Analysis Center
Statistical Tool Group
20th November 2008

Gfitter – A Generic Fitting Package for HEP Model Testing

Martin Goebel
DESY/ Universität Hamburg

Gfitter group: H. Flächer (CERN), M. G. (Univ. Hamburg, DESY), J. Haller (Univ. Hamburg), A. Höcker (CERN), K. Mönig (DESY), J. Stelzer (DESY)

The Gfitter project

- Gfitter: A Generic Fitter Project for HEP Model Testing
- aim: provide a reliable framework for involved fitting problems in the LHC era (and beyond).
- software:
  - abstract object-oriented code in C++ using ROOT functionality
  - core package:
    - tools for data handling, fitting, statistical analyses
  - physics: plug-in packages
    - **GSM**: Library for the Standard Model fit to the electroweak precision data
    - **G2HDM**: Library for the 2HDM extension of the SM
    - **GSUSY**: Library for supersymmetric extensions of the SM (in preparation)

The Gfitter project

- Gfitter features:
  - consistent treatment of statistical, systematic and theoretical errors, correlations, and inter-parameter dependencies
    - theoretical uncertainties: Rfit prescription [A Höcker et al., EPJ C21, 225 (2002)]
      - theory uncertainties included in χ^2 estimator with flat likelihood in allowed ranges
  - fitting:
    - several minimization algorithms available, e.g. TMinuit, genetic fitter
    - Next steps: inclusion of lvmini
  - caching of computation results between fit steps
    - only theory predictions are recalculated that depend on modified parameters
    - substantial speed improvement
  - advanced statistical analyses (frequentist approach):
    - e.g. parameter scans, contours, MC toy analyses, goodness-of-fit p-value, etc.

SM fit: theoretical predictions

- first theoretical library implemented in Gfitter framework: SM predictions of electroweak precision observables
- state-of-the art calculations (OMS scheme); in particular:
  - M_W and $\sin^2\theta^l_{eff}$: full two-loop + leading beyond-two-loop correction

    [M. Awramik et al., Phys. Rev D69, 053006 (2004 and ref.] [M. Awramik et al., JHEP 11, 048 (2006) and refs.]
  - radiator functions: N^{3}LO of the massless QCD Adler function

    [P.A. Baikov et al., Phys. Rev. Lett. 101 (2008) 012022]
- calculations thoroughly cross-checked against ZFitter (Fortran) package → excellent agreement
- free fit parameters:
  - M_Z, M_H, m_t, $\Delta\alpha_{had}^{(5)}(M_Z^2)$, $\alpha_S(M_Z^2)$, $\overline{m}_c$, $\overline{m}_b$
  - parameters for theoretical uncertainties on M_W (δM_W=4-6GeV), $\sin^2\theta^l_{eff}$ ($\delta\sin^2\theta^l_{eff}$ =4.7·10^{-5}) (and the electroweak form factors ρ_Z^f, κ_Z^f)

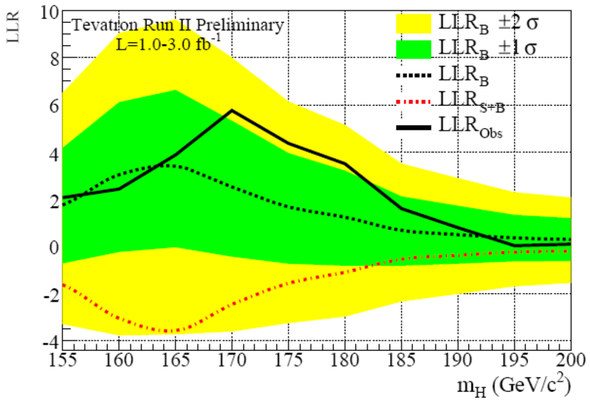
SM fit: experimental input

Parameter	Input value	Free in fit
M_Z [GeV]	91.1875 ± 0.0021	yes
Γ_Z [GeV]	2.4952 ± 0.0023	–
$\sigma^0_{\rm had}$ [nb]	41.540 ± 0.037	–
R^0_ℓ	20.767 ± 0.025	–
$A^{0,\ell}_{\rm FB}$	0.0171 ± 0.0010	–
$A_\ell\ ^{(\star)}$	0.1499 ± 0.0018	–
A_c	0.670 ± 0.027	–
A_b	0.923 ± 0.020	–
$A^{0,c}_{\rm FB}$	0.0707 ± 0.0035	–
$A^{0,b}_{\rm FB}$	0.0992 ± 0.0016	–
R^0_c	0.1721 ± 0.0030	–
R^0_b	0.21629 ± 0.00066	–
$\sin^2\theta^\ell_{\rm eff}(Q_{\rm FB})$	0.2324 ± 0.0012	–
M_H [GeV] $^{(\circ)}$	Likelihood ratios	yes
M_W [GeV]	80.398 ± 0.025	–
Γ_W [GeV]	2.106 ± 0.050	–
$\overline{m}_c$ [GeV]	1.25 ± 0.09	yes
$\overline{m}_b$ [GeV]	4.20 ± 0.07	yes
m_t [GeV]	172.4 ± 1.2	yes
$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)\ ^{(\dagger\Delta)}$	2769 ± 22	yes
$\alpha_s(M_Z^2)$	–	yes
$\delta_{\rm th} M_W$ [MeV]	$[-4, 4]_{\rm theo}$	yes
$\delta_{\rm th} \sin^2\theta^\ell_{\rm eff}\ ^{(\dagger)}$	$[-4.7, 4.7]_{\rm theo}$	yes
$\delta_{\rm th} \rho^f_Z\ ^{(\dagger)}$	$[-2, 2]_{\rm theo}$	yes
$\delta_{\rm th} \kappa^f_Z\ ^{(\dagger)}$	$[-2, 2]_{\rm theo}$	yes

