(II) THE HIGGS AS A GOLDSTONE BOSON

. Vacuum misalignment

After the general introduction it is now time to enter more into the quantitative details of the CH scenario. As we saw, a key ingredient of the modern incarnation of the CH idea is the Goldstone-boson nature of the Higgs, which is at the origin of the CH properties.

An important concept related to the Goldstone structure is the vacuum misalignment mechanism, which is the basic reason why a composite Goldstone can generate EW symmetry breaking and effectively behave as an elementary Higgs.

We consider a composite new sector endowed with a global invariance $\mathcal{G}$. The vacuum state of this sector, when considered in isolation, is only invariant under a subgroup $\mathcal{H} \subset \mathcal{G}$, leading to the $\mathcal{G} \to \mathcal{H}$ spontaneous breaking and to the appearance of massless Nambu-Goldstone bosons (NGB's) in the coset $\mathcal{G}/\mathcal{H}$.

As we discussed before $\mathcal{H}$ should contain the SM EW group, $G_{EW} = SU(2)_L \times U(1)_Y$, moreover the $\mathcal{G}/\mathcal{H}$ coset should give rise to at least one Higgs doublet.

We split the $\mathcal{G}$ generators T^A ($A = 1, \dots, \dim[\mathcal{G}]$) into

$$\begin{array}{llll} \text{"unbroken"} & T^a & a = 1, \dots, \dim[\mathcal{H}] & \Rightarrow \text{generating } \mathcal{H} \\ \text{"broken"} & \hat{T}^{\hat{a}} & \hat{a} = 1, \dots, \dim[\mathcal{G}/\mathcal{H}] . & \end{array} \qquad (2.1)$$

We also introduce a reference vacuum field configuration $\vec{F}$ that describes one of the degenerate vacua of the composite sector. We choose it to satisfy

$$T^a \vec{F} = 0 \quad , \quad \hat{T}^{\hat{a}} \vec{F} \neq 0 . \qquad (2.2)$$

Notice that the above choices are conventional, since when $\mathcal{G}$ is an exact symmetry we can always redefine our definition of $\mathcal{H}$ inside $\mathcal{G}$. Any choice is equivalent and does not imply any assumption on the $\mathcal{G} \to \mathcal{H}$ breaking pattern.

As well known, the NGB fields are local transformations in the direction of the broken generators $\{\hat{T}^{\hat{a}}\}$, corresponding to

$$\vec{\Phi}(x) = e^{i\theta^{\hat{a}}(x)\hat{T}^{\hat{a}}} \vec{F} . \qquad (2.3)$$

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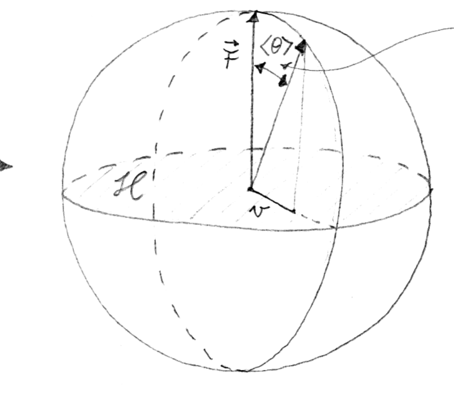
If $\mathcal{G}$ is unbroken a constant vacuum expectation value (VEV) for the NGB's is unphysical, since we can always get rid of it by a global $\mathcal{G}$ transformation

$$\vec{\Phi} \to \exp[-i\langle\theta^{\hat{a}}\rangle \hat{T}^{\hat{a}}]\,\vec{\Phi}\,. \qquad (2.4)$$

In this way we can set $\langle\theta^{\hat{a}}\rangle = 0$.

When we include a breaking of $\mathcal{G}$, the θ's become pseudo-NGB's and the situation changes. First of all θ acquires a potential and its VEV is not arbitrary any more. Moreover $\langle\theta\rangle$ becomes observable. Its physical effect is to break G_{EW} embedded into $\mathcal{H}$, giving rise to EW symmetry breaking. This can be seen geometrically

corresponds to an $SO(3)\to SO(2)$ scenario



$\langle\theta\rangle$ measures the misalignment angle between $\vec{F}$ and the true vacuum

The EW symmetry breaking effects are controlled by the projection of

$$\exp[i\langle\theta^{\hat{a}}\rangle \hat{T}^{\hat{a}}]\,\vec{F}$$

on the $\mathcal{H}$ plane, whose magnitude we denote by v, in formulas

$$v = f\sin\langle\theta\rangle \quad , \text{ where } \quad f = |\vec{F}|. \qquad (2.5)$$

The parameter f is the scale of the $\mathcal{G}\to\mathcal{H}$ spontaneous breaking.

