PRESENTATION

The **New Relationship** between Inflation & Gravitational Waves

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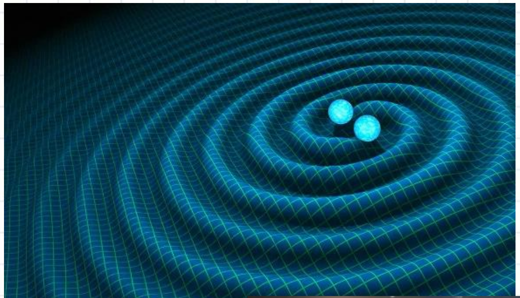
Oct/24/2016 Cosmology Seminar@DESY

GW detected!

- It exists!

  aLIGO detected GW from BH binary



- Primordial GW soon?

  PGW observed by the CMB B-mode

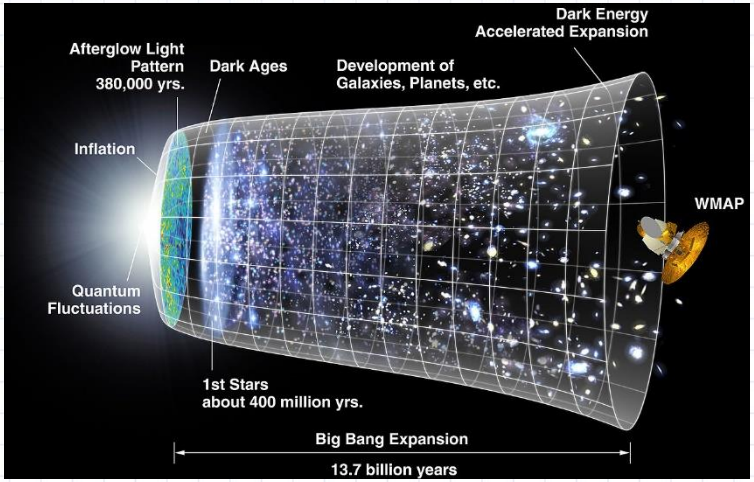
- Projects are on-going

  e.g. LiteBIRD (Japan), CMB-S4(US), PRISM(?)

Why PGW?

- Universal prediction of inflation

Inflation



- Inflation : $H \approx \mathrm{const.}$

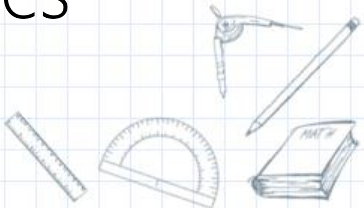
  Accelerating expansion era in primordial universe.

- It generates fluctuations seen in CMB/LSS

  Perturbations of all the light fields are produced.

- Mechanism is unknown = New physics

  ρ_{inf} = energy scale of BSM physics.



Why PGW?

- Universal prediction of inflation
- Vacuum GW: $r_{\rm vac} \approx 10^{-3} \left(\frac{H_{\rm inf}}{10^{13}{\rm GeV}}\right)^2$

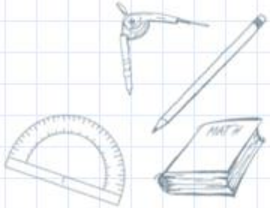
Vacuum fluctuation

- Inflation generates fluctuations from vacuum

$$h_k'' + \left(k^2 - \frac{2}{\tau^2}\right) h_k = 0, \qquad h_k = \frac{2}{\sqrt{2k}} \frac{e^{-ik\tau}}{M_{Pl}}.$$

- $\delta\phi$ & $M_{Pl} h_{ij}$ are produced with amplitude $\mathcal{O}(H_{\text{inf}})$ because H_{inf} is the only dimensionful quantity.

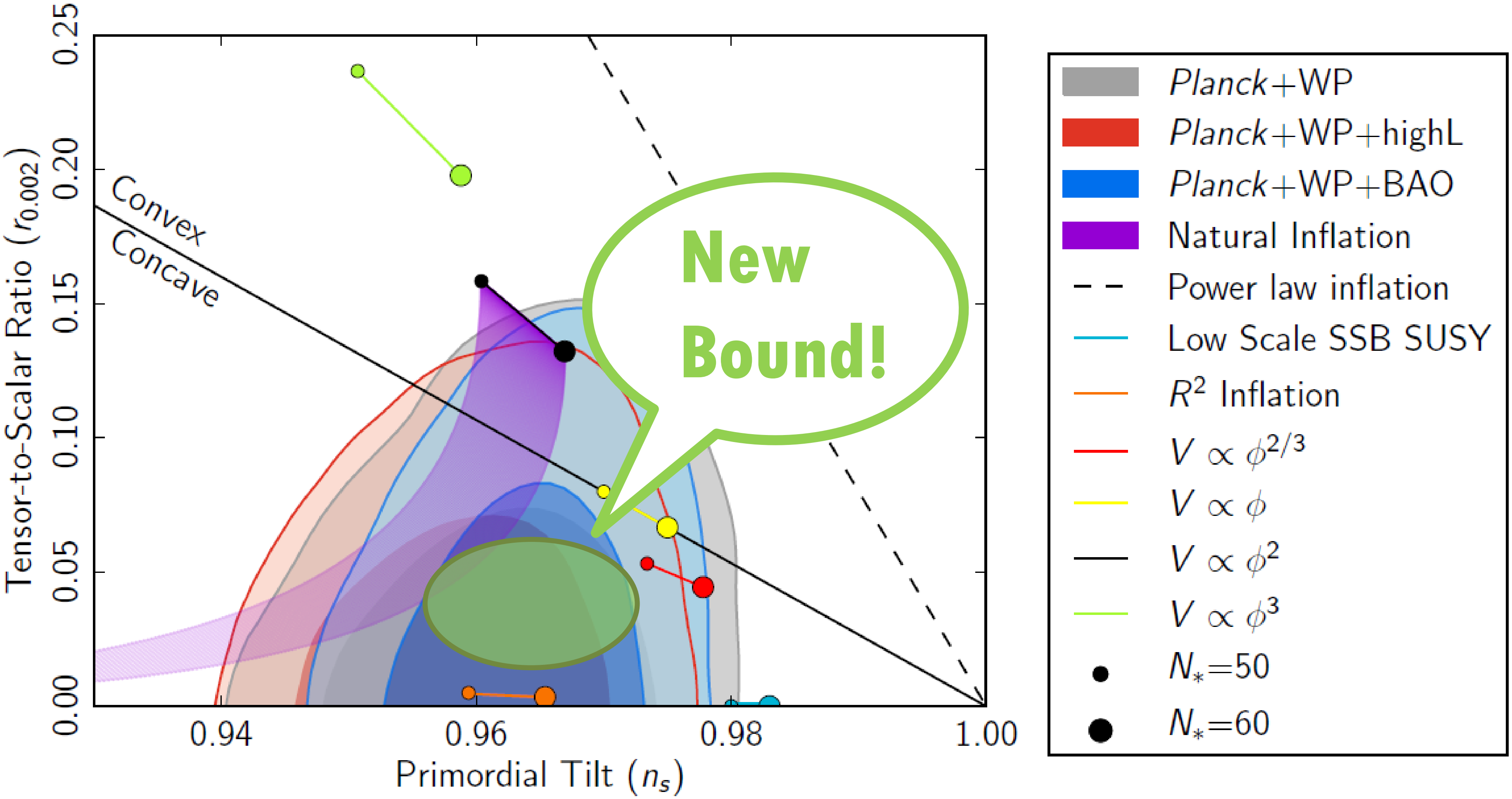
$$\boxed{\mathcal{P}_h^{\text{vac}} = \frac{2H_{\text{inf}}^2}{\pi^2 M_{Pl}^2} \Longrightarrow r \equiv \frac{\mathcal{P}_h}{\mathcal{P}_\zeta} \propto H_{\text{inf}}^2}$$



Why PGW?