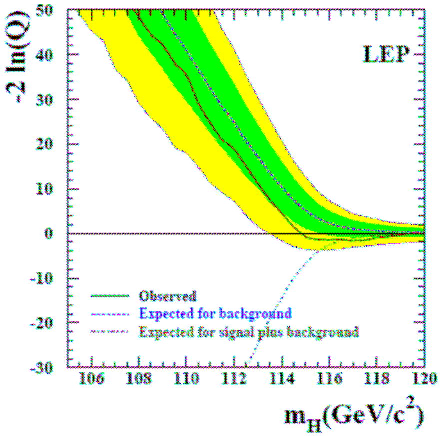
† in units of 10^{-5}

- usage of latest experimental results:
  - **Z-pole observables**: LEP/SLD results [ADLO+SLD, Phys. Rept. 427, 257 (2006)]
  - **M_W and Γ_W**: weighted mean of LEP + Tevatron [ADLO, hep-ex/0612034] [CDF, Phys Rev. D77, 112001 (2008)] [CDF, Phys. Rev. Lett. 100, 071801 (2008)] [CDF+D0, Phys. Rev. D 70, 092008 (2004)]
  - **m_c, m_b**: world averages [PDG, J. Phys. G33,1 (2006)]
  - **m_t**: latest Tevatron average [arXivx:0808.1089 [hep-ex]]
  - **$\Delta\alpha_{had}^{(5)}(M_Z^2)$**: [K. Hagiwara et al., Phys. Lett. B649, 173 (2007)] + Gfitter rescaling mechanism to account for α_S-dependency
- fits are performed in two versions:
  - *Standard fit*: all data except results from direct Higgs searches
  - *Complete fit*: all data including results from direct Higgs searches at LEP [ADLO: Phys. Lett. B565, 61 (2003)] and Tevatron [CDF+D0: arXiv:0804.3423, CDF+D0: arXiv:0808.0534]

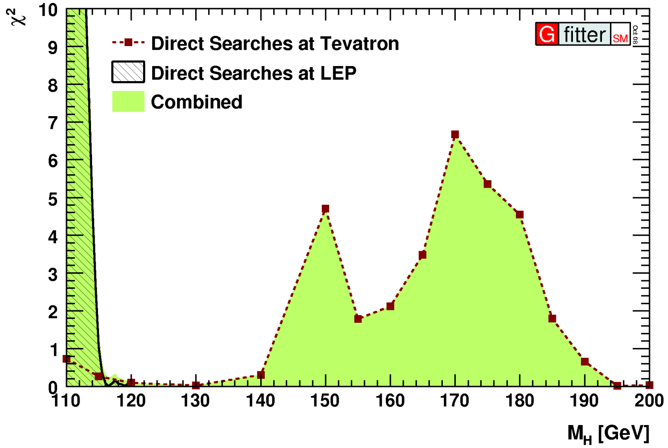
Tevatron



LEP



contribution to χ^2 estimator



- <u>Usage of CL_{S+B}</u>:
  - describe probability of upwards fluctuations of the test statistics (LLR, -2lnQ)
  - transform one-sided CL_{S+B} into a two-sided CL
  - contribution to χ^2 estimator obtained via inverse error function
  - in optimal case: equivalent to χ^2 interpretation of test statistics