The CH construction becomes interesting when $\langle\theta\rangle \ll 1$. This condition is usually stated as

$$\xi \equiv \frac{v^2}{f^2} = \sin^2\langle\theta\rangle \ll 1. \qquad (2.6)$$

In the limit $\xi\to 0$, at fixed v, the composite sector decouples, since $f\to\infty$. In this limit only the Goldstone states remain in the composite sector spectrum, while all the other composite states decouple. In this limit the CH theory reduces to the SM and the Higgs behaves like an elementary state. This means that going to small ξ we can systematically reduce all the deviations from the SM, as required by the experimental data.

going to small ξ, however, is not "for free". In generic models there is no reason for which $\langle\theta\rangle$ should be small, so we typically expect $\xi \sim 1$, thus leading to large deviations from the SM.
A small value for ξ can be instead obtained by a clever model building and/or by some amount of fine-tuning, as we will discuss later on.

· The CCWZ construction

An efficient way to write down the most general description of a Goldstone dynamics is to use the Callan - Coleman - Wess - Zumino (CCWZ) construction.

The key object which we will use to describe the NGB dynamics is the Goldstone matrix $U[\Pi]$, where Π denote the Goldstone fields. In analogy with what we saw in the vacuum misalignment discussion (see eq. (2.3)), the Goldstone matrix is defined by the following equations

$$\vec{\Phi}(x) = U[\Pi]\, \vec{F} \qquad (2.7)$$

with

$$U[\Pi] = \exp\left[i \frac{\sqrt{2}}{f} \Pi_{\hat{a}}(x) \hat{T}^{\hat{a}} \right], \qquad (2.8)$$

where $\Pi_{\hat{a}}(x)$ denotes the canonically normalized Goldstones. Obviously the Goldstone modes correspond to the broken generators and span the G/H coset.

Under a G transformation g, we assume $\vec{\Phi}(x)$ to transform linearly, namely

$$\vec{\Phi}(x) \to g\,\vec{\Phi}(x) = g\, U[\Pi]\, \vec{F}. \qquad (2.8)$$

We want to find the $\Pi \to \Pi^{(g)}$ transformation that correspond to the action of g, namely

$$\vec{\Phi}(x) \to g\,\vec{\Phi}(x) = U[\Pi^{(g)}]\, \vec{F}. \qquad (2.9)$$

Naively applying a linear transformation we would find

$$U[\Pi] \to g\, U[\Pi] = \exp\left[i \frac{\sqrt{2}}{f} \tilde{\Pi}_{\hat{a}}(x) \hat{T}^{\hat{a}} + i \frac{\sqrt{2}}{f} \tilde{\Gamma}_a T^a \right], \qquad (2.10)$$

but the new expression for U does not match with the form in eq. (2.8), since other components along the unbroken T^a generators are turned on.

To compensate this mismatch we can apply a suitable transformation in $\mathcal{H}$, so that

$$g\, U[\Pi] = U[\Pi^{(g)}] \cdot h[\Pi; g] \qquad (2.11)$$

where $h[\Pi; g]$ has the generic form

$$h[\Pi; g] = \exp[i\, \zeta_a[\Pi; g]\, T^a] .$$

In this way $U[\Pi^{(g)}]$ is recast to the standard form (2.8). Equation (2.11) can be equivalently written as

$$U[\Pi] \to U[\Pi^{(g)}] = g \cdot U[\Pi] \cdot h^{-1}[\Pi; g] \qquad (2.12)$$

which provides the transformation rule for the Goldstone matrix under a generic $\mathcal{G}$ transformation. Since $\vec{F}$ is invariant under $\mathcal{H}$, we get $h^{-1}[\Pi; g]\vec{F} = \vec{F}$, thus we recover the correct transformation for $\vec{\Phi}$:

$$\begin{aligned}\vec{\Phi} = U[\Pi]\vec{F} \to U[\Pi^{(g)}]\vec{F} &= g\, U[\Pi]\, h^{-1}\vec{F} \\ &= g\, U[\Pi]\, \vec{F} \\ &= g\, \vec{\Phi}\end{aligned}$$

It is important to notice that eq. (2.12) provides a full representation of $\mathcal{G}$ since it respects the multiplication rule

$$\Pi^{(g_1 \cdot g_2)} = \left(\Pi^{(g_2)}\right)^{(g_1)} . \qquad (2.13)$$

However this representation is non-linear.

Under $\mathcal{H}$ the Goldstone bosons instead transform linearly. For a transformation

$$g_{\mathcal{H}} = e^{i\alpha_a T^a} \in \mathcal{H}$$

the Goldstones transform as

$$\Pi_{\hat{a}} \to \Pi_{\hat{a}}^{(g_{\mathcal{H}})} = \left(\exp[i\alpha_a t_\Pi^a]\right)_{\hat{a}}^{\;\hat{b}}\, \Pi_{\hat{b}} , \qquad (2.14)$$

where t_Π^a are the $\mathcal{H}$ generators in the representation r_Π in which the Goldstone transform under $\mathcal{H}$. The r_Π representation can be easily obtained by decomposing the adjoint representation of $\mathcal{G}$ in terms of $\mathcal{H}$ representations

$$Ad_{\mathcal{G}} = Ad_{\mathcal{H}} \oplus r_\Pi . \qquad (2.15)$$

For instance in the $SO(N)/SO(N-1)$ cosets, r_Π is simply the fundamental representation of $SO(N-1)$.

