EPTA: Searching for gravitational waves in nano-Hertz band.

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for EPTA collaboration

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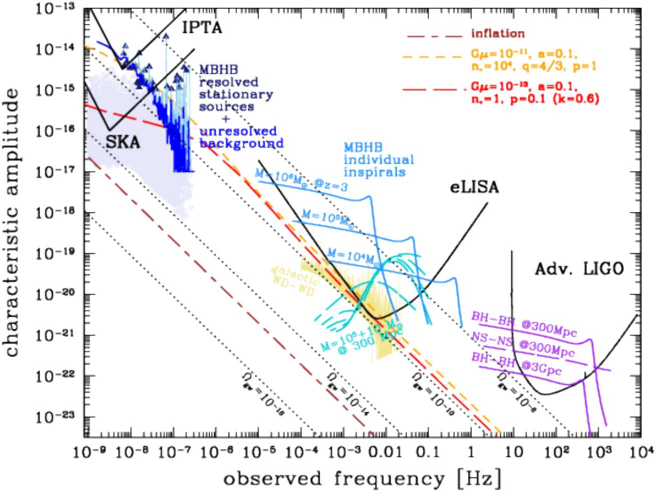
Outline

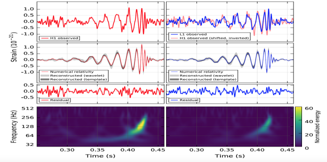
- Gravitational wave landscape
- Millisecond pulsars as cosmic clocks
- GW signal in the nano-Hz band
- Upper limit on GW signal with EPTA

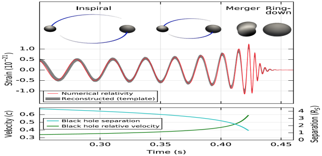
GW landscape

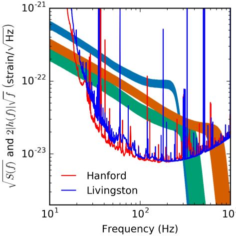


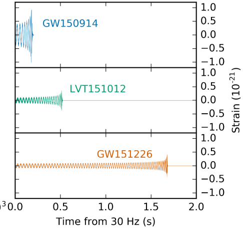
[credits: A. Sesana]

Detection of Gravitational wave signal from the merging black hole binary.



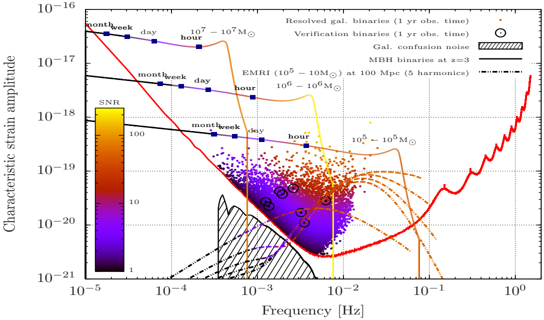






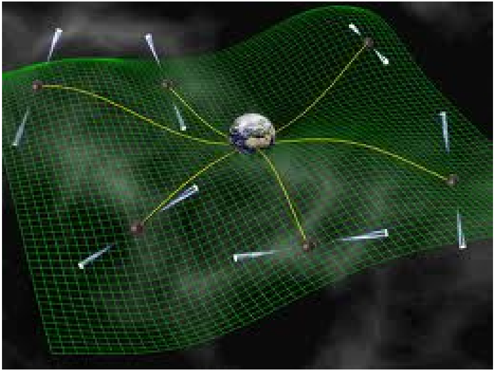
eLISA: GW observatory in space

Gravitational wave in mHz: eLISA mission to be launched 20...



Pulsar Timing Array

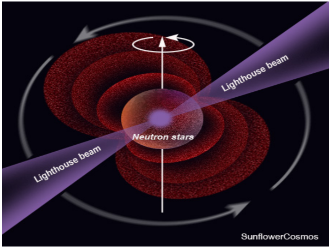
The main idea behind pulsar timing array (PTA) is to use ultra-stable millisecond pulsars as beacons for detecting GW in the nano-Hz range ($10^{-9} - 10^{-7}$ Hz).

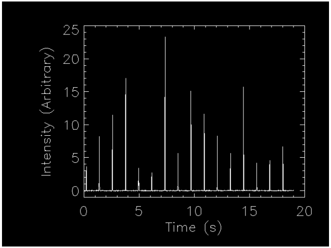


[credits D. Champion]

Milisecond Pulsars

Pulsars are neutron stars with rapid rotation and strong magnetic field. Period from few seconds to few milliseconds (MSP). MSP - usually old, recycled pulsars, often in binaries.