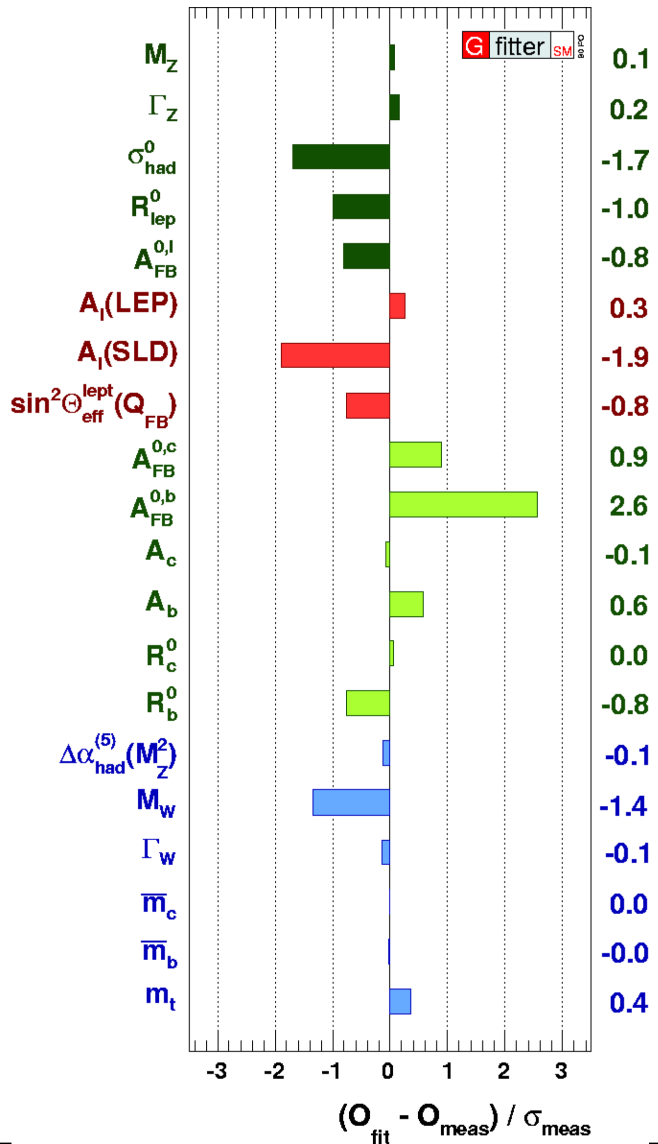
$$\chi^2(M_H) = \frac{(O_{expected,SB} - O_{observed})^2}{\sigma_{band}^2}$$

SM fit: fit results

- convergence and naïve p-values:
  - *standard fit*: $\chi^2_{min}=16.4 \rightarrow \mathrm{Prob}(\chi^2_{min},13)=0.23$
  - *complete fit*: $\chi^2_{min}=18.0 \rightarrow \mathrm{Prob}(\chi^2_{min},14)=0.21$
- α_S from *complete fit*: $\alpha_S(M_Z^2) = 0.1193^{+0.0028}_{-0.0027} \pm 0.0001$
  - first error is experimental fit error
  - second error due to missing QCD orders:
    - incl. variation of renorm. scale from $M_Z/2$ to $2M_Z$ and massless terms of order/beyond $\alpha_S^5(M_Z)$ and massive terms of order/beyond $\alpha_S^4(M_Z)$
  - excellent agreement with recent N^3LO result from τ decay [M. Davier et al., arXiv:0803.0979]

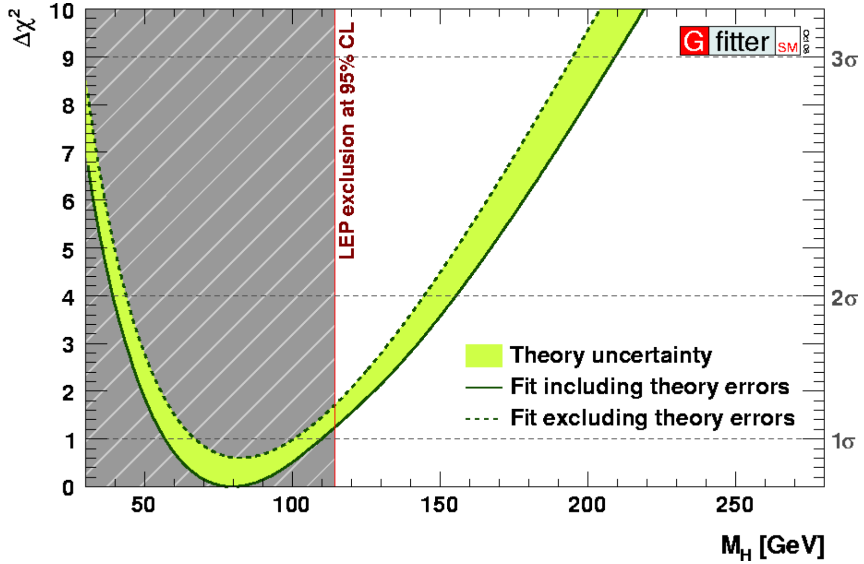
$$\alpha_S(M_Z^2) = 0.1212 \pm 0.0011$$

- pull values of *complete fit*
  - no value exceeds 3σ
  - known tension: leptonic and hadronic asymmetries