At this point we can use the Goldstone matrix to construct the effective Lagrangian describing the NGB's dynamics. For this purpose it is useful to introduce two fundamental objects with "simple" transformation rules under $\mathcal{G}$.

We start from the Maurer-Cartan form constructed from U and we decompose it in the $\mathcal{G}$ algebra

$$i\, U[\Pi]^{-1} \cdot \partial_\mu U[\Pi] = d_\mu^{\hat{a}} \hat{T}^{\hat{a}} + e_\mu^{a} T^{a} \equiv d_\mu + e_\mu , \qquad (2.14)$$

where we introduced the two objects d_μ and e_μ.

It can be shown that the d and e symbols transform as

$$\begin{cases} d_\mu[\Pi] \to h[\Pi; g]\, d_\mu[\Pi]\, h[\Pi; g]^{-1} \\ e_\mu[\Pi] \to h[\Pi; g]\, (e_\mu + i\partial_\mu)\, h[\Pi; g]^{-1} \end{cases} \qquad (2.15)$$

When written in components $d_\mu^{\hat{a}}$ transform by a simple rotation of the $\hat{a}$ index in the r_Π representation

$$d_\mu^{\hat{a}} \to d_\mu^{(g)\,\hat{a}} = R_{\Pi\,\hat{a}}^{\ \ \hat{b}}\, d_{\mu\,\hat{b}} ,$$

while e_μ^{a} have the index a in the adjoint of $\mathcal{H}$ and transforms as if they were gauge fields associated to a local $\mathcal{H}$ invariance.

It is however important to keep in mind that the d_μ and e_μ symbols still transform under the whole $\mathcal{G}$ group, although in a highly non-linear way. So they can be used to construct operators invariant under the full $\mathcal{G}$.

The simplest invariant operator we can construct is obtained by using two $d_\mu^{\hat{a}}$ symbols and contracting the $\hat{a}$ index:

$$\mathcal{L}^{(2)} = \frac{f^2}{4}\, d_{\mu,\hat{a}}\, d^{\mu,\hat{a}} . \qquad (2.16)$$

This is an operator containing two ∂_μ derivatives and provides the kinetic term for the Goldstones

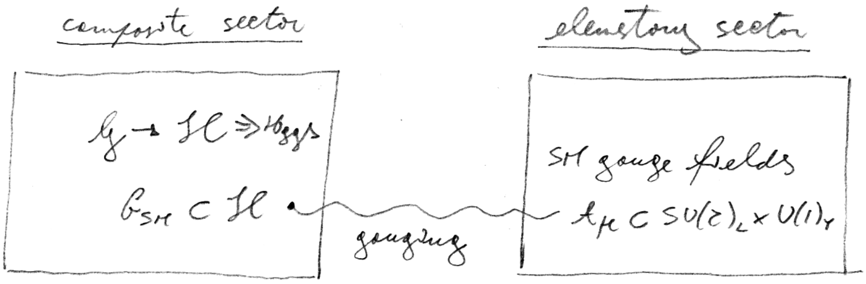
$$\mathcal{L}^{(2)} = \frac{1}{2} \partial_\mu \Pi^{\hat{a}} \partial^\mu \Pi_{\hat{a}} + \sum_n \mathcal{O}\left((\partial\Pi)^2 \cdot \Pi^n / f^n\right).$$

In addition it provides an infinite set of two-derivative interactions among the Goldstones, which are all fixed by the symmetry and controlled by the unique parameter f.

· The SM gauge fields.

So far we considered the Goldstones "in isolation", however a key ingredient to build successful CH models is to couple the composite sector with the SM fields included in the elementary sector.

For the gauge fields this is straightforwardly done by weak gauging: the SM fields gauge an $SU(2)_L \times U(1)_Y$ subgroup of $\mathcal{H}$.



The gauging is easily included in the CCWZ construction and, as we will see, can be done in a unique way, that is the dynamics of the gauge fields and their interaction with the Goldstones is fully determined by the $\mathcal{G} \to \mathcal{H}$ invariance.

First of all we embed the SM gauge fields in $\mathcal{G}$:

$$A_\mu = A_{\mu,A} T^A = g\, W_\mu^\alpha T_L^\alpha + g' B_\mu T_R^3 ,$$

where W_μ^α and B_μ are the $SU(2)_L$ and $U(1)_Y$ gauge fields and T_L^α, T_R^3 are the corresponding generators inside $\mathcal{H} \subset \mathcal{G}$. $A_{\mu,A}$ is used as a shorthand notation; some of the $A_{\mu,A}$ components, indeed, are zero, while the others coincide with the SM fields.