- Universal prediction of inflation
- Vacuum GW: $r_{\mathrm{vac}} \approx 10^{-3}\left(\frac{H_{\mathrm{inf}}}{10^{13}\mathrm{GeV}}\right)^2$
- New physics & Model selection

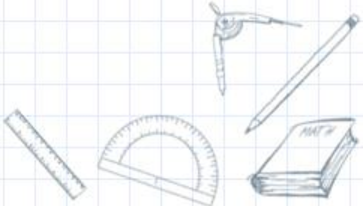
What if r is detected?



Why PGW?

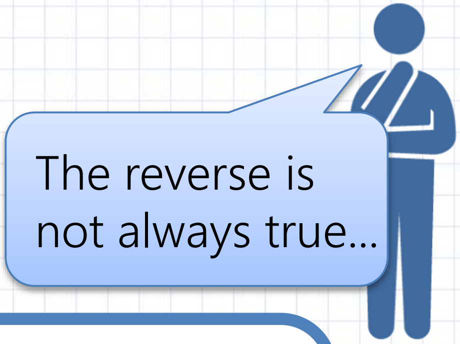
Written in textbook

- Universal prediction of inflation
- Vacuum GW: $r_{\mathrm{vac}} \approx 10^{-3}\left(\frac{H_{\mathrm{inf}}}{10^{13}\mathrm{GeV}}\right)^2$
- New physics & Model selection



Are You Sure??

Is it Robust?

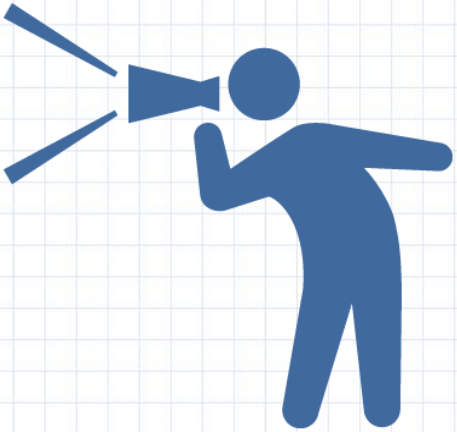


$$H_{\mathrm{inf}} = 10^{13}\mathrm{GeV} \Rightarrow r_{\mathrm{vac}} \approx 10^{-3}$$

$$r_{\mathrm{obs}} \approx 10^{-3} \Rightarrow H_{\mathrm{inf}} = 10^{13}\mathrm{GeV}$$

$r_{\mathrm{model}} < r_{\mathrm{obs}}$ ⇒ model dies?

No!



That's not necessarily true.

$r_{\text{obs}} \approx 10^{-3}$ ⇒ $H_{\text{inf}} = 10^{13}\text{GeV}$

$r_{\text{model}} < r_{\text{obs}}$ ⇒ model dies?

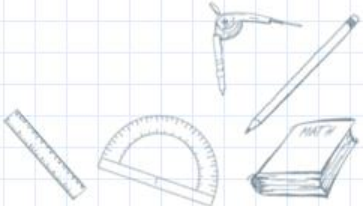
Counter example

If we have alternative GW production mechanism

$$r = r_{\rm vac} + \ \ r_{\rm alt}$$

and alternative GW is larger than vacuum one,
the relationship btw $r_{\rm obs}$ and $H_{\rm inf}$ is changed

$$r_{obs} \xcancel{=} r_{\rm vac} \approx 10^{-3} \left(\frac{H_{\rm inf}}{10^{13}{\rm GeV}}\right)^2$$



Counter example

Main message

GW larger than vacuum fluctuation can be produced by spectator sector during inflation.

In principle, detectable GW ($r = 10^{-3}$) can be generated even if $H_{\mathrm{inf}} = 10^{-13}\mathrm{GeV}$ or lower.

Our scenario

New mechanism to generate non-vacuum GW

$$\mathcal{L}_{\mathrm{inflaton}} + \mathcal{L}_{\mathrm{spectator}}$$

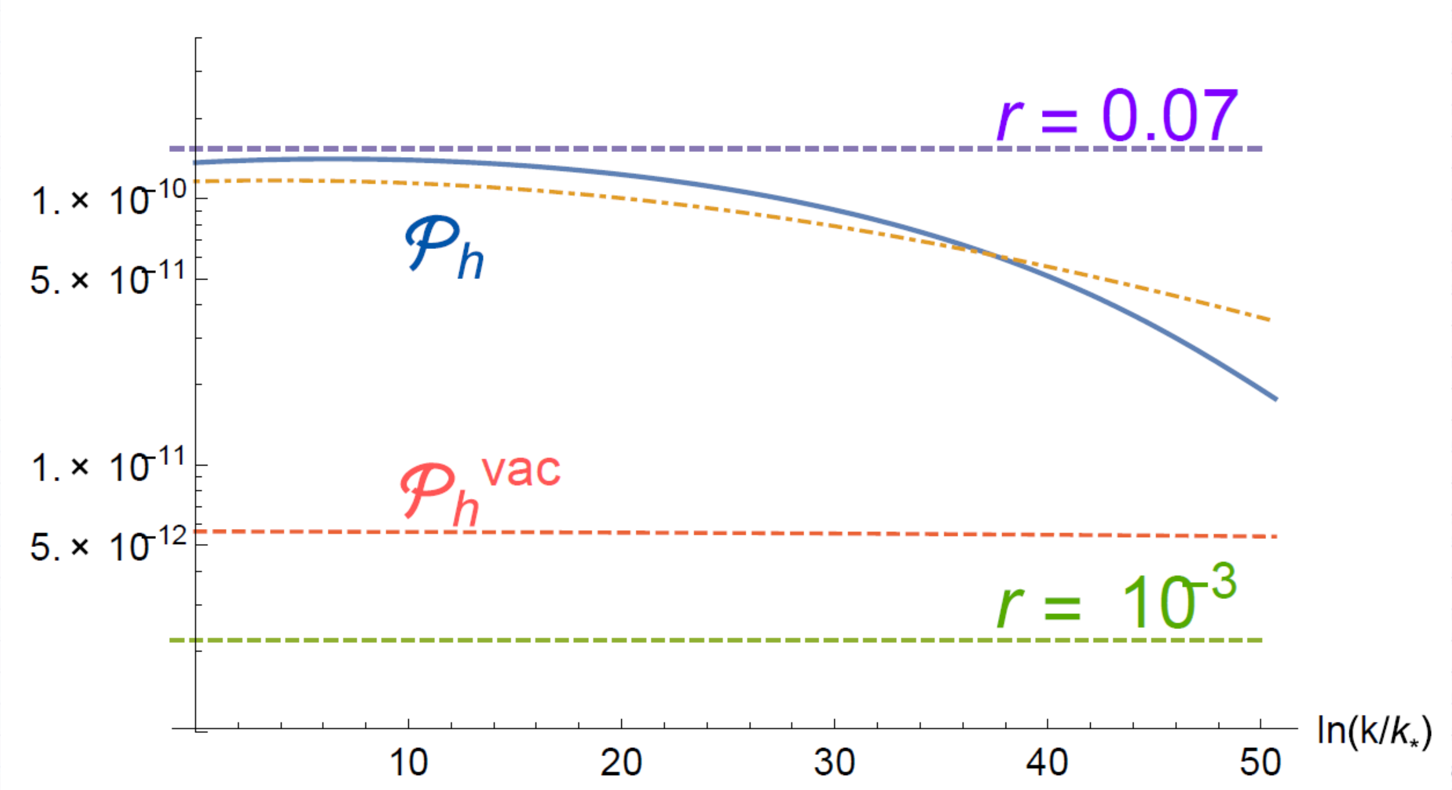
$$\begin{tikzcd}
\rho_{\mathrm{inf}} \arrow[d, "H_{\mathrm{inf}}"'] \arrow[r, phantom, "\gg"] & \rho_{\mathrm{spec}} \arrow[d, "\text{Source}"] \\
\mathcal{P}_{\mathrm{GW}}^{\mathrm{vac}} \arrow[r, phantom, "\ll"] & \mathcal{P}_{\mathrm{GW}}^{\mathrm{spec}}
\end{tikzcd}$$

$\mathcal{L}_{\mathrm{inf}}$ is arbitrary and responsible for ζ generation.