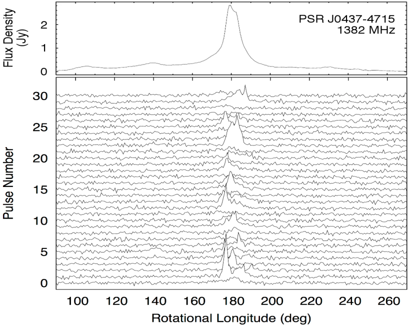


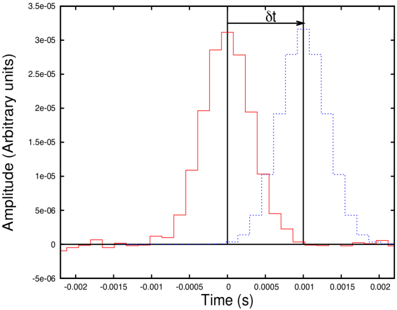


IPTA/EPTA

- International Pulsar Timing Array (IPTA) consortium of consortia: PPTA (Australian), EPTA (European), NanoGrav (North American).
- EPTA: consist of 5 radio telescopes (coherent observations; LEAP-project)

Timing pulsars

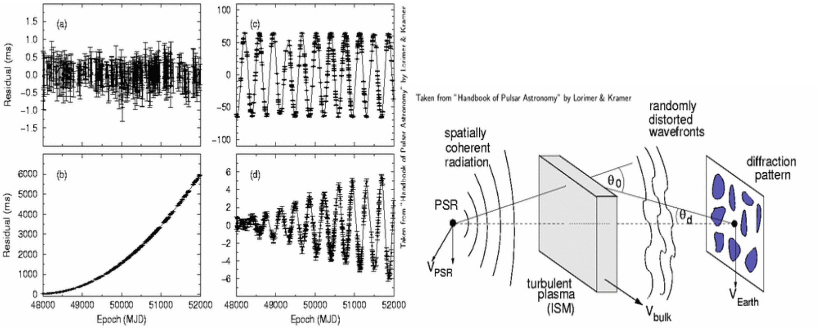




[Figs. credits S. Burke-Spolar and L. Lentati]

- Each pulse profile has a lot of micro-structure, but averaged over hour it is stable
- We use average pulse profile to get time-of-arrival (TOA) for the pulses
- We know well the spin of pulsars: can predict TOAs and subtract them from measured: residuals

Timing pulsars



- We need to add a timing model: to adjust the theoretical TOAs taking into account various physical effects
- We need to take into account dispersion and time dependent variations of dispersion, need to fit for rate of change of rotation (b), for position of a pulsar (c), proper motion of a pulsar (d) to get clean residuals

Timing residuals

The complete timing model for times-of-arrival depends on many parameters:

$$t_{toa} = t_{toa}(P, \dot{P}, \ddot{P}, \Delta_{clock}, \Delta_{DM}(L), \Delta_{\odot-\oplus}, \Delta_E, \Delta_S)$$

$P, \dot{P}, \ddot{P}$ - period of pulsar, its spin-down, glitches
Δ_{clock} difference in local clock and terrestrial standard
$\Delta_{DM}(L)$ delay caused by ISM
$\Delta_{\odot-\oplus}$ translation from the SSB to observatory frame
Δ_E accounts for the time dilation from the moving pulsar (and observatory) and the gravitational redshift caused by the Sun and planets or the binary companion
Δ_S is the extra time required by the pulses to travel through the curved space-time containing the Sun/planets/companions

$$dt = t^p_{toa} - t^o_{toa} = dt_{errors} + \delta\tau_{GW} + noise$$

Response to GW

- Assume TT-gauge. On the ground (LIGO) or in space (eLISA) we use Michelson interferometer to measure the phase (or frequency) difference between the laser beams travelled along arms in the tidal field of GW.
- PTA can be seen as a multi-arm detector where e/m signal travels only in one direction (from a pulsar to the Earth). Pulsar plays role of an accurate clock, and we measure change in phase (frequency) of arriving pulses (similar to the frequency (phase) of the laser light)
- Important quantity in the response function is $\epsilon = (2\pi f L/c)$: if $\epsilon << 1$ - long wavelength approximation $R \sim h_{ij} x^i x^j$
- Introduce $f_* : \quad \epsilon(f_*) = 1$, LIGO: $f_* = 12000$Hz, LISA: $f_* = 0.05$ Hz, PTA: $f_* = 0.002$nHz

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Response to GW

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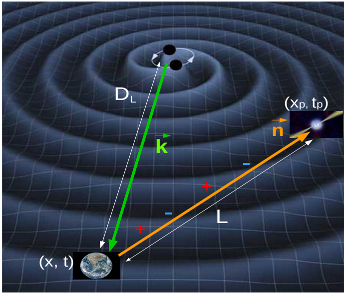
The response to GW is given as

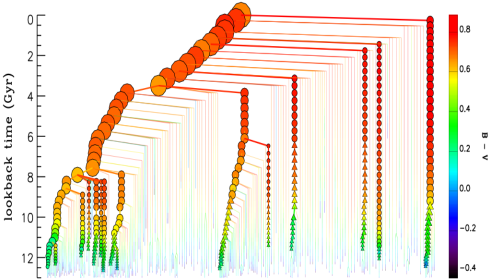
$$\delta\tau_{GW} = r(t) = \int_0^t \frac{\delta\nu}{\nu}(t')dt'; \qquad \frac{\delta\nu}{\nu} = \frac{1}{2}\frac{\hat{n}^i\hat{n}^j}{1+\hat{n}.\hat{k}}\Delta h_{ij}$$



$\Delta h_{ij} = h_{ij}(t_p = t - L(1+\hat{n}.\hat{k})) - h_{ij}(t)$
Since the pulsars are not correlated (t_p, the emission time of the pulse detected at the time t on the Earth, is different for all pulsars) the "pulsar" terms do not add up coherently.