We now modify our definition of the d and e symbols by starting from a slightly different Maurer-Cartan form:

$$U[\Pi]^{-1} \cdot (A_\mu + i\partial_\mu) \cdot U[\Pi] \equiv d_\mu[\Pi, A] + e_\mu[\Pi, A]. \qquad (2.17)$$

It is easy to check that the new d and e transform as the old ones

$$\begin{cases} d_\mu[\Pi, A] \to h[\Pi; g]\, d_\mu[\Pi, A] \cdot h[\Pi; g]^{-1} \\ e_\mu[\Pi, A] \to h[\Pi; g]\, (e_\mu[\Pi, A] + i\partial_\mu) \cdot h[\Pi; g]^{-1} \end{cases} \qquad (2.18)$$

we can thus use the new objects exactly as the old ones. In particular the Goldstone kinetic term is still given by

$$\mathcal{L}^{(2)} = \frac{f^2}{4} d_{\mu \hat{a}}[\Pi, A]\, d^{\mu \hat{a}}[\Pi, A]. \qquad (2.19)$$

This Lagrangian, however, now also contains the interactions with the SM gauge fields!

· Corrections to the Higgs couplings to gauge fields.

We can now use the result we just got to analyze the Higgs couplings to the SM gauge fields. For definiteness I will specialize the discussion to the "minimal" coset : $SO(5) \to SO(4)$.

This is the minimal symmetry pattern that gives rise to only one Higgs doublet and contains an additional custodial protection, which we will discuss better in a while.

The decomposition of the $Ad_{SO(5)}$ is very simple

$$\begin{aligned} Ad_{SO(5)} &= Ad_{SO(4)} \oplus fund_{SO(4)} \\ 10 &= 6 \oplus 4 \end{aligned} \tag{2.20}$$

4 real d.o.f.'s corresponding to the Higgs composite.

The SM gauge group is embedded into $SO(4)$. To see this it is useful to recall that the $SO(4)$ algebra is equivalent to

$$SO(4) \simeq SU(2)_L \times SU(2)_R$$

so that we can embed

$$\begin{cases} W_\mu^\alpha,\ SU(2)_L^{SM} \longrightarrow T_L^\alpha \text{ of } SU(2)_L \\ B_\mu,\ U(1)_Y^{SM} \longrightarrow T_R^3 \text{ of } SU(2)_R \end{cases} \tag{2.21}$$

The $d_\mu^{\hat{a}}$ symbol can be explicitly computed and it has the structure

$$d_\mu^{\hat{a}} = -\frac{\sqrt{2}}{\Pi} \sin\frac{\Pi}{f} D_\mu \Pi^{\hat{a}} + \text{terms not involving } A_\mu \tag{2.22}$$

where

$$D_\mu \Pi \equiv \left(\partial_\mu - i W_\mu^\alpha T_L^\alpha - i B_\mu T_R^3\right)\Pi . \tag{2.23}$$

We thus get in full generality

$$\mathcal{L}^{(2)} = \frac{f^2}{4} d_\mu^{\hat{a}} d^{\mu\hat{a}} = \frac{f^2}{2|H|^2} \sin^2\frac{\sqrt{2}|H|}{f} (D_\mu H)^\dagger D^\mu H + \dots \tag{2.24}$$

In the unitary gauge $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$ so we get

$$\mathcal{L}^{(2)} = \frac{1}{2}(\partial_\mu h)^2 + \frac{g^2}{4} f^2 \sin^2\frac{v+h}{f}\left(|W|^2 + \frac{1}{2c_w^2} Z^2\right). \tag{2.25}$$

We immediately read the gauge boson masses

$$m_W = c_W m_Z = \frac{1}{2} g f \sin\frac{V}{f} \equiv \frac{1}{2} g v , \qquad (2.26)$$

where $v \simeq 246$ GeV is the usual EW symmetry breaking scale.

Notice that the SM relation

$$m_W = c_W m_Z$$

is respected in our scenario. This is due to the presence of the custodial symmetry, namely $SO(4) \simeq SU(2)_L \times SU(2)_R$ symmetry. After EW symmetry breaking a subgroup is still preserved by the Higgs VEV

$$SO(4) \longrightarrow SO(3)_c$$

and this protects the relation between the W and Z mass. This protection translates in a protection of the EW T parameter, which is constrained experimentally at the 0.1% level.

From eq. (2.25) we can also extract the Higgs couplings to the gauge fields

$$\frac{g^2 v^2}{4}\left(|W|^2 + \frac{1}{2c_W^2}|Z|^2\right)\left[2\sqrt{1-\xi}\,\frac{h}{v} + (1-2\xi)\frac{h^2}{v^2} + \cdots\right] \qquad (2.27)$$

where we used the ξ parameter

$$\xi = \sin^2\frac{V}{f} = \frac{v^2}{f^2} .$$

Exactly as in the SM we find single- and double-Higgs vertices, but with modified couplings

$$k_V \equiv \frac{g^{CH}_{hVV}}{g^{SM}_{hVV}} = \sqrt{1-\xi} < 1 , \qquad \frac{g^{CH}_{hhVV}}{g^{SM}_{hhVV}} = 1 - 2\xi . \qquad (2.28)$$

Moreover higher-order interactions, involving 3 or more Higgses, are also present. Notice that, as expected from our discussion on the vacuum misalignment mechanism, the deviations from the SM are controlled by ξ, so that in the $f \to \infty$ ($\xi \to 0$) limit we recover the usual SM relations.

