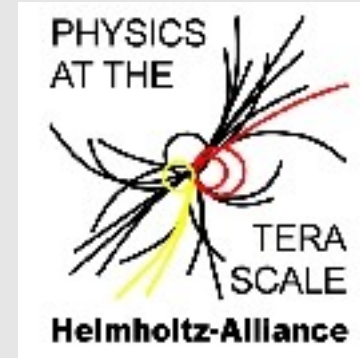




BAT

A Bayesian Analysis Toolkit



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Outline:

- Motivation and scope
- Technical aspects
- Examples
- Summary and outlook

Measurements ↔ Data analysis

What does the data tell us about our models?

→ **Estimate parameters**

Which model is favored by the data?

→ **Compare models**

Is the chosen model in agreement with the data?

→ **Judge validity of models**

...from simple line fitting to cosmological model testing!

→ ***Need tools to extract information***



$$p(\lambda|\vec{x}) = \frac{p(\vec{x}|\lambda)p_0(\lambda)}{\int p(\vec{x}|\lambda)p_0(\lambda)d\lambda}$$

Should be able to ...

- (1) phrase arbitrary problems
- (2) interface to HEP-software
- (3) estimate parameters
- (4) extract pdfs for single parameters and correlations
evaluate limits;
- (5) propagate uncertainties
- (6) compare models
- (7) check validity of a models

Specification:

- **C++ framework** based on ROOT core functionality;
- Define modules for numerical tasks (optimization, etc.)
- Define base class for models which then need to be specified by the user.

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Specification:

- Use optimization methods, e.g. Interface to TMinuit. Working on simulated annealing.
- Use **Markov Chain Monte Carlo** for marginalization and error propagation

Key element: MCMC

Should be able to ...

- (1) phrase arbitrary problems
- (2) interface to HEP-software
- (3) estimate parameters
- (4) extract pdfs for single parameters and correlations
evaluate limits;
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- (7) check validity of a models

Specification:

- Model comparison using common definitions (e.g. Direct comparison of probabilities, Bayes information criterion)
- Model validity is a difficult topic. Use *p-value* to judge (generalization of χ^2 -probability).