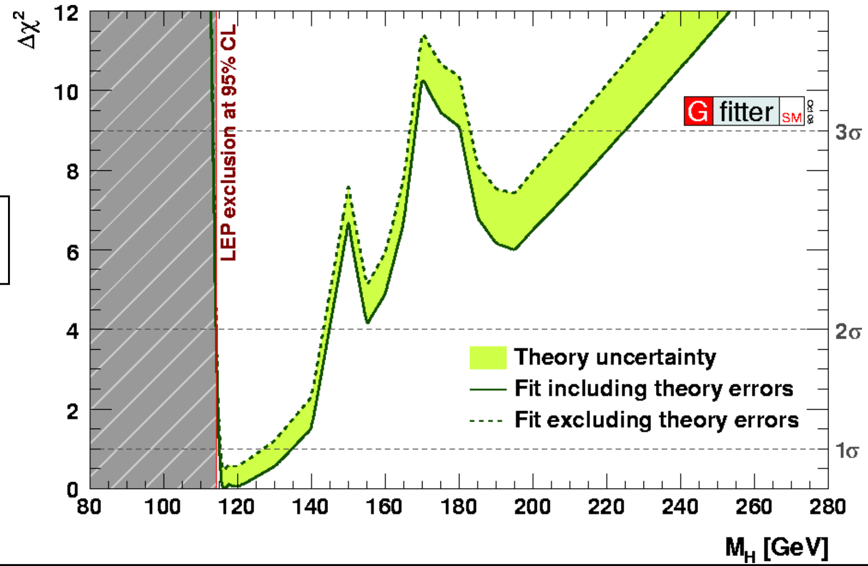


SM fit: results on M_H

- M_H from *standard fit*:
  - central value ±1σ: $M_H = 80^{+30}_{-23}\ GeV$
  - 2σ interval: [39, 155] GeV
  - 3σ interval: [26, 209] GeV
  - theory errors with Rfit scheme ->smaller n_{dof} and smaller χ^2



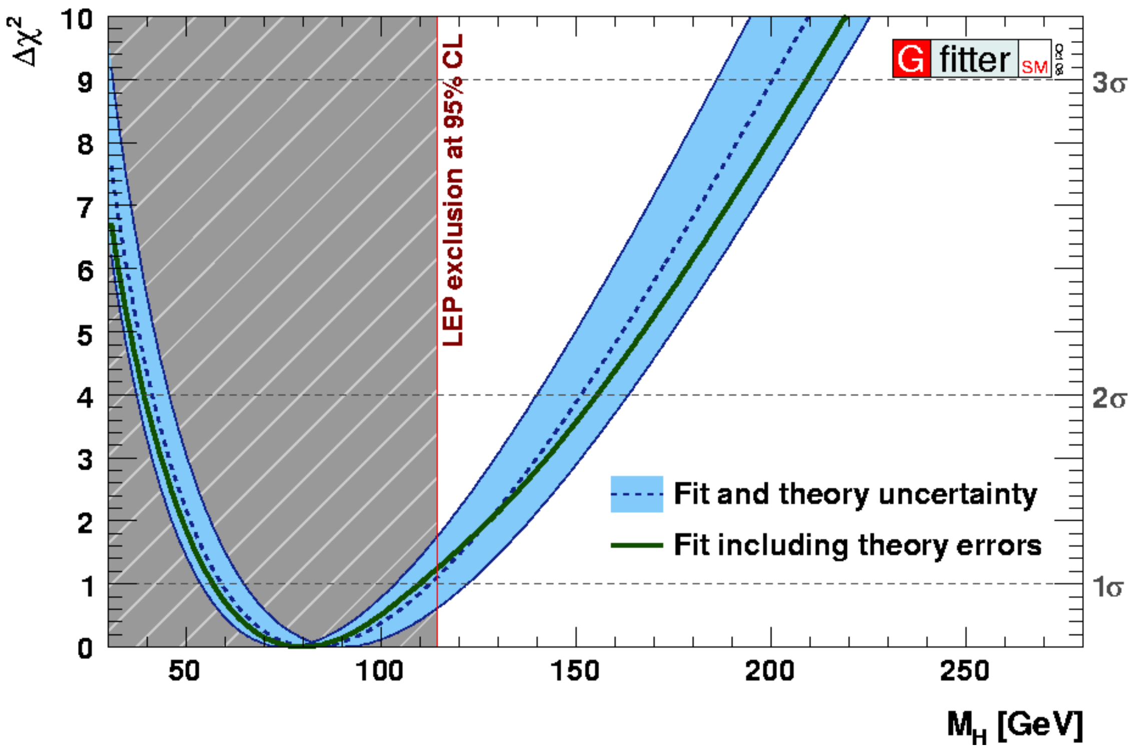
- M_H from *complete fit* (i.e. incl. direct Higgs searches):
  - central value ±1σ: $M_H = 116.4^{+18.3}_{-1.3}\ GeV$
  - 2σ interval: [114, 145] GeV



Treatment of theo. uncertainties

Uncertainties for Theory-Prediction (two main sources)

$$M_W \pm \Delta M_W(theo) \qquad \sin^2\Theta_{eff}^{lept} \pm \Delta\sin^2\Theta_{eff}^{lept}(theo)$$



Old Treatment:

Band was done by **shifting** the predictions by these uncertainties **redoing** the scan and **choosing** the worst cases