• Partial fermion compositeness

Now it is time to introduce in our construction the SM fermions. Obviously, in order to be able to generate the necessary interactions of the SM fermions with the Higgs, we need to couple them to the composite dynamics. An interesting way to do that is to rely on the "partial compositeness" hypothesis.

We assume that the SM fermions are linearly mixed with some operators coming from the composite dynamics. This mixing arises at high energy, Λ_{UV}, and then evolves at low energy through the RG evolution. Different scaling dimensions of the composite operators can give rise to hierarchies of mixings and, ultimately, translate into hierarchies of Yukawa couplings.

$$
\begin{array}{l|l}
\Lambda_{UV} & \mathcal{L}_{int}[UV] \sim \dfrac{\zeta}{\Lambda_{UV}^{d-5/2}}\,\bar{q}\,\mathcal{O}_F \\
 & \qquad\qquad \downarrow \\
E_{cs} & \mathcal{L}_{int}[E_{cs}] \sim \dfrac{\zeta[E_{cs}]}{E_{cs}^{d-5/2}}\,\bar{q}\,\mathcal{O}_F \\
EW &
\end{array}
$$

At high energy the composite sector is fully $\mathcal{G}$ invariant, so the $\mathcal{O}_F$ operators must appear in complete $\mathcal{G}$ representations. The representations we can consider, however, are not fully fixed, so that this part of the CH construction has some arbitrariness. The only requirement is that the $\mathcal{O}_F$ operators must have suitable quantum numbers to be able to couple with the SM fermions:

- they must have spin $1/2$
- must have some components with the SM fermions quantum numbers under $SU(3)_c \times SU(2)_L \times U(1)_Y$.

I will give an explicit example of this later on.

First of all, I want to discuss an interesting consequence of the mixing I introduced for the SM fermions. At low energy (below Λ_{cs}) the $\mathcal{O}_F$ operators can be associated to resonances with the same quantum numbers of the operators. This means that fermionic resonances Ψ (usually called fermionic "partners") are present. At low energy the interaction Lagrangian gives rise to a mass mixing between the SM fermions and the partners

$$\zeta\,\bar{q}\,\mathcal{O}_F \longrightarrow \zeta\,\bar{q}\,\Psi \qquad (2.29)$$

The partners are expected to be massive and to have a mass of order

$$m_* \sim \Lambda_{cs}\,. \qquad (2.30)$$

We thus get a full mass Lagrangian

$$-\lambda\,\bar q\,\Psi - m_*\,\bar\Psi\,\Psi\,. \qquad (2.31)$$

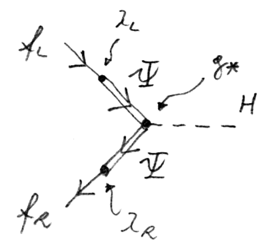
This implies that the mass eigenstates are no more given simply by q and Ψ, but instead they are an admixture of elementary and composite states

$$|phys_i\rangle = \cos\theta_i\,|elem_i\rangle + \sin\theta_i\,|comp_i\rangle. \qquad (2.32)$$

In particular the SM mass eigenstates acquire a component along the composite partners. The larger the mixing λ is, the larger the amount of composite component becomes. This structure explains why we called this construction "partial compositeness".

. The fermion Yukawa's

From the partial compositeness construction it is easy to understand how the SM Yukawa's are generated. They arise from diagrams like

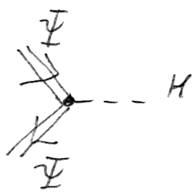
$y_* \sim$ 

$\sim \dfrac{\lambda_L \lambda_R}{g_*}\,. \qquad (2.33)$

To derive the estimate we rewrite the mixing terms as

$$\mathcal{L}_{mass} = -m_*\,\bar\Psi\,\Psi - \frac{\lambda_f}{g_*}\,m_*\,(\bar q\,\Psi + h.c), \qquad (2.34)$$

where g_* is the typical coupling strength among the states in the composite sector



$\sim g_*\,.$

We can be more quantitative and also extract information about the Higgs couplings to the SM fermions by applying the CCWZ techniques to the fermions.

For this purpose the first step is to understand how the SM fermions fit within the $\mathcal{G}/\mathcal{H}$ symmetry structure. For simplicity we will focus on the $SO(5)/SO(4)$ minimal coset and we consider only the top quark, which due to its large Yukawa is the state with the largest mixing to the composite sector.