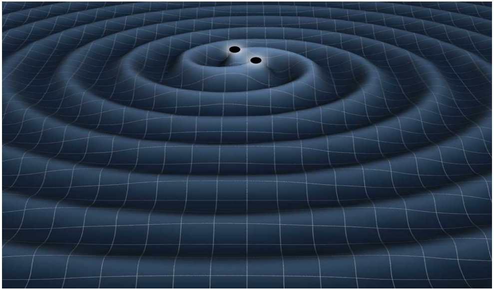
Super massive black holes (SMBHs)

We believe that (super) massive black holes reside in the nuclei of every galaxy. SMBHs are formed from relatively small initial seeds (popIII stars, direct collapse of a giant protocloud) and acquired mass through accretion and major mergers



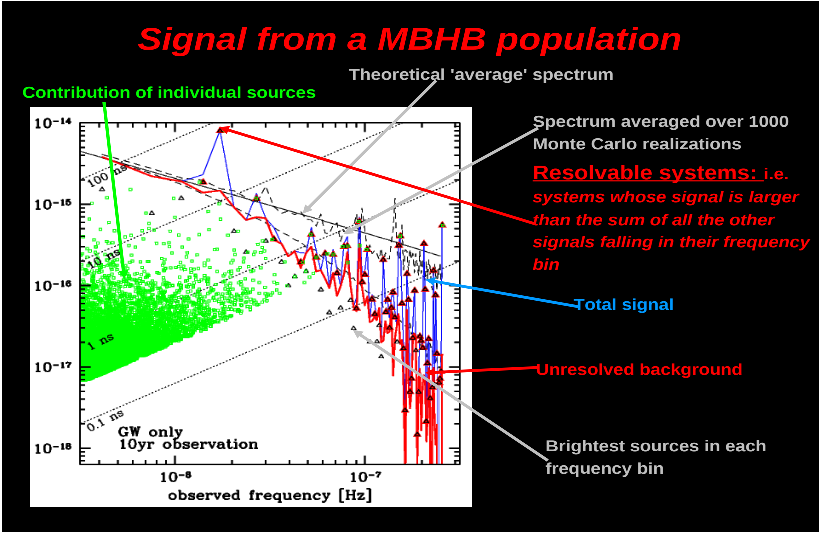
[credits: G. De Lucia]

SMBH binaries as GW sources



- Main sources are super-massive ($10^7 - 10^{10} M_\odot$) black hole binaries
- PTA is sensitive to binaries in the broad orbits (period $\sim$ year)
- The GWs remove orbital energy and angular momentum from the binaries $dE/dt \sim \eta(M/r)^5$: signal is almost monochromatic at nHz orbital frequency

Population of SMBH binaries



[credits: A. Sesana]

GW signal

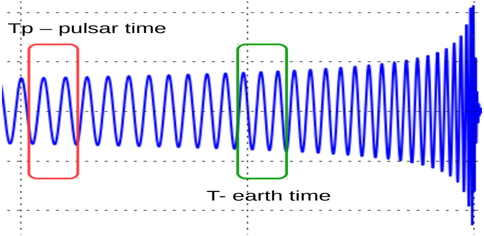
Consider non-spinning SMBH binaries in circular orbit.

- each pulsar and earth term are monochromatic signals
- frequency of pulsar term might or might not coincide with the earth term $t_p = t - L(1 + \hat{n}.\hat{k})$
- amplitude of a pulsar term is larger ($\sim \omega^{-1/3}$)

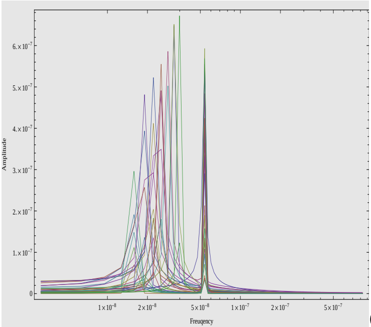
$$
\begin{aligned}
s_\alpha \quad = \quad & F_\alpha^{+}(\hat{k}, \hat{n}_\alpha)\left[\frac{h_+(t_p^\alpha, \omega_\alpha)}{2\pi f_\alpha} - \frac{h_+(t, \omega)}{2\pi f}\right] + \\
& F_\alpha^{\times}(\hat{k}, \hat{n}_\alpha)\left[\frac{h_\times(t_p^\alpha, \omega_\alpha)}{2\pi f_\alpha} - \frac{h_\times(t, \omega)}{2\pi f}\right]
\end{aligned}
$$

Earth-term and pulsar-term

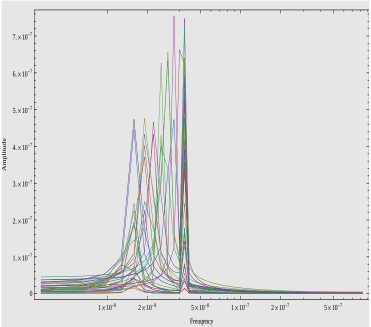
Pulsar and earth terms "see" different part of the GW signal.