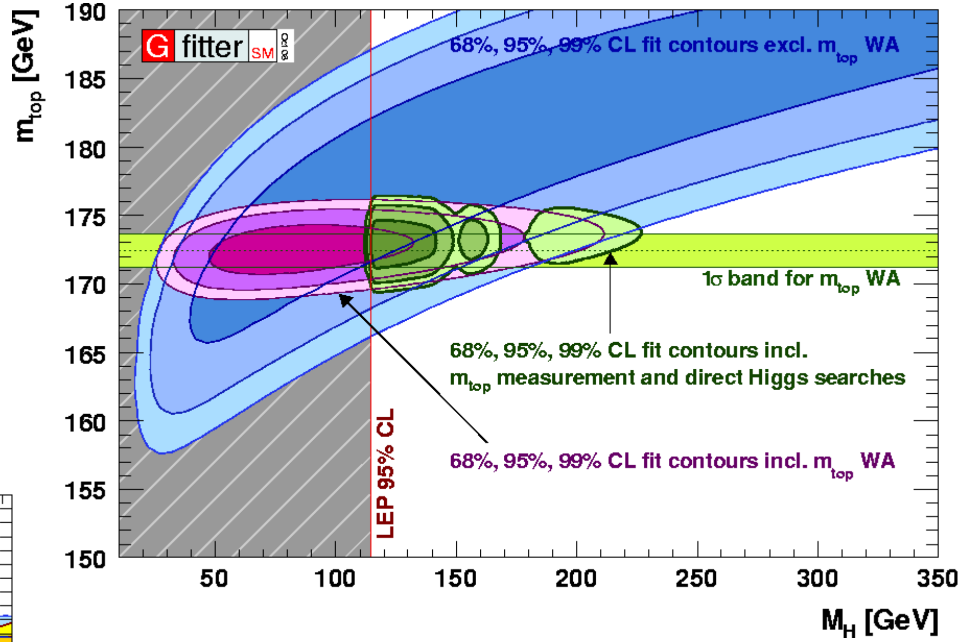
New Treatment:
(à la Rfit [CKMFITTER])

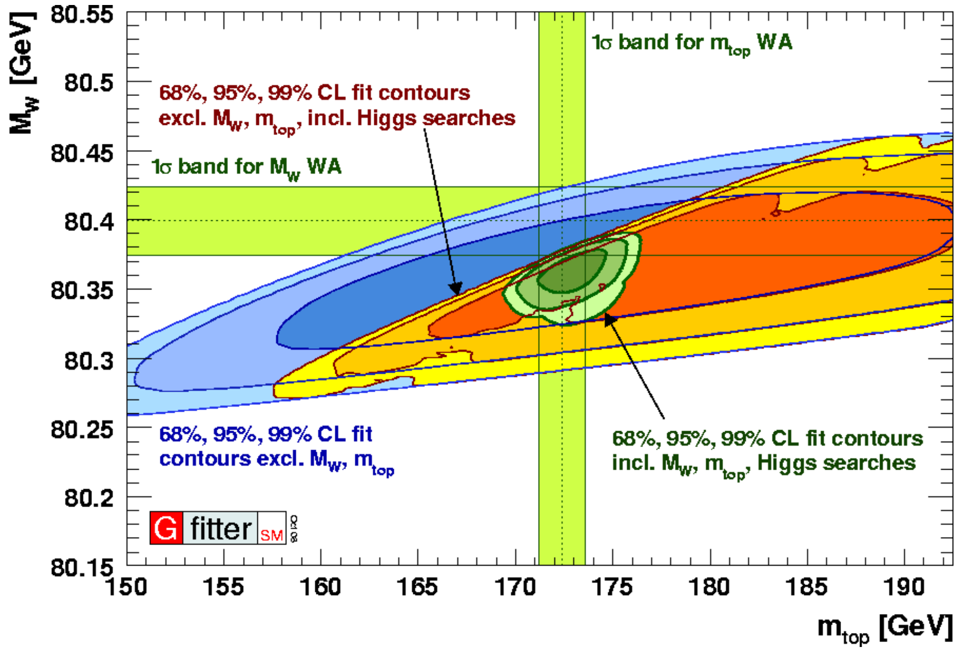
if measurement

- within theory uncertainty: **no contribution** to χ^2.
- outside theory uncertainty: χ^2 determined by **distance** between **measurement** and **prediction ± uncertainty**

SM fit: 2-dim results

- Gfitter allows 1-dim, 2-dim scans and contour plots
- different types of fits e.g.:
  - indirect (i.e. excluding the respective measurements)
  - including the *real* measurements
  - including in addition the results from direct Higgs searches





