(IV) THE FLAVOR STRUCTURE

We saw in the previous lectures that an important ingredient of the CH construction is partial compositeness, which allow to mix the SM fermions with the composite dynamics. An interesting byproduct of this structure is a straightforward implementation of the flavor structure, known as the "anarchic" flavor structure.

This construction allows to naturally suppress flavor-violating and CP-violating new-physics effects involving the light quarks, thus dramatically improving the compatibility with the experimental constraints. Indeed, in the absence of a suitable flavor protection, some classes of BSM flavor effects (the $\Delta F = 2$ 4-fermion operators) need to be suppressed by an energy scale $\Lambda \gtrsim 10^5$ TeV. The anarchic flavor protection allows to suppress enough all these effects in such way that a new-physics scale around a few TeV can be compatible with the present bounds.

· The general structure

As we saw, in partial compositeness the elementary fermion mixing with the composite sector arises at high energy through operators of the form

$$\mathcal{L}_{int}^{UV} = \frac{\lambda^{ij}_{u_R}}{\Lambda_{UV}^{d^j_{u_R} - 5/2}} \bar{u}^i_R \, \mathcal{O}_F^{u_R j} + h.c. \qquad d^j_{u_R} \equiv \dim\left[\mathcal{O}_F^{u_R j}\right]$$

and analogous ones for the q_L and d_R fields.

$\lambda^{ij}_{u_R}$ controls the strength of the couplings in the UV, and is naturally expected to be an anarchic matrix with entries all of the same order.

At low energy, through the RG running, one finds

$$\lambda^{ij}_{u_R}[\Lambda_{cs}] \simeq \lambda^{ij}_{u_R}\left(\frac{\Lambda_{cs}}{\Lambda_{UV}}\right)^{d^j_{u_R} - 5/2} \equiv c^{ij}_{u_R}\, \zeta^j \,, \quad \text{where } \zeta^j \equiv \left(\frac{\Lambda_{cs}}{\Lambda_{UV}}\right)^{d^j_{u_R} - 5/2}$$

where $c^{ij}_{u_R}$ is an anarchic matrix. Notice that at low energy the $\lambda^{ij}_{u_R}[\Lambda_{cs}]$ matrix can have a strongly hierarchical structure if the $d^j_{u_R}$ dimensions associated to the various generations are different.

At low energy we can diagonalize the $c^{ij}_{u_R}$ matrix by rotating the O_F operators(*). To understand the result we need first of all a simple theorem. Let's consider a matrix with entries

$$M^{ij} \sim \zeta^i_L \zeta^j_R \qquad \text{with } O(1) \text{ coefficients not specified.}$$

we order the $\zeta_{L,R}$ so that $\zeta^i_L \leq \zeta^j_L$ for $i<j$ (analogously for ζ^i_R).
One gets that M can be diagonalized as

$$M^{ij} = U_L^{ik}\, m^{kl} (U_R^+)^{lj}$$

where m is diagonal with entries

$$m^{ii} \sim \zeta^i_L \zeta^i_R$$

and $U_{L/R}$ are unitary transformations with entries of order

$$U_L^{ij} \sim \begin{cases} \zeta^i_L/\zeta^j_L & \text{for } i<j \\ 1 & \text{for } i=j \\ \zeta^j_L/\zeta^i_L & \text{for } i>j \end{cases}$$

For hierarchical ζ^i_L or ζ^i_R the result is highly non-trivial and leads to a hierarchical structure for m and for $U_{L/R}$.

Applying the result to our Lagrangian we get

$$\underline{\mathcal{L}_{int} = \tilde{\lambda}^i\, \bar{u}^i_R\, O_F^{u_R i} + h.c}$$

where

$$\underline{\tilde{\lambda}^i \sim c\, \zeta^i.} \qquad \text{where } c \sim c^{ij}_{u_R} \ \forall i,j$$
since $c^{ij}_{u_R}$ is anarchic.

This tells us that the hierarchy in ζ^i translated directly in an hierarchy of mixings of the elementary fermions with the composite operators $O_F^{u_R i}$.

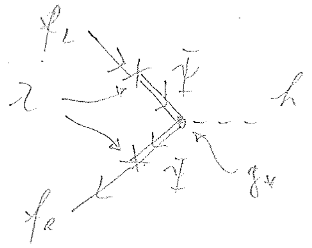
(*) Notice that this can not be done in general in the UV, since the theory is conformal and $d^i_{u_R}$ are conserved. In the IR the conformal invariance is instead broken.