Earth-term and pulsar term



or



Detection statistic and search algorithm

- The likelihood function (likelihood of the signal with given parameters is present in the observed data) is given by

$$P(\vec{\delta t}, \vec{\theta}) = \frac{1}{\sqrt{(2\pi)^n det(C)}} \exp\left(-\frac{1}{2}(\vec{\delta t} - \vec{s})^T C^{-1} (\vec{\delta t} - \vec{s})\right),$$

- $\vec{\delta t}$ are observed timing residuals from all pulsars packed in a single array of a total size n
- $\vec{s}$ is a model for a GW contribution from individual resolvable signals.
- C is a variance-covariance matrix:

$$C_{\alpha i,\beta j} = C^{wn}\delta_{\alpha\beta}\delta_{ij} + C^{rn}_{ij}\delta_{\alpha\beta} + C^{dm}_{ij}\delta_{\alpha\beta} + C^{GW}_{\alpha i,\beta j}.$$

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Characterizing signal and noise

- Individual signals are characterized by: Earth term $\mathcal{A}, \iota, \psi, \Phi_0$, f, θ, ϕ, pulsar term L_α, $\mathcal{M}_c$: $(8 + N_p$ parameters)
- GW background (GWB) signal is characterized by amplitude and slope $A(f_0), \gamma$ (for population of binaries $\gamma = 13/3$: (2 parameters)
- "Red" noise and DM variations are characterized by amplitude and slope (similar to GWB), DM variations depend on measurement (radio) frequency. White noise: measurement error σ is scaled $+$ systemtics: $\alpha_i \sigma^2 + \beta_i^2$: $(4 \times N_p + (2 \times N_b) \times N_p)$ parameters).

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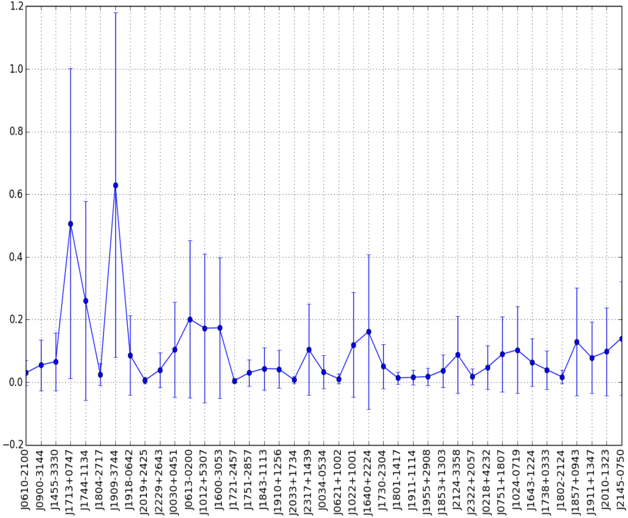
Analysing EPTA data

- Analysing 41 pulsar (or 6 best pulsars for most computationally expensive searches)
- Noise: the noise parameters should not correlate across pulsars (only GWB), so we can estimate parameters of the noise for each pulsar separately (single pulsar analysis). $\rightarrow$ use either maximum likelihood estimators for the noise parameters or posterior distribution to sample from.
- Data analysis tools: Multi-modal genetic algorithm, MCMC Hammer, MultiNest.

Analysing EPTA data



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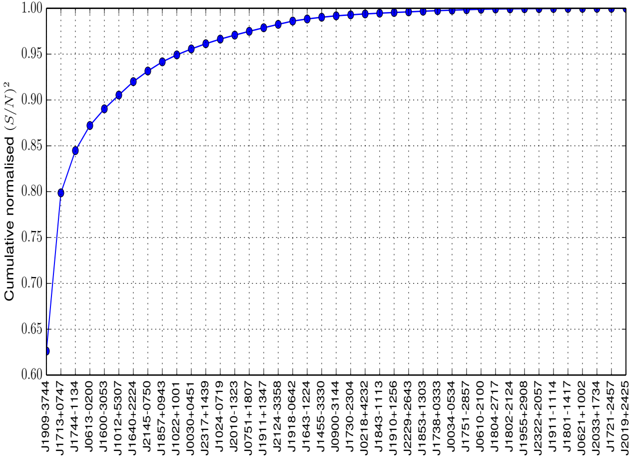
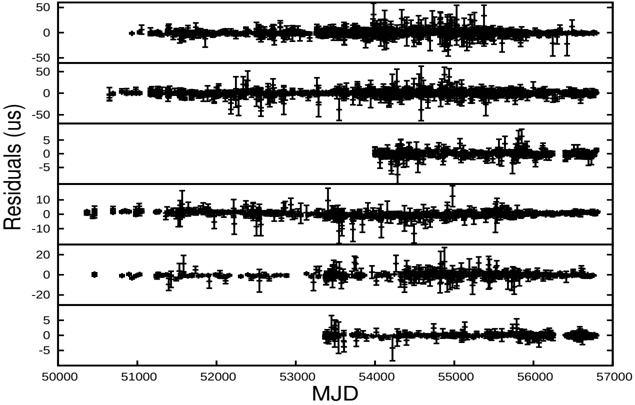