$\mathcal{L}_{\mathrm{spec}}$ is added just to produce GW during inf.

Result

$$H_* = 1.3 \times 10^{13}\,\mathrm{GeV}$$



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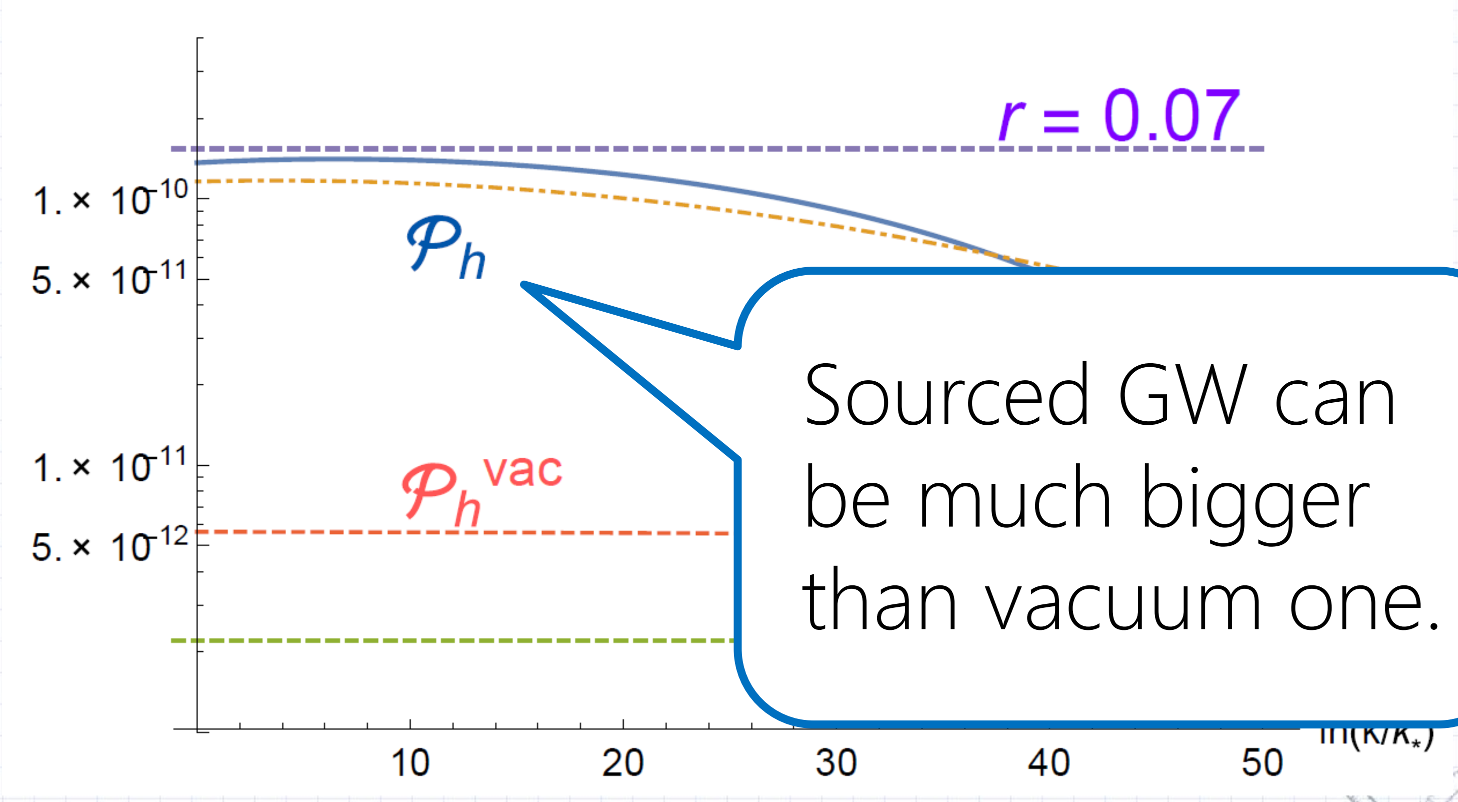


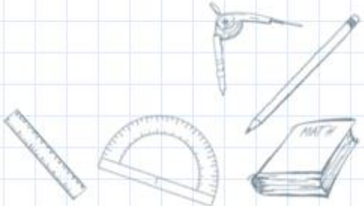
Sourced GW can be much bigger than vacuum one.

How’s it work?

Adding axion-SU(2) gauge spectator sector

$$
\begin{aligned}
\mathcal{L} = \mathcal{L}_{\text{inflaton}} \\
&+ \frac{1}{2}(\partial\chi)^2 - \mu^4\left(\cos\frac{\chi}{f} + 1\right) \\
&- \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{\lambda}{4f}\chi F^a_{\mu\nu}\tilde{F}^{a\mu\nu}
\end{aligned}
$$

[cf. Chromo-natural inflation: Adshead&Wyman(2012)]



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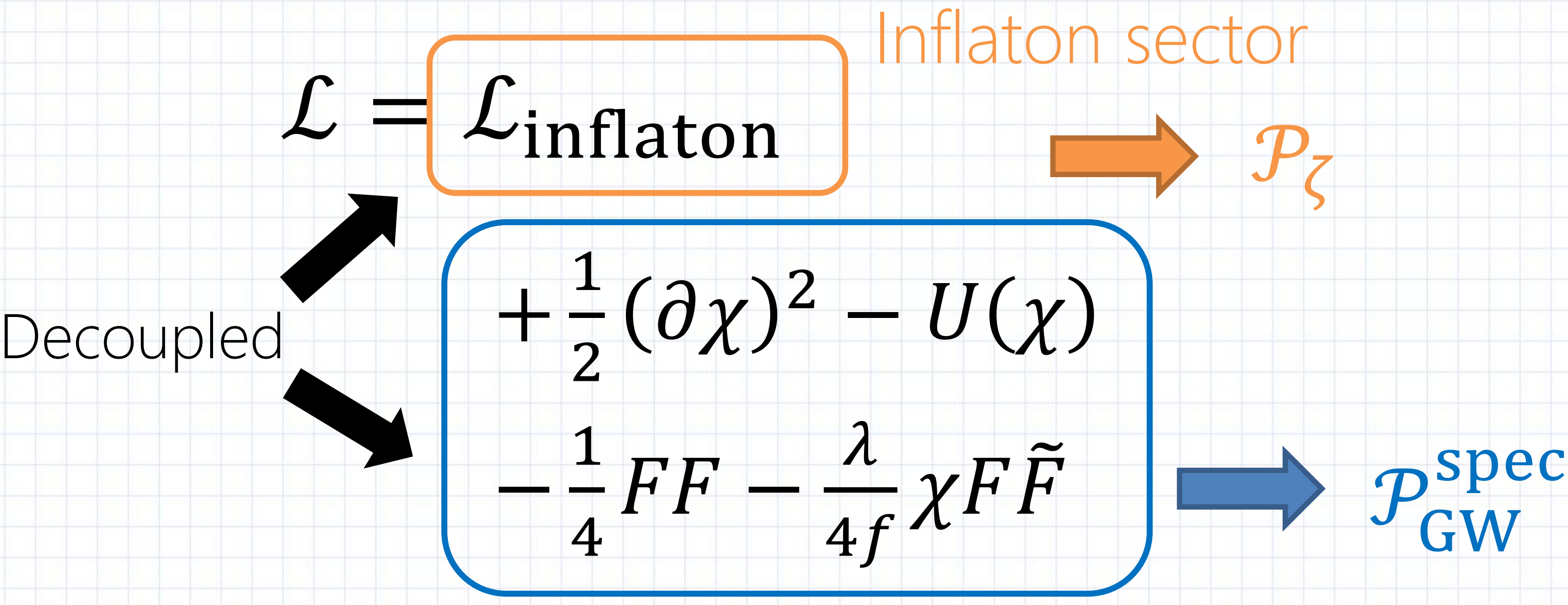