Analogously we can obtain the mixing of q_L^i and d_R^i, so that

$$\mathcal{L}_{int} = \lambda_{q_L}^i \, \bar{q}_L^i \, O_F^{q_L^i} + \lambda_{u_R}^i \, \bar{u}_R^i \, O_F^{u_R^i} + \lambda_{d_R}^i \, \bar{d}_R^i \, O_F^{d_R^i} + h.c.$$

We can now see how the Yukawa's are generated.

As we discussed before, the Yukawa's come from diagrams like



thus they have the structure

$$y_u^{ij} = \frac{\lambda_{q_L}^i \lambda_{u_R}^j}{g_*} c^{ij} \qquad y_d^{ij} = \frac{\lambda_{q_L}^i \lambda_{d_R}^j}{g_*} c'^{ij},$$

with $c^{ij} \sim c'^{ij} \sim 1$ anarchic matrices.

We can now apply again the diagonalization theorem with

$$\lambda_{q_L}^1 \ll \lambda_{q_L}^2 \ll \lambda_{q_L}^3, \quad \lambda_{u_R, d_R}^1 \ll \lambda_{u_R, d_R}^2 \ll \lambda_{u_R, d_R}^3.$$

In this way we find

$$y_u = U_L \, y_u^D \, U_R^\dagger, \quad y_d = D_L \, y_d^D \, D_R^\dagger$$

where

$$y_u^{D\,ii} \sim \frac{\lambda_{q_L}^i \lambda_{u_R}^i}{g_*}, \quad y_d^{D\,ii} \sim \frac{\lambda_{q_L}^i \lambda_{d_R}^i}{g_*}.$$

Moreover the rotation matrices $U_{L,R}$ and $D_{L,R}$ are all hierarchical:

$$(U_L)_{ij} \sim \begin{cases} \lambda_{q_L}^i / \lambda_{q_L}^j & i<j \\ 1 & i=j \\ \lambda_{q_L}^j / \lambda_{q_L}^i & i>j \end{cases}.$$

The L rotations are related to the CKM matrix:

$$V_{CKM} = U_L^\dagger D_L,$$

therefore we find an estimate for the CKM elements:

$$V_{CKM}^{ij} \sim \lambda_{q_L}^i / \lambda_{q_L}^j \quad \text{for } i<j.$$

By the simple choice

$$\frac{\lambda^1_{qL}}{\lambda^2_{qL}} \sim \lambda_c \qquad \frac{\lambda^2_{qL}}{\lambda^3_{qL}} \sim \lambda_c^2$$

we can reproduce the correct form of the CKM matrix

$$V_{CKM} \sim \begin{pmatrix} 1-\lambda_c^2/2 & \lambda_c & \lambda_c^3 \\ \lambda_c & 1-\lambda_c^2/2 & \lambda_c^2 \\ \lambda_c^3 & \lambda_c^2 & 1 \end{pmatrix}, \quad \lambda_c \sim 0.2$$

The size of the other mixings $\lambda_{uR,dR}$ are fixed by the quark masses:

$$\frac{m_u}{m_c} \sim \frac{\lambda^1_{qL}}{\lambda^2_{qL}} \frac{\lambda^1_{uR}}{\lambda^2_{uR}} \sim \lambda_c \frac{\lambda^1_{uR}}{\lambda^2_{uR}} \qquad \frac{m_c}{m_t} \sim \frac{\lambda^2_{qL}}{\lambda^3_{qL}} \frac{\lambda^2_{uR}}{\lambda^3_{uR}} \sim \lambda_c^2 \frac{\lambda^2_{uR}}{\lambda^3_{uR}}$$

$$\frac{m_d}{m_s} \sim \frac{\lambda^1_{qL}}{\lambda^2_{qL}} \frac{\lambda^1_{dR}}{\lambda^2_{dR}} \sim \lambda_c \frac{\lambda^1_{dR}}{\lambda^2_{dR}} \qquad \frac{m_s}{m_b} \sim \frac{\lambda^2_{qL}}{\lambda^3_{qL}} \frac{\lambda^2_{dR}}{\lambda^3_{dR}} \sim \lambda_c^2 \frac{\lambda^2_{dR}}{\lambda^3_{dR}}.$$

