The QCD Axion: Knowns and Unknowns

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arXiv:1511.02867

E.H.
arXiv:1609.00208

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Motivation

$$\mathcal{L} \supset \frac{\alpha_s}{8\pi}\theta_0 G\tilde{G}$$

cryoEDM

Experiment: $\theta = \theta_0 + \arg\,\det M_q \lesssim \mathcal{O}(10^{-10})$

Other phases in Yukawa matrices order 1

Non-decoupling contributions from new CP violating physics at arbitrarily high scales

Effects on large distance physics irrelevant for $\theta \lesssim 10^{-1} \div 10^{-2}$

Begs for a dynamical explanation!

Standard Model + extra pseudo-goldstone boson with coupling

$$\mathcal{L} \supset \frac{\alpha_s}{8\pi}\left(\theta + \frac{a}{f_a}\right) G\tilde{G}$$

Strong coupling $\longrightarrow$ Axion potential & axion VEV that removes θ

$\longrightarrow$ Axion mass $\mathcal{O}(m_