Scale-invariant inflationary models: A geometrical interpretation

Javier Rubio

based on G. Karananas, JR, Phys.Lett. B 761 (2016) 223-228

UNIVERSITÄT HEIDELBERG
Zukunft. Seit 1386.

Rethinking Quantum Field Theory
DESY, Hamburg

Flatness from scale invariance

$$x^\mu \to \alpha^{-1} x^\mu \qquad \Phi_i(x) \to \alpha^{d_i} \Phi_i(\alpha^{-1} x)$$

J-frame

$$\frac{\mathcal{L}}{\sqrt{g}} = \frac{\xi_h h^2}{2} R - \frac{1}{2}(\partial h)^2 - \frac{\lambda}{4} h^4$$

with $\xi_h > 0$ and $\lambda > 0$

E-frame

$$\frac{\mathcal{L}}{\sqrt{g}} = \frac{M_P^2}{2} R - \frac{1}{2}(\partial \phi)^2 - \frac{\lambda M_P^4}{4\xi_h^2} \, .$$

with $\phi = -\frac{M_P}{2\sqrt{|\kappa_c|}} \log \frac{M_P^2}{\xi_h h^2}$ $\qquad \kappa_c \equiv -\frac{\xi_h}{1 + 6\xi_h}$

No graceful inflationary exit

Two perspectives

Classical
but not
Quantum

Classical
and
Quantum
(with SSB)

Meissner, Nicolai
Wetterich
Iso, Okada, Orikasa
Boyle, Farnsworth, Fitzgerald, Schade
Salvio, Strumia ….

W.A.Bardeen,
Englert, Truffin, Gastmans, 1976
Zenhäusern, Shaposhnikov
Armillis, Monin, Shaposhnikov
Gretsch, Monin
Herrero-Balea
D.M. Ghilencea, Z. Lalak, P. Olszewski …

“Simplest” two-field SI model

All the scales are generated by SSB of global scale invariance

$$\frac{\mathcal{L}}{\sqrt{g}} = \frac{1}{2}\left(\xi_\chi \chi^2 + \xi_h h^2\right) R - \frac{1}{2}(\partial h)^2 - \frac{1}{2}(\partial \chi)^2 - U(h,\chi)$$

$$\text{with} \quad U(h,\chi) = \frac{\lambda}{4}\left(h^2 - \alpha \chi^2\right)^2$$

The dilaton is the new mass donor
It gives mass to the Higgs and defines the Planck scale.

A singlet under the SM group
No couplings with SM particles

We *won't try to be UV complete*
We treat *this as a low-energy effective theory*

M. Shaposhnikov, D. Zenhausern, Phys.Lett. B671 (2009) 187-192
J. García-Bellido, JR, M. Shaposhnikov, D. Zenhausern, Phys.Rev. D84 (2011) 123504