Residuals of 6 best EPTA pulsars

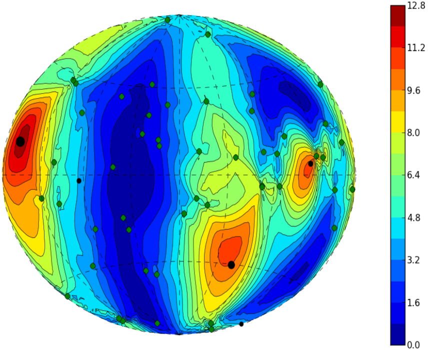
From top to bottom these are PSRs: J0613-0200, J1012+5307, J1600-3053, J1713+0747, J1744-1134, and J1909-3744



Assumptions and setup

- Assume non-spinning SMBHs in circular orbit
- Model contains a single GW signal
- Separate searches: (i) using earth-term only (ii) using full non-evolving signal ($f_p = f_e$) (iii) using full evolving signal
- Methods: We use frequentist and Bayesian methods for setting upper limit on the strain of monochromatic GW source

- What do we expect to see if we use a “single-GW-source" model?
- Consider simulated data: 5 GW sources, and 50 pulsars. The sky map assuming a single GW source.



With 1-source model we resolve three strongest sources (the size of circle ∼ strength)

Simulated data [SB, A. Sesana, 2012]

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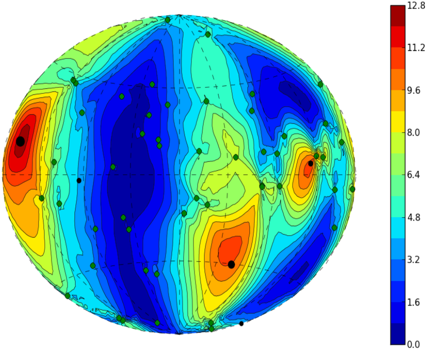

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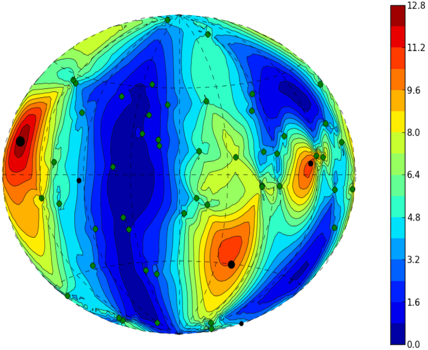

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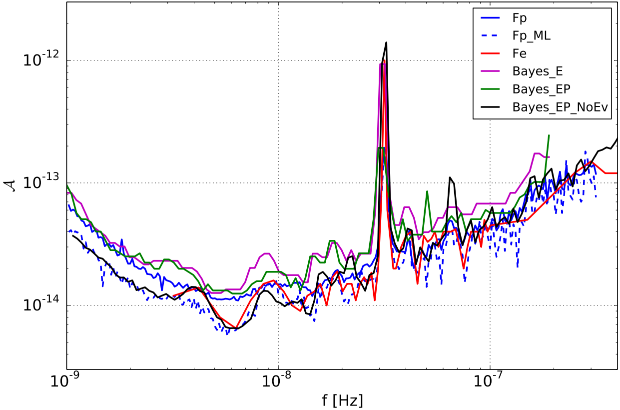
The upper limit on the strain of continuous GW signal with EPTA data, [Babak+ 2015]

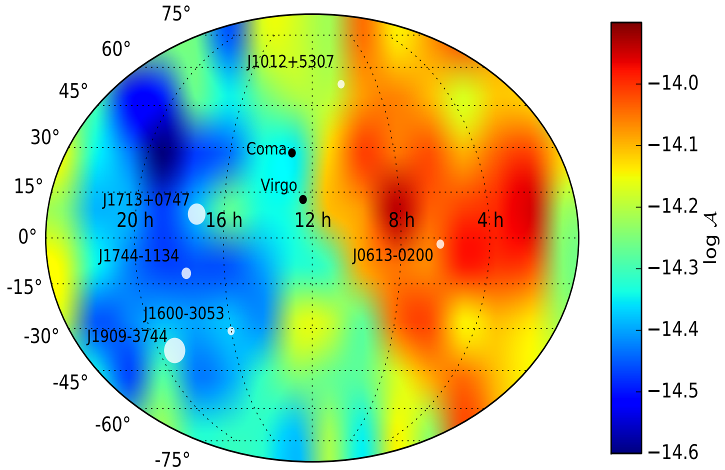
The upper limit on the strain of continuous GW signal:

$6 \times 10^{-15} < \mathcal{A} < 1.5 \times 10^{-14}$



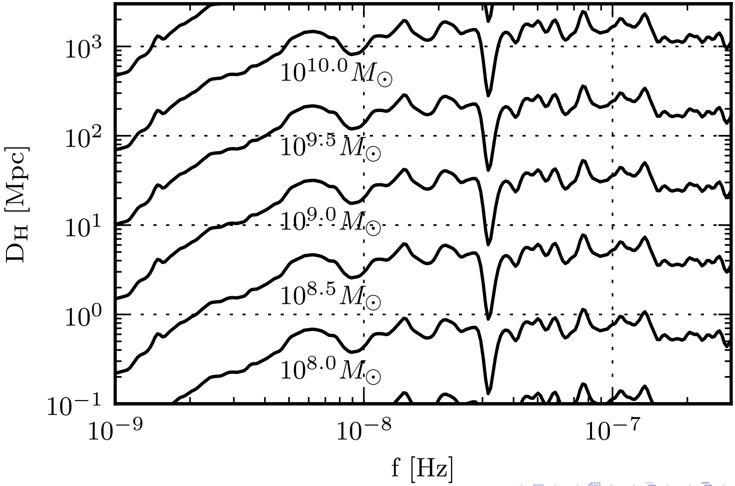
The upper limit on the strain of continuous GW signal with EPTA data. [Babak+ 2015]

Directional dependence of the upper limit (at 7nHz)



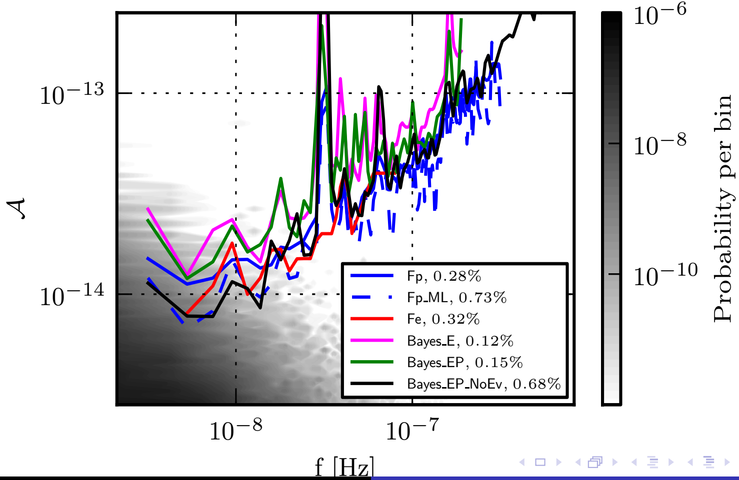
The upper limit on the strain of continuous GW signal with EPTA data. [Babak+ 2015]

Horizon distance: max distance we are sensitive to.



The upper limit on the strain of continuous GW signal with EPTA data. [Babak+ 2015]

Given current EPTA sensitivity: what probability that we would detect a GW signal?



Stochastic GW background

- Once more the likelihood:

$$P(\vec{\delta t}, \vec{\theta}) = \frac{1}{\sqrt{(2\pi)^n det(C)}} \exp\left(-\frac{1}{2}(\vec{\delta t} - \vec{s})^T C^{-1} (\vec{\delta t} - \vec{s})\right),$$

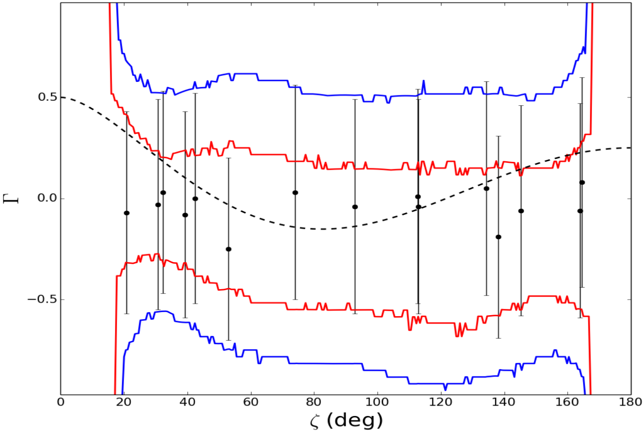
where

$$C_{\alpha i, \beta j} = C^{wn}\delta_{\alpha\beta}\delta_{ij} + C^{rn}_{ij}\delta_{\alpha\beta} + C^{dm}_{ij}\delta_{\alpha\beta} + C^{GW}_{\alpha i, \beta j}.$$

- The GW signal is a noise like, but noise which is the same in each pulsar data. If it comes from superposition of multiple MBH binaries then it has power-law spectral shape: $S(f) \sim A_{gw} f^{-13/3}$.
- The red noise (jitter of the NS spin vector) has similar shape but the noise is not correlated between pulsars $S_{rn} \sim A_{rn} f^{-\gamma}$, where $\gamma < 3$.

