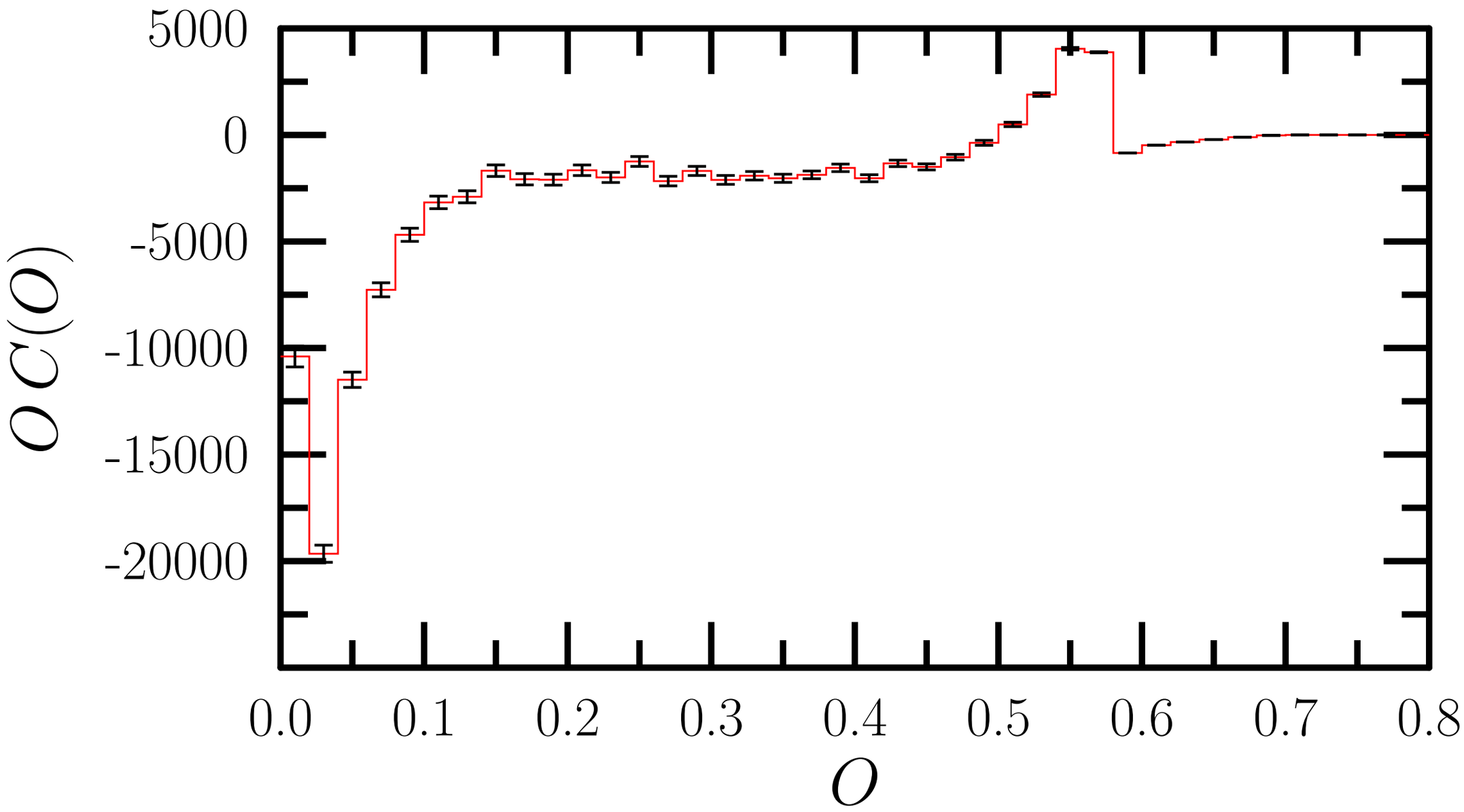


NNLO QCD correction to $1 - T$ distribution

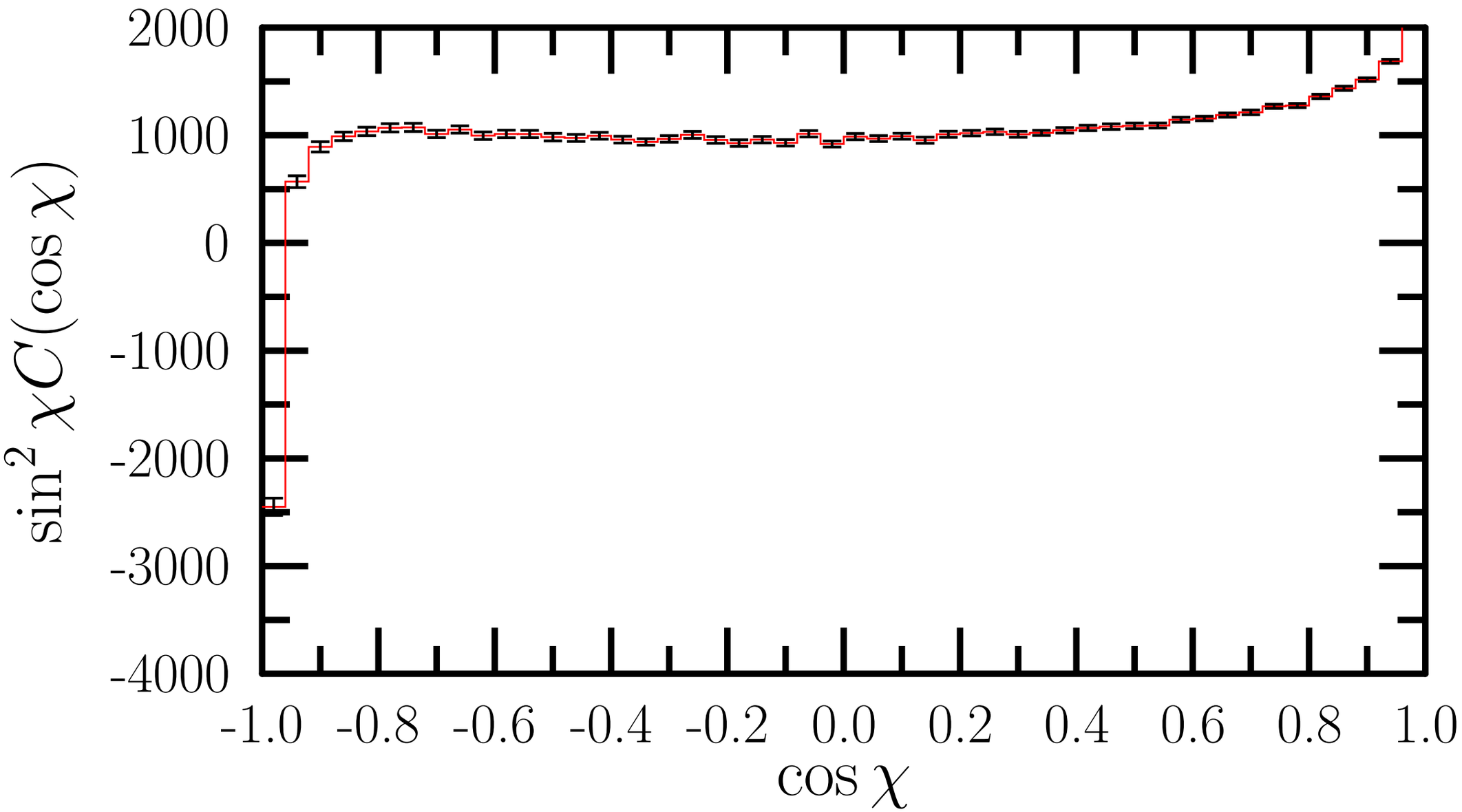