Inflationary observables

Scalar spectral tilt

$$n_s\,(k^*) \simeq 1 - 8\xi_\chi \coth\left(4\xi_\chi N^*\right)$$

Amplitude

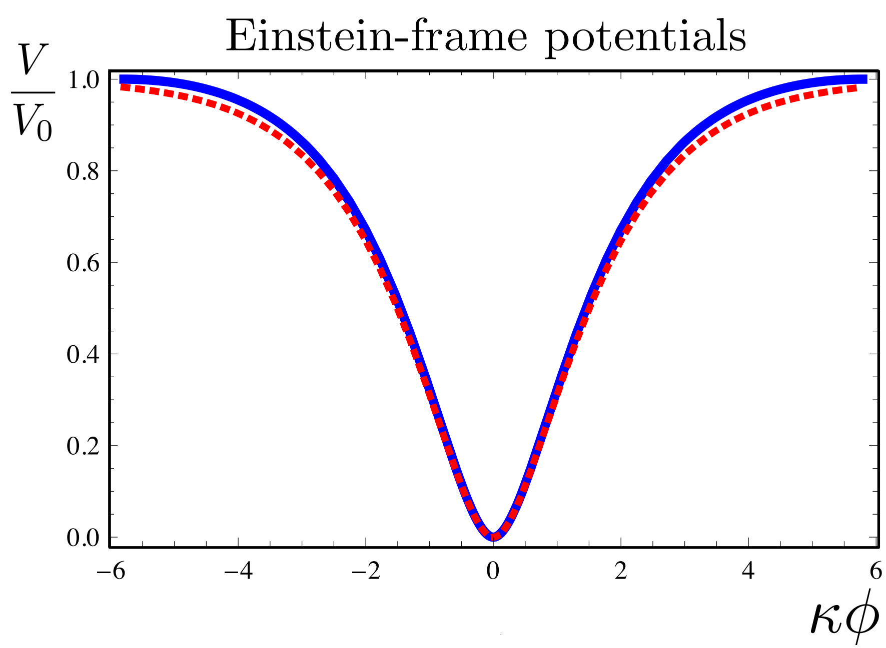
$$\Delta^2_\zeta(k^*) \simeq \frac{\lambda \sinh^2\left(4\xi_\chi N^*\right)}{1152\pi^2\xi_\chi^2\xi_h^2}$$

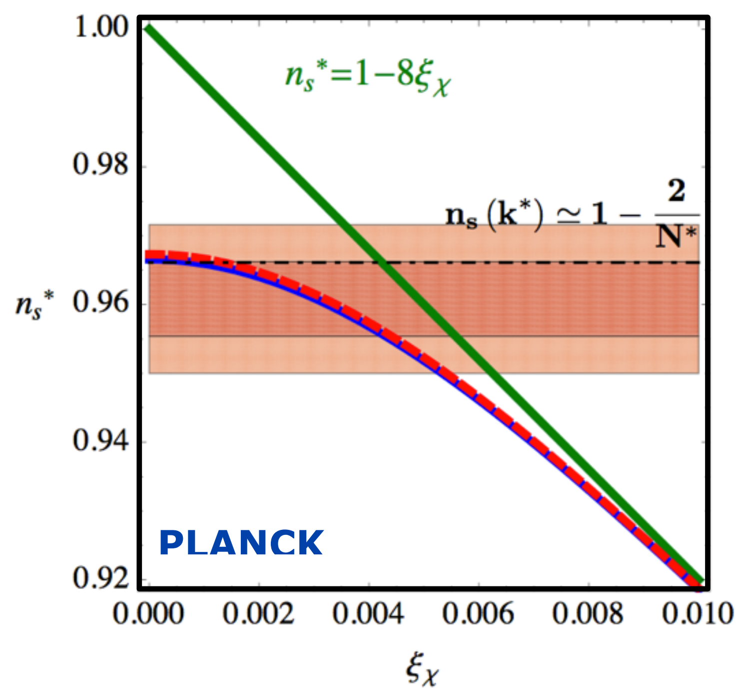
Running of the tilt

$$\alpha_\zeta(k^*) \simeq -32\xi_\chi^2 \sinh^{-2}\left(4\xi_\chi N^*\right)$$

Tensor-to-scalar ratio

$$r(k^*) \simeq 192\xi_\chi^2 \sinh^{-2}\left(4\xi_\chi N^*\right)$$





J. García-Bellido, JR, M. Shaposhnikov, D. Zenhausern, Phys.Rev. D84 (2011) 123504
See also: Manuel Trashorras et al. Phys.Rev. D94 (2016) no.6, 063511

The dilaton is introduced ad hoc

Can it appear “naturally”?

