Rigid Superconformal Theories in Three Dimensions

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MOTIVATIONS

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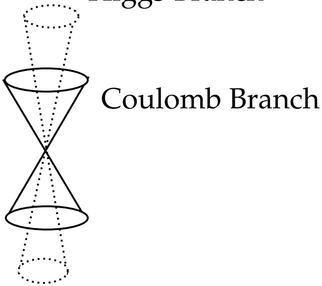
- The Higgs branch is the one in which the scalars in the hypermultiplet get expectation values.
- The Coulomb branch is the one in which the scalars in the vector multiplet get expectation values.
- As a consequence of SUSY, both are hyper-Kähler spaces.
- The R-symmetry is $SU(2)_H \times SU(2)_C$. The hypermultiplet (vector multiplet) scalars transform as a triplet under $SU(2)_H$ ($SU(2)_C$).

THE GEOMETRIC QUESTION



- We have a pair of singular non-compact hyper-Kähler spaces.
- The pair of vacuum moduli spaces share a singular point. This is where SCFT sits. (From the EFT at low energies : Singularities ∼ new massless particles)
- Warning : Unlike what is shown in the schematic picture above, the actual singularities that we will discuss are generically not of orbifold type.

Domain Wall Theories $T^{\rho}[G]$

- Introduced by Gaiotto-Witten in their study of boundary conditions in $\mathcal{N} = 4$ SYM. Vigorously studied since then.
- $T[SU(2)]$ is a quiver gauge theory corresponding to the quiver $(1) - [2]$ (Intrilligator-Seiberg).
- $T[SU(N)]$ is a quiver gauge theory corresponding to the quiver $(1) - (2) - (3) - \cdots - (N-1) - [N]$.

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- More generally, $T^{\rho}[G]$ for G classical sometimes admits useful Quiver Descriptions. But, we will not be using these descriptions.
- Instead, study them using their (defining) relation to infinite dimensional hyper-Kähler quotients (solutions to Nahm's equations, Hitchin's equations).

PROPERTIES OF $T^{\rho}[G]$

- The theory $T[G]$ has a product of nilpotent cones $\mathcal{N}_{\mathfrak{g}} \times \mathcal{N}_{\mathfrak{g}^{\vee}}$ as its vacuum moduli space. The SCFT has a $G \times G^{\vee}$ global symmetry.
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- Example : For $T[SU(2)]$, both moduli spaces happen to be $\mathbb{C}^2/\mathbb{Z}_2$.
- For $T^{\rho}[G]$, the Higgs branch is some stratum inside $\mathcal{N}_{\mathfrak{g}}$ (a Slodowy Slice) and the Coulomb branch is some stratum inside $\mathcal{N}_{\mathfrak{g}^{\vee}}$ (a Nilpotent Orbit Closure).
- The important question : Which strata naturally pair up as Higgs and Coulomb branches ? Answer : There is a duality d between $\rho, \rho^{\vee}$. It is a duality that takes "big" nilpotent orbits to "small" nilpotent orbits.

Relation to 6d

- These domain wall theories arise from four dimensional (or codimension-two) defects of the 6d $(0,2)$ SCFTs ("Theory $X[\mathfrak{j}]$")
- In other dimensional reductions from six dimensions, these defects are of great utility in constructing (old and new) 4d $\mathcal{N}=2$ theories (Gaiotto, Gaiotto-Moore-Neitzke) - the Class $\mathcal{S}[\mathfrak{j}, C_{g,n}]$ construction.
- In this realization, the properties of these theories determines the local data at punctures on $C_{g,n}$.
- For example, the residue of the Higgs field of the associated Hitchin system is determined by the Coulomb Branch of $T^{\rho}[G]$.

Mass deformations in type A

- The following rule is known to be sufficient : Deform the residue of the Higgs field in the Hitchin System to be semi-simple instead of nilpotent.
- But, outside of type A, this rule covers only a proper subset of codimension two defects.
- In recent work (w/ J. Distler, an earlier proposal of Chacaltana-Distler-Tachikawa), we were able to understand mass deformations for arbitrary $T^{\rho}[G]$.
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- We uncovered some new physical results in the process.
- Specifically, we found that the standard lore regarding mass deformations is too simplistic.