User defined

Definition of models

- Parameters
- Conditional probabilities
- Prior probabilities

Data set definition

- Read in from file
(.txt, .root, histogram,
user defined)
- Create on-the-fly

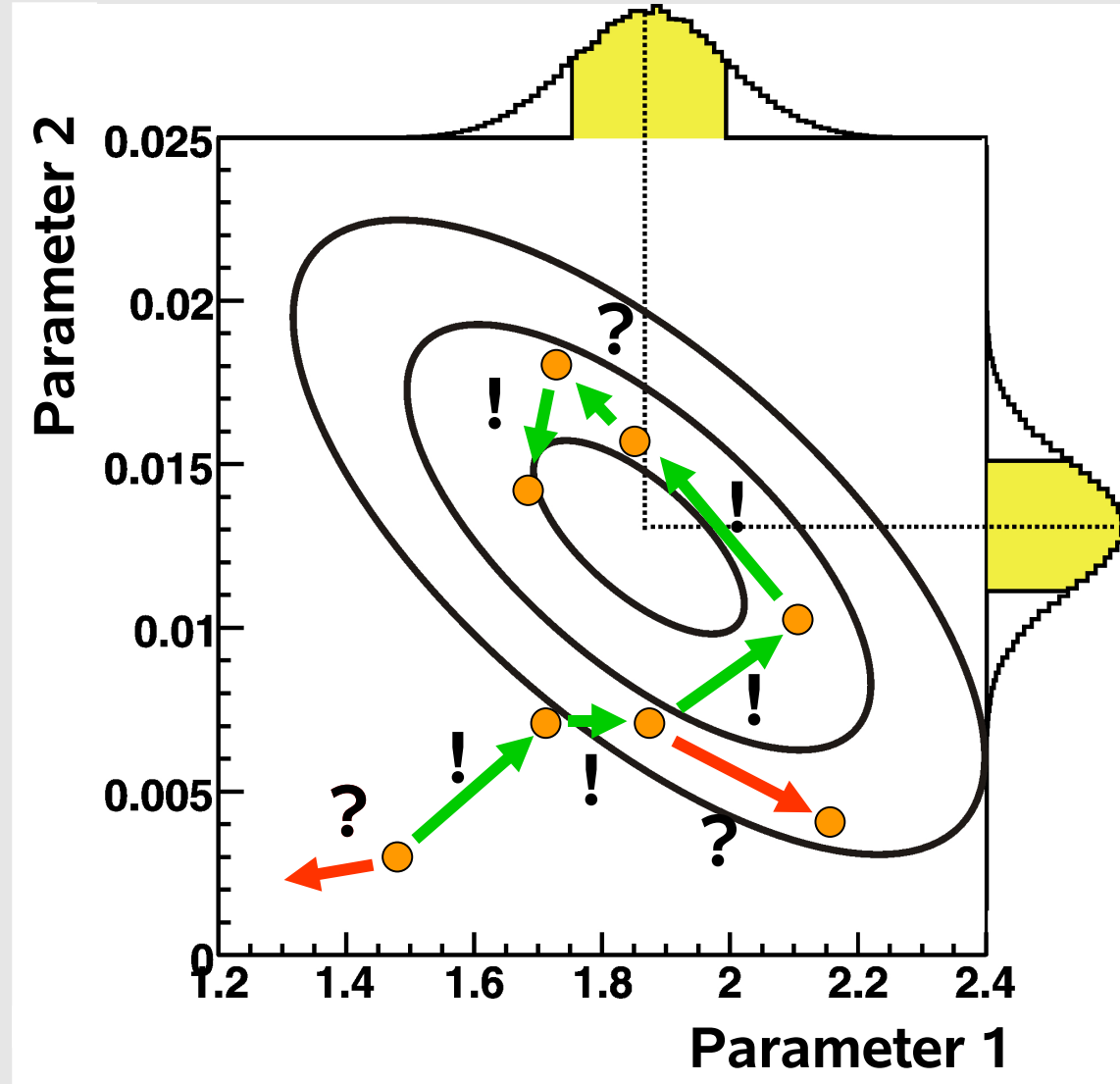
Common tasks

Analysis

- Normalize (Integration)
- Maximize (“Minimization”)
- **Marginalize (Sampling)**
- **Goodness-of-fit**
- **Model comparison**
- **Error propagation**
- Write output file
- Graphical output

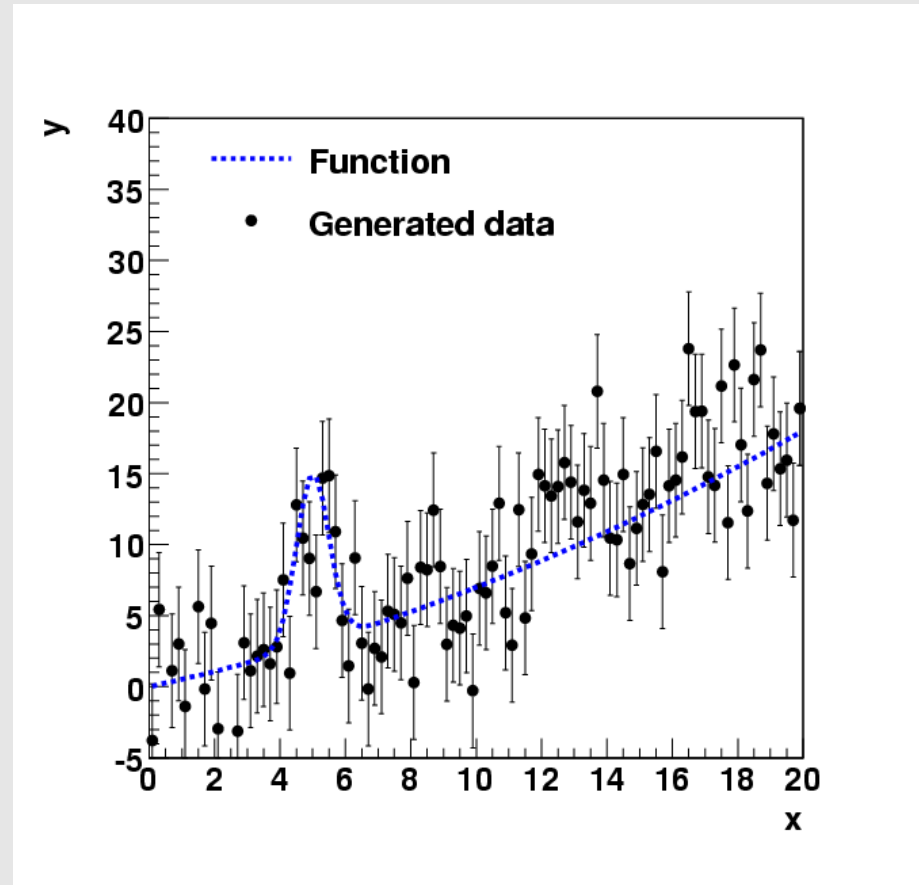
Markov Chains:

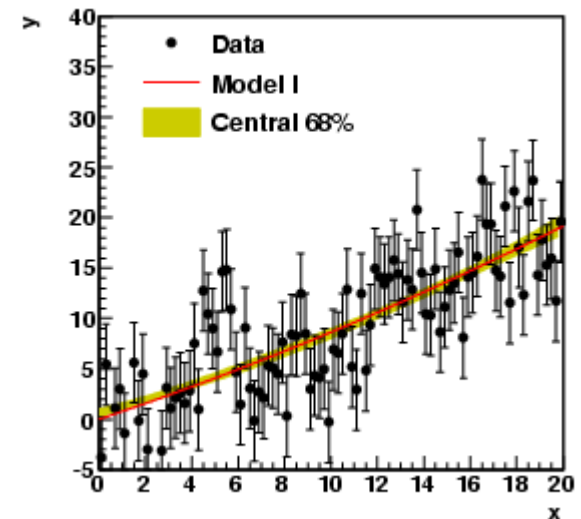
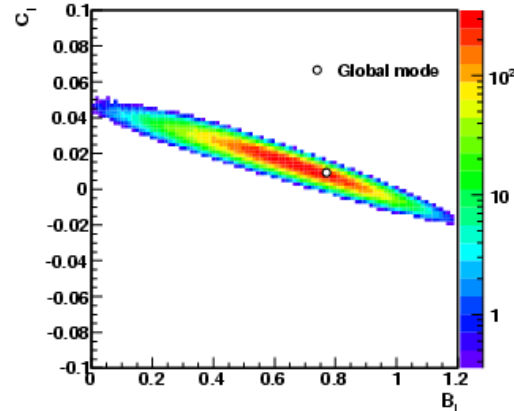
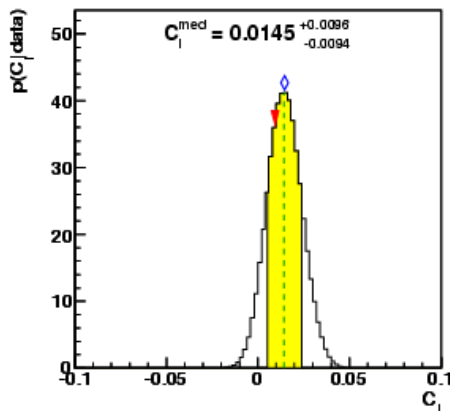
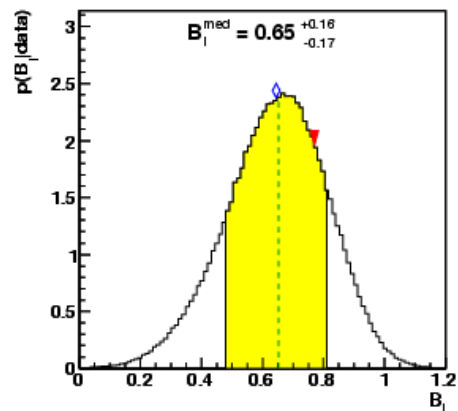
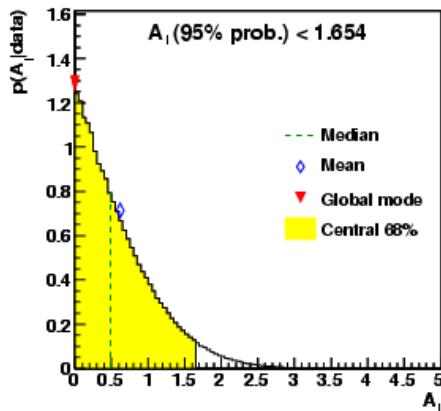
- Sample parameter space by migrating to regions of larger probability
- Converge towards underlying distribution
- Marginalize pdf while walking
- Calculate any function of parameters while walking (uncertainty propagation)
- Find global maximum