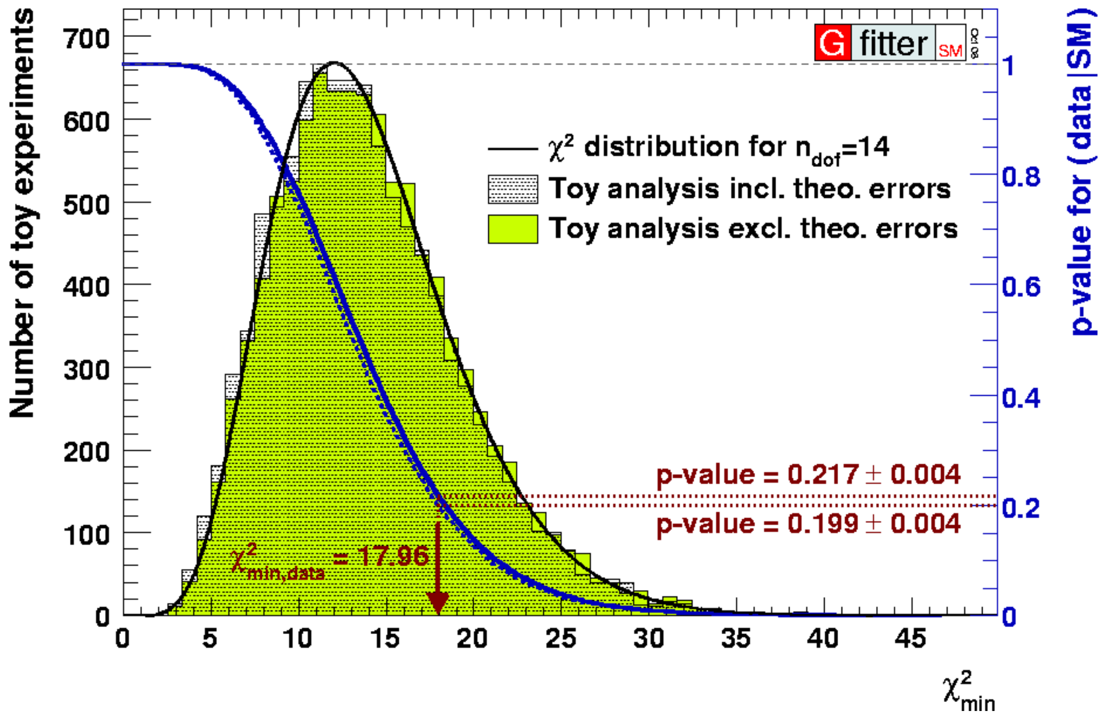
- indirect fit results agree with experimental values
- results from Higgs searches significantly reduce the allowed parameter space
- good probe of SM, if M_H is measured

Goodness of global fit

by using toy analysis

- execute the SM fit
- generate toy sample by random sampling from Gaussian distributions around initial fit results (Correlations are taken into account)
- refit with new values for observables, achieve a new χ^2

10,000 toy experiments



Testing the SM

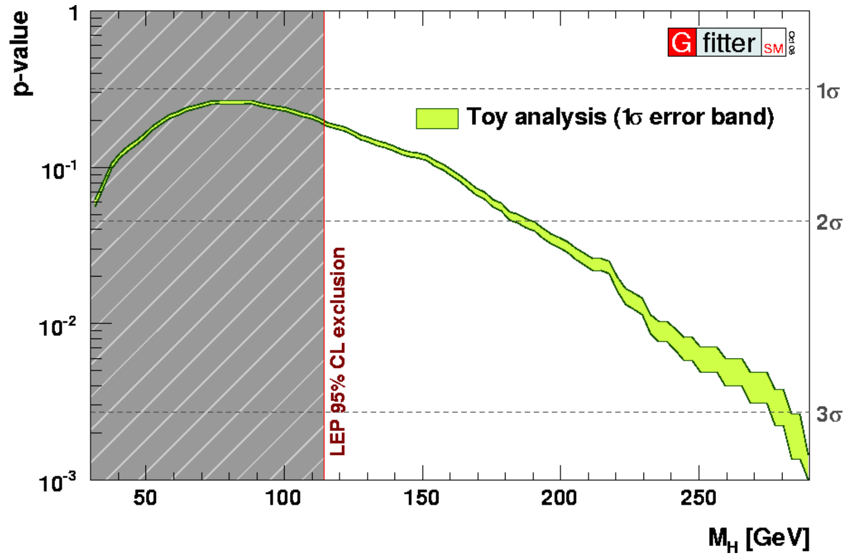
p-value: Probability

- for wrongly rejecting the SM
- for getting a χ^2 larger than the χ^2 of the fit