In these way we fixed the size of almost all $\lambda_{qL,uR,dR}$ parameters. The only freedom is encoded into the ratio of L and R compositeness (mixing) for the top:

$$x_t \equiv \frac{\lambda^3_{qL}}{\lambda^3_{uR}}.$$

This ratio is related to the top mass

$$y_t \simeq g_* \frac{\lambda^3_{qL}}{g_*} \frac{\lambda^3_{uR}}{g_*}$$

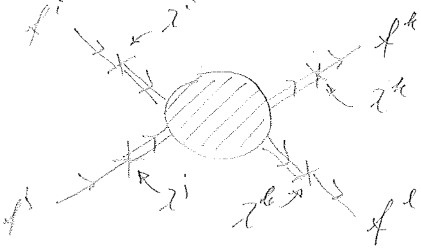
given that $\lambda^3_{qL}, \lambda^3_{uR} < g_*$ we find

$$y_t/g_* \lesssim x_t \lesssim g_*/y_t,$$

so x_t can only vary in a finite range.

Constraints

The anarchic structure implies a good suppression of flavor-changing effects. For instance let's consider the 4-fermion operators



$$\sim \frac{\lambda^i \lambda^j \lambda^k \lambda^l}{g_*^2 \Lambda_{cs}^2} f^i f^j f^k f^l$$

the prefactor is suppressed for the light fermions since their λ's are small. This tells us that new-physics effects involving the light quarks, which are the most strongly constrained ones, are efficiently suppressed.

There are a few classes of operators that give relevant bounds on Λ_{cs}.

- $\underline{\Delta F = 2}$ mediated by 4-fermion interactions

$$\mathcal{L}_{\Delta F=2} \sim \frac{\lambda_i \lambda_j \lambda_k \lambda_l}{g_*^2 \Lambda_{cs}^2} (\bar{f}_i \gamma^\mu f_j)(\bar{f}_k \gamma_\mu f_l)$$

  - $\underline{K\bar{K}}$ ds operators $\underline{\Lambda_{cs} \gtrsim 10 \text{ TeV}}$
  - $\underline{B_d, B_s}$ bd and bs operators $\underline{\Lambda_{cs} \gtrsim 8 x_t \text{ TeV}}$
  - $\underline{D}$ uc operators $\Lambda_{cs} \gtrsim 1.2\, x_t \text{ TeV}$

- $\underline{\Delta F = 1}$ mediated by

  typically loop generated

  - $\underline{\text{dipoles}}$ $\mathcal{L}_{\Delta F=1} \sim \left(\frac{g_*^2}{16\pi^2}\right) \frac{\lambda_i \lambda_j}{g_*} \frac{v}{\Lambda_{cs}^2} \bar{f}_i \sigma_{\mu\nu} g_{SM} F^{\mu\nu} f_j$

    - $b \to s\gamma$ transitions (EW dipoles) $\underline{f \gtrsim 0.5 \text{ TeV}}$
    - $K^0 \to 2\pi$, $\mathrm{Re}(\varepsilon'_K/\varepsilon_K)$ transitions (QCD dipoles) $\underline{f \gtrsim 1.2 \text{ TeV}}$

  - $\underline{Z \text{ and } W \text{ couplings modifications}}$

$$\mathcal{L}_{\Delta F=1} \sim \frac{\lambda_i \lambda_j}{\Lambda_{cs}^2} \bar{f}_i \gamma^\mu f_j \, i H^\dagger \overleftrightarrow{D}_\mu H$$

$$\sim \frac{\lambda_i \lambda_j}{\Lambda_{cs}^2} \bar{u}_R^i \gamma^\mu d_R^j \, H^{c\dagger} D_\mu H$$

- EDM's mediated by dipole operators

  - Neutron EDM receives contributions from d, u, c, b, t dipoles

$$\begin{cases} d \text{ quark} \to f \gtrsim 4.5 \text{ TeV} \\ u \text{ quark} \to f \gtrsim 2 \text{ TeV} \end{cases}$$

$$\begin{cases} c \text{ quark} \to f \gtrsim 1 \text{ TeV} \\ b \text{ quark} \to f \gtrsim 0.5 \text{ TeV} \\ t \text{ quark} \to f \gtrsim 0.5 \text{ TeV} \end{cases}$$