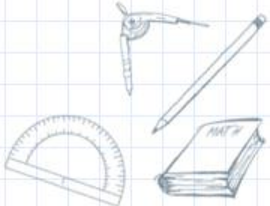
Inflaton sector

$$\mathcal{L} = \boxed{\mathcal{L}_{\text{inflaton}}} \Rightarrow \mathcal{P}_\zeta$$

Decoupled

$$\boxed{\begin{aligned} &+\frac{1}{2}(\partial\chi)^2 - U(\chi) \\ &-\frac{1}{4}FF - \frac{\lambda}{4f}\chi F\tilde{F} \end{aligned}} \Rightarrow \mathcal{P}_{\text{GW}}^{\text{spec}}$$

Axion- SU(2) gauge spectator sector

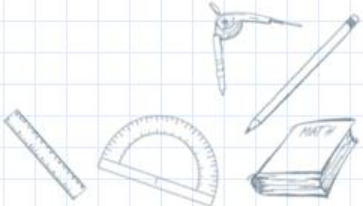


Why SU(2)?

SVT Decomposition Theorem:

At the 1st order cosmological perturbation, scalar, vector and tensor are decoupled.

$$\delta S, \delta V_i \xrightarrow[\text{Source}]{} \delta T_{ij}$$



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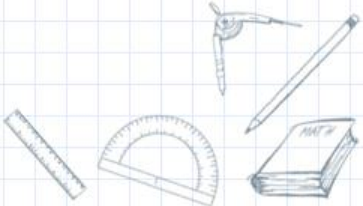


Why SU(2)?

Using the 2nd order pert., GW can be sourced.
But it's hard to generate $\mathcal{P}_{\mathrm{GW}}^{\mathrm{spec}} \gg \mathcal{P}_{\mathrm{GW}}^{vac}$.

$$\partial_i \delta S \partial_j \delta S,\ \delta V_i \delta V_j \xrightarrow[\text{Source}]{} \delta T_{ij}$$

[Biagetti et al.(2013), Mukohyama et al.(2014), TF et al.(2015),
Biagetti et al.(2015), Choi et al.(2015), Namba et al.(2016).]

Way Out

Background vector field V_i^{BG} helps.

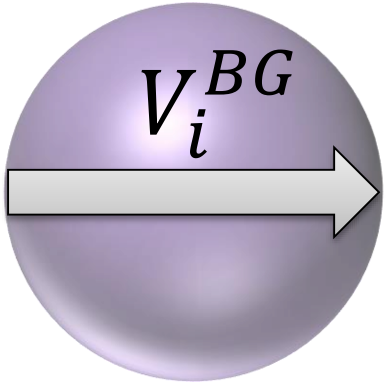
$$V_i^{BG}\delta V_j \xrightarrow[\text{Source}]{} \delta T_{ij}$$

CMB says the universe is isotropic.

U(1) gauge → Anisotropic BG

SU(2) gauge → Isotropic BG is Attractor.

Isotropy is violated



[Maleknejad&Erfani(2014)]

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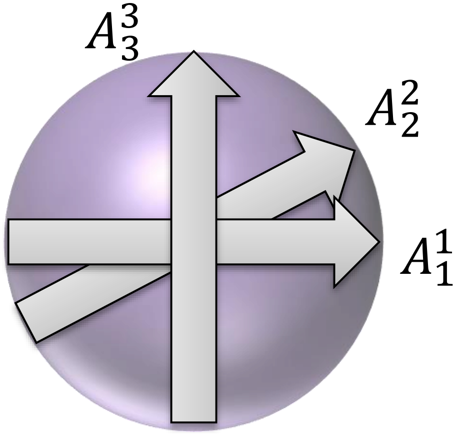
Isotropy is conserved



$$A_i^a = aA^{BG}(t)\delta_i^a$$

Coupling of Tensor perturbations

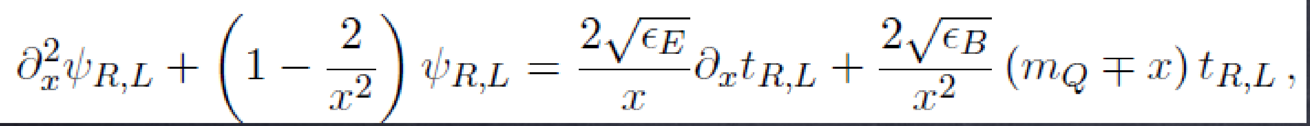
The T.T. component of $A_i \delta A_j$ is named t_{ij}

$$A^{BG}_{(i} \delta A^a_{j)} \supset t_{ij}, \qquad \partial_i t_{ij} = t_{ij} = 0$$

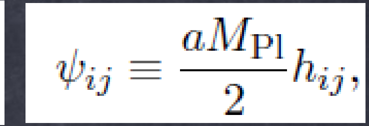
The EoM for GW mode function gets source terms

$$h_k'' + \left(k^2 - \frac{2}{\tau^2}\right) h_k = \boxed{\mathcal{O}\left(\Omega_A^{1/2}\right) t_k}$$

Now GW has the inhomogeneous solution!



$$\partial_x^2 \psi_{R,L} + \left(1 - \frac{2}{x^2}\right) \psi_{R,L} = \frac{2\sqrt{\epsilon_E}}{x} \partial_x t_{R,L} + \frac{2\sqrt{\epsilon_B}}{x^2} \left(m_Q \mp x\right) t_{R,L} \,,$$



$$\psi_{ij} \equiv \frac{aM_{\rm Pl}}{2} h_{ij},$$

$$x \equiv k/aH$$

Beat vacuum

We can source h_{ij} at linear order. However…

$$A_i^{BG}\delta A_j \supset t_{ij} \xrightarrow[\text{Source}]{} h_{ij}, \qquad h_{ij}^{\mathrm{vac}}$$

Both are 1st order. No reason for $h^{\mathrm{source}} \gg h^{\mathrm{vac}}$.
We need to amplify the former, but how?

Instability of tensor pert. of SU(2)

Instability of Chiral Tensor

Background χ and Q break the parity symmetry (SSB),

$$0 \neq \langle \chi(t,x) \rangle \overset{P}{\to} \langle -\chi(t,x) \rangle$$

$$0 \neq \langle A_i^a \rangle = aA^{BG} \delta_i^a \overset{P}{\to} -aA^{BG} \delta_i^a$$

Therefore, when we use left/right-handed polarization

$$t_{R/L} \equiv \frac{1}{\sqrt{2}}(t^+ \pm it^\times), \quad h_{R/L} \equiv \frac{1}{\sqrt{2}}(h^+ \pm ih^\times)$$

t_R and t_L behave in different ways.