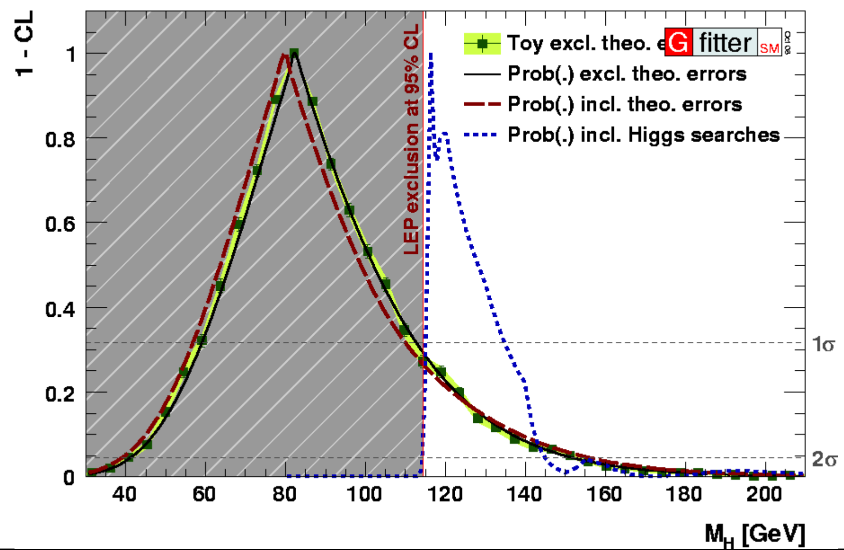
good agreement with ideal χ^2 function

SM fit: statistical analysis

- p-values for fixed Higgs masses using toy experiment
- here: p-value is greater than for fit with free Higgs mass
  - Higgs mass fixed
  - n_{dof} increased by one



- Gfitter allows statistical analysis of fit results
- example: study of the Gaussian properties of the $\Delta\chi^2$ estimator
  - good agreement of CL from MC toy with Gaussian approximation using Prob().



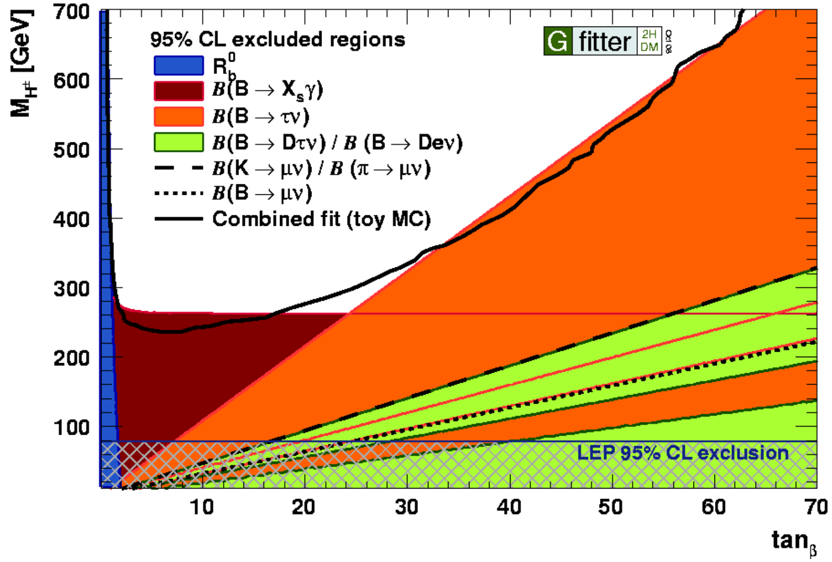
2HDM fit: exp. input and theory

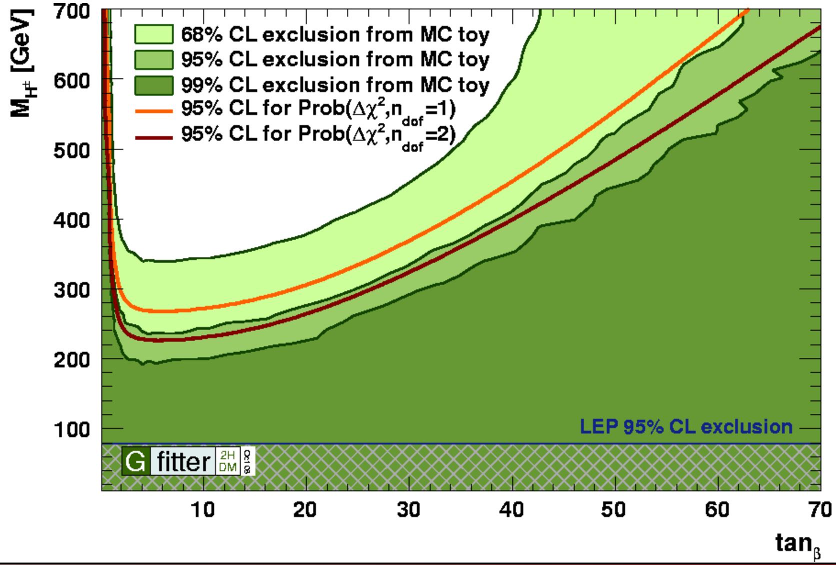
- 2HDM (Type-II)
  - additional Higgs doublet
  - one doublet couples to u-type, one doublet couples to d-type quarks
  - 6 free parameters → $M_{H\pm}$, M_{A0}, M_{H0}, M_h, $\tan\beta$, $|\alpha|$
- so far: only looked at processes sensitive to charged Higgs → $M_{H\pm}$, $\tan\beta$