Dilaton as part of the metric

- **The minimal gauge group required to construct a metric theory including spin-2 polarizations is TDiff**

$$x^\mu \mapsto \tilde{x}^\mu(x), \text{ with } J \equiv \left|\frac{\partial \tilde{x}^\mu}{\partial x^\nu}\right| = 1 \qquad with \qquad \delta x^\mu = \xi^\mu, \quad \partial_\mu \xi^\mu = 0$$

- **A TDiff theory is not a scalar-tensor theory but rather UG plus a propagating TDiff scalar: the metric determinant g (exceptions: GR & UG).**

- **The TDiff action contain arbitrary (theory-defining) functions of g**

$$\frac{\mathcal{L}_{\text{TDiff}}}{\sqrt{g}} = \frac{\rho^2 f(g)}{2}R - \frac{1}{2}\rho^2 G_{gg}(g)(\partial g)^2 - \frac{1}{2}G_{\rho\rho}(g)(\partial\rho)^2 - G_{\rho g}(g)\rho\,\partial g\cdot\partial\rho - \rho^4 v(g)$$

$$\text{invariant under} \qquad g_{\mu\nu}(x) \mapsto g_{\mu\nu}(\lambda x) \qquad \rho(x) \mapsto \lambda\rho(\lambda x)$$

- **A equivalent Diff version can be obtained using the Stückelberg trick**

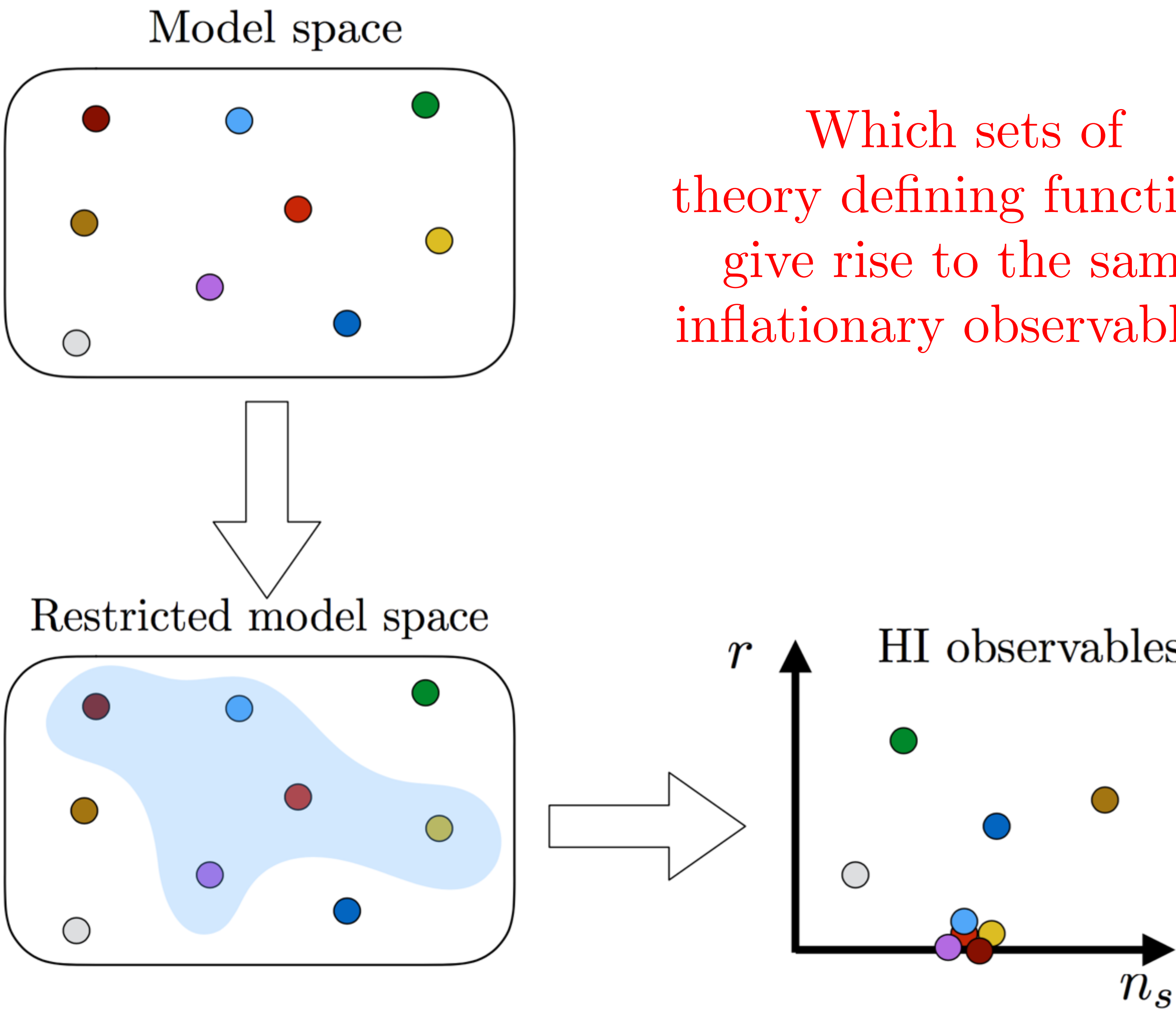
$$\frac{\mathcal{L}_{\text{Diff}}}{\sqrt{g}} = \frac{\rho^2 f(\theta)}{2}R - \frac{1}{2}\rho^2 G_{gg}(\theta)(\partial\theta)^2 - \frac{1}{2}G_{\rho\rho}(\theta)(\partial\rho)^2 - G_{\rho g}(\theta)\rho\,\partial\theta\cdot\partial\rho - \rho^4 v(\theta)$$