Instability of Chiral Tensor

The EoMs for tensor perturbations are

$$h''_{R,L} + \left(1 - \frac{2}{x^2}\right) h_{R,L} = \mathcal{O}\left(\Omega_A^{1/2}\right) t_{R,L}$$

$$t''_{R,L} + \left(1 + \frac{2m_Q\xi}{x^2} \mp \frac{2}{x}\left(m_Q + \xi\right)\right) t_{R,L} = \mathcal{O}\left(\Omega_A^{1/2}\right) h_{R,L}$$

$m_Q \equiv gA^{BG}/H$

$\xi \equiv \lambda\dot{\chi}/2fH$

$A_i^a \equiv a\delta_i^a A^{BG}$

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$$FF \supset g\epsilon^{ijk} A^i A^j \partial A^k, \qquad \chi F\tilde{F} \supset \dot{\chi}\epsilon_{ijk} A_i \partial_j A_k$$

where we have used $\epsilon_{ijk} k_i e_{jl}^{R,L}(\hat{\boldsymbol{k}}) = \mp k e_{kl}^{R,L}(\hat{\boldsymbol{k}})$.

ϵ_{ijk} **terms make t_R instable (but not t_L)**

⇨ $h_R \gg h_L$ **: Chiral GW is generated!**

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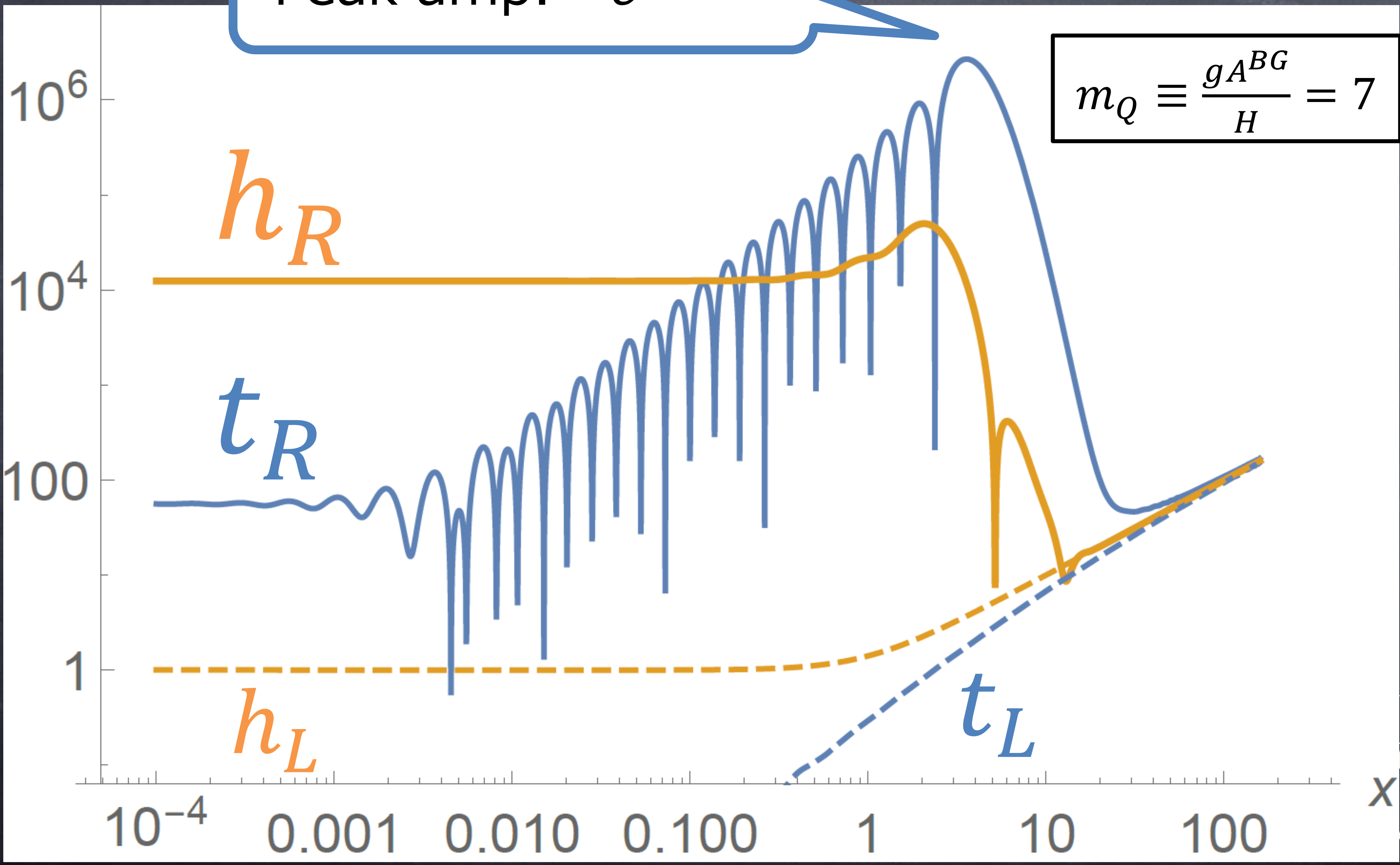
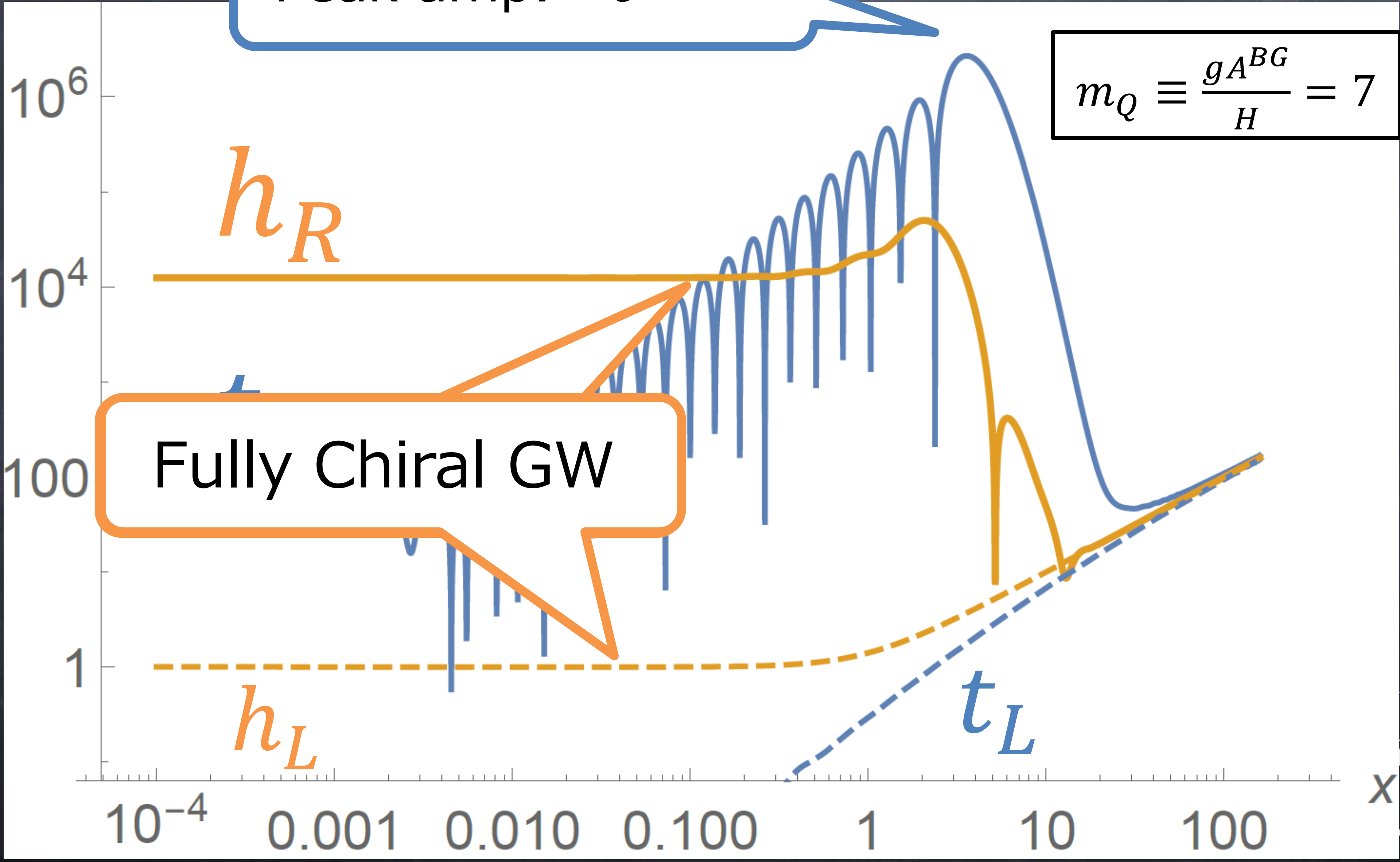
Insta nsor