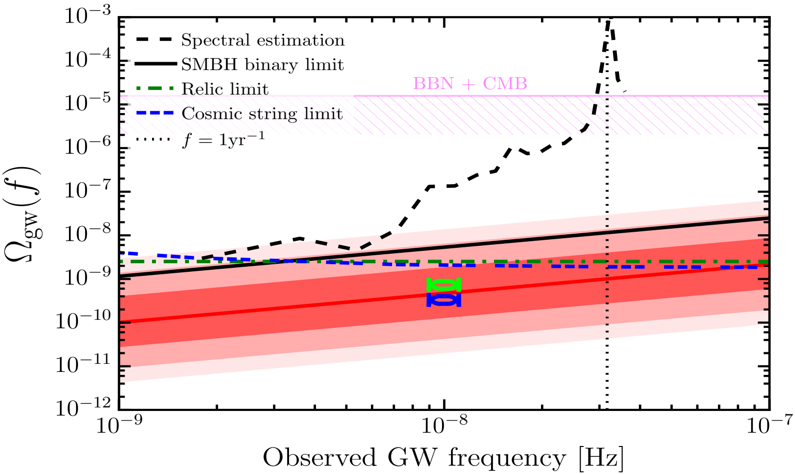
Searching GWB with EPTA pulsars. [Lentati+ 2015]

Estimation of the correlation coefficients and comparison to Hellings-Downs correlation (dashed curve)



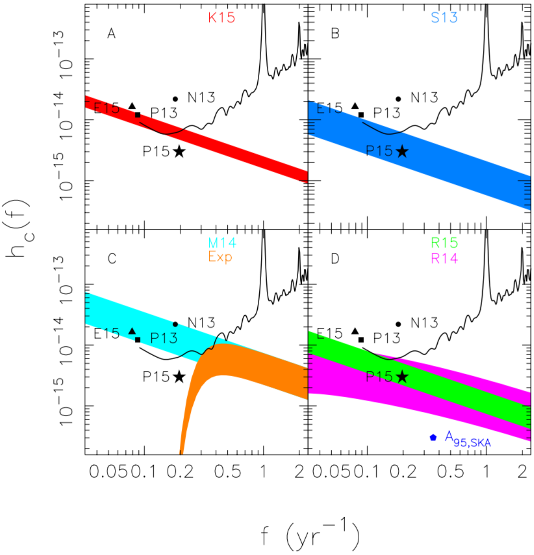
Upper limit on GWB with EPTA pulsars & astrophysical implication. [Lentati+ 2015]

Results in terms of $\Omega_{gw}(f)$ as a function of GW frequency, with $H_0 = 70 kms^{-1} Mpc^{-1}$



PPTA limit oh stochastic GWs [Shannon et al. 2015]

The best current limit on stochastic GW background from PPTA

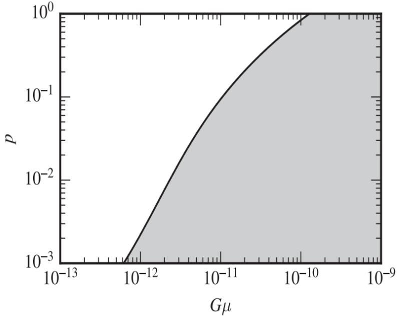


data:

Limiting parameters of strings from stochastic GWs

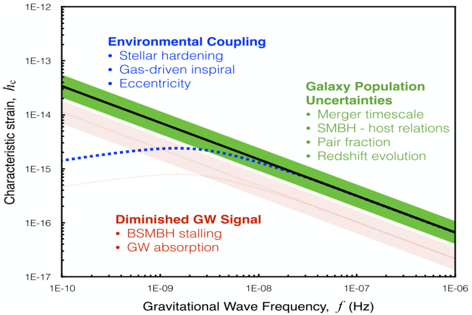
[Arzoumanian et al. 2015]

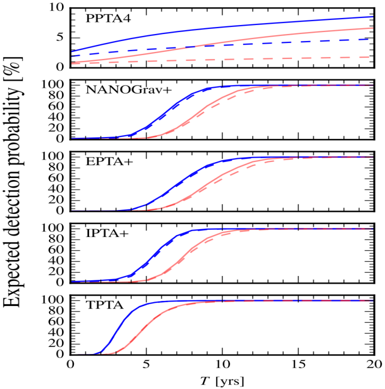
Assume that the stochastic GW background is created by cosmic (super)strings, then the upper limit can be translated into restrictions on the string parameters: tension (μ) vs. probability reconnection and/or tension vs char. size at birth (α).



Astrophysical uncertainties and future perspectives [S. Burke-Spolaor 2015]

There are many uncertainties which come into predictions of astrophysical GW background (talk by A. Sesana)





Estimation of significance

- We use Bayesian approach to the detection and parameter estimation
- Bayesian model selection (model that the signal is present in the data vs. model where we have noise only) is based on computation of the Bayes factor (ratio of evidence for each hypothesis)
- Need to quantify if it is significant or not

K	dHart	bits	Strength of evidence
$< 10^0$	< 0		negative (supports M_2)
10^0 to $10^{1/2}$	0 to 5	0 to 1.6	barely worth mentioning
$10^{1/2}$ to 10^1	5 to 10	1.6 to 3.3	substantial
10^1 to $10^{3/2}$	10 to 15	3.3 to 5.0	strong
$10^{3/2}$ to 10^2	15 to 20	5.0 to 6.6	very strong
$> 10^2$	> 20	> 6.6	decisive

$2 \ln K$	K	Strength of evidence
0 to 2	1 to 3	not worth more than a bare mention
2 to 6	3 to 20	positive
6 to 10	20 to 150	strong
>10	>150	very strong

Estimation of significance

- LIGO: Time shift of H1 and L1 data > light propagation time guarantees that there is no signal in coincidence (low event environment)
- eLISA: No need, usually high SNR events, the problem is the signal confusion
- PTA: ???, low SNR signal, signal is stochastic correlated noise + (maybe) individual signals.

In PTA we can destroy correlation (Hellings-Downs) of the signal across pulsars: (1) scramble the sky position of pulsars, (2) mess up the phase across frequency bands

Estimation of significance

- We need to produce realistic data which has no GW signal (or we destroy the GW properties (correlation) in the data)
- Alternative: Search for a "wrong" signal in the data
- "wrong" $\rightarrow$ (1) using wrong position of pulsars on the sky (2) using uncorrelated phase at different frequencies

Let us look at "SNR" in the frequency domain ([Anholm et.al. 2008]):

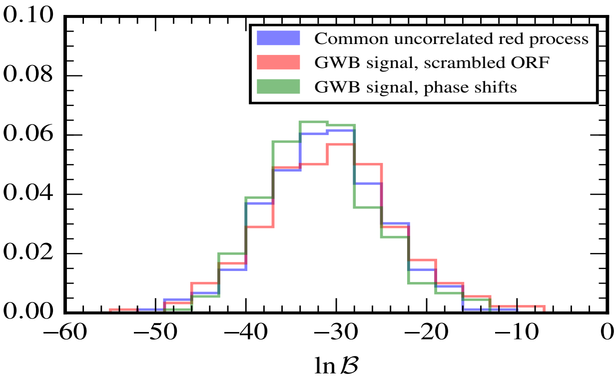
$$\rho \sim \sum_{\alpha,\beta} \sum_{k} f_k^{-13/3} \Gamma_{\alpha\beta}(f_k, \theta_{\alpha\beta}) \frac{S_{\alpha\beta}(f_k, \theta_{\alpha\beta})}{P_\alpha(f_k) P_\beta(f_k)}$$

- $S_{\alpha\beta}(f_k, \theta_{\alpha\beta})$ observed cross-correlation between each pair of pulsars α, β, and $P_\alpha(f_k), P_\beta(f_k)$ is PSD for each pulsar data at frequency f_k
- $f_k^{-13/3} \Gamma_{\alpha\beta}(f_k, \theta_{\alpha\beta})$ expected amplitude (spectral) and correlation of GWB

Instead of touching the data (which is also possible), we modify the "expected signal": $\Gamma_{\alpha\beta}(f_k, \theta_{\alpha\beta}) \rightarrow \Gamma_{\alpha\beta}(f_k, \tilde{\theta}_{\alpha\beta})$ - changing the sky position of pulsars; or $\Gamma_{\alpha\beta}(f_k, \theta_{\alpha\beta}) \rightarrow \Gamma_{\alpha\beta}(f_k, \theta_{\alpha\beta}) \cos(\phi_{\alpha,k} - \phi_{\beta,k})$ - changing the phase at different f_k.

Estimation of significance

- We have simulated the data with GWB: Bayes factor 43
- We have performed both (1) sky position scramble (red) and (2) freq. dependent phase shift: The Bayes factor is consistent with common *uncorrelated* red noise



[S. R. Taylor, L. Lentati, S. Babak, P. Brem, J. R. Gair, A. Sesana, A. Vecchio 2016]

Summary

- The SMBH binaries in the local Universe will create GW signal which could be seen as a stochastic signal plus (may be) few resolvable signals standing above the background
- Several search methods were developed: (i) search for isotropic stochastic GWB (ii) search for multiple individual signals (iii) combined single source(s) plus isotropic background
- No detection. The upper limits on the stochastic GWB:
  EPTA: $A < 3 \times 10^{-15}, \ \ \Omega_{gw} h^2 < 1.1 \times 10^{-9}$,
  Nanograv: $A < 1.5 \times 10^{-15}, \ \ \Omega_{gw} h^2 < 4.2 \times 10^{-10}$
  PPTA: $A < 10^{-15}, \ \ \Omega_{gw} < 2.3 \times 10^{-10}$
  and individual sources become astrophysically interesting (we rule out some models).

Summary

- The GW signal in PTA is a result of long-term integration: will gradually emerge
- We have discussed two methods how to make destroy the properties of the GWB in the data and estimate significance of any a GW candidate