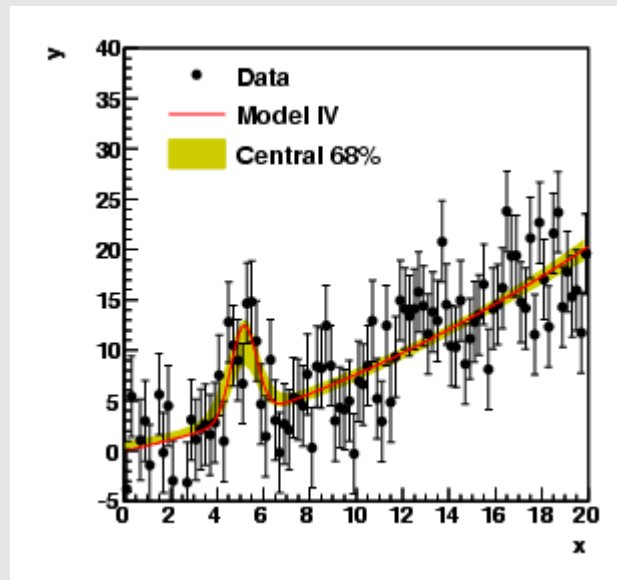
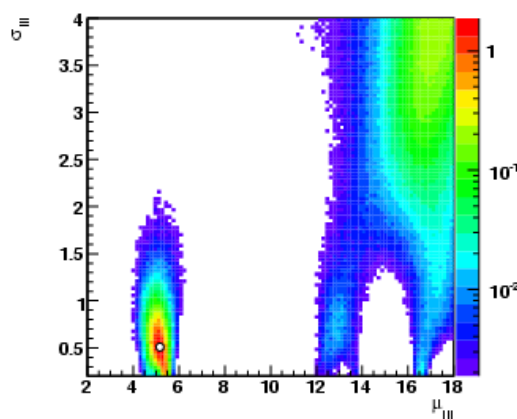
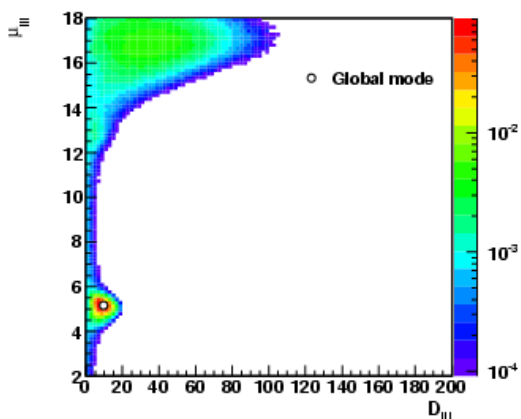
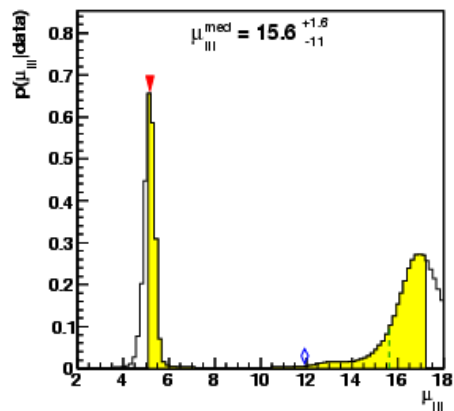
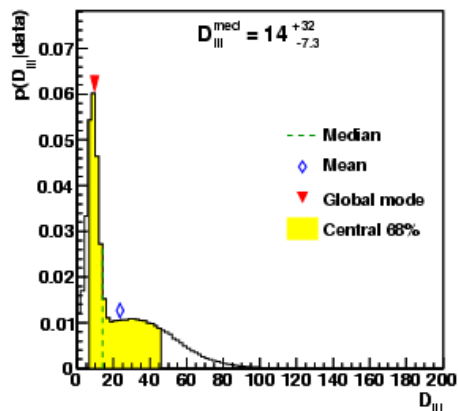
Example for fitting:

- Generate spectrum with 2nd order polynomial plus Gaussian at $x = 5$
- Test different hypotheses:
 - 2nd order polynomial
 - Gaussian plus constant
 - Gaussian plus line
 - Gaussian plus 2nd order polynomial
- Flat prior probabilities
- Peak position in $[2, 18]$, width < 4

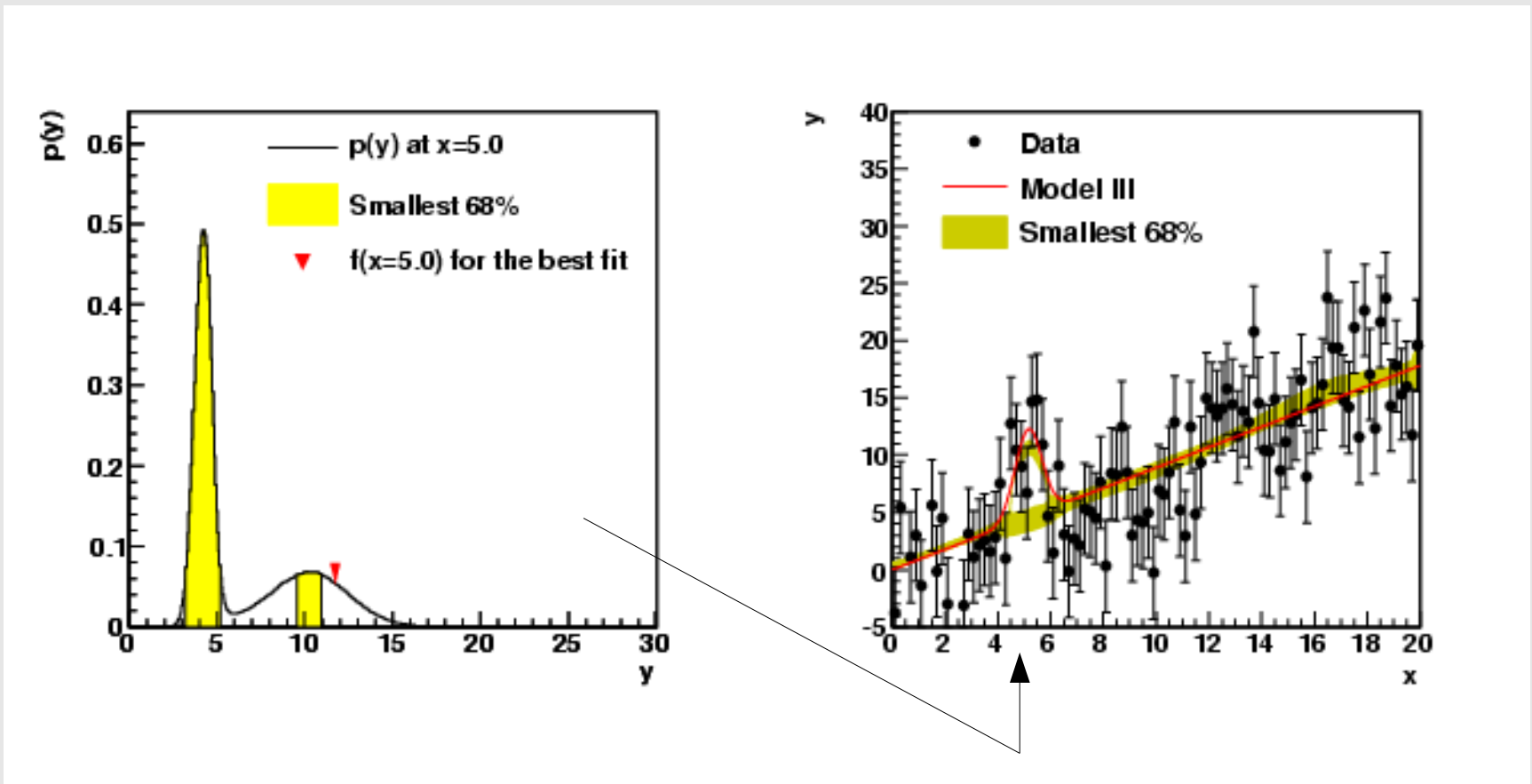




- Full information in Markov chain (written to file).
- Project sampled distribution onto one (or two) parameters. Integrate over all others.