Peak amp.$\sim e^{1.8 m_Q}$

$$m_Q \equiv \frac{gA^{BG}}{H} = 7$$

Insta nsor



Peak amp.~ $e^{1.8m_Q}$

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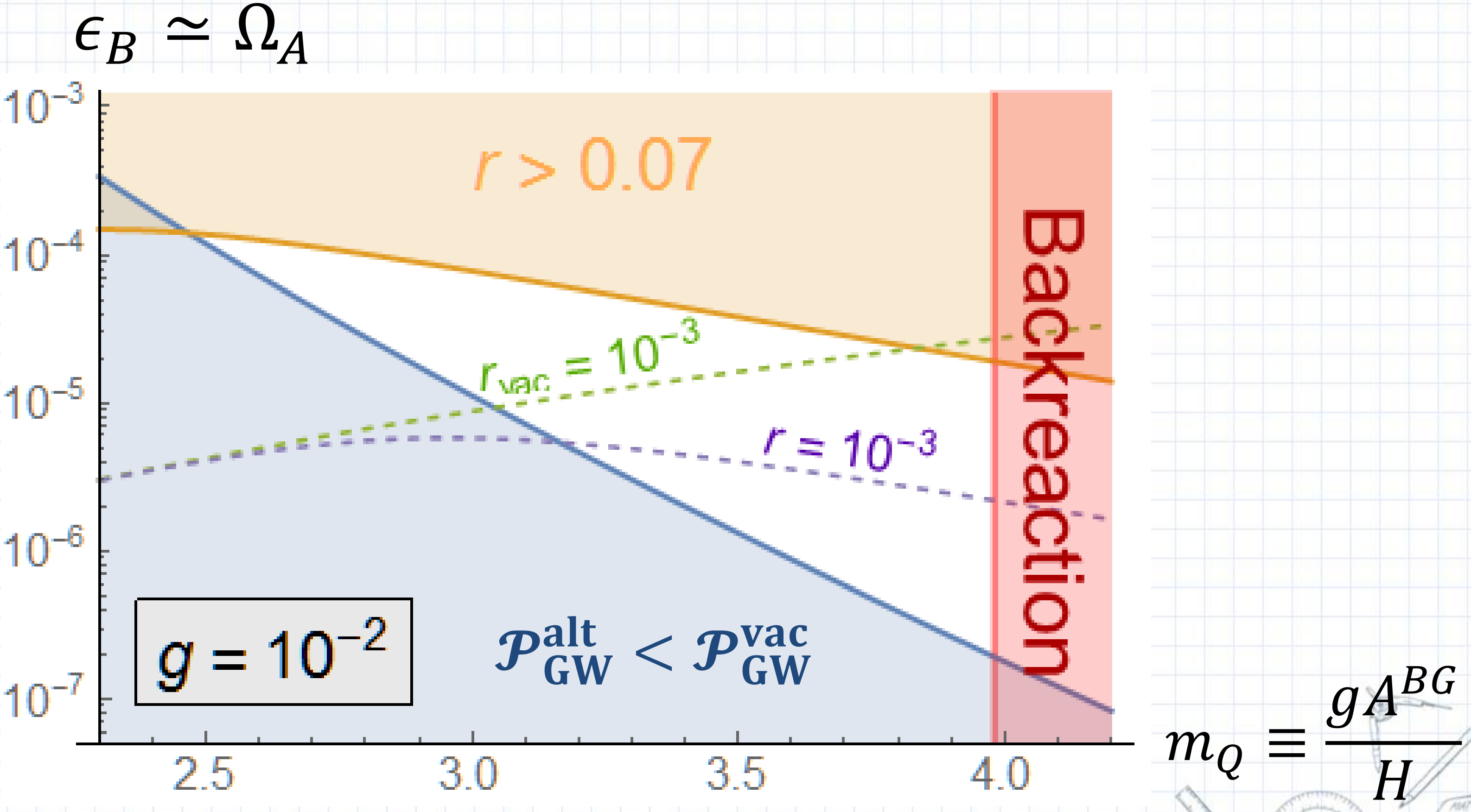
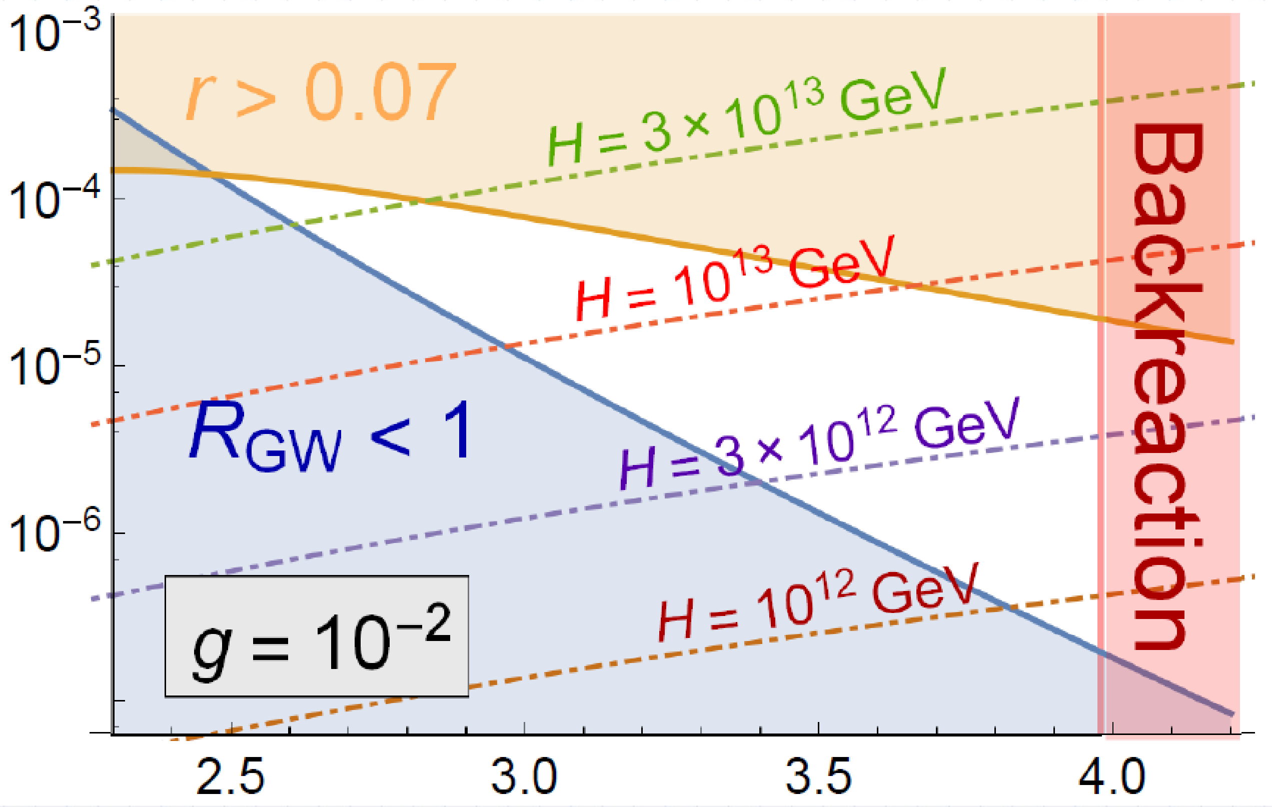
Fully Chiral GW

When $\mathcal{P}_{\mathrm{GW}}^{\mathrm{alt}} > \mathcal{P}_{\mathrm{GW}}^{\mathrm{vac}}$?



