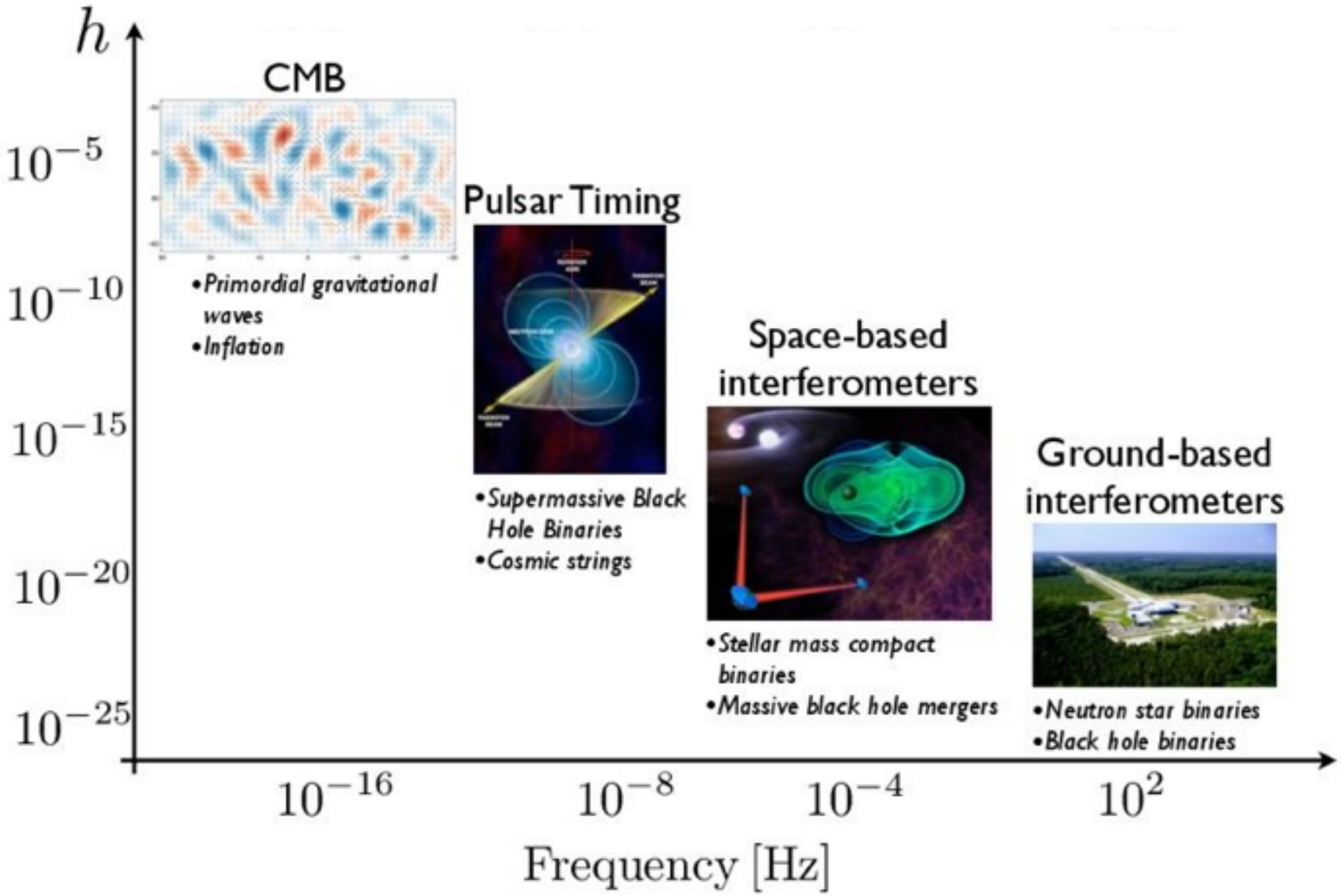
Detecting Stochastic Gravitational Wave Signals



eXtreme Gravity Institute

Neil Cornish

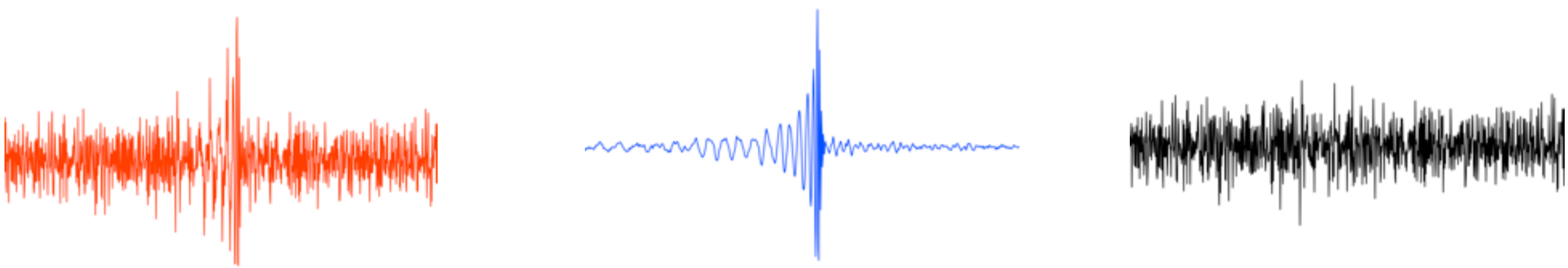
MONTANA STATE UNIVERSITY

Outline

- Gravitational Wave Data Analysis 101
- Noise and Signal Models
- Multiple Detector Cross-correlation
- Single Detector Spectral Separation
- Alternative Gravity Tests with PTAs
- Detection with LISA

Romano & Cornish 1608.06889, Cornish & Romano 1305.2934,
Adams & Cornish 1002.1291, Adams & Cornish 1307.4116

Signal Detection 101

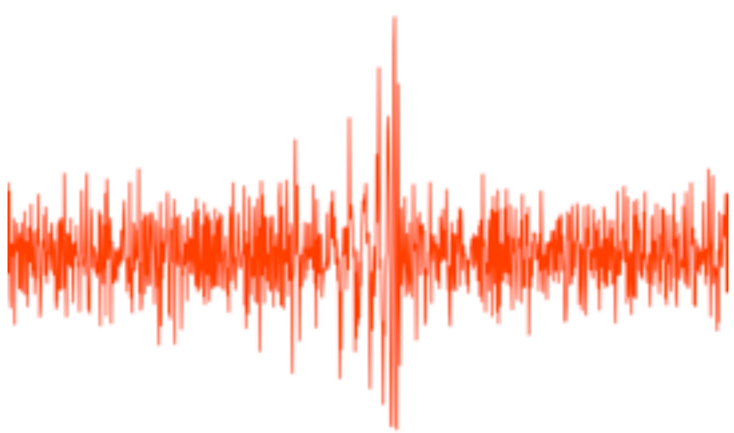


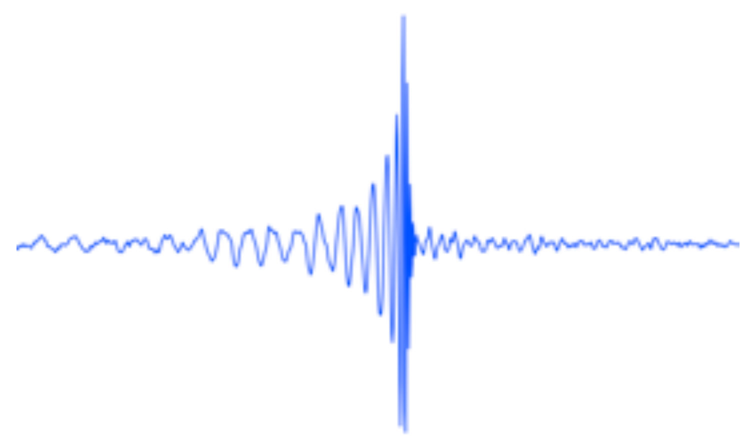
data = signal + noise

$$d = h + n$$

(randomly selected aLIGO data from 14 September 2015)

Signal Detection 101

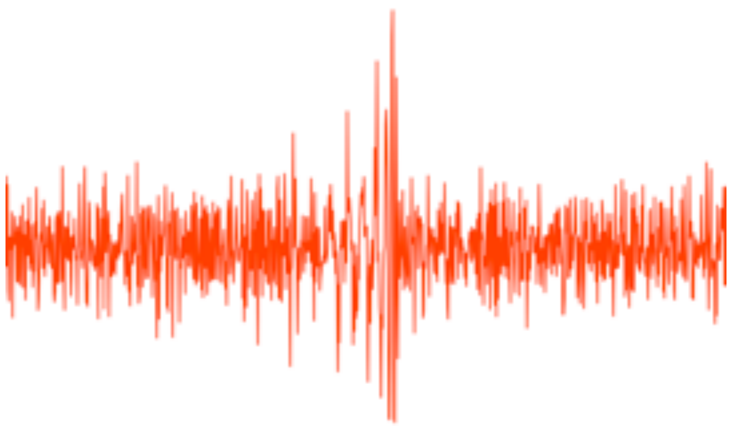


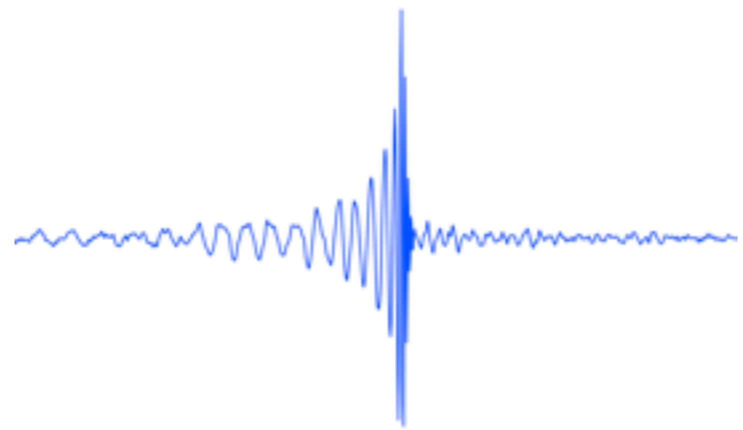


data - signal = noise

$$d - h = n$$

Signal Detection 101





data - signal = noise

$$d - h = n$$

$$p(d|h) = p(d-h) = p(n)$$

Signal Detection 101

$$p(d|h) = p(d-h) = p(n)$$

The likelihood is our statistical model for the noise

Noise Models

$$p(d|h) = p(d-h) = p(n)$$

Example: Stationary, colored, Gaussian noise

$$p(n, S_n(f)) = \prod_f \frac{1}{2\pi S_n(f)} e^{-\frac{\tilde{n}_f \tilde{n}_f^*}{S_n(f)}}$$

Noise Models

$$p(d|h) = p(d-h) = p(n)$$

Example: Stationary, colored, Gaussian noise

$$p(n, S_n(f)) = \prod_f \frac{1}{2\pi S_n(f)} e^{-\frac{\tilde{n}_f \tilde{n}_f^*}{S_n(f)}}$$