$$\text{invariant under} \qquad g_{\mu\nu}(x) \mapsto g_{\mu\nu}(\lambda x) \qquad \rho(x) \mapsto \lambda\rho(\lambda x) \qquad \theta(x) \mapsto \theta(\lambda x)$$

Goldstone

D. Blas, M. Shaposhnikov, D. Zenhausern Phys.Rev. D84 (2011) 044001

SI TDiff theory contains a massless dilaton.



Which sets of theory defining functions give rise to the same inflationary observables?

Revisiting the simplest model

J-frame

$$\frac{\mathcal{L}}{\sqrt{g}} = \frac{1}{2}\left(\xi_\chi \chi^2 + \xi_h h^2\right) R - \frac{1}{2}(\partial h)^2 - \frac{1}{2}(\partial \chi)^2 - U(h,\chi)$$

with $U(h,\chi) = \frac{\lambda}{4}\left(h^2 - \alpha\chi^2\right)^2$

Conformal transformation + **field redefinition**

$$\tilde{g}_{\mu\nu} = M_P^{-2}(\xi_\chi \chi^2 + \xi_h h^2) g_{\mu\nu}$$

$$r^2 = \xi_h h^2 + \xi_\chi \chi^2$$

$$\theta^{-1} = 1 + \frac{\xi_h h^2}{\xi_\chi \chi^2}$$

E-frame

$$\frac{\mathcal{L}}{\sqrt{g}} = \frac{M_P^2}{2} R - \frac{M_P^2}{2}\Big[K_{\theta\theta}(\theta)(\partial\theta)^2 + 2K_{\theta r}(\theta)(\partial\theta)(\partial \log r/M_P) + K_{rr}(\theta)(\partial \log r/M_P)^2\Big] - V(\theta)$$

Inflationary dynamics

$$\log\frac{r}{M_P} \to \log\frac{r}{M_P} - \varphi(\theta)\ , \qquad \text{with} \qquad \varphi'(\theta) = \frac{K_{\theta r}(\theta)}{K_{rr}(\theta)}$$

After diagonalization

$$\frac{\mathcal{L}}{\sqrt{g}} = \frac{M_P^2}{2}R - \frac{M_P^2}{2}\left[K(\theta)(\partial\theta)^2 + K_{rr}(\theta)(\partial\log r/M_P)^2\right] - V(\theta)$$

$$\text{with} \qquad K(\theta) = \frac{1}{4\theta(\theta-\zeta)}\left[6 - \frac{1+6\kappa_0}{\kappa_0}\frac{1}{1-\theta}\right]$$