As we already mentioned the composite operators that mix with the SM fermions must have suitable quantum numbers under the SM gauge group. The quarks are in the fundamental representation of $SU(3)_c$ and have $SU(2)_L \times U(1)_Y$ quantum numbers

$$q_L \subset 2_{1/6} \quad , \quad t_R \subset 1_{2/3} \qquad \left(b_R \subset 1_{-1/3} \right).$$

From this we derive an important piece of information, namely the $\mathcal{O}_F$ operators must carry QCD charges. This in turn implies that the composite sector must be charged under QCD. This fact has important impact on the composite resonances phenomenology, as we will discuss later on.

The $\mathcal{O}_F$ operators must also be in $SO(5)$ representations that include the $2_{1/6}$ and $1_{2/3}$ G_{EW} representations. However in $SO(5)$ there are no suitable representations satisfying this requirement. The simplest way out is to slightly enlarge the $\mathcal{G}$ group by adding an unbroken $U(1)_X$ factor

$$SO(5) \times U(1)_X \to SO(4) \times U(1)_X . \qquad (2.35)$$

The SM hypercharge $U(1)_Y$ is now generated by

$$Y = T^3_R + X ,$$

that is it is a combination of the third generator of $SU(2)_R \subset SO(4)$ and of the $U(1)_X$ generator X.

Notice that the extra $U(1)_X$ factor commutes with $SO(5)$, so all the discussion we presented for the Goldstones is not affected by the change.

It is now easy to find suitable $\mathcal{G}$ representations for $\mathcal{O}_F$. A simple choice is

$$5_{2/3} \xrightarrow{SO(4)\times U(1)_X} 4_{2/3} \oplus 1_{2/3} \xrightarrow{SU(2)_L\times U(1)_Y} 2_{7/6} \oplus \underset{q_L}{\underset{\uparrow}{2_{1/6}}} \oplus \underset{t_R}{\underset{\uparrow}{1_{2/3}}} . \qquad (2.36)$$

This representation contains the correct ingredients to mix with the q_L and t_R fields.

For the t_R it is easy to write down the interaction Lagrangian:

$$\mathcal{L}_{int}^{t_R} = \lambda_{t_R}\, \bar{t}_R \left(\mathcal{O}_F^R\right)_5 + h.c. = \lambda_{t_R} \left(\bar{T}_R\right)^I \left(\mathcal{O}_F^R\right)_I + h.c. \qquad (2.37)$$

where the subscript 5 denotes the 5th component of the 5 representation, which in our conventions is a singlet under SO(4). As we will see, it is also useful to formally embed the t_R field in the $5_{2/3}$ representation:

$$T_R = \{0,0,0,0,\, t_R\}^T. \qquad (2.38)$$

Rewriting the interaction in terms of T_R is very useful. Indeed as far as we consider the elementary fields as external "spectators" (or better sources) and we do not consider their effects in loops, we can <u>formally</u> treat T_R as a complete SO(5) multiplet that transforms linearly under $\mathcal{G}$:

$$(T_R)_I \to g_I{}^J (T_R)_J\,, \qquad (2.39)$$

with $U(1)_X$ charge $2/3$ (and $SU(3)_c$ color).

We can thus derive the operators in the effective Lagrangian by just imposing the $\mathcal{G}$ invariance and using the CCWZ techniques.

Analogously we can proceed for $q_L = \{t_L, b_L\}$, whose embedding is

$$Q_{t_L} = \frac{1}{\sqrt{2}} \{-i b_L,\, -b_L,\, -i t_L,\, t_L,\, 0\}^T. \qquad (2.40)$$

leading to the interaction Lagrangian

$$\mathcal{L}_{int}^{t_L} = \lambda_{t_L} \left(\bar{Q}_{t_L}\right)^I \left(\mathcal{O}_F^L\right)_I + h.c. \qquad (2.41)$$

We can now build the effective Lagrangian describing the top sector and the couplings to the Goldstones. We need to use the T_R and Q_{t_L} multiplets as well as the Goldstone matrix U. Recalling that U transforms as

$$U \to g\, U\, h[\Pi; g]$$

we can construct objects that transform in SO(4) representations

$$\left\{\begin{array}{lll} U^{-1} T_R & \nearrow 4_{2/3} & (U^{-1}T_R)_4 \equiv T_R^4 \\ & \searrow 1_{2/3} & (U^{-1}T_R)_1 \equiv T_R^1 \\ U^{-1} Q_{t_L} & \nearrow 4_{2/3} & (U^{-1}Q_{t_L})_4 \equiv Q_{t_L}^4 \\ & \searrow 1_{2/3} & (U^{-1}Q_{t_L})_1 \equiv Q_{t_L}^1 \end{array}\right. \qquad (2.42)$$