Note: This is a parametrized model, depends on the unknown spectrum $S_n(f)$

GW Posterior Distribution

Noise model

Signal model

Posterior probability for waveform h

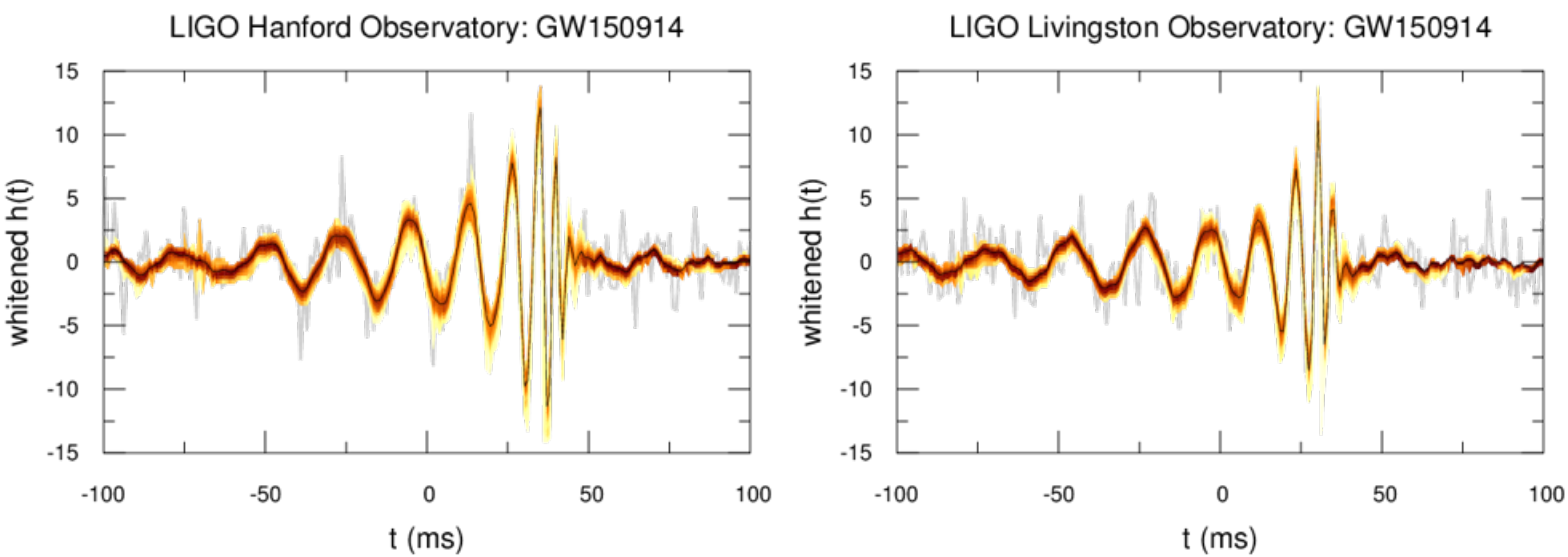
$$p(h|d) = \frac{p(d|h)p(h)}{p(d)}$$

Normalization - model evidence

GW Posterior Distribution

Posterior probability for waveform h

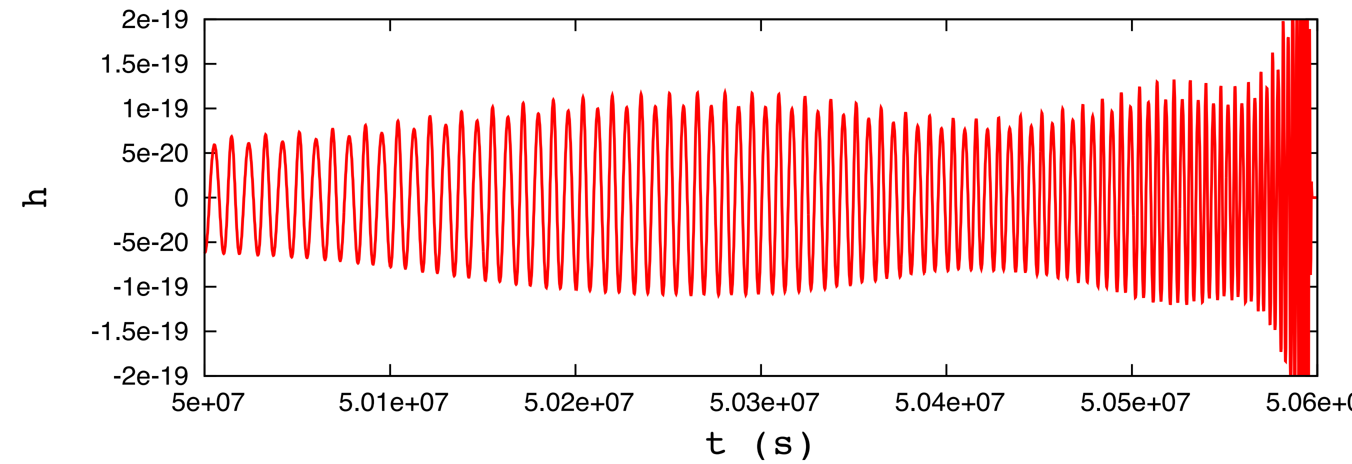
$$p(h|d) = \frac{p(d|h)p(h)}{p(d)}$$



Signal Models

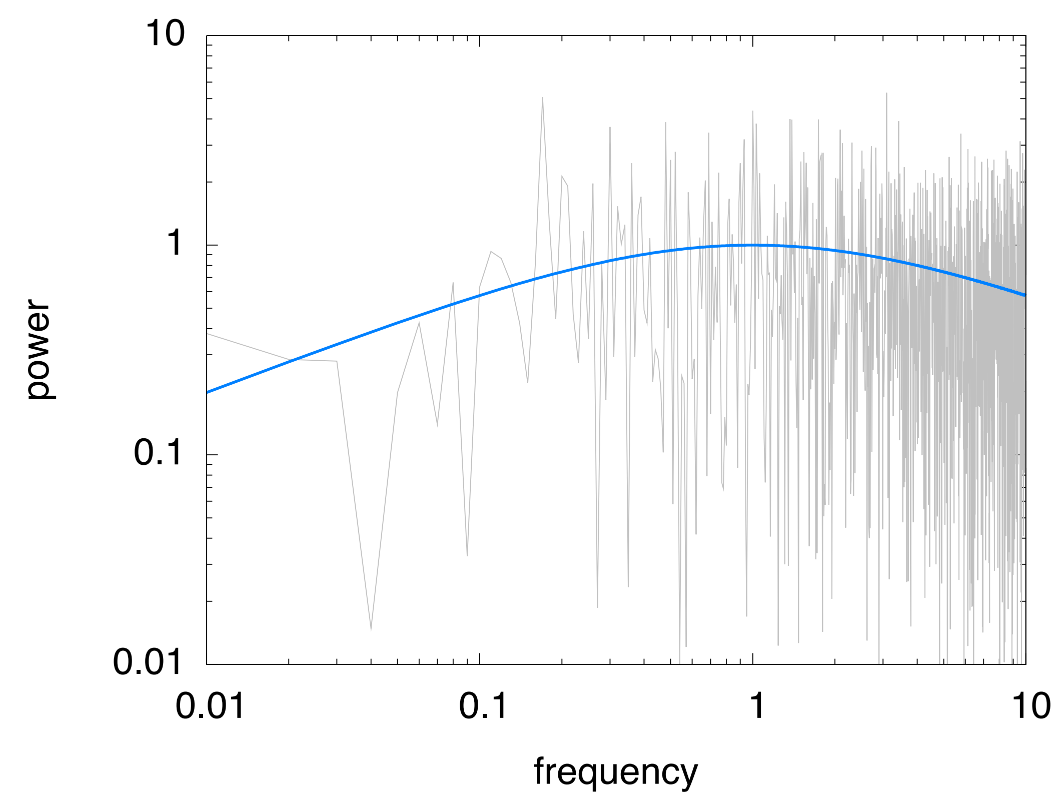
GW Template

$$p(h, \vec{\lambda}) = p(\vec{\lambda})\, \delta(h - h(\vec{\lambda}))$$



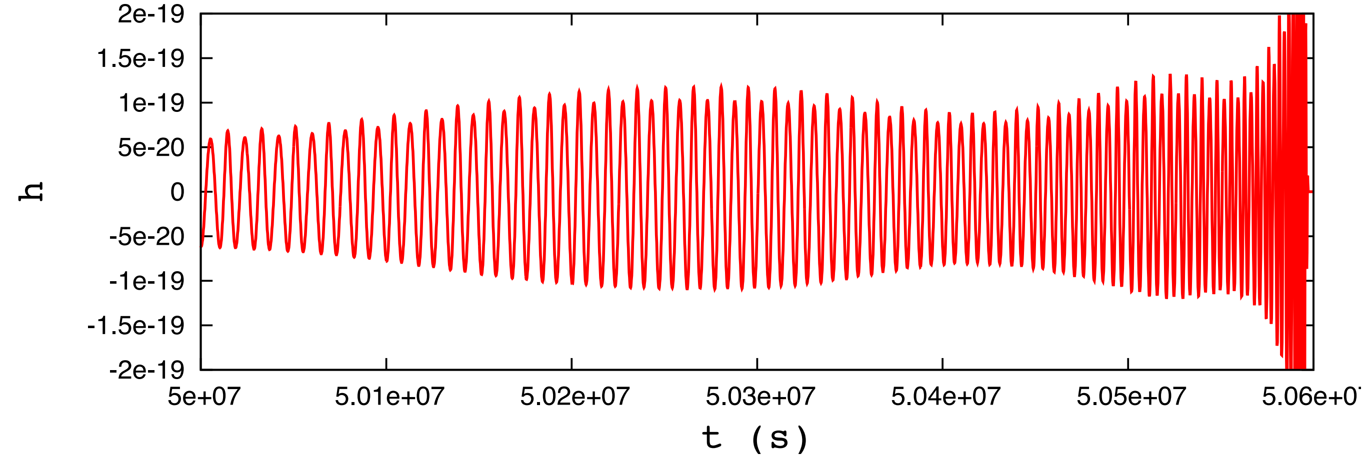
Stochastic GW

$$p(h, S_h(f)) = \prod_f \frac{1}{2\pi S_h(f)} e^{-\frac{\tilde{h}_f \tilde{h}_f^*}{S_h(f)}}$$



Template Based Models

$$p(h, \vec{\lambda}) = p(\vec{\lambda})\, \delta(h - h(\vec{\lambda}))$$



Marginalize over h(t). Converts posterior for waveform into posterior for physical parameters

$$p(\vec{\lambda}|d) = \int \frac{p(d|h)p(h, \vec{\lambda})}{p(d)}\, dh$$

$$= \frac{p(d|\vec{\lambda})p(\vec{\lambda})}{p(d)}$$

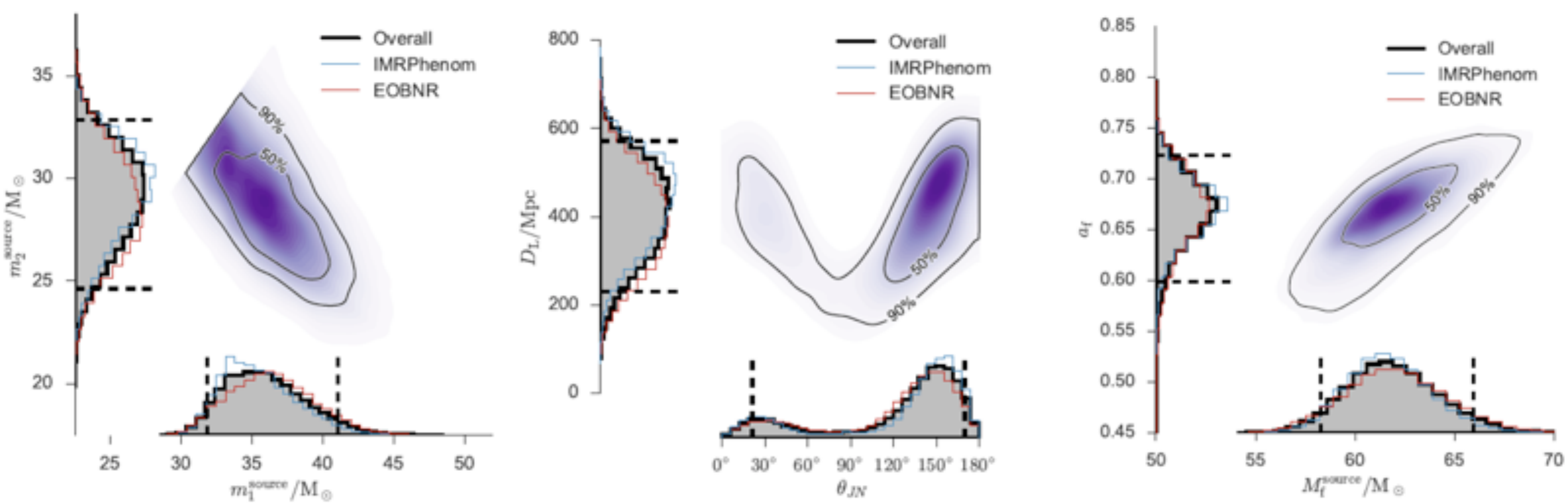
Template Based Models

$$p(d|\vec{\lambda}, S_n(f)) = \prod_f \frac{1}{2\pi S_n(f)}\, e^{-\frac{(\tilde{d}_f - \tilde{h}_f(\vec{\lambda}))(\tilde{d}^* - \tilde{h}^*_f(\vec{\lambda}))}{S_n(f)}}$$

Template Based Models

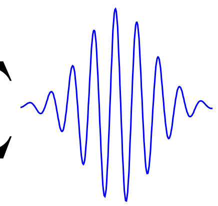
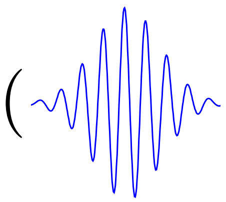
$$p(d|\vec{\lambda}, S_n(f)) = \prod_f \frac{1}{2\pi S_n(f)} e^{-\dfrac{(\tilde{d}_f - \tilde{h}_f(\vec{\lambda}))(\tilde{d}^* - \tilde{h}^*_f(\vec{\lambda}))}{S_n(f)}}$$

$$p(\vec{\lambda}|d)$$

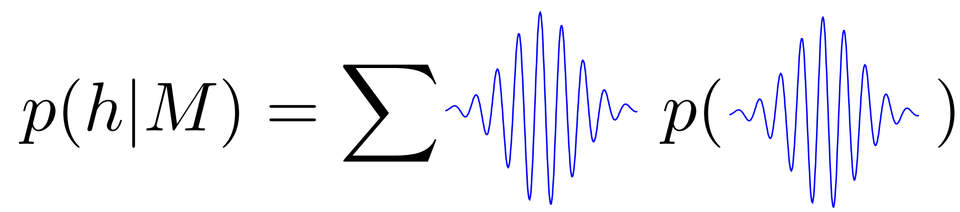


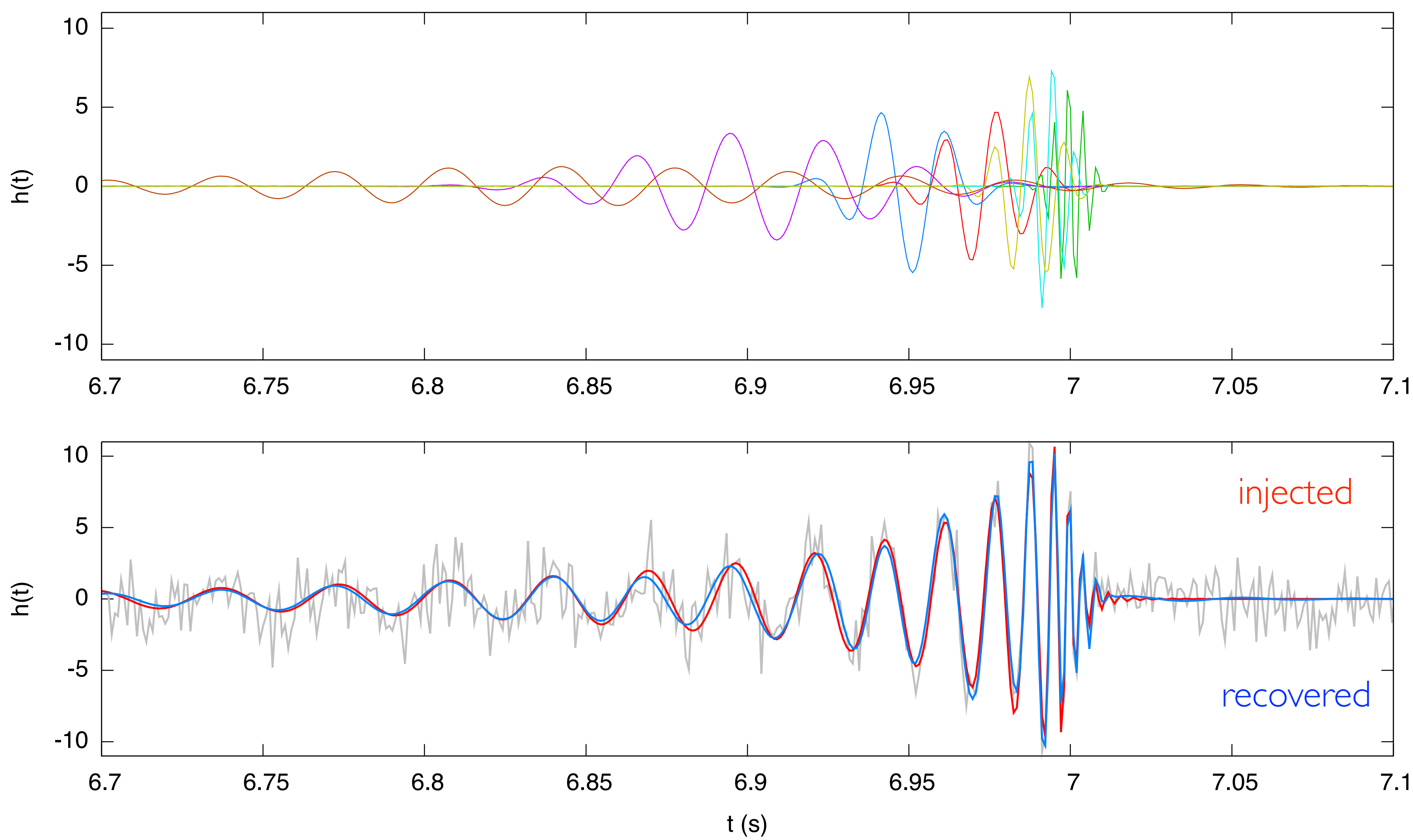
Example of 1-d and 2-d GR IMR model posteriors for GW150914 [arXiv:1602.03840]

Wavelet Based Models

$p(h|M) = \sum$  $p($  $)$

Wavelet Based Models

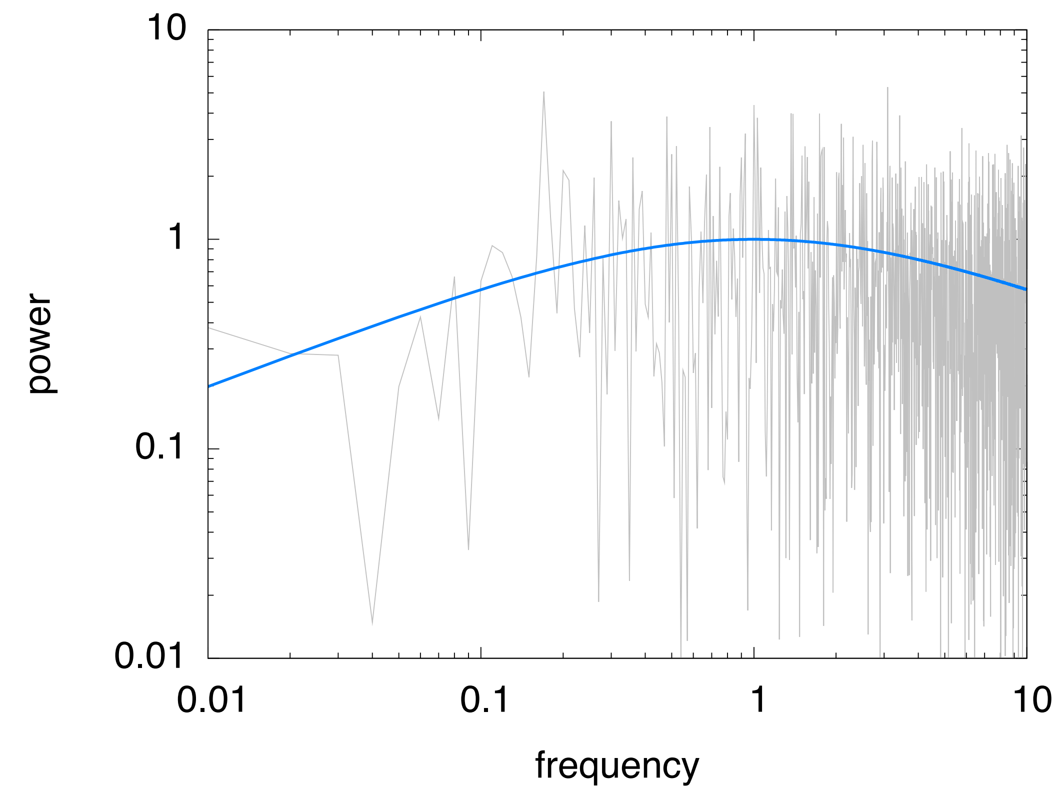




Cornish & Littenberg 1410.3835

Stochastic Signal Models

$$p(h, S_h(f)) = \prod_f \frac{1}{2\pi S_h(f)} e^{-\frac{\tilde{h}_f \tilde{h}_f^*}{S_h(f)}}$$



Marginalize over h(f). Converts posterior for waveform into posterior for GW power spectrum

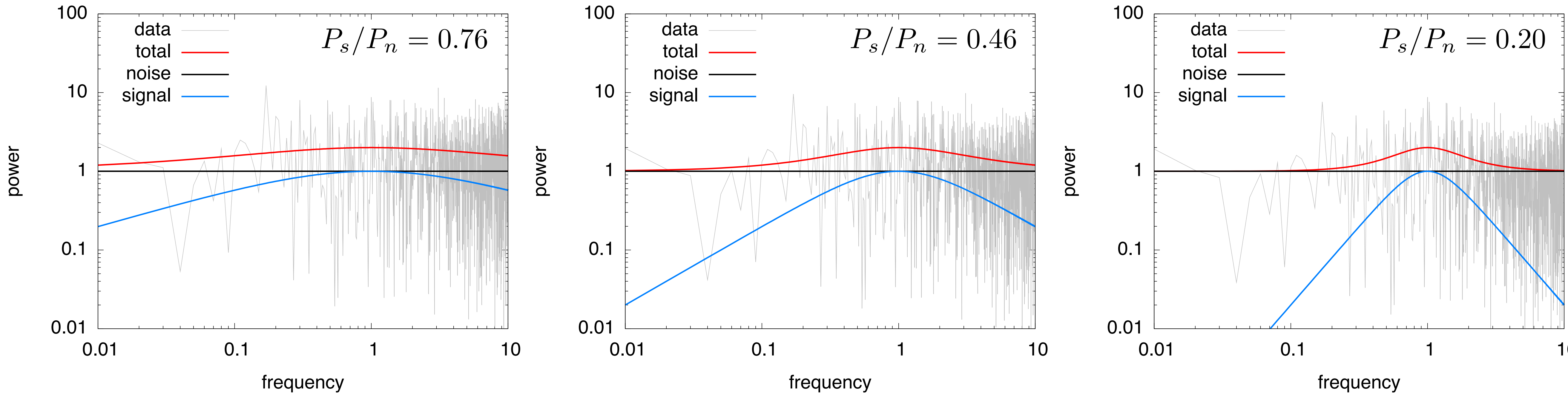
e.g. Single Detector

$$p(d, S_h(f), S_n(f)) = \prod_f \frac{1}{2\pi(S_h(f) + S_n(f))} e^{-\frac{\tilde{d}_f \tilde{d}_f^*}{(S_h(f) + S_n(f))}}$$