Inflationary pole — **Unreachable pole** — **Minkowski pole**

K_{rr} as dynamical variable

Using $\theta(K_{rr})$ and rescaling the dilaton field as

$$\rho \equiv \sqrt{\left|\frac{\kappa_0}{\kappa_c}\right|} \log \frac{r}{M_P} \qquad K_{\rho\rho}(\theta) \equiv \left|\frac{\kappa_c}{\kappa_0}\right| K_{rr}(\theta)$$

we get

$$\frac{\mathcal{L}}{\sqrt{g}} = \frac{M_P^2}{2} R - \frac{M_P^2}{2}\left[- \frac{(\partial K_{\rho\rho})^2}{4\, K_{\rho\rho}(\kappa_0 K_{\rho\rho} + c)} + K_{\rho\rho}(\partial\rho)^2 \right] - V(K_{\rho\rho})$$

with $\kappa_0 \equiv -\dfrac{\xi_h - \xi_\chi}{1 + 6\xi_h}$ $\qquad c \equiv c(\kappa, \kappa_0) \propto \xi_\chi > 0$

$K_{\rho\rho} > 0 \qquad \kappa_0 K_{\rho\rho} + c < 0$

The meaning of κ_0

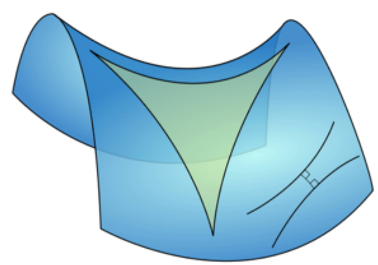
κ_0 is the Gaussian curvature (in units of M_P) of the manifold spanned by $\varphi_1 \equiv K_{\rho\rho}$ and $\varphi_2 \equiv \rho$

$$\frac{\mathcal{L}}{\sqrt{g}} = \frac{M_P^2}{2} R - \frac{1}{2} \gamma_{ab}(\varphi_1) g^{\mu\nu} \partial_\mu \varphi^a \partial_\nu \varphi^b - V(\varphi_1)$$