Physical prediction to $1 - T$ distribution

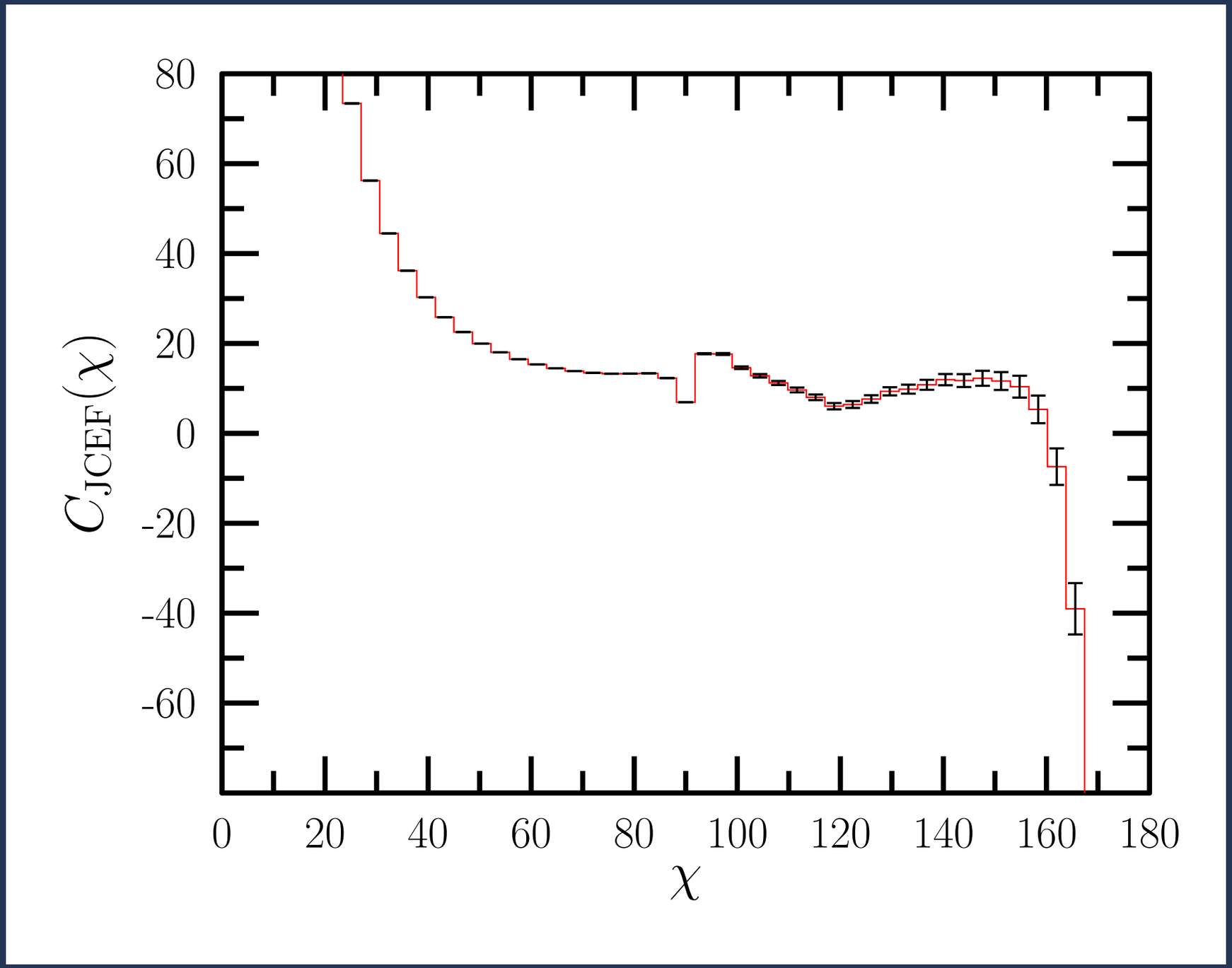
NNLO QCD corrections to "new" event shapes



NNLO QCD correction to oblateness



NNLO QCD correction to energy-energy correlation



NNLO QCD correction to jet cone energy fraction