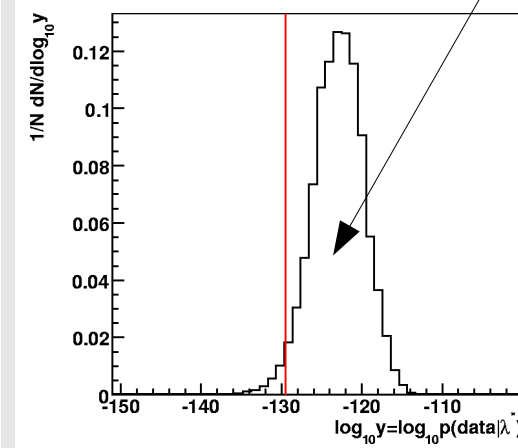
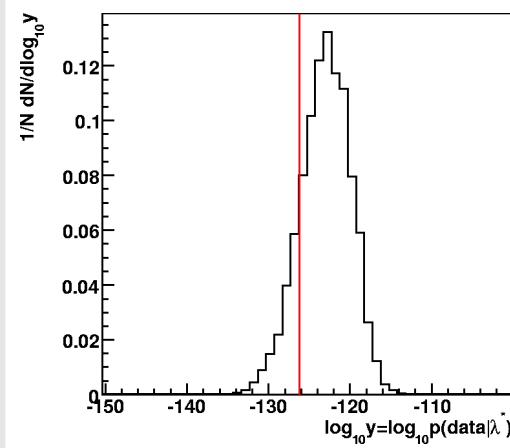


- Central interval not always optimal
- Multiple maxima in parameter space
- MCMC follow complicated distributions



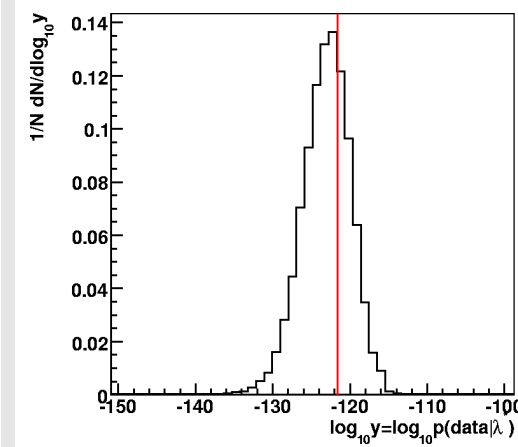
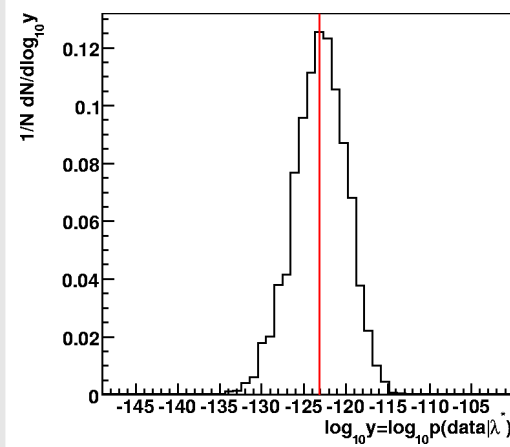
Use posterior pdf to calculate $p(y|x)$ and thus uncertainty band on y .