(1) THE $SO(3)_c$ SYMMETRY (i.e. the custodial symmetry)

Let's consider the Higgs sector of the SM "in isolation". The Higgs doublet

$$H = \begin{pmatrix} h_u \\ h_d \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \pi^2 + i\pi^1 \\ \pi^4 - i\pi^3 \end{pmatrix} \quad \in 2_{1/2} \text{ of } G_{EW}$$

can be equivalently described by 4 real components

$$H = \{\pi^1, \pi^2, \pi^3, \pi^4\}.$$

It is easy to see that the Higgs kinetic term and the Higgs potential are invariant under an $SO(4)$ rotation of the 4 real components π^i. Obviously the G_{EW} group should be a subgroup of $SO(4)$ and indeed this can be seen by rewriting the $SO(4)$ algebra in the equivalent form

$$SO(4) \simeq SU(2)_L \times SU(2)_R .$$

In this new notation the Higgs can be rewritten as a 2×2 matrix:

$$\Sigma = \left(H^c, H\right) = \frac{1}{\sqrt{2}}\left(i\sigma_\alpha \pi^\alpha + \mathbb{1}_{2\times 2}\, \pi^4\right), \qquad \text{where } H^c = i\sigma_2 H^* \quad (1.1)$$

under $SU(2)_L \times SU(2)_R$ the Higgs matrix transforms as

$$\Sigma \to g_L \Sigma g_R^\dagger .$$

We can now understand how the SM group is embedded in $SO(4)$. The $SU(2)_L^{SM}$ group is identified with $SU(2)_L \subset SO(4)$, while $U(1)_Y \subset SU(2)_R$, in particular $U(1)_Y$ is generated by the T^3_R generator of $SU(2)_R$.

When the Higgs gets a VEV, $\langle \pi^i \rangle = v\,\delta_{i4}$, the $SO(4)$ invariance is broken, but an $SO(3)$ subgroup survives. In fact

$$\langle \Sigma \rangle = \frac{v}{\sqrt{2}} \mathbb{1}_2 ,$$

so that the vector part of $SU(2)_L \times SU(2)_R$ is still unbroken. This is the custodial $SO(3)_c$.

Let's now discuss what happens with the G_{EW} gauging. The W's fully gauge $SU(2)_L$ and thus preserve $SO(4)$ provided we embed them into the $(3,1)$ representation of $SU(2)_L \times SU(2)_R$. The hypercharge gauging instead breaks $SO(4)$ and the custodial $SO(3)_c$.

The cancellation of the $\hat{T}$ parameter

$$\hat{T} = \frac{4}{v^2}\left(\Pi_{W^3W^3} - \Pi_{W^1W^1}\right)\Big|_{p^2=0}$$

immediately follows from the symmetry structure. The $\hat{T}$ parameter corresponds to a mass term for the W's, however the only term compatible with an unbroken $SO(3)_C$ is

$$\mathcal{L}_{mass} = \frac{g^2 v^2}{8} W^\alpha_\mu W^\mu_\alpha ,$$

which contributes in the same way to $\Pi_{W^3W^3}$ and $\Pi_{W^1W^1}$.

A non-vanishing $\hat{T}$ would correspond to a term

$$\frac{v^2}{8} S_{\alpha\beta} W^\alpha_\mu W^{\beta\mu}$$

where $S_{\alpha\beta}$ is a symmetric traceless tensor in the $(5,1)$ representation of $SU(2)_L \times SU(2)_R$.

Hypercharge gauging breaks the custodial invariance, however the effects of the breaking can only affect the W dynamics at loop level (since the B_μ field should not appear in the external legs). Thus at tree level $\hat{T}$ still vanishes. The mass terms get generalized at tree-level as

$$\mathcal{L}_{mass} = \frac{v^2}{8}\left((gW^1)^2 + (gW^2)^2 + (gW^3 - g'B)^2\right).$$

This term just coincides with the usual SM gauge mass term and respects the usual relation between the W and Z mass, namely

$$m_Z = \frac{1}{c_W} m_W \quad \Rightarrow \quad \rho = 1 .$$

Another source of breaking of the custodial invariance arises from the fermions, which do not fill complete $SO(4)$ representations (in particular the L multiplets). However also these effects contribute to $\hat{T}$ only at loop level.