From these building blocks we can construct invariant operators. Notice however that

$$(\bar{Q}^4_{t_L})^i (T^4_R)_i + \bar{Q}^1_{t_L} T^1_R = (\bar{Q}_{t_L})^I \left[U_I{}^i U^{+}_i{}^J + U_I{}^5 U^{+}_5{}^J \right] (T_R)_J$$
$$= (\bar{Q}_{t_L})^I (T_R)_I = 0 ,$$

thus only one invariant combination can be formed. One can easily work out the explicit form

$$\mathcal{L}^t_{Yuk} = -c\, \lambda_{t_L} \lambda_{t_R}\, \bar{Q}^1_{t_L} T^1_R + h.c.$$

$$= -c\, \lambda_{t_L} \lambda_{t_R} \frac{1}{2\sqrt{2}\,|H|} \sin \frac{2\sqrt{2}\,|H|}{f}\; \bar{q}_L H^c t_R + h.c. \qquad (2.43)$$

where $H^c = i\sigma^2 H^*$.

We can fix the c coefficient by relating the operator (2.43) to the top mass. In this way we find

$$\mathcal{L}^t_{Yuk} = -m_t \bar{t} t - \frac{1-2\xi}{\sqrt{1-\xi}} \frac{m_t}{v} h \bar{t} t + 2\xi \frac{m_t}{v^2} h^2 \bar{t} t + \dots \qquad (2.44)$$

The above result shows that the top Yukawa coupling is modified as

$$k^5_t = \frac{g^{CH}_{htt}}{g^{SM}_{htt}} = \frac{1-2\xi}{\sqrt{1-\xi}} . \qquad (2.45)$$

Moreover new non-renormalizable operators are present, the first one being

$$2\xi \frac{m_t}{v^2} h^2 \bar{t} t \equiv -c_2 \frac{m_t}{v^2} h^2 \bar{t} t \qquad (2.46)$$

Notice that the deviation in the top Yukawa k^5_t depends on the representation we choose for the O_F operators coupled to the top quark. This result is thus less "universal" than the one for the Higgs couplings to the gauge fields k_V and k_{2V}, which only depend on the G/H symmetry pattern.

Similar considerations hold for the bottom quark and the other SM fermions. In the following table I list the Yukawa modifications for some common embeddings of the fermions in SO(5)

reps.	Top		Bottom
$5 \oplus 5$	$k_t = \frac{1-2\xi}{\sqrt{1-\xi}}$	$c_2 = -2\xi$	$k_b = \frac{1-2\xi}{\sqrt{1-\xi}}$
$4 \oplus 4$	$k_t = \sqrt{1-\xi}$	$c_2 = -\xi/2$	$k_b = \sqrt{1-\xi}$
$14 \oplus 1$	$k_t = \frac{1-2\xi}{\sqrt{1-\xi}}$	$c_2 = -2\xi$	$k_b = \frac{1-2\xi}{\sqrt{1-\xi}}$

The deviations in the Higgs couplings are a robust prediction of the CH scenario, so they are a good experimental target to test the CH idea. As we saw the deviations in the couplings to gauge fields depend only on $l_g/\mathcal{H}$, thus they are somewhat "universal". The deviations in the fermion Yukawa's instead depend on the choice of the fermion embedding so there considerable more freedom is present.

In several minimal scenarios the top and bottom Yukawa's are modified in the same way, $k_t = k_b = k_F$. Although it is quite easy to choose embedding that break this rule. For instance in the original $MCHM_4$ and $MCHM_5$ holographic scenarios the fermion embedding is $4+4$ or $5+5$, thus $k_t = k_b$.

Present experimental results give bounds on the deviations in k_V and in the top and bottom Yukawa's under the assumption $k_t = k_b = k_F$. In this case the bounds translate into

$$\begin{cases} \xi < 0.12 & \text{in the } 4+4 \text{ case} \\ \xi < 0.1 & \text{in the } 5+5 \text{ case} \end{cases}$$

· See coupling strength measurement plot in the slides.

· The Higgs potential, top partners and tuning

An additional theoretical aspect of the CH scenario it is important to discuss is the origin of the Higgs potential, together with its structure, which is at the root of EW symmetry breaking.

We will assume, as usually done in most explicit realizations of the CH idea, that the only explicit breaking of the $\mathcal{G}$ symmetry is due to the interactions with the SM fields. As a consequence, when the composite sector is considered "in isolation", the $\mathcal{G}$ invariance is preserved and the Higgs is an exact Goldstone, i.e. it has no potential and, thus, is massless.

The SM field content, on the other hand, does not respect the $\mathcal{G}$ symmetry, thus its interactions with the composite dynamics necessarily break the global invariance and make the Higgs a pseudo-NGB. This mechanism gives rise to the Higgs potential, which eventually controls EW symmetry breaking.

It is easy to understand that the largest contributions to the Higgs potential come from the largest interaction terms of the SM fields with the composite sector. In general it is natural to expect that the largest mixing is the one related to the top quark, since the top is the heaviest state in the SM. This expectation is indeed verified in a large class of CH models. In the following we will thus focus on the top sector only and neglect (subleading) contributions coming from the other SM fermions and from the gauge fields.