The probability of the measured data, for the best fit parameters, should not be in the tail of the distribution.



const +
Gauss
 p -value = 0.041

Model not
appropriate



2nd order pol. +
Gauss
 p -value = 0.778

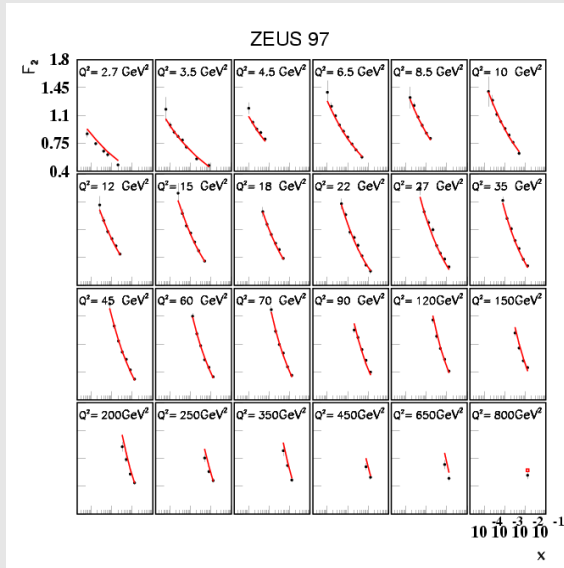
Model well
suited to
describe data

Your choice! Data not „strong“ enough for clear distinction

2nd order pol.
 p -value = 0.232
Model OK

1st order pol. +
Gauss
 p -value = 0.540
Model well
suited to
describe data

Structure Function fits

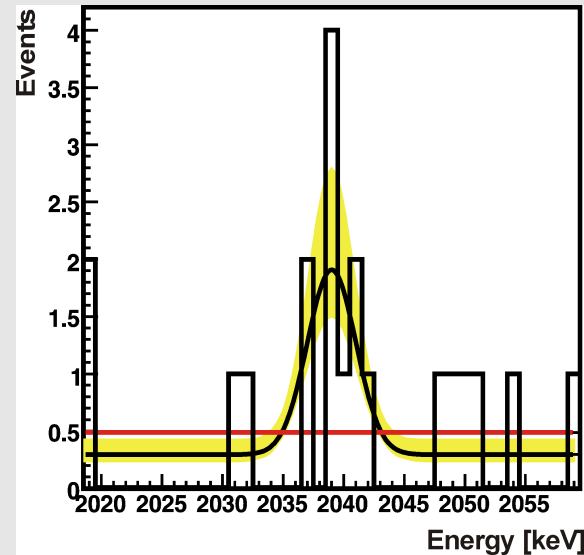


Parameterization of F_2 structure function

Here: ZEUS 97 F_2 data with parameterization superimposed.

33 parameters

GERDA ($0\nu\beta\beta$)

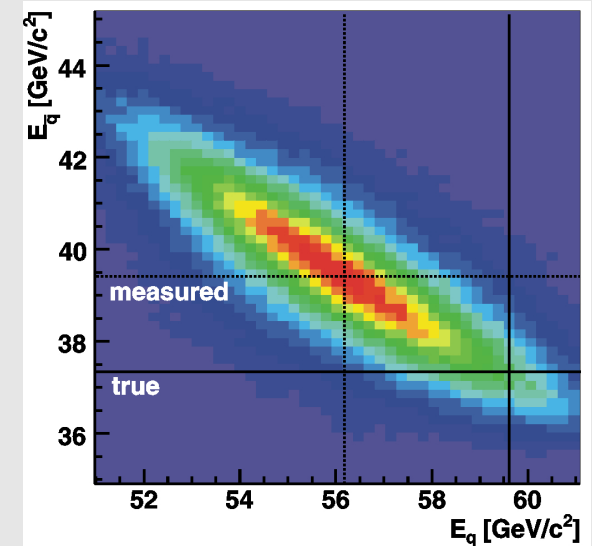


Line search in $0\nu\beta\beta$ -experiments

Here: Energy spectrum in the region of interest using simulated data. The model contains signal and flat background

low-statistics

ATLAS (HEP)



Kinematic fitting in top events.

Here: Correlation between two light jets coming from a W.

correlated parameters

- Toolkit tailored for Bayesian data analysis in HEP
- Parameter estimate, model comparison, model validity
- Strong numerical tools and interfaces implemented
- A variety of possible applications in physics: low-statistics experiments, kinematic fitting, structure function fitting, ...
- Visit our website: www.mppmu.mpg.de/bat (beta-release available)
- Publication submitted to “Computer Physics Communications”
[\[arxiv:0808:2552\]](https://arxiv.org/abs/0808.2552)
- Future plans: interface to ROOT statistics package, implementation of numerical modules, ...
- **Forming of a formal collaboration**
- **Developers and beta-testers welcome!**