Only the sum of the signal and noise spectra constrained

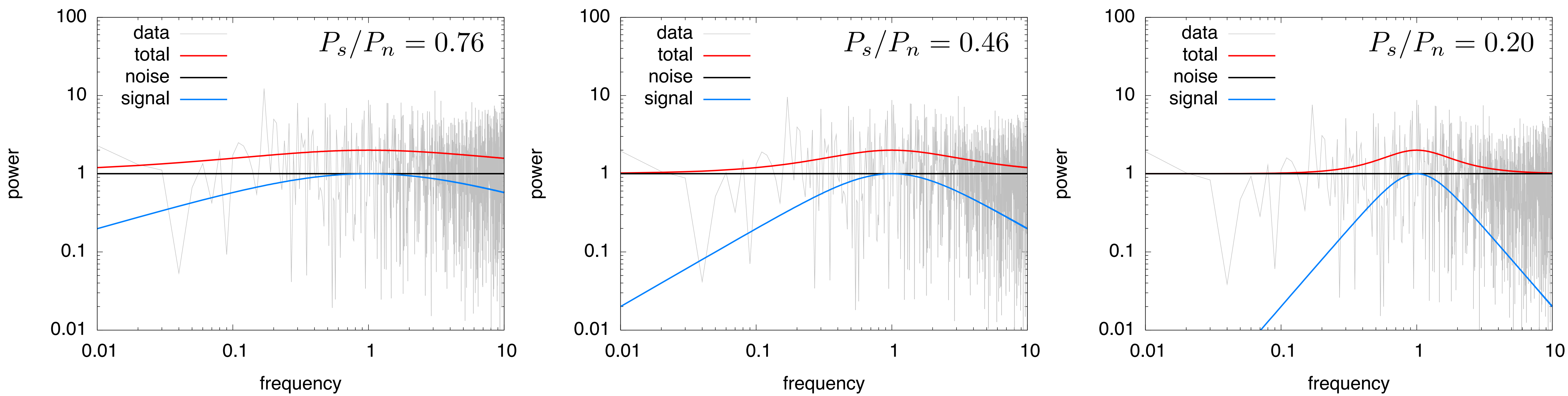
But, can separate them if we have strong priors on the spectra

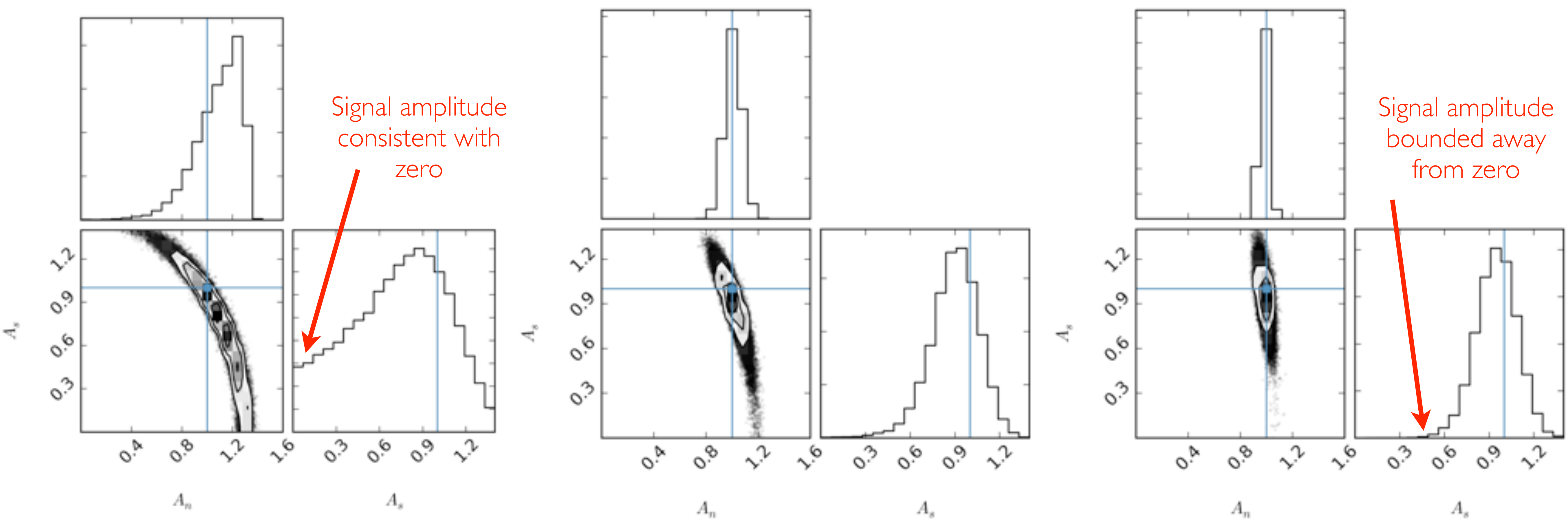
Single Detector Spectral Component Separation



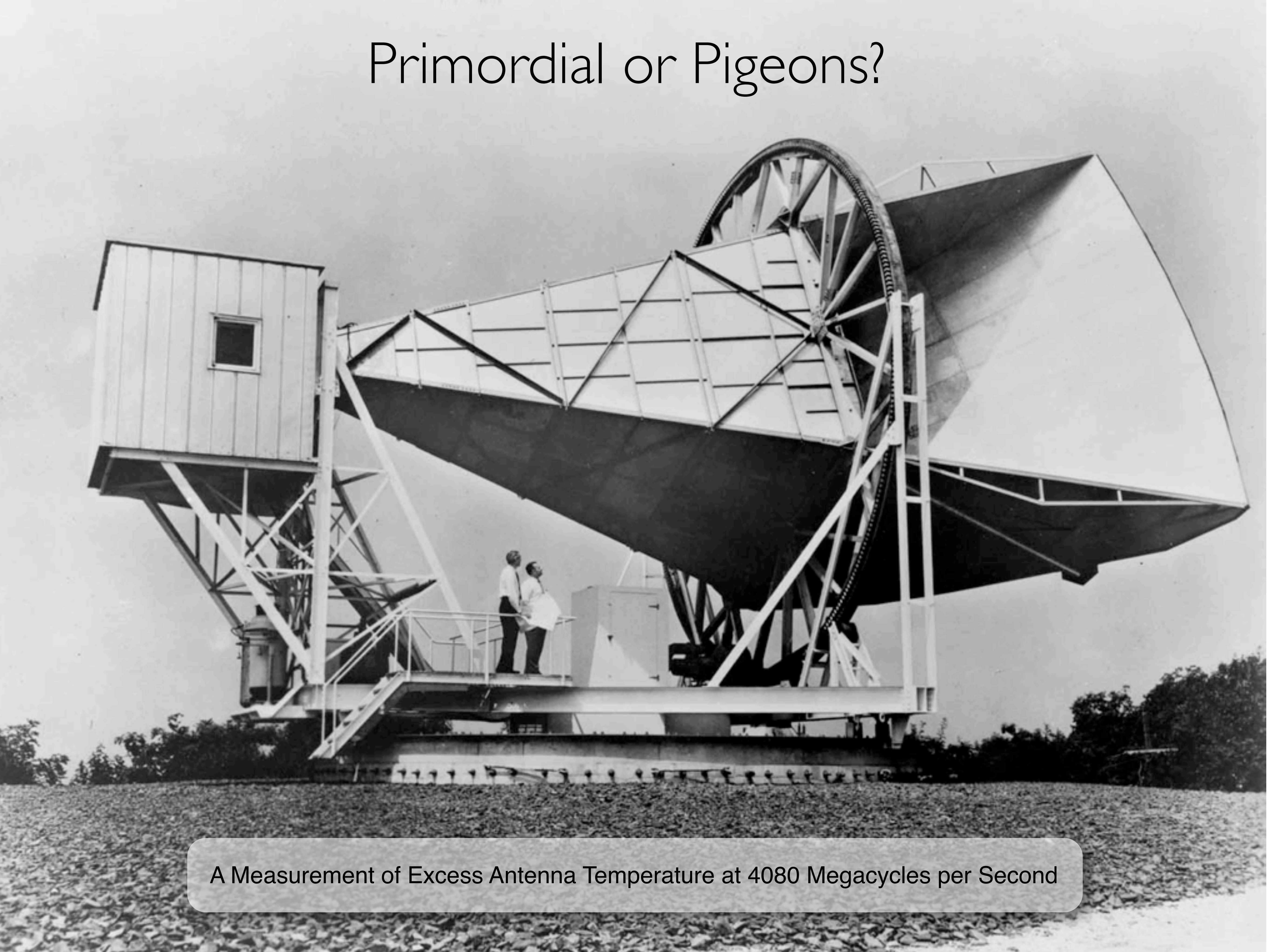
In this model the signal and noise shapes are assumed to be know, only the amplitudes unknown

Single Detector Spectral Component Separation



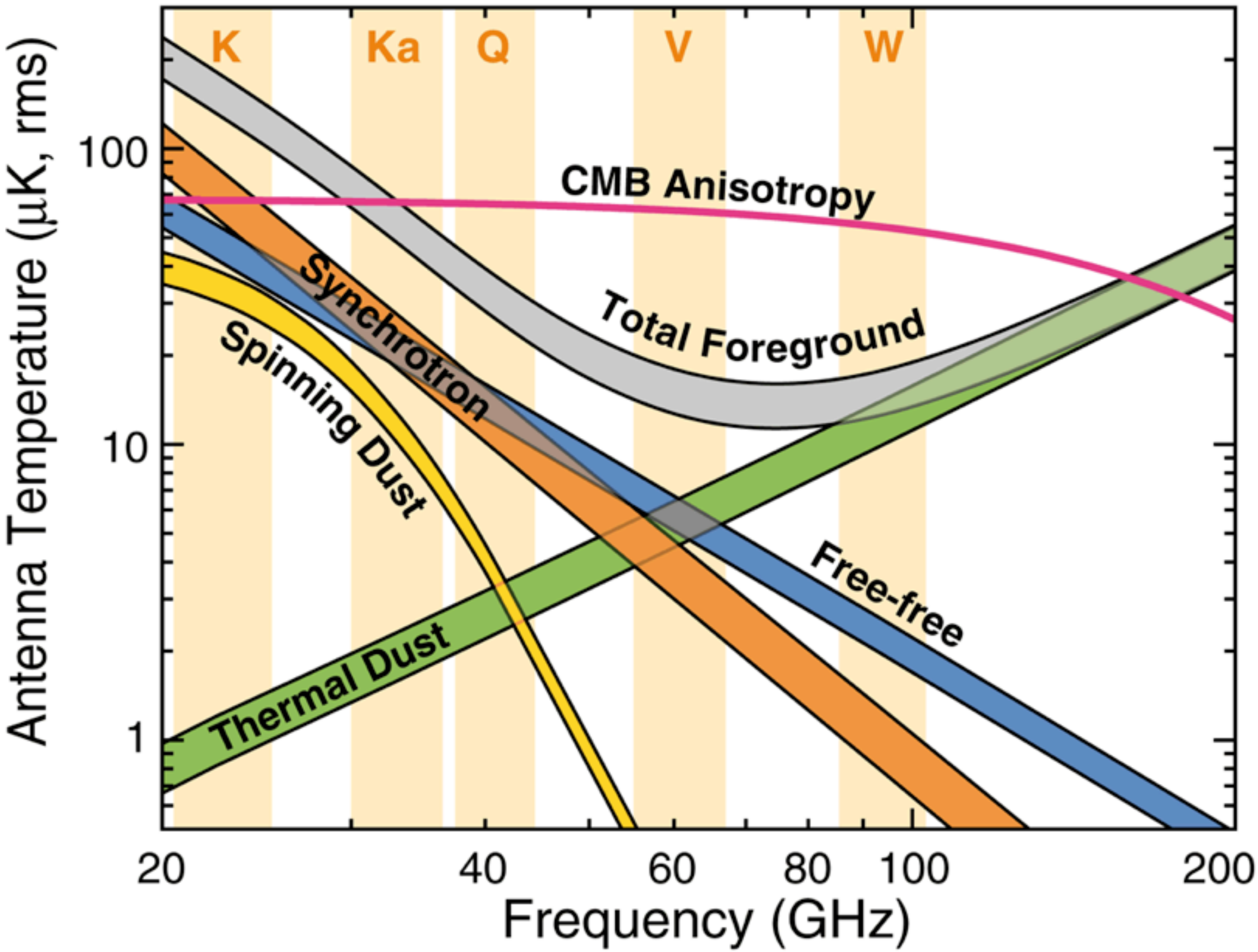


Primordial or Pigeons?