RELEVANT DEFORMATIONS OF 3D $\mathcal{N} = 4$ SCFTS

- There are three different kinds of relevant deformations for 3d $\mathcal{N} = 4$ SCFTs (see [Cordova, Dumitrescu, Intrilligator], there is also earlier work of [Dolan]).

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- Twisted mass deformations ($\sim$ F.I parameters).
- Universal Mass deformation.

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- Universal Mass deformation.
- In this talk, we are concerned only about Mass deformations.

HYPER-KÄHLER SINGULARITIES AND THEIR RESOLUTIONS

- The Coulomb branch associated to $T^{\rho}[G]$ is a special nilpotent orbit $\mathcal{O}$ of $\mathfrak{g}^{\vee}$. This is a hyper-Kähler space.
- In type A, we know that deforming the nilpotent residue to a semi-simple residue provides a hyper-Kähler resolution of $\overline{\mathcal{O}}$. It follows that it is a resolution that preserves the holomorphic symplectic structure. Such resolutions are called Symplectic Resolutions (Beauville, Fu, Namikawa).
- The mass parameters are identified with the eigenvalues of the residue.
- So, let us try the same trick.

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- Question : Is this always possible ?
- Answer : No! Outside of type A, there exist several examples of nilpotent orbits which can't be deformed to a semi-simple orbit in a $dim(orbit)$ preserving way (Theory of Sheets).
- It turns out that the following are the three possibilities. Let a be the residue for the Higgs field in the Hitchin system. Then, we can have
  1. $a_n \rightarrow a_{ss}$ such that $dim(a_n) = dim(a_{ss})$ ("Smoothable").
  2. $a_n \rightarrow a_{ss} + a'_n$ such that $dim(a_n) = dim(a_{ss}) + dim(a'_n)$. Here, a'_n is a non zero nilpotent orbit in a suitable subalgebra of $\mathfrak{g}^\vee$. ("Malleable").
  3. a_n has no deformation. ("Rigid").

Consequence for mass deformations

This has the following interpretation in terms of mass deformations

- If the orbit is "Smoothable", then eigenvalues of a_{ss} are the mass parameters.
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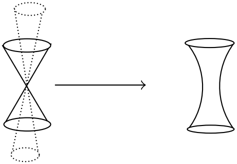
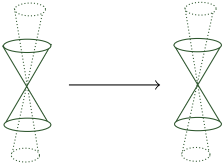
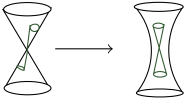
- If the orbit is "Rigid", then there are no mass deformations.
- Note : In Type A, Rigid or Malleable orbits don't exist.
- From a physics perspective, it is most natural to apply the Smoothable, Malleable, Rigid classification directly to the SCFT. This gives a refinement of the well known mathematical story.

A SCHEMATIC VIEW OF THE GEOMETRY

- The "Smoothable" case : 
- The "Rigid" case : 
- The "Malleable" case : 

EXAMPLES

Here are two examples from the D-series

- $T^{[5,3]}[SO(8)]$ is a Rigid SCFT.
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As we go to higher rank, these occur fairly regularly. Rigid and Malleable $T^{\rho}[G]$ are completely classified (including the case where $G = G_2, F_4, E_6, E_7, E_8$).

Takeaway for physics

Two important conclusions arise from a detailed study of deformation properties of nilpotent orbit. Here, there is also related conjectures of [Braden-Licata-Proudfoot-Webster] under the theme of Symplectic Duality. (Other aspects of this relation have been explored by Bullimore-Dimofte-Gaiotto-Hilburn-Kim)

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- It follows that the standard lore that “masses kill the Higgs branch and resolve the Coulomb branch" is not true in general.
- In our work, we were able to give a complete description of all mass deformations of $T^{\rho}[G]$ theories.

APPLICATIONS

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- Taking scaling limits of such geometries would lead to asymptotically free theories and (possibly) many new 4d $\mathcal{N} = 2$ SCFTs.