When $\mathcal{P}_{\mathrm{GW}}^{\mathrm{alt}} > \mathcal{P}_{\mathrm{GW}}^{\mathrm{vac}}$?

$\epsilon_B \simeq \Omega_A$



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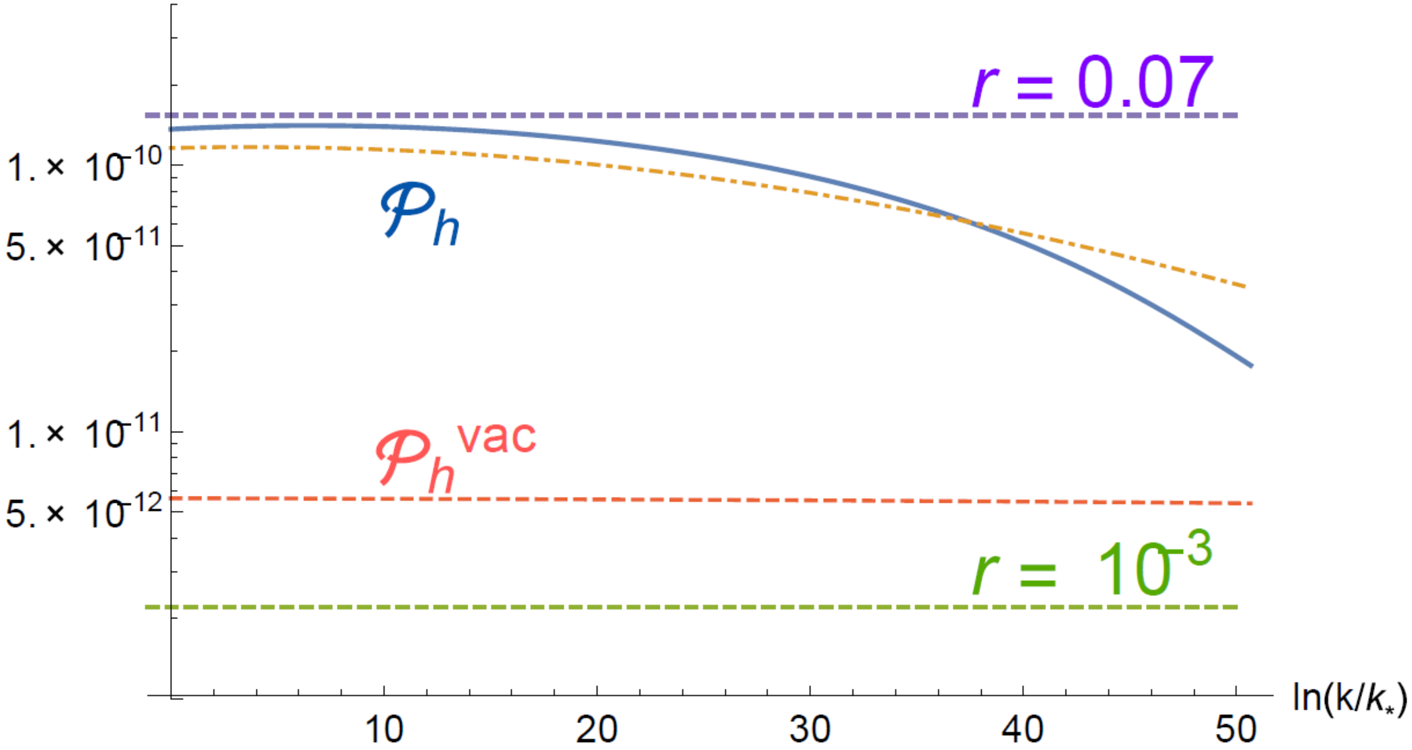
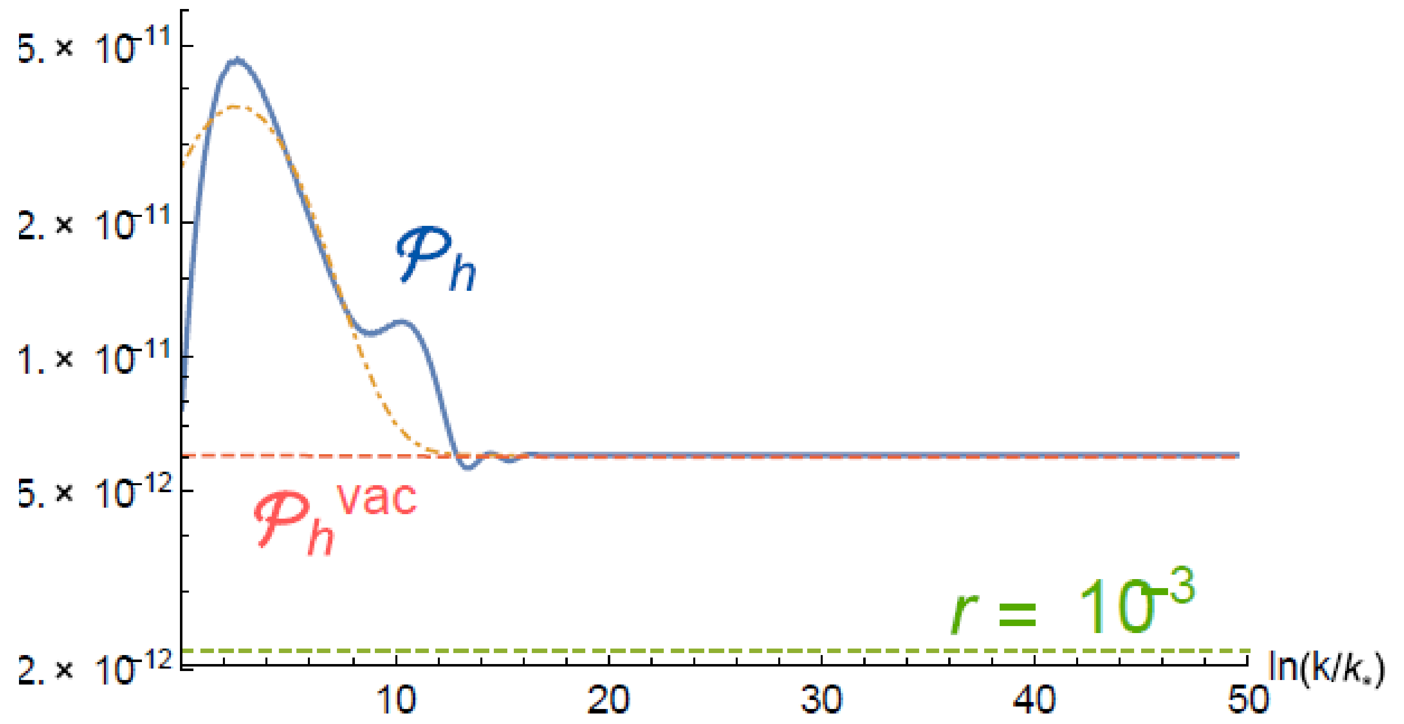
Scale-invariant



$$g = 1.11 \times 10^{-2}, \quad \lambda = 500, \quad \chi_* = \frac{\pi}{2} f = 6.28 \times 10^{16} \mathrm{GeV},$$
$$H_* = 1.28 \times 10^{13} \mathrm{GeV}, \quad \mu = 1.92 \times 10^{15},$$