observable	input value	exp. ref	calculation
R_b^0	0.21629 ± 0.00066	[ADLO, Phys. Rept.427, 257 (2006)	[H. E. Haber and H. E. Logan, Phys. Rev. D62, 015011 (2000)]
BR (B->$X_s\gamma$)	$(3.52 \pm 0.23 \pm 0.09) \cdot 10^{-4}$	[HFAG, latest update]	[M. Misiak et al., Phys. Rev. Lett. 98, 022002 (2007)]
BR (B->$\tau\nu$)	$(1.51 \pm 0.33) \cdot 10^{-4}$	[P.Chang, Talk at ICHEP 2008]	[W. S. Hou, Phys. Rev. D48, 2342 (1993)]
BR (B->$\mu\nu$)	$(-5.7 \pm 6.8 \pm 7.1) \cdot 10^{-4}$	[E. Baracchini, Talk at ICHEP 2008]	[W. S. Hou, Phys Rev. D48, 2342 (1993)]
BR (K->$\mu\nu$)/ BR(π->$\mu\nu$)	1.004 ± 0.007	[FlaviaNet,, arXiv:0801.1817]	[FlaviaNet, arXiv:0801.1817]
BR(B->D$\tau\nu$)/ BR(B->Deν)	$0.416 \pm 0.117 \pm 0.052$	[Babar, Phys. Rev. Lett 100, 021801 (2008)]	[J. F. Kamenik and F. Mescia, arXiv:0802.3790]

2HDM fit: fit results

- Overlay of individual 95% CL excluded regions
  - assuming n_{dof}=1 and 2-sided limits
- Combined fit:
  - excluded area depends on assumptions [Prob($\Delta\chi^2$,n_{dof}=1), Prob($\Delta\chi^2$,n_{dof}=2)]
  - resolved by MC toy study
  - 2-sided limits
  - χ^2_{min}=3.9 at M_H=858 GeV and tanβ=6.8
- Excluded at 95% CL:
  - small tanβ
  - for all tanβ
    - M_H < 240 GeV
    - M_H < (8.6 tanβ) GeV





Summary

- Gfitter is a framework for involved fitting problems
- First theory package: Revisit of the electroweak fit of the SM
  - latest theoretical calculations and experimental results
  - advanced studies of the statistical properties of the fit
  - inclusion of direct Higgs searches
- Example for SM extension: 2HDM (Type-II)
- Next steps:
  - implementation of more theories, e.g. SUSY models, little Higgs
  - improve framework, e.g. lvmini
- More information:
  - http://cern.ch/Gfitter
  - paper submitted to Eur. Phys. J. C, (arXiv:0811.0009)

SM fit: prospects

- LHC, ILC (+GigaZ)
  - exp. improvement on M_W, m_t, $\sin^2\theta^l_{eff}$, R_l^0
- improved $\Delta\alpha_{had}^{(5)}(M_Z^2)$, e.g. $\sigma(\Delta\alpha_{had}^{(5)}) \sim 7 \cdot 10^{-5}$
  [F. Jegerlehner, hep-ph/0105283]

Quantity	Expected uncertainty			
	Present	LHC	ILC	GigaZ (ILC)
M_W [MeV]	25	15	15	6
m_t [GeV]	1.2	1.0	0.2	0.1
$\sin^2\theta^\ell_{\rm eff}$ $[10^{-5}]$	17	17	17	1.3
R^0_ℓ $[10^{-2}]$	2.5	2.5	2.5	0.4
$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ $[10^{-5}]$	22 (7)	22 (7)	22 (7)	22 (7)
$M_H(=120\ {\rm GeV})$ [GeV]	${}^{+56}_{-41}$ $({}^{+52}_{-39})$ $[{}^{+39}_{-31}]$	${}^{+45}_{-35}$ $({}^{+42}_{-33})$ $[{}^{+30}_{-25}]$	${}^{+42}_{-33}$ $({}^{+39}_{-31})$ $[{}^{+28}_{-23}]$	${}^{+26}_{-23}$ $({}^{+20}_{-18})$ $[{}^{+8}_{-7}]$
$\alpha_s(M_Z^2)$ $[10^{-4}]$	28	28	28	6

[ATLAS, Physics TDR (1999)][CMS, Physics TDR (2006)][A. Djouadi et al., arXiv:0709.1893]
[I. Borjanovic, EPJ C39S2, 63 (2005)][S. Haywood et al., hep-ph/0003275]
[R. Hawkings, K. Mönig, EPJ direct C1, 8 (1999)]
[A. H. Hoang et al., EPJ direct C2, 1 (2000)][M. Winter, LC-PHSM-2001-016]

- Fits:
  - not used: α_S, M_H measurements
  - assume M_H=120 GeV
  - improvement of M_H prediction
    - to be confronted with direct measurement → goodness-of-fit
    - broad minima: Rfit treatment of theo. uncertainties
  - GigaZ: significant improvement for $\alpha_S(M_Z^2)$