During inflation, the field space of the simplest Higgs-Dilaton model is

MAXIMALLY SYMMETRIC

$$\kappa_0 \equiv -\frac{\xi_h - \xi_\chi}{1 + 6\xi_h}$$

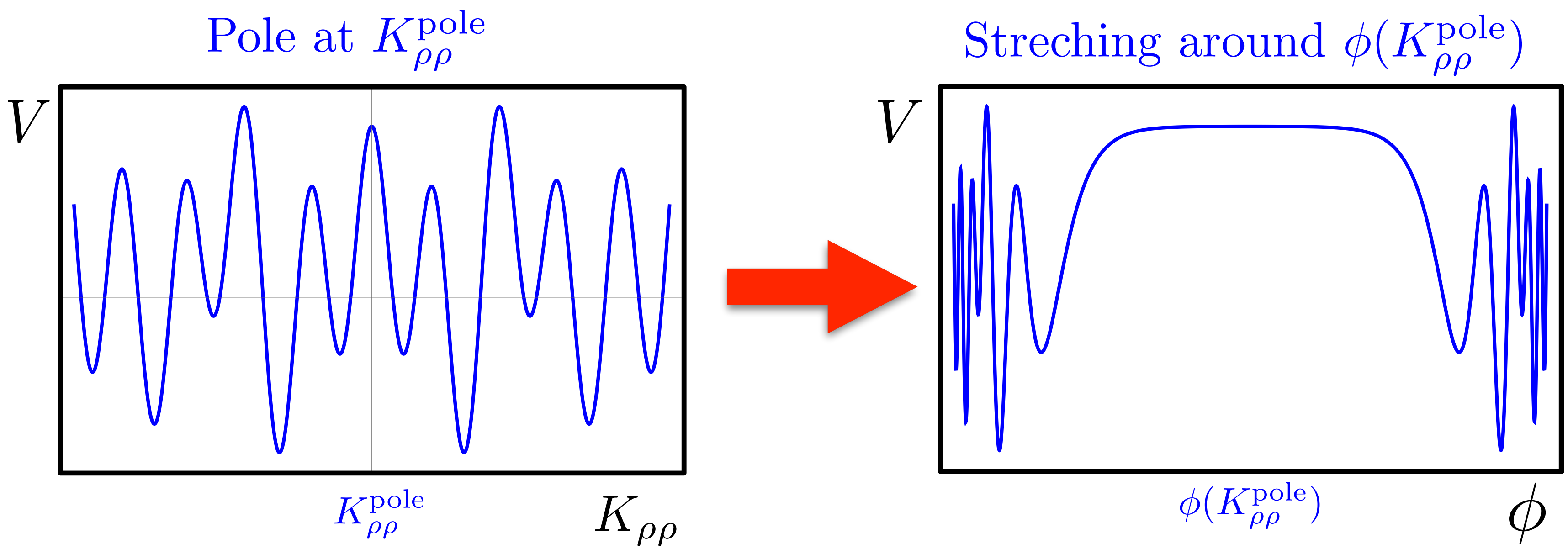


Canonical field stretching

$$\frac{\mathcal{L}}{\sqrt{g}} = \frac{M_P^2}{2} R - \frac{M_P^2}{2} \left[- \frac{(\partial K_{\rho\rho})^2}{4\, K_{\rho\rho}(\kappa_0 K_{\rho\rho} + c)} + K_{\rho\rho} (\partial \rho)^2 \right] - V(K_{\rho\rho})$$

Canonically normalized field

$$\phi = \int \frac{d K_{\rho\rho}}{\sqrt{4\ K_{\rho\rho}(|\kappa_0| K_{\rho\rho} - c)}}$$



G. Karananas, JR, Phys.Lett. B761 (2016) 223-228
See also M. Galante, R. Kallosh, A. Linde, D. Roest, Phys.Rev.Lett. 114 (2015) no.14, 141302

Inflationary observables

$$\frac{\mathcal{L}}{\sqrt{g}} = \frac{M_P^2}{2} R - \frac{M_P^2}{2}\left[- \frac{(\partial K_{\rho\rho})^2}{4\, K_{\rho\rho}(\kappa_0 K_{\rho\rho} + c)} + K_{\rho\rho}(\partial\rho)^2 \right] - V(K_{\rho\rho})$$

$K_{\rho\rho} > 0 \qquad \kappa_0 K_{\rho\rho} + c < 0$

$\xi_\chi \to 0,\ \lvert c\rvert \to 0$ $K_{\rho\rho} \to \lvert c/\kappa_0\rvert \to 0$	Quadratic pole HI predictions $n_s \simeq 1 - \frac{2}{N} \qquad r \simeq \frac{2}{\lvert\kappa_0\rvert N^2}$
$\xi_\chi \neq 0,\ \lvert c\rvert \neq 0$ $K_{\rho\rho} = 0$ unreachable	Linear pole $\mathcal{O}\left(c^2/\kappa_0\right)$ $n_s \approx 1 - 4\lvert c\rvert \qquad r \approx 32\lvert c\rvert^2\, e^{-4\,\lvert c\rvert\, N}$