Since the Higgs boson is a NGB its potential is in general a trigonometric function of h/f. Its structure is generically of the form

$$V(h) \simeq -\alpha f^2 \sin^2 \frac{h}{f} + \beta f^2 \sin^4 \frac{h}{f}, \qquad (2.47)$$

where the coefficients α and β can be computed within explicit models. In order to obtain the correct Higgs mass and a small enough value for $\xi = v^2/f^2$, we need to appropriately choose α and β, namely

$$\begin{cases} \alpha = 2\xi\beta \\ m_h^2 = 8\xi(1-\xi)\beta = 4(1-\xi)\alpha \end{cases} \qquad (2.48)$$

As we will see both conditions cost some tuning.

Let's start from the first one. From this relation we find that for generic values of the parameters

$$\xi = \frac{\alpha_{\text{expected}}}{2\beta_{\text{expected}}}$$

where α_{expected}, β_{expected} denote the typical value of the α and β coefficients generally found in the parameter space of the model. In all existing models

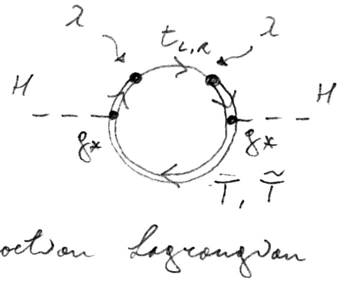
$$\alpha_{\text{expected}} \gtrsim \beta_{\text{expected}}$$

thus one usually expects $\xi \sim 1$. Enforcing $\xi \ll 1$, instead, requires some tuning, which is at least of order

$$\Delta = \frac{(\alpha/\beta)_{\text{expected}}}{\alpha/\beta} \geq \frac{1}{2\xi}\,. \qquad (2.49)$$

The current bound $\xi < 0.1$ already corresponds to a certain amount of fine-tuning. Notice moreover that the estimate in (2.49) should be interpreted as a sort of lower bound on the tuning. In some minimal models (noticeably the ones with SM in the $5+5$ or $4+4$) one has $\alpha_{\text{expected}} > \beta_{\text{expected}}$, thus the amount of tuning related to ξ is even higher.

Let us now consider the second condition in eq. (2.48), the one related to the Higgs mass. To work out the implications of this condition we need to derive an estimate for the coefficients of the potential, in particular for α, which is always greater or of the order of β.
The largest contributions to α come from loops of top and top partners



Starting from the interaction Lagrangian

$$\mathcal{L}_{mix} \sim \frac{\lambda_L}{g_*} m_* \bar{t}_L \underbrace{T}_{\text{partners}} + \frac{\lambda_R}{g_*} m_* \bar{t}_R \underbrace{\widetilde{T}}_{\text{partners}} + h.c. \qquad (2.50)$$

we can easily derive the estimate

$$\alpha \sim a_L N_c \frac{\lambda_L^2}{16\pi^2} m_*^2 + a_R N_c \frac{\lambda_R^2}{16\pi^2} m_*^2\,, \qquad (2.51)$$

where a_L, a_R are numerical coefficients, in general of $\mathcal{O}(1)$, and $N_c = 3$ is the number of QCD colors.

If m_* is large, obtaining the correct Higgs mass, requires a cancellation between the L and R contributions, obtained by tuning the α_L and α_R coefficients. This implies a tuning

$$\Delta = \frac{3\lambda^2}{4\pi^2}\left(\frac{m_*}{m_h}\right)^2 \simeq \lambda^2\left(\frac{m_*}{450\,\mathrm{GeV}}\right)^2 \qquad (2.52)$$

where we assumed $\lambda_L \simeq \lambda_R \equiv \lambda$, which is the configuration that minimizes the tuning.

We can also go a step further and recall the relation between $\lambda_{L,R}$ and the top mass, namely

$$y_t \simeq \frac{\lambda_L \cdot \lambda_R}{g_*}, \qquad (2.53)$$

which gives

$$\lambda \simeq \sqrt{y_t \cdot g_*} > \sqrt{y_t}. \qquad (2.54)$$

Substituting this back into (2.52) we finally find

$$\Delta \simeq y_t\, g_* \left(\frac{m_*}{450\,\mathrm{GeV}}\right)^2 \gtrsim \left(\frac{m_*}{450\,\mathrm{GeV}}\right)^2. \qquad (2.55)$$

This result is very interesting and important, indeed it tells us that the amount of tuning in CH models is directly related to the mass of the top partners. If we want a Natural theory top partners must be relatively light, $m_* \lesssim$ TeV.

Directly searching for these states at the LHC (and possibly future collider machines) is thus a powerful way to test the CH idea.

Note that similar considerations apply to the partners of the other SM fermions and to the partners of the gauge bosons. Due to their smaller interactions with the composite dynamics, however, they are allowed to be substantially heavier than top partners in Natural theories.