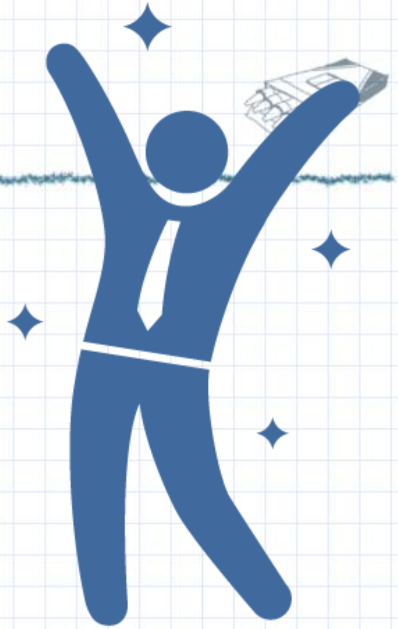
Small coupling



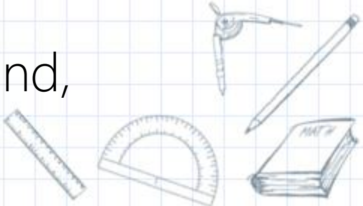
$$g = 10^{-2}, \quad \lambda = 50, \quad \chi_* = \frac{5}{4} f = 6.32 \times 10^{15} \mathrm{GeV},$$

$$H_* = 1.32 \times 10^{13} \mathrm{GeV}, \quad \mu = 1.10 \times 10^{15} \mathrm{GeV},$$

Summary



- Axion-SU(2) gauge field spectator sector can produce dominant GW → r doesn't fix ρ_{inf}
- SU(2) BG Attractor enables loophole of SVT decomposition and source GW at 1st order without anisotropy.
- Parity-breaking BG amplifies either of R/L polarization mode and fully chiral GW is predicted.
- Rich Phenomenology: Small-field inflation model, various features in spectrum, break $n_T = -r/8$ & Lyth bound, Non-gaussianity of GW, Curvaton, etc…



Lowest H for $r = 10^{-3}$

What's the lowest possible $H_{\rm inf}$ generating detectable r ??

$H_{\rm inf}$ can be reduced to the BBN bound, $H_{\rm inf} \approx 10^{-22}{\rm GeV}$

Analytic estimate: $\mathcal{P}_h^{(s)} \sim \Omega_A e^{3.6 m_Q} \times \mathcal{P}_h^{\rm vac}$,

Lowering H and increasing m_Q, we can keep $r = 10^{-3}$.

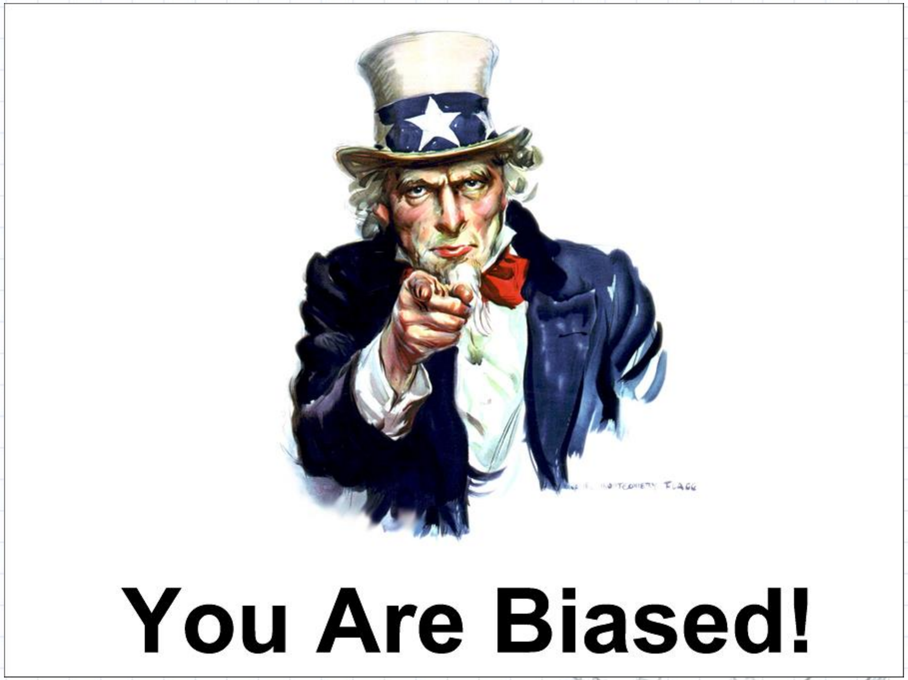
Counter-intuitive?

Usually low energy inflation predicts smaller GW.

$$r_{\text{vac}} \approx 10^{-3} \left(\boxed{\frac{H_{\text{inf}}}{10^{13}\text{GeV}}} \right)^2$$

Now even inflation with $H_{\text{inf}} \approx 10^{-22}\text{GeV}$ is OK.

Why?

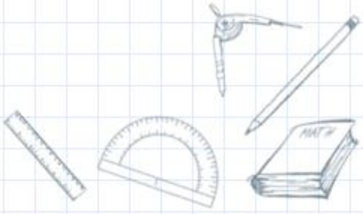


Energy transfer vs Vacuum

Energy of vacuum fluctuation is always $\mathcal{O}(H^4)$.

$$\rho_{GW} \sim H^4$$

t_R is produced because energy is transferred from $\chi\&Q$



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t_R is produced because energy is transferred from $\chi\&Q$

$$\rho_\chi, \rho_A \xrightarrow{\text{Source}} \rho_{t_R}$$

It is not so hard to transfer a certain fraction energy (say 1%) from Background field to GW.

It’s Easy to get $r = 10^{-3}$

r is proportional to energy fraction of GW

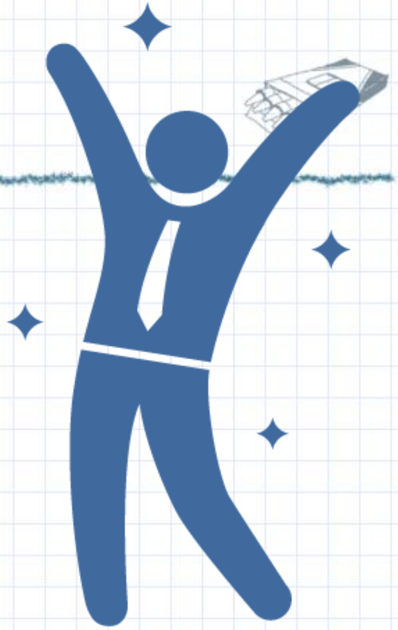
$$r \sim \Omega_{GW}$$

In vacuum case, Ω_{GW} is proportional to H^2

$$\Omega_{GW} \simeq \frac{H^4}{3M_{Pl}^2 H^2} \propto H^2$$

But if Ω_{GW} is sourced by other fields, low H does not makes it harder.

$$\boxed{\Omega_A \Rightarrow \Omega_{GW}}$$



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