G. Karananas, JR, Phys.Lett. B761 (2016) 223-228

A universality class

Any TDiff embedding of the Higgs-Dilaton idea

$$\frac{\mathcal{L}}{\sqrt{g}} = \frac{M_P^2}{2}R - \frac{M_P^2}{2}\left[K_{\theta\theta}(\theta)(\partial\theta)^2 + 2K_{\theta\rho}(\theta)(\partial\theta)(\partial\log\rho/M_P) + K_{\rho\rho}(Z)(\partial\log\rho/M_P)^2\right] - V(\theta)$$

constructed out of a sufficiently well-behaved potential and (arbitrary) functions $K_{\theta\theta},\, K_{\theta\rho},\, K_{\rho\rho}$ giving rise to an approximately constant field-space curvature

$$\kappa(\theta) = \frac{K'_{\rho\rho}(\theta)F'(\theta) - 2F(\theta)K''_{\rho\rho}(\theta)}{4F^2(\theta)} \qquad F(\theta) \equiv K(\theta)K_{\rho\rho}(\theta)$$

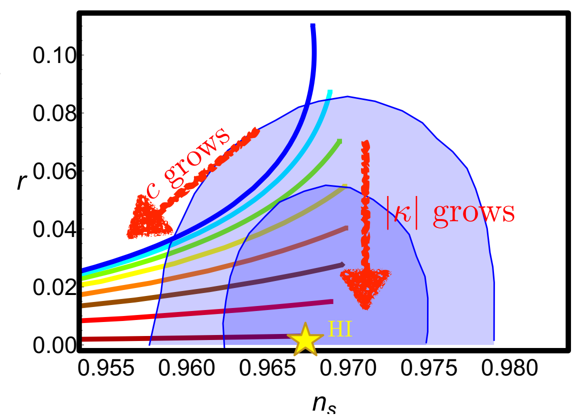
can be written as

$$\frac{\mathcal{L}}{\sqrt{g}} = \frac{M_P^2}{2}R - \frac{M_P^2}{2}\left[-\frac{(\partial K_{\rho\rho})^2}{4\,K_{\rho\rho}(\kappa K_{\rho\rho}+c)} + K_{\rho\rho}(\partial\rho)^2\right] - V(K_{\rho\rho})$$

The inflationary observables of this class of models approach

$$n_s \simeq 1 - \frac{2}{N} \qquad r \simeq \frac{2}{|\kappa| N^2}$$

in the large curvature/ large number of e-folds limit



Conclusions

Higgs-Dilaton Cosmology: A SI + UG extension of SM

- ✓ Massless dilaton: unique source for masses / scales.
- ✓ It naturally gives rise to inflation with a graceful exit.
- ✓ Excellent agreement with observations.

Natural embedding in a TDiff framework: the dilaton as a metric d.o.f

- ✓ Robust inflationary predictions for theory-defining functions giving rise to a maximally symmetric field spaces in the Diff invariant formulation.
- ✓ No non-gaussianities / No isocurvature perturbations due to SI.

The pole structure

$$\frac{\mathcal{L}}{\sqrt{g}} = \frac{M_P^2}{2} R - \frac{M_P^2}{2} \left[- \frac{(\partial K_{\rho\rho})^2}{4\, K_{\rho\rho}(\kappa_0 K_{\rho\rho} + c)} + K_{\rho\rho}(\partial\rho)^2 \right] - V(K_{\rho\rho})$$

$\xi_\chi \to 0,\ \lvert c \rvert \to 0$ $K_{\rho\rho} \to \lvert c/\kappa_0 \rvert \to 0$	Quadratic pole Asymptotic flatness $$K_{\rho\rho} = e^{-2\sqrt{\lvert\kappa_0\rvert}\frac{\phi}{M_P}}$$
$\xi_\chi \neq 0,\ \lvert c \rvert \neq 0$ $K_{\rho\rho} = 0$ unreachable	Linear pole Restricted flatness $$K_{\rho\rho} = \frac{c}{-\kappa_0} \cosh^2\left(\frac{\sqrt{-\kappa_0}\,\phi}{M_P}\right)$$ $\frac{M_P}{\sqrt{-\kappa_0}}$ non-compact analog of axion decay constant