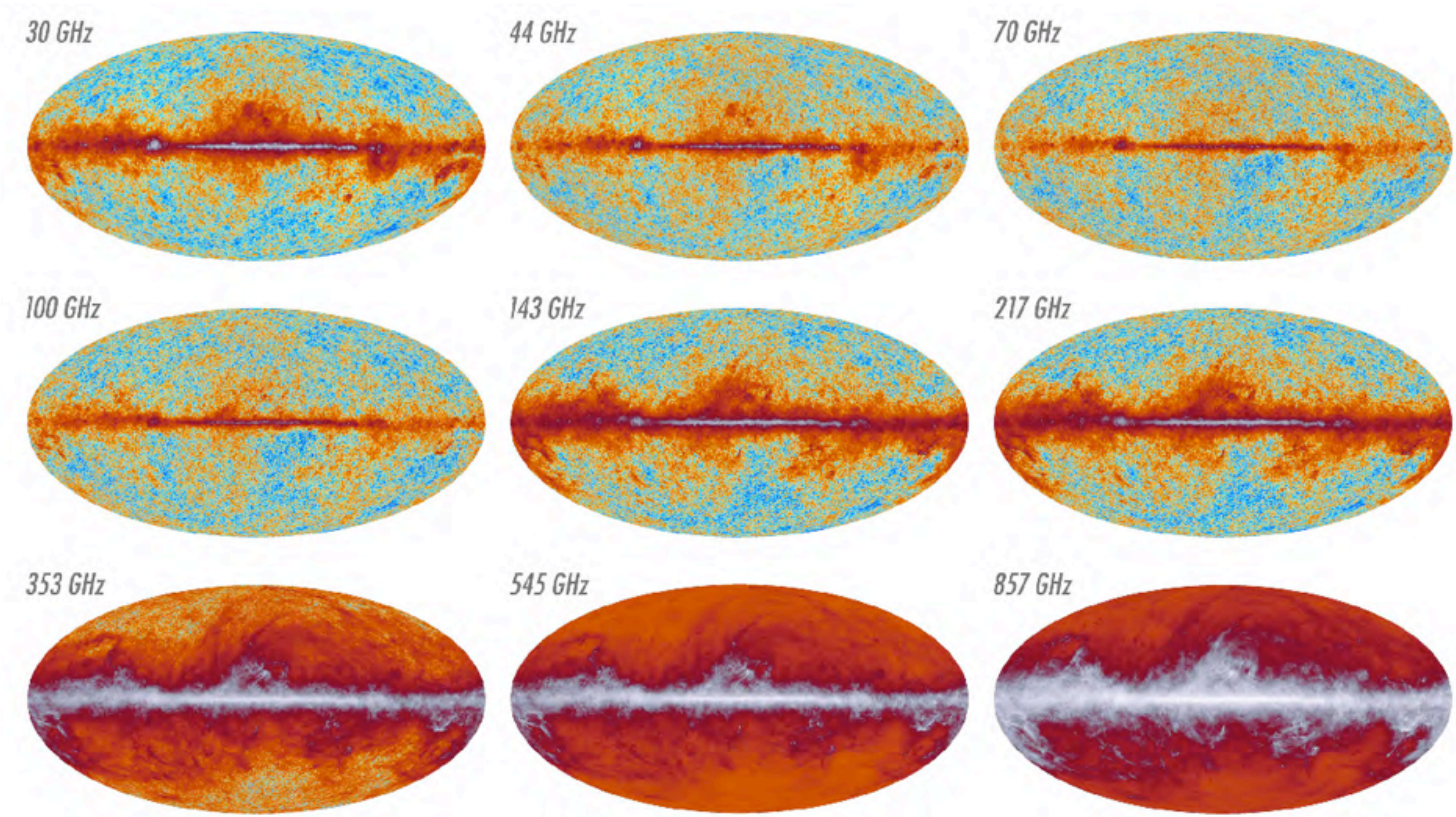


A Measurement of Excess Antenna Temperature at 4080 Megacycles per Second

CMB Spectral Component Separation



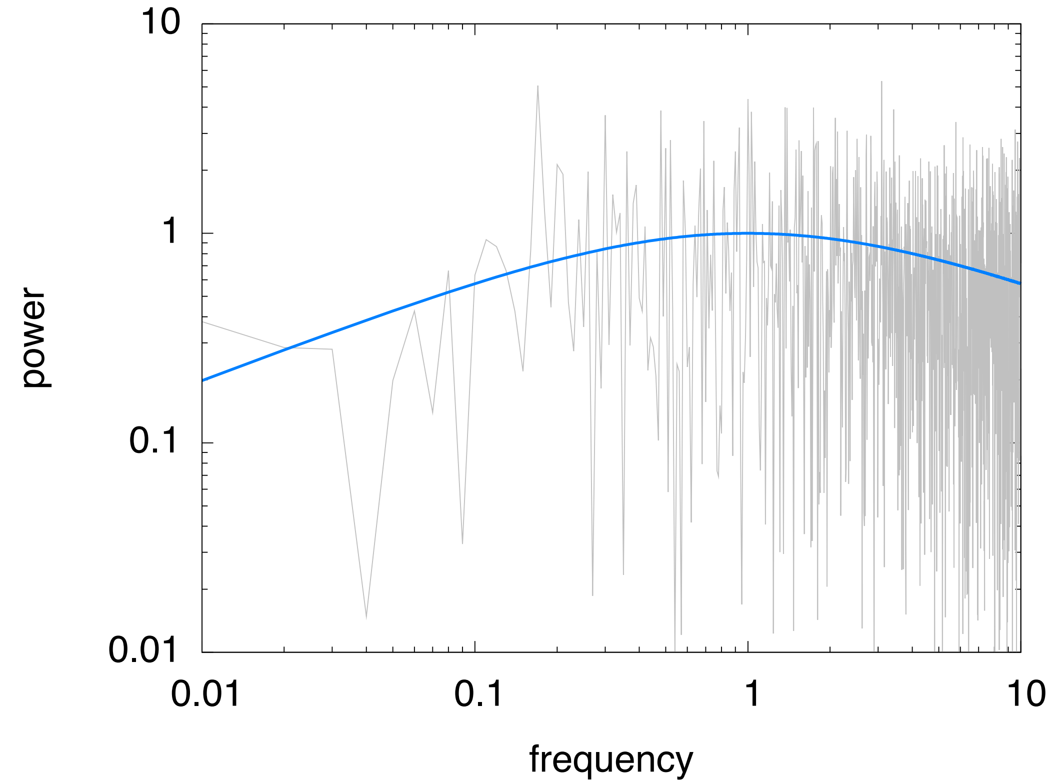
CMB Spectral Component Separation



Stochastic Signal Models

$$p(h, S_h(f)) = \prod_f \frac{1}{2\pi S_h(f)} e^{-\frac{\tilde{h}_f \tilde{h}^*_f}{S_h(f)}}$$

Marginalize over h(f). Converts posterior for waveform into posterior for GW power spectrum



Multiple Detectors labeled by i,j

$$p(d|S_h(f), S_n(f)) = \prod_f \frac{1}{2\pi \det C(f)} e^{-(\tilde{d}_f)_i C^{-1}_{ij} (\tilde{d}^*_f)_j}$$

$$C_{ij}(f) = \delta_{ij}\, S_{n,i}(f) + \gamma_{ij}\, S_h(f)$$

Cornish & Romano 1305.2934

Stochastic Signal Models

$$p(d|S_h(f), S_n(f)) = \prod_f \frac{1}{2\pi\,\det C(f)} e^{-(\tilde{d}_f)_i C^{-1}_{ij} (\tilde{d}^*_f)_j}$$

$$C_{ij}(f) = \delta_{ij}\, S_{n,i}(f) + \gamma_{ij}(f)\, S_h(f)$$

Noise uncorrelated
between detectors

Signal correlated
between detectors

Stochastic Signal Models

$$p(d|S_h(f), S_n(f)) = \prod_f \frac{1}{2\pi \det C(f)} e^{-(\tilde{d}_f)_i C^{-1}_{ij} (\tilde{d}^*_f)_j}$$

$$C_{ij}(f) = \delta_{ij}\, S_{n,i}(f) + \gamma_{ij}(f)\, S_h(f)$$

Noise uncorrelated between detectors

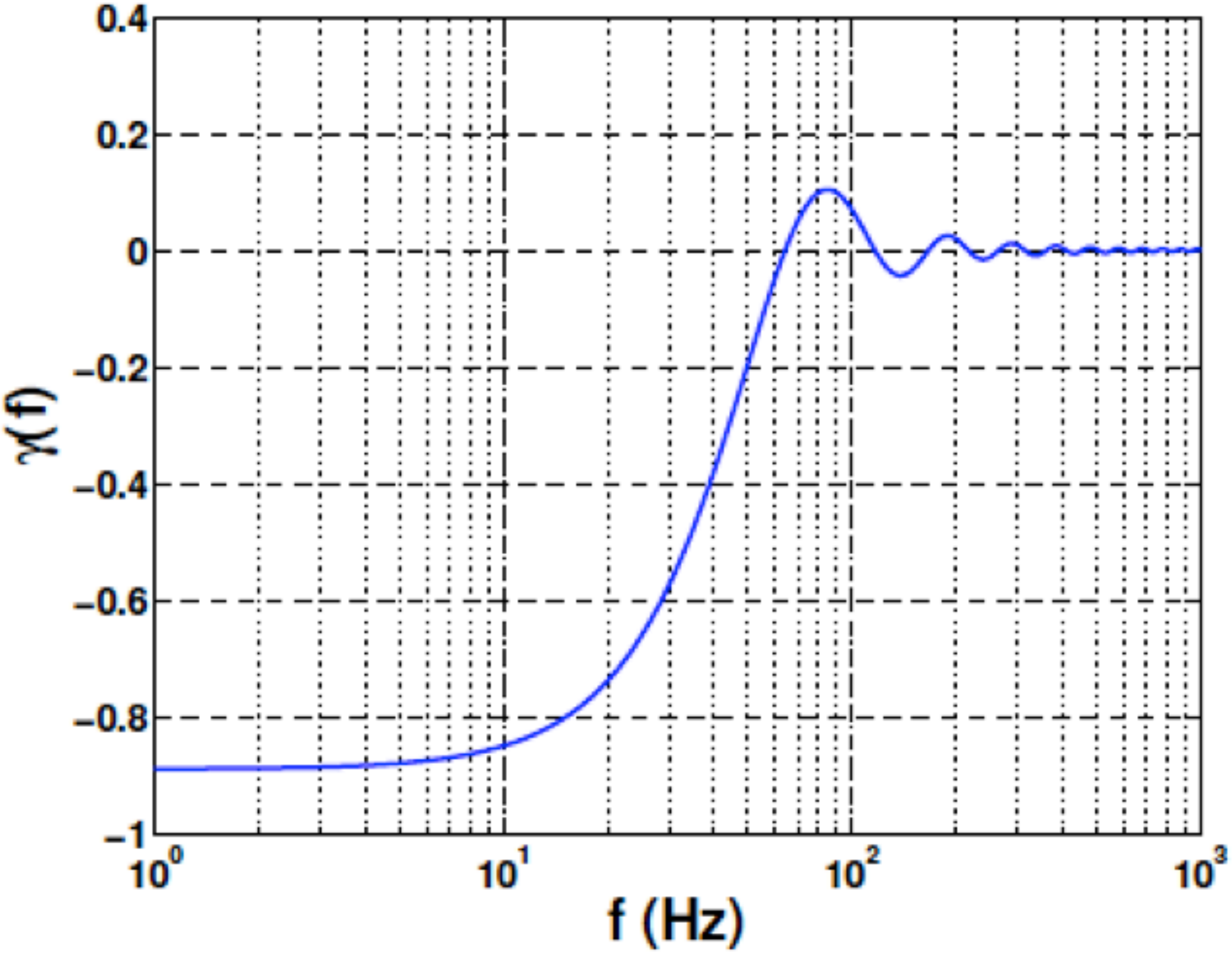
Signal correlated between detectors

Overlap-reduction function, aka Hellings-Downs curve

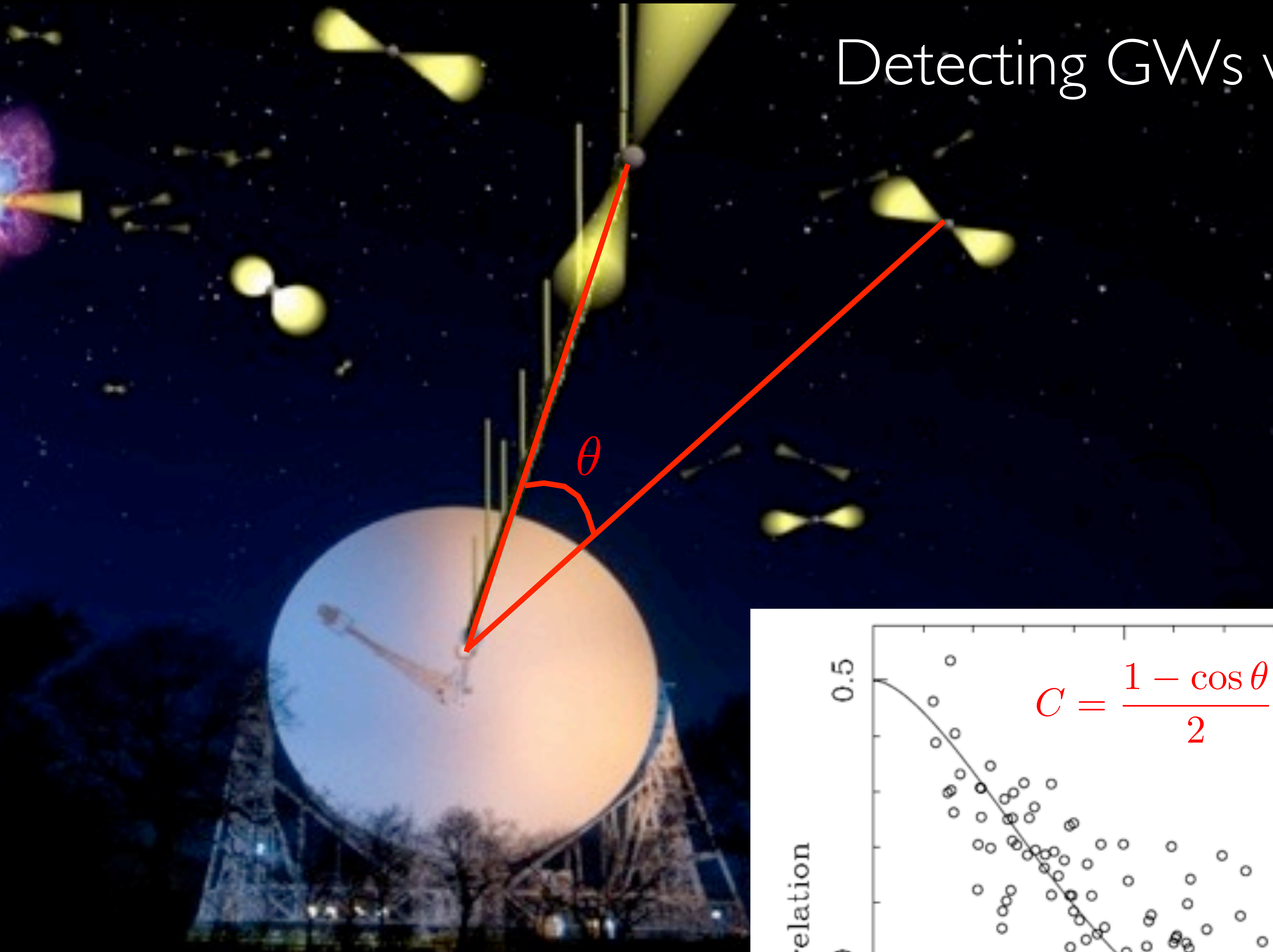
$$\gamma_{ij}(f) = \frac{1}{4\pi} \int \left(F_i^+(\hat{n}) F_j^+(\hat{n}) + F_i^\times(\hat{n}) F_j^\times(\hat{n})\right) e^{2\pi i f (\vec{x}_i - \vec{x}_j)\cdot \hat{n}}\, d\Omega_{\hat{n}}$$

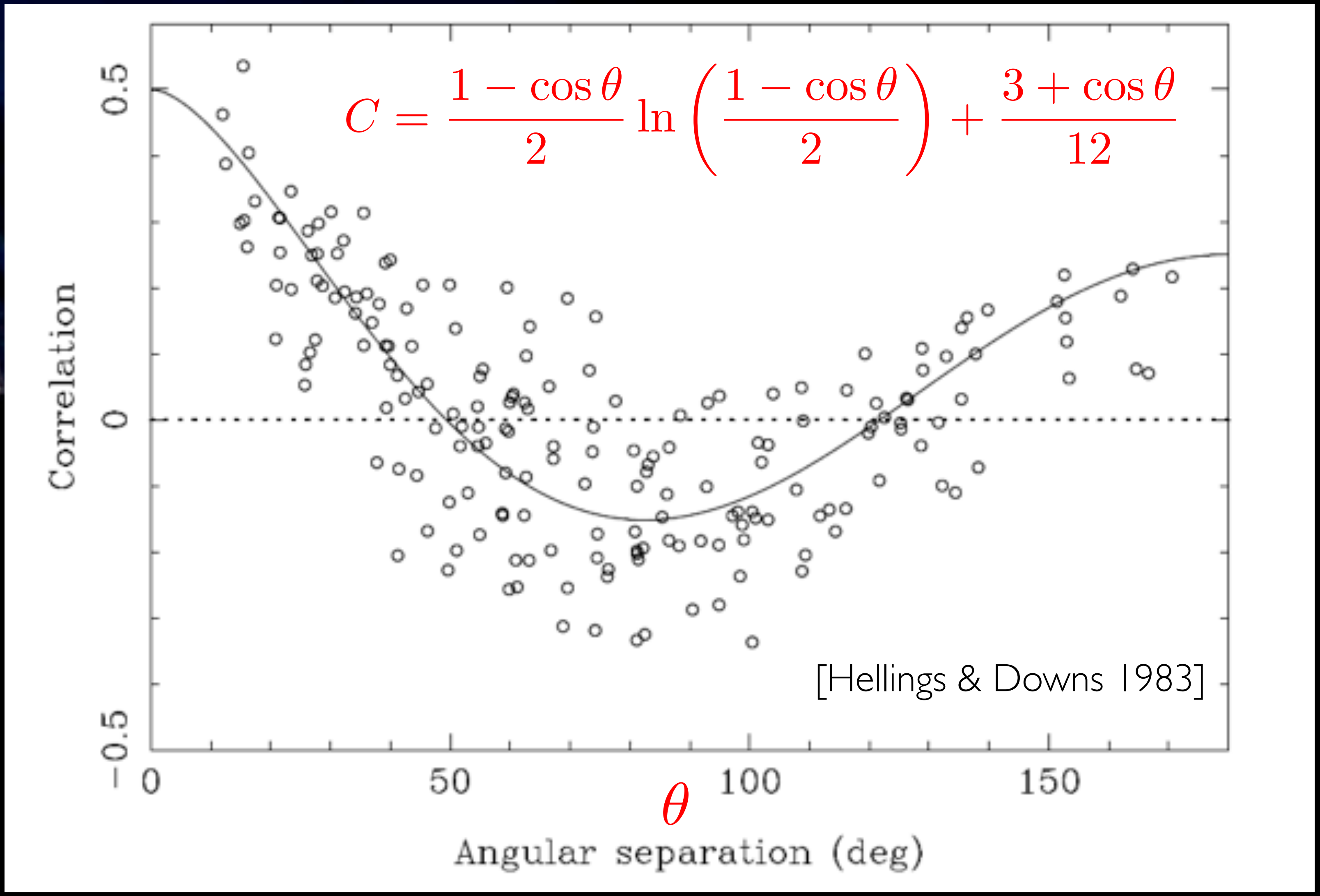
Correlation Function

LIGO Hanford x Livingston

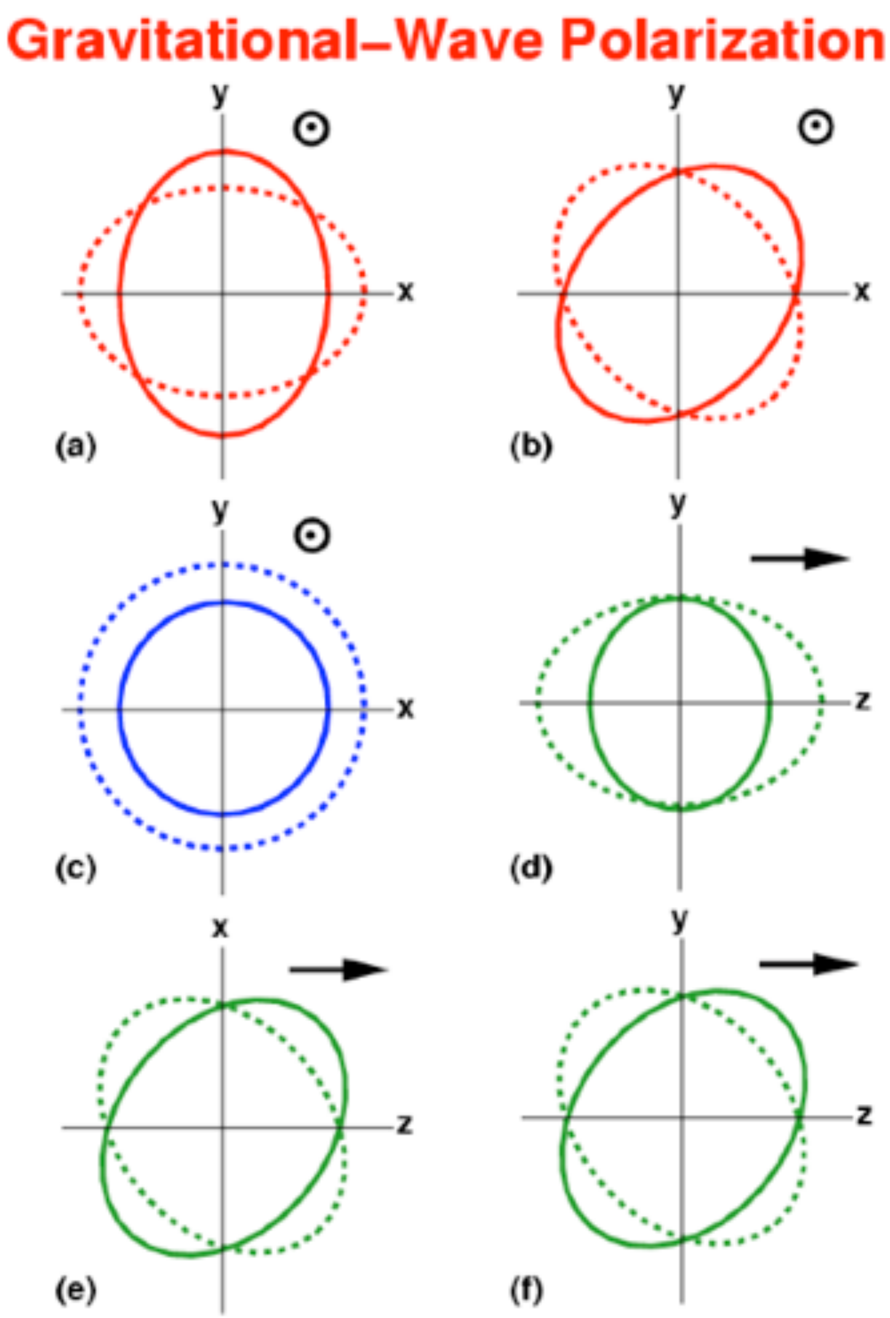


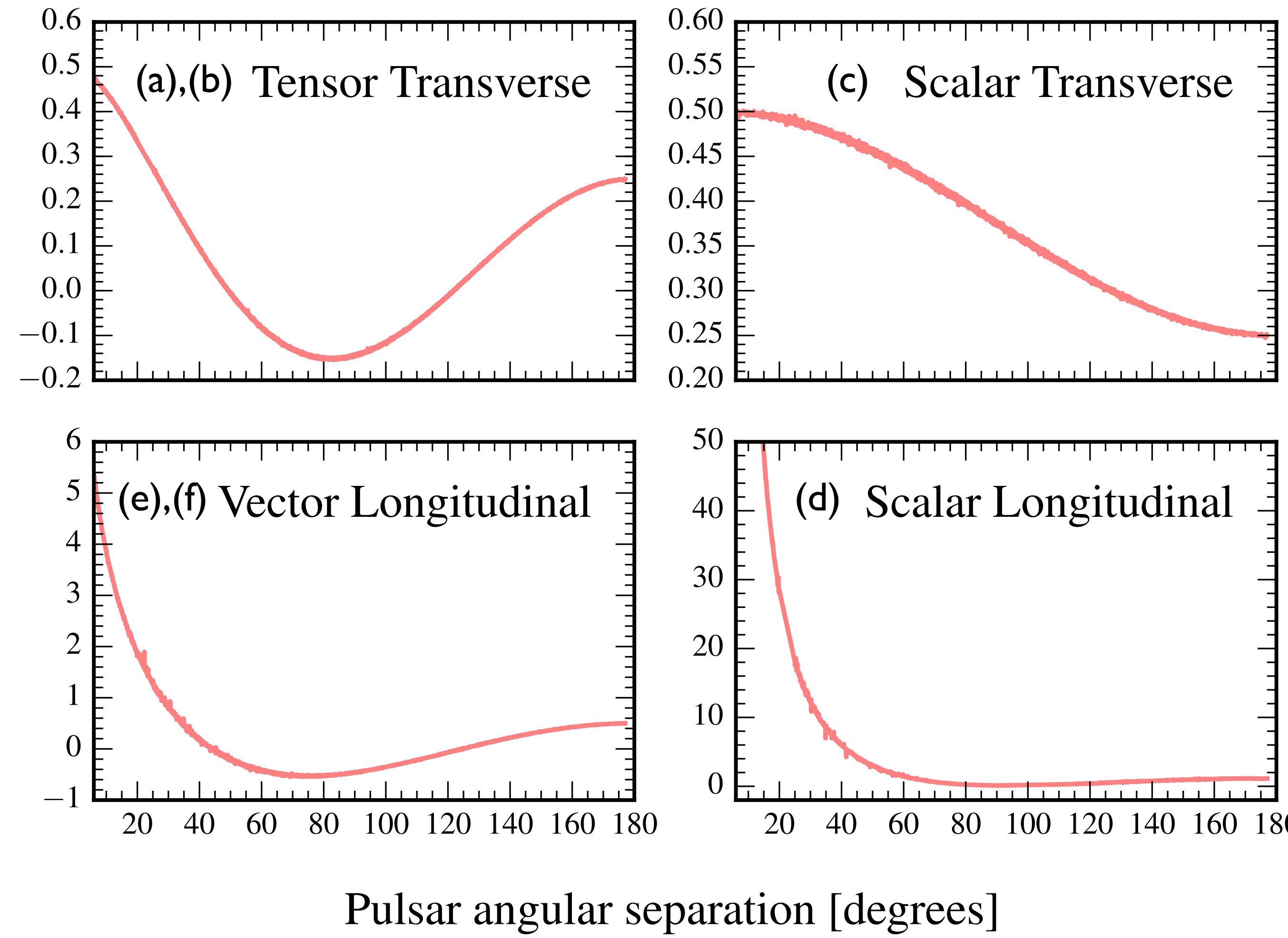
Detecting GWs with Pulsar Timing





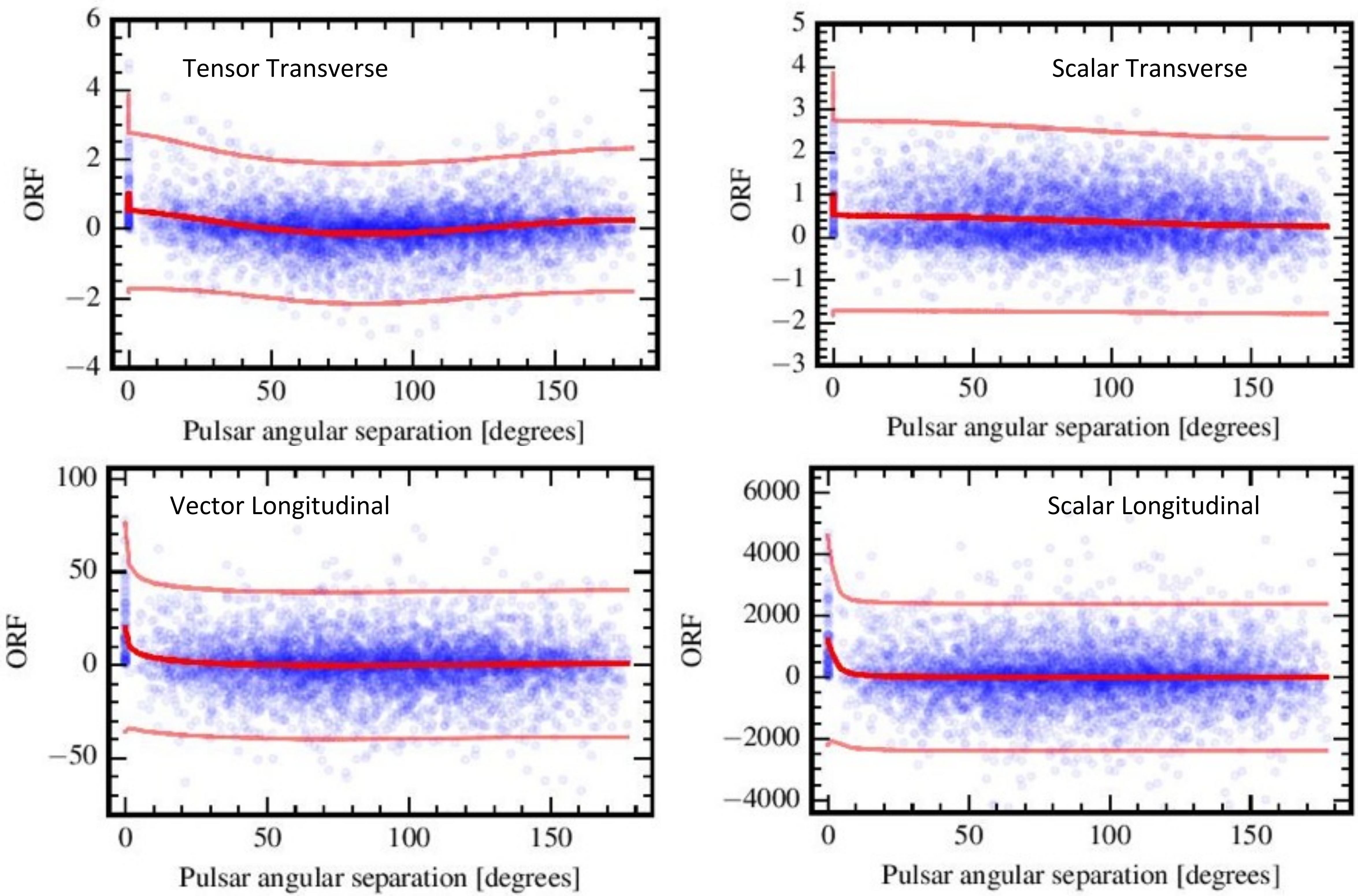
Alternative Polarization States





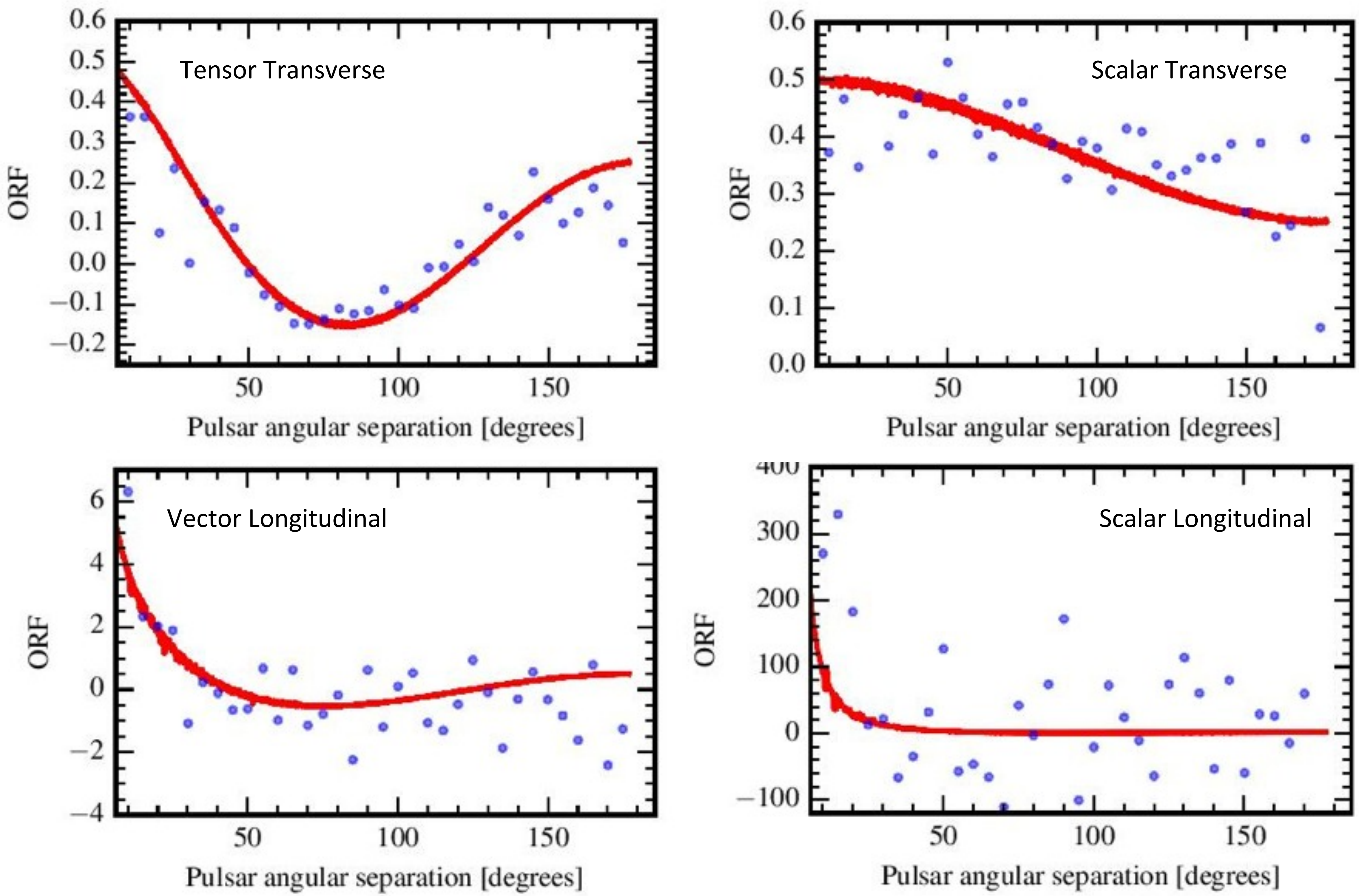
O'Bierne, Cornish, Taylor, Yunes, Sampson in prep

100 Pulsars, no timing noise



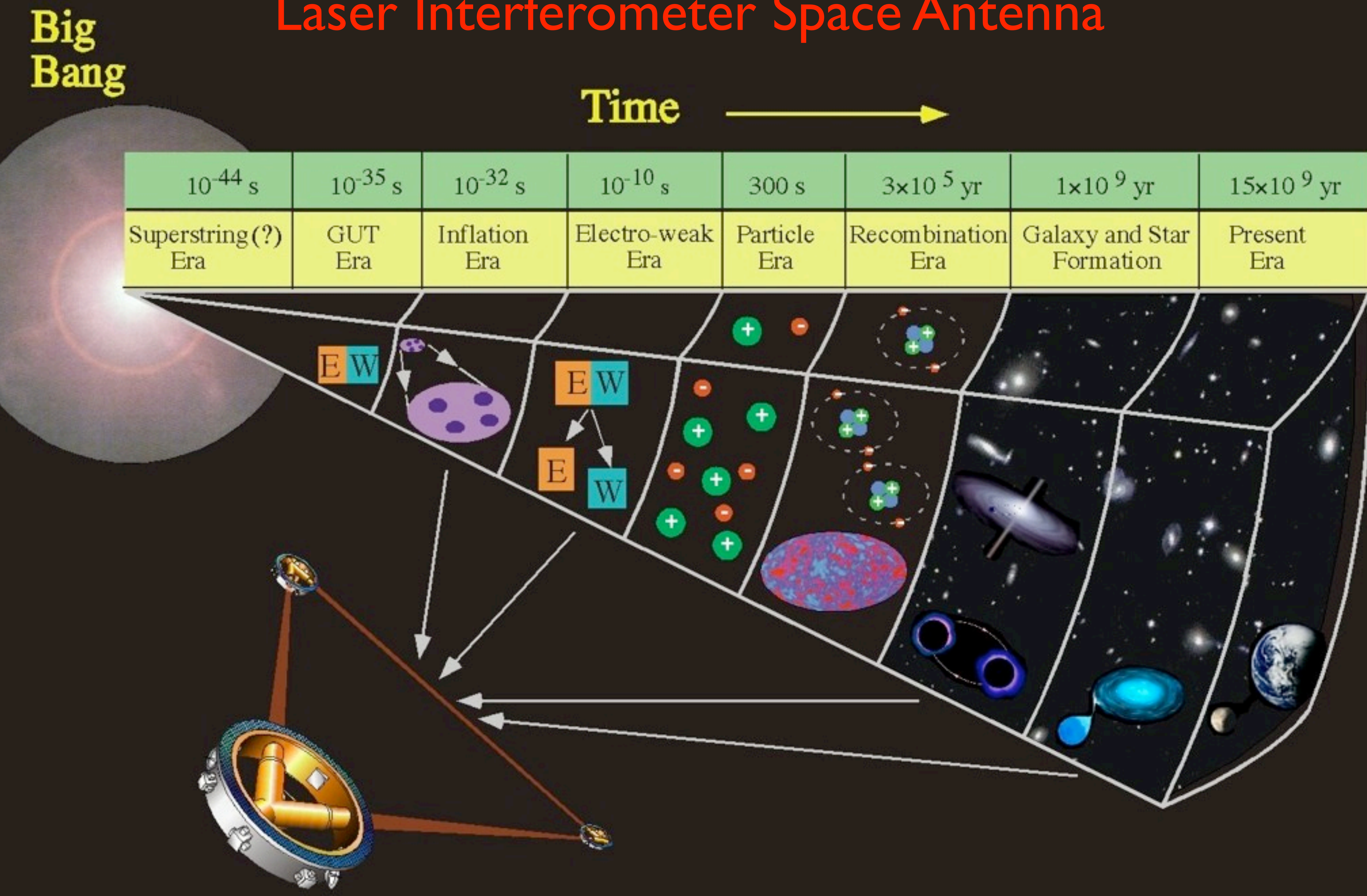
$$\sigma^2(\alpha) = C(0)^2 + C(\alpha)^2$$

100 Pulsars, no timing noise, 5 degree bins

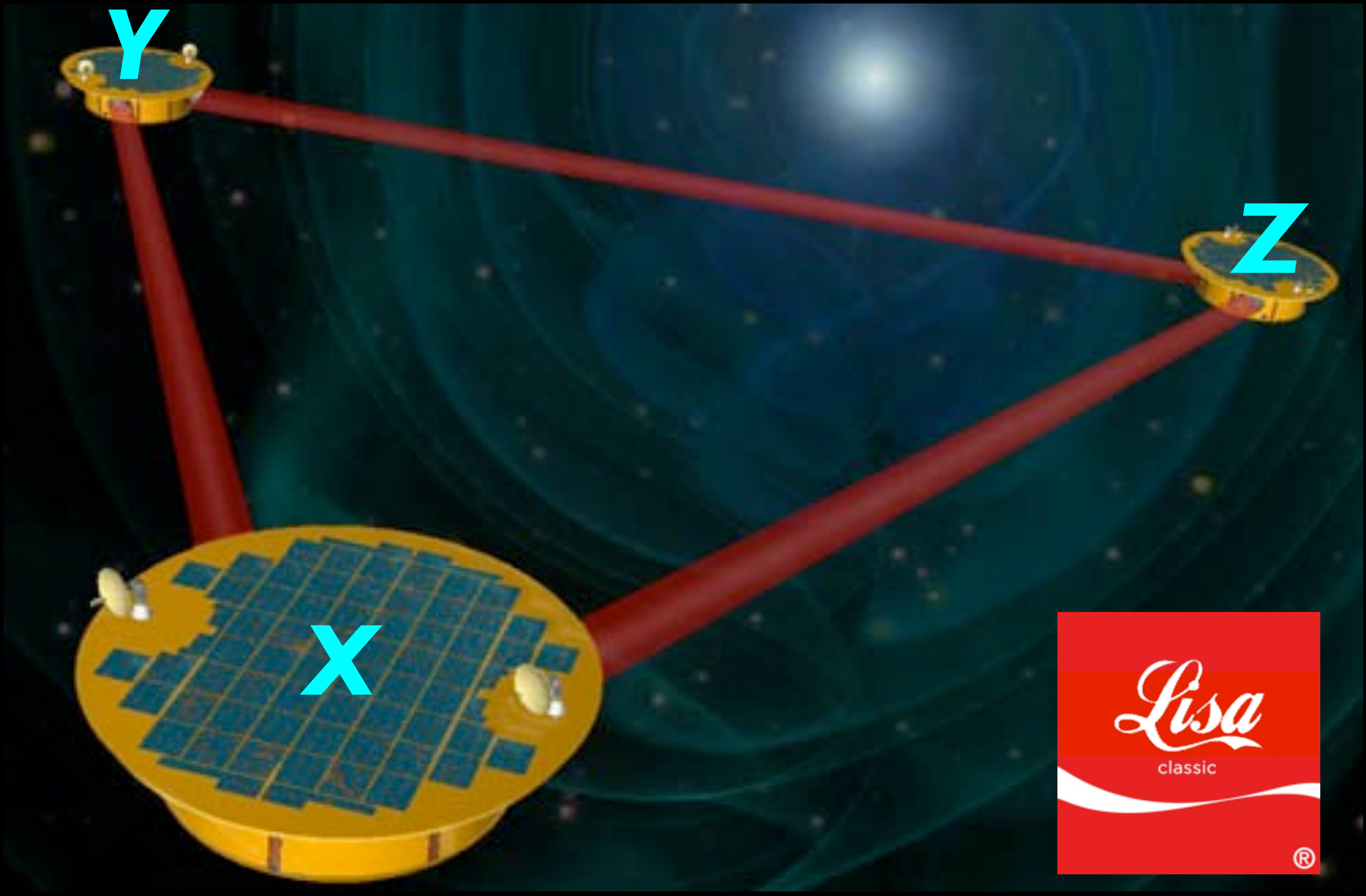


Large variance of longitudinal modes makes them very difficult to detect

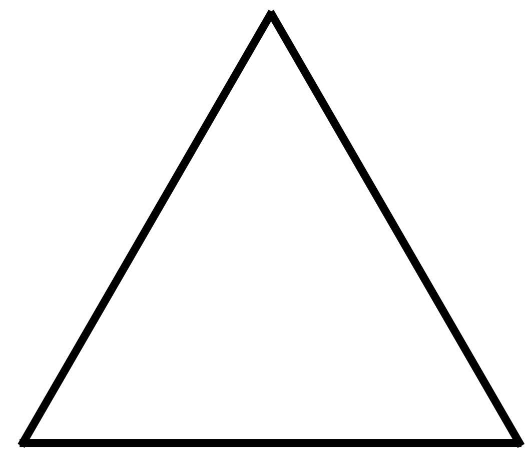
Laser Interferometer Space Antenna



LISA Detector(s)



Signal Orthogonal Combinations



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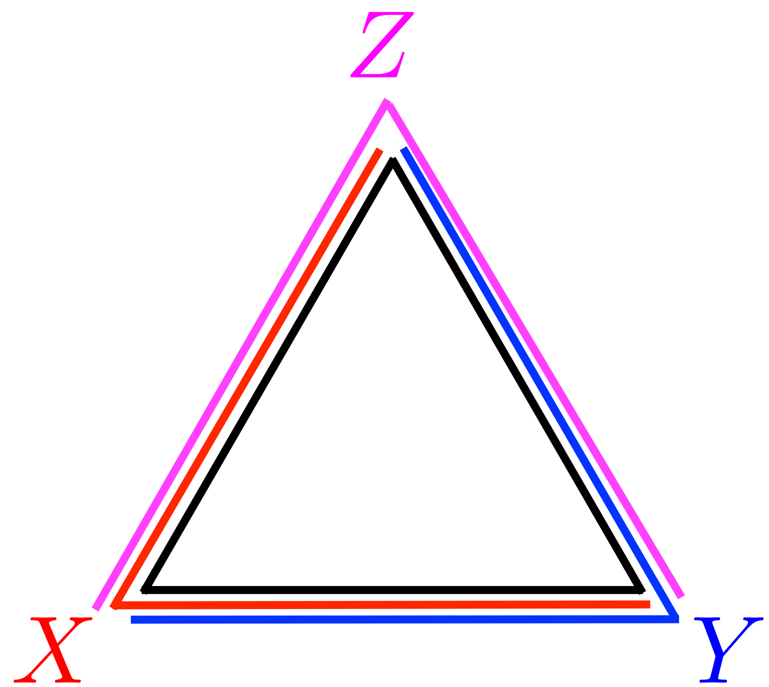
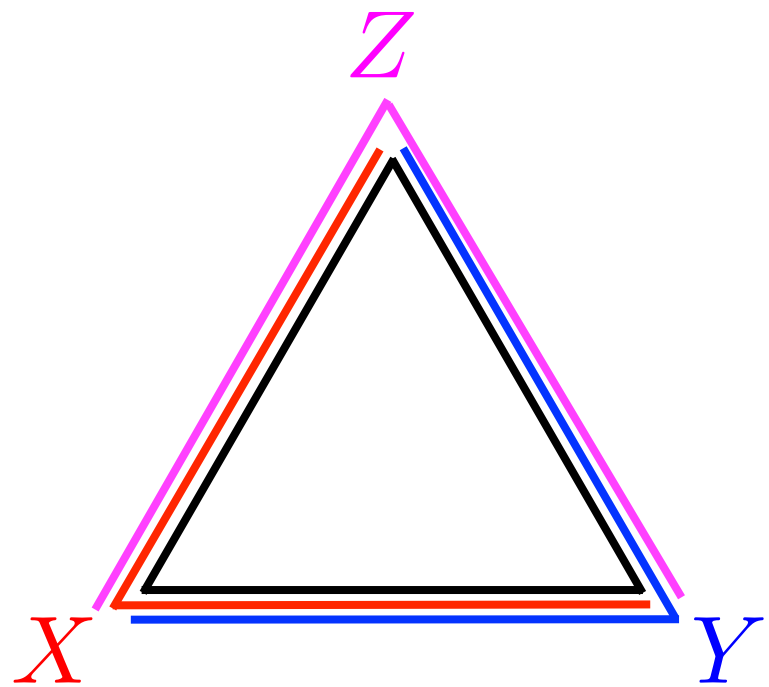
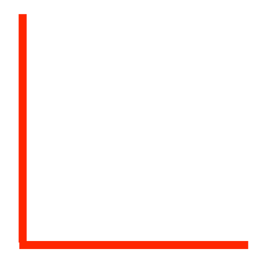


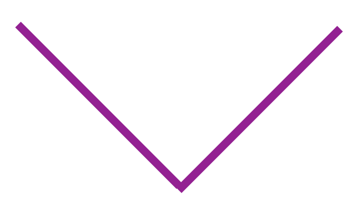
Signal Orthogonal Combinations



$$A = \frac{\sqrt{3}}{2} X \quad \Rightarrow$$

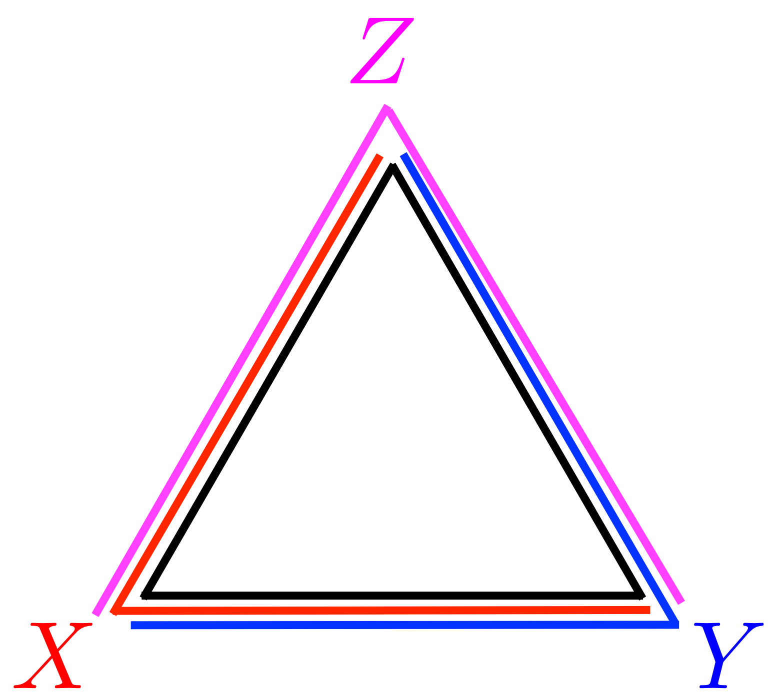


$$E = \frac{1}{2}(X + 2Y) \quad \Rightarrow$$

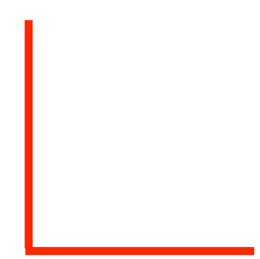


Instantaneous measurement of both polarization states and increased signal-to-noise

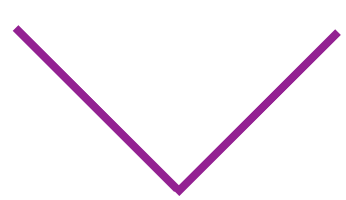
Signal Orthogonal Combinations



$$A = \frac{\sqrt{3}}{2} X \Rightarrow$$



$$E = \frac{1}{2}(X + 2Y) \Rightarrow$$

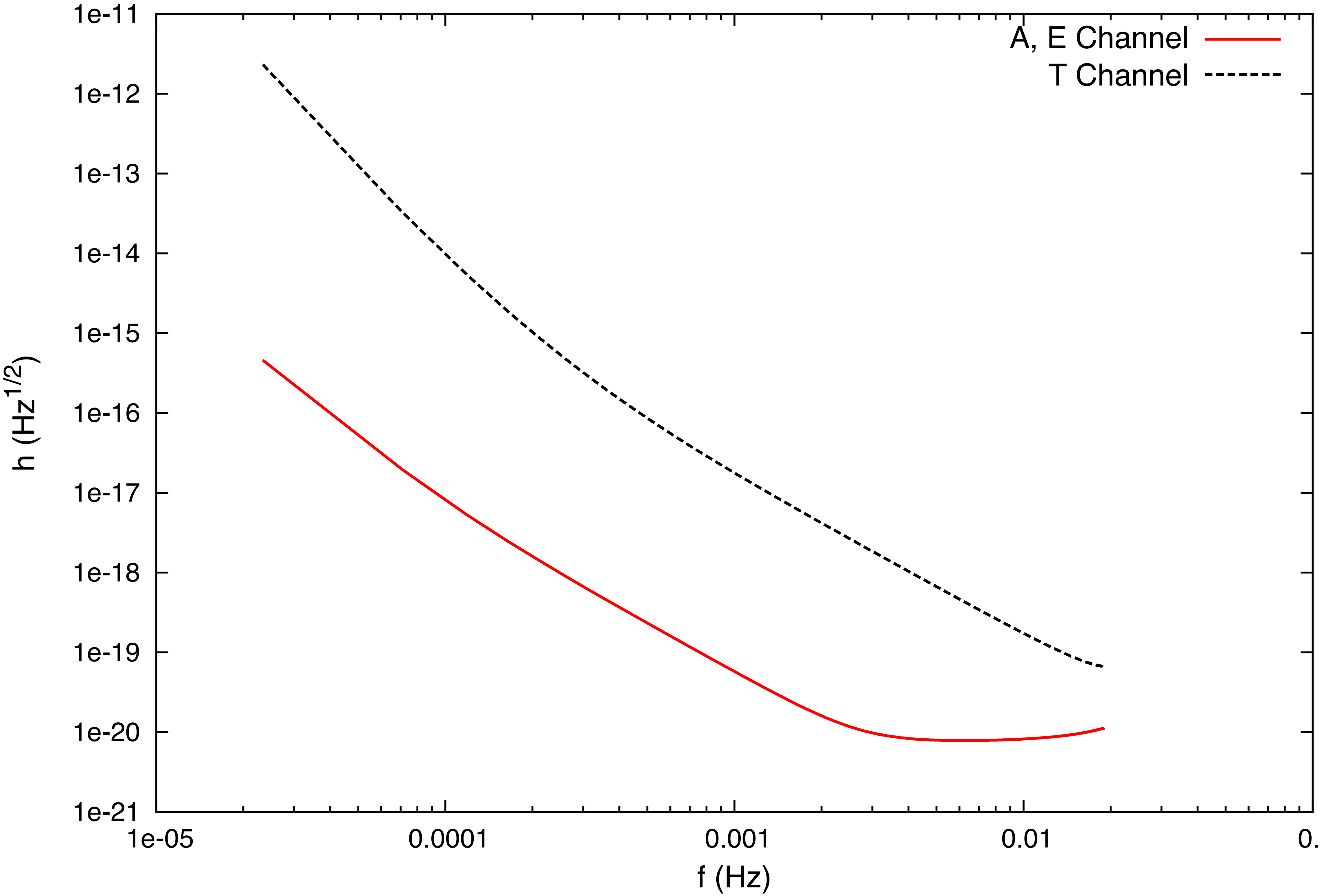


Instantaneous measurement of both polarization states and increased signal-to-noise

$$T = \frac{1}{3}(X + Y + Z) \Rightarrow$$

Null channel to monitor average low frequency instrument noise

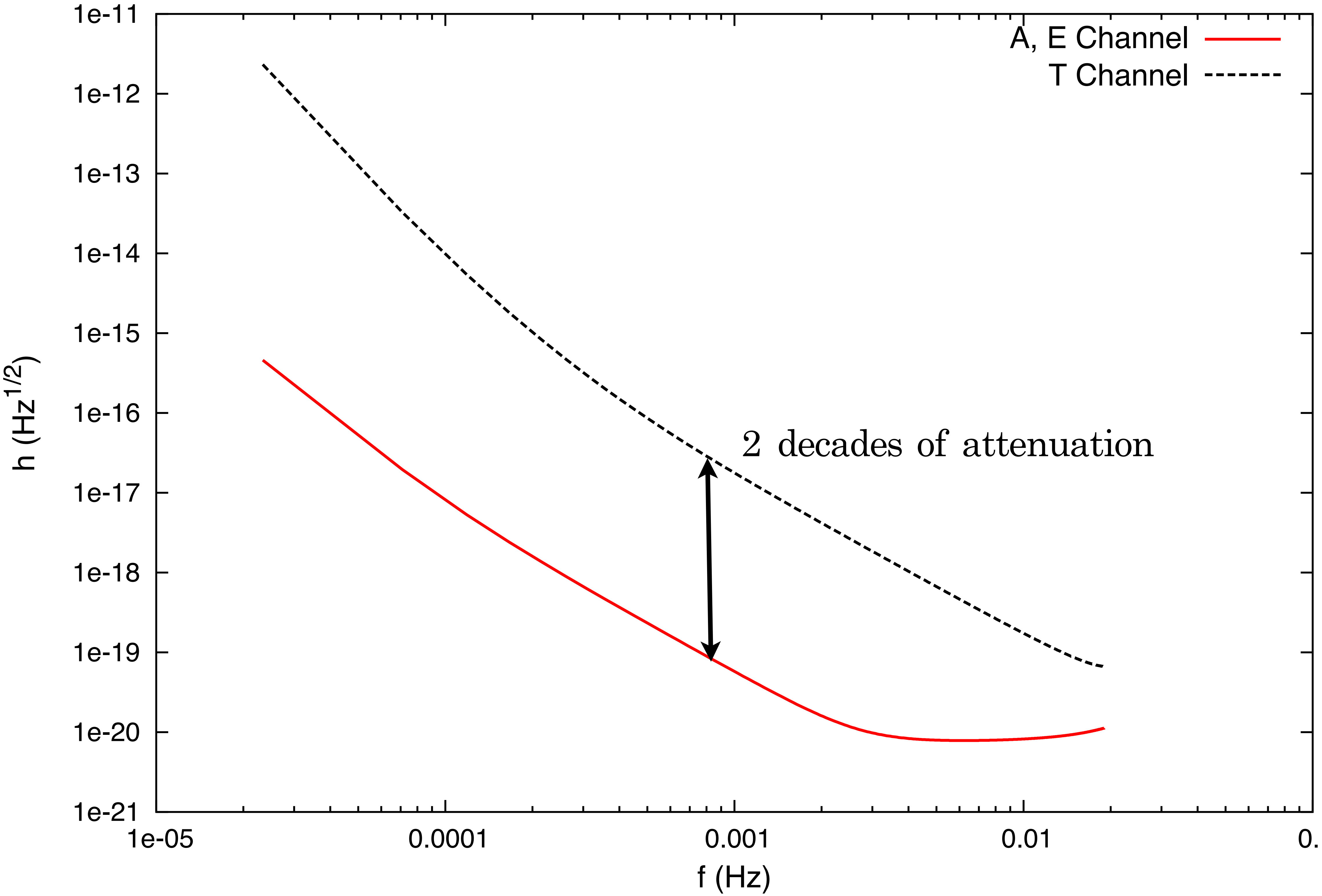
LISA Sensitivity Curves



Tinto, Armstrong & Estabrook, Phys.Rev.D63:021101,2001

Hogan & Bender, Phys. Rev. D64 062002 (2001)

LISA Sensitivity Curves



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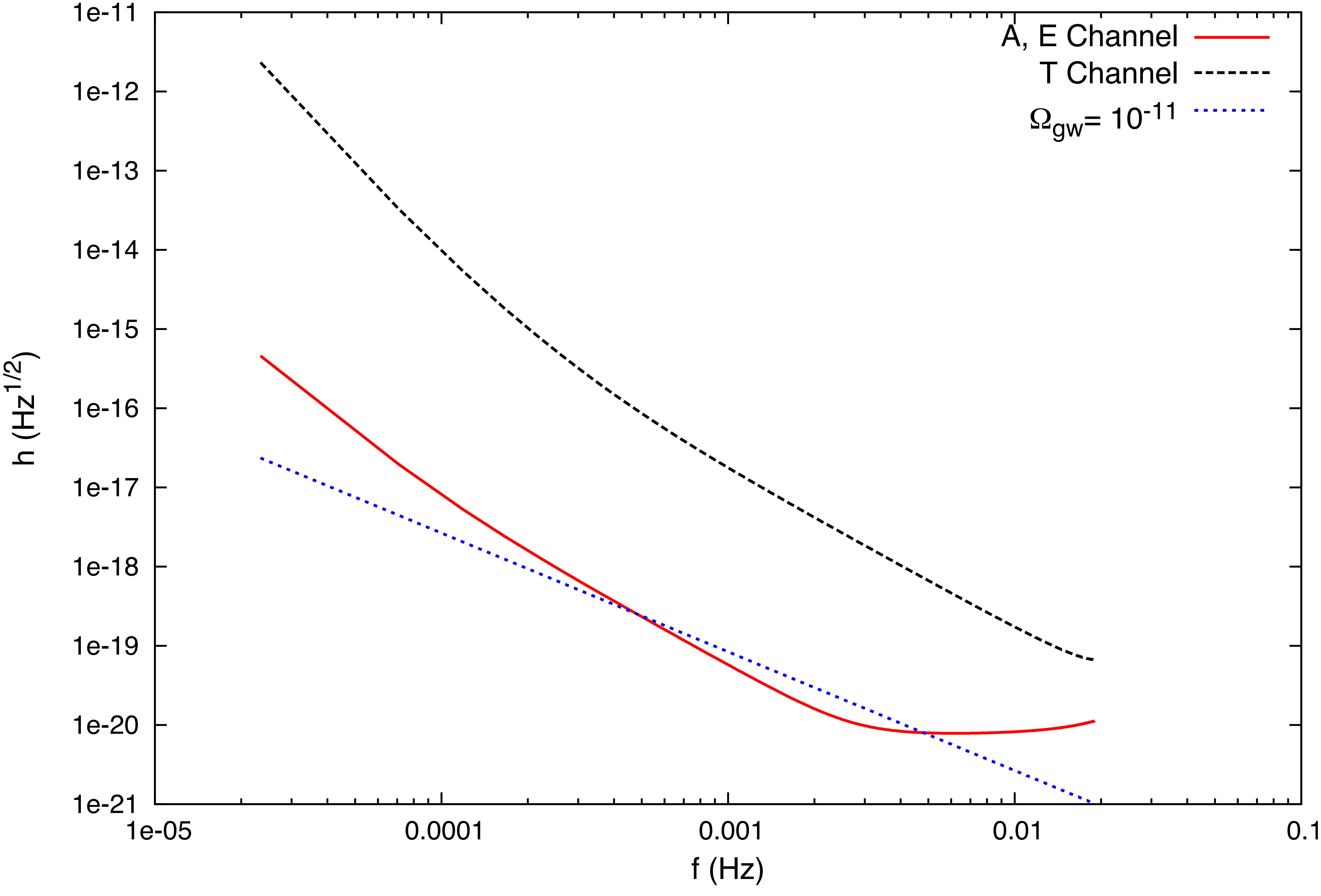
LISA Sensitivity Curves



Instrument Noise Model

The phase measurement at a detector is given by:

$$\Phi_{ij}(t) = C_i(t - L_{ij}) - C_j(t) + \psi_{ij}(t) + n^p_{ij} - \hat{x}_{ij} \cdot \left(\hat{n}^a_{ij}(t) - \hat{n}^a_{ij}(t - L_{ij})\right)$$

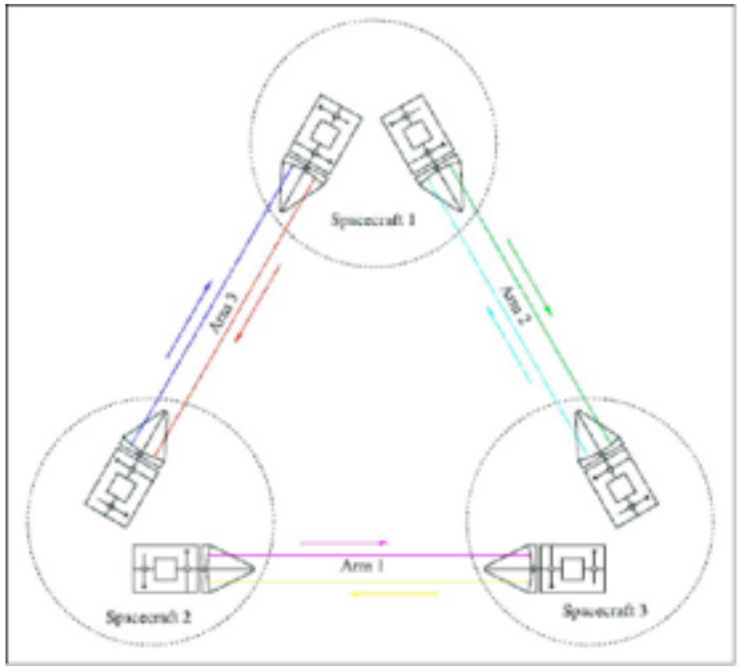
Construct a Michelson signal by:

$$M_1(t) = \Phi_{12}(t - L_{12}) + \Phi_{21}(t) - \Phi_{12}(t - L_{13}) - \Phi_{31}(t)$$

Move to Frequency Domain and form power spectra $$S_{ij}(f) = \langle n_{ij}(f) n^*_{ij}(f) \rangle$$

Include 12 Noise Parameters:

$$S^p_{12}(f), S^p_{21}(f), S^p_{12}(f), S^p_{21}(f), S^p_{32}(f), S^p_{32}(f)$$
$$S^a_{12}(f), S^a_{21}(f), S^a_{12}(f), S^a_{21}(f), S^a_{32}(f), S^a_{32}(f)$$



Stochastic Background Model

The LISA transfer functions are determined by the geometry of the constellation:

$$R_{ij}(f) = \sum_P \int \frac{d\Omega}{4\pi} F_i^P(\hat{\Omega}, f) F_j^{P^*}(\hat{\Omega}, f)$$

The spectral density from a uniform stochastic background is:

$$S_h = \frac{3H_0^2}{4\pi^2} \frac{\Omega_{gw}}{f^3}$$

The power seen in the LISA channels is:

$$\langle S_i(f), S_j(f) \rangle = S_h(f) R_{ij}(f)$$

We included a spectral slope parameter giving a total of two parameters to describe the stochastic background

$$\Omega_w, n$$

Transfer functions for noise and signal

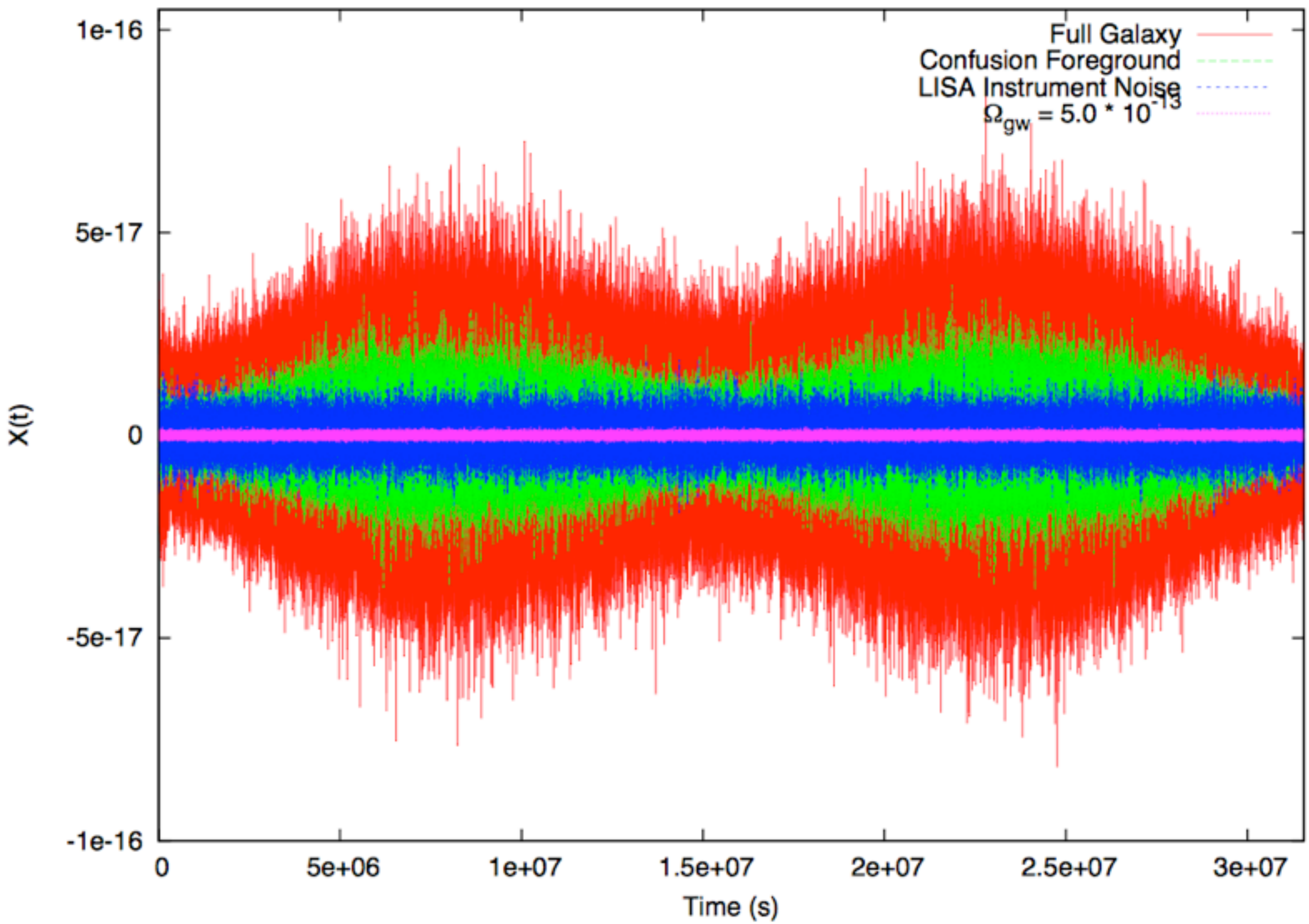
$$\langle AA^* \rangle_p = \frac{4}{9}\sin^2\left(\frac{f}{f_*}\right)\cos\left(\frac{f}{f_*}\right)\left(\left[4(S^p_{21}+S^p_{12}+S^p_{13}+S^p_{31})-2(S^p_{23}+S^p_{32})\right]\right.$$
$$\left.+5(S^p_{21}+S^p_{12}+S^p_{13}+S^p_{31})+2(S^p_{23}+S^p_{32})\right)$$

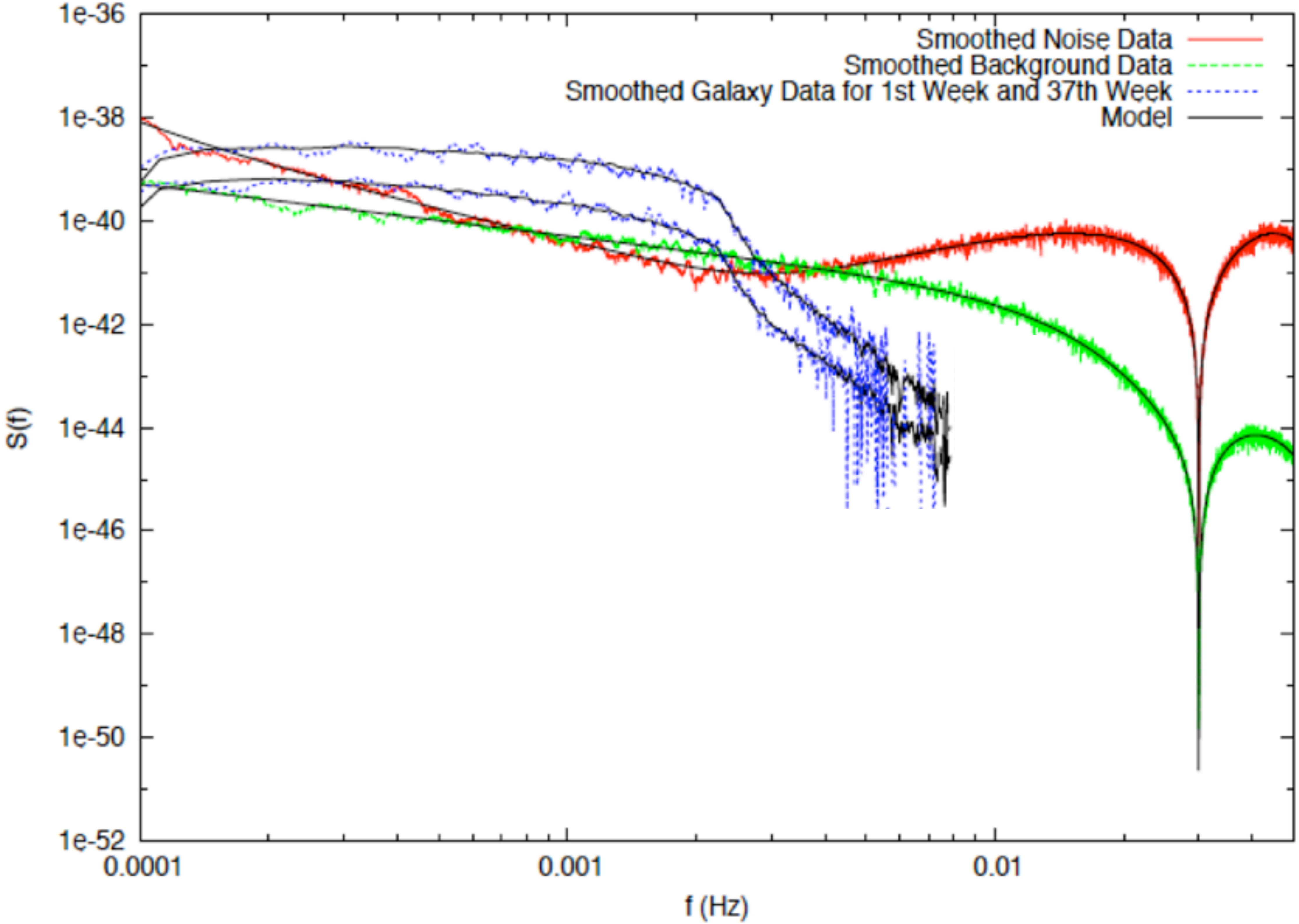
$$\langle AA^* \rangle_a = \frac{16}{9}\sin^2\left(\frac{f}{f_*}\right)\left\{(\cos\left(\frac{f}{f_*}\right)\left[4(S^a_{12}+S^a_{13}+S^a_{31}+S^a_{21})-2(S^a_{23}+S^a_{32})\right]\right.$$
$$+\cos\left(\frac{f}{f_*}\right)\left[\frac{3}{2}(S^a_{12}+S^a_{13}+S^a_{23}+S^a_{32})+2(S^a_{31}+S^a_{21})\right]$$
$$\left.+\frac{9}{2}(S^a_{12}+S^a_{13})+3(S^a_{31}+S^a_{21})+\frac{3}{2}(S^a_{23}+S^a_{32})\right\}$$

$$\langle AA \rangle_s = \left\{\frac{3}{10}-\frac{169}{1680}\left(\frac{f}{f_*}\right)^2+\frac{85}{6048}\left(\frac{f}{f_*}\right)^4-\frac{178273}{159667200}\left(\frac{f}{f_*}\right)^6+\frac{19121}{24766560000}\left(\frac{f}{f_*}\right)^8+\dots\right\}S_h(f)$$

Also have $\langle AE \rangle$, $\langle TT \rangle$, etc.

Adams & Cornish (2009)


Full Galaxy
Confusion Foreground
LISA Instrument Noise
$\Omega_{gw} = 5.0 * 10^{-13}$
1e-16
5e-17
0
-5e-17
-1e-16
X(t)
0
5e+06
1e+07
1.5e+07
2e+07
2.5e+07
3e+07
Time (s)



Smoothed Noise Data
Smoothed Background Data
Smoothed Galaxy Data for 1st Week and 37th Week
Model
S(f)
f (Hz)
1e-36
1e-38
1e-40
1e-42
1e-44
1e-46
1e-48
1e-50
1e-52
0.0001
0.001
0.01

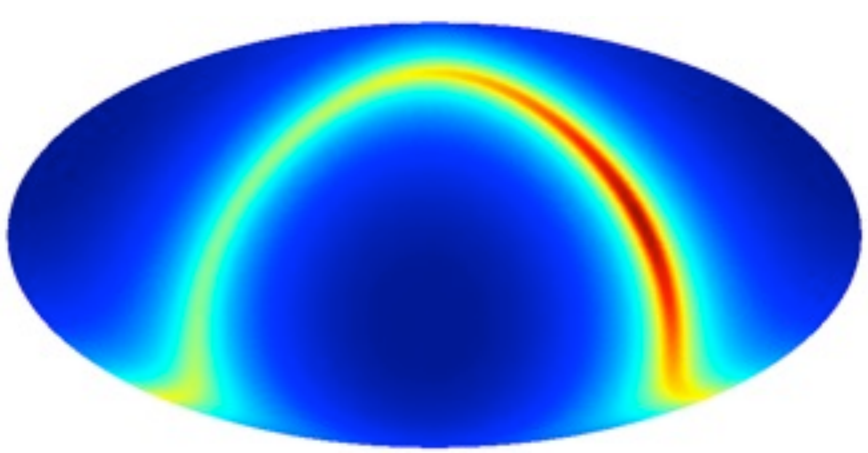
Bayesian Inference

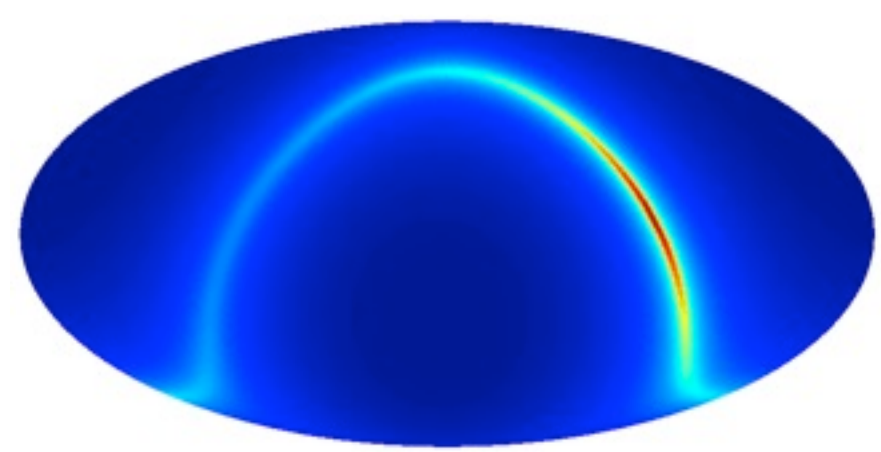
Likelihood $$p(X|S_a, S_p, S_h) = \prod_f \frac{1}{(2\pi)^{3/2}|C|} e^{-(X_i C_{ij}^{-1} X_j)/2}$$

$$\text{where} \quad C(f) = \begin{pmatrix} \langle AA \rangle & \langle AE \rangle & \langle AT \rangle \\ \langle EA \rangle & \langle EE \rangle & \langle ET \rangle \\ \langle TA \rangle & \langle TE \rangle & \langle TT \rangle \end{pmatrix}$$

Spectral Priors $$p(S_a),\ p(S_p)\ p(S_h)$$

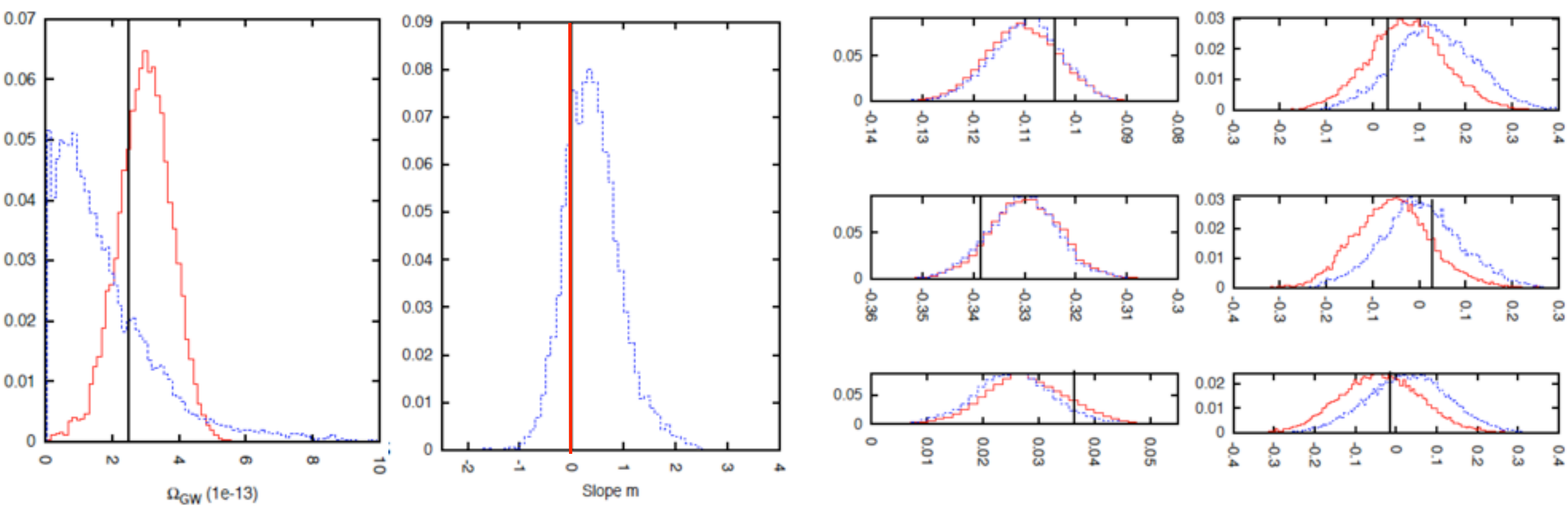
Galaxy Shape Priors $$\rho(x, y, z) = \rho_0\, e^{-\sqrt{x^2+y^2}/R_d}\, \mathrm{sech}^2(z/Z_d)$$





Bayesian Inference

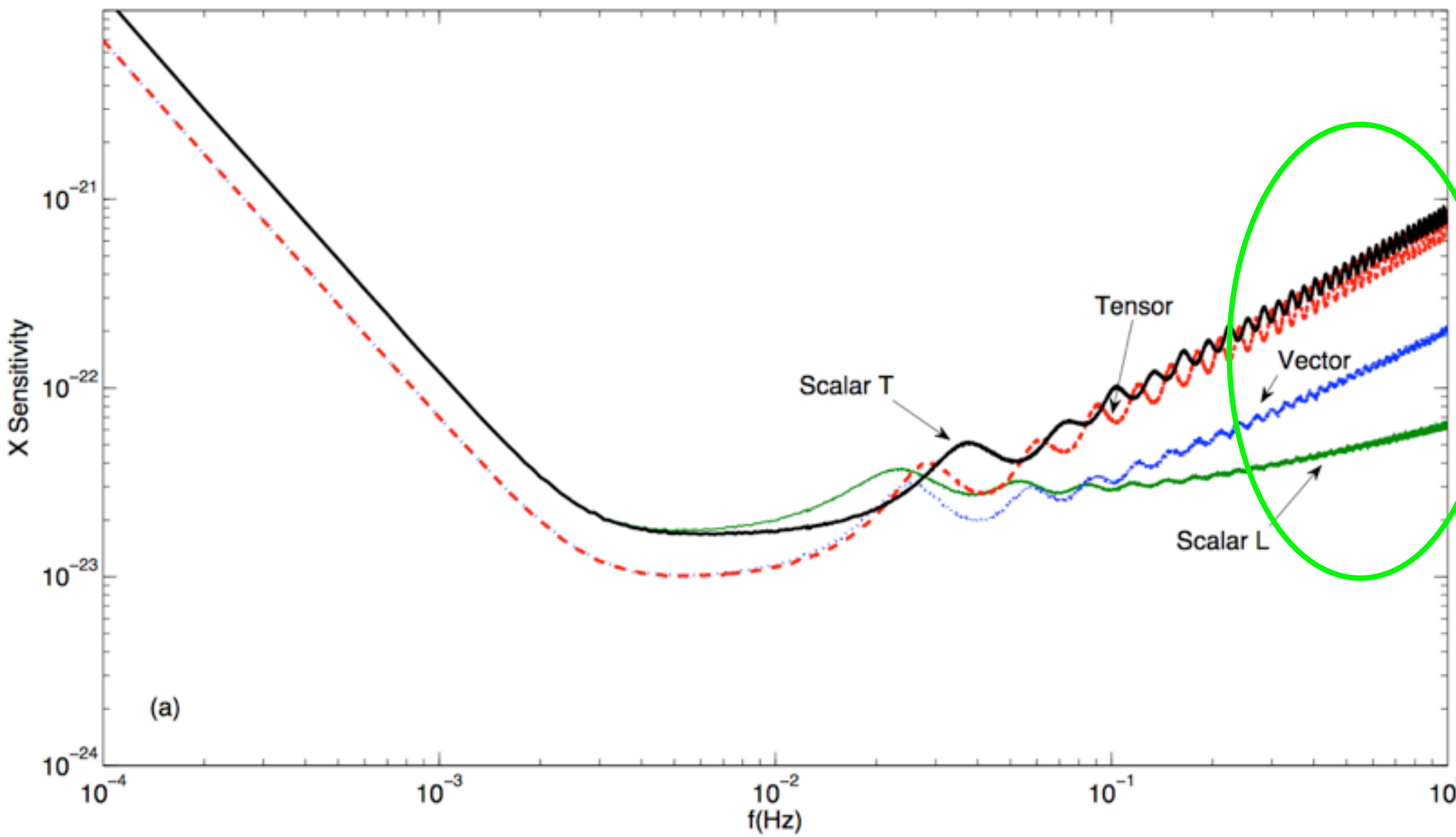
Simultaneously solve for amplitude of instrument noise, stochastic background and galactic white dwarf density and distribution



GW amplitude | GW slope | Position noise | Acceleration noise

Adams & Cornish 1307.4116

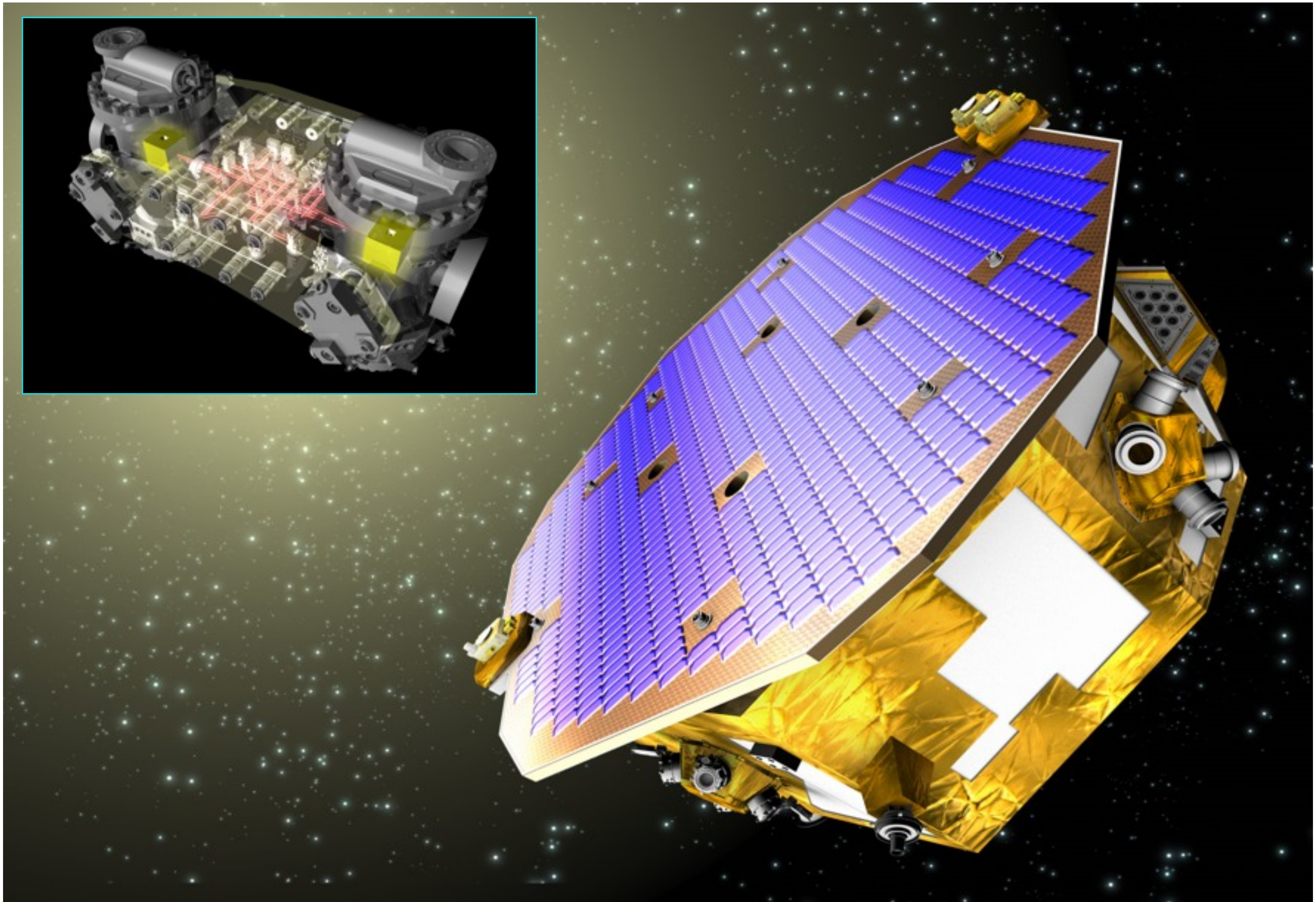
LISA sensitivity to alternative polarization states



(Tinto, da Silva Alves 2010)

Additional slides on estimating LISA noise

LISA Instrument Characterization = LISA Pathfinder V2



How well can we characterize the instrument noise?

- Position Noise
  - Easy to study since at high frequencies
  - Enhance by lowering laser power
  - Enhance by warming optical bench
  - Enhance by increasing electronic noise in phase meter
- Acceleration Noise
  - Hard to study since at low frequency
  - Many couplings to study/enhance

How well can we characterize the instrument noise?

- Proof mass couplings to spacecraft
    - viscous gas flow around the test mass
    - patch fields coupling to changing charge on the TM and the electrode housing
    - radiometer effect (needs thermal gradient)
    - radiation pressure from the laser used in the TM-bench readout
    - gravity noise due to thermo-elastic deformation of the spacecraft
    - noise in the capacitive actuation
    - residual magnetic fields
    - readout noise coupled in via control system
    - thruster noise coupled in via capacitive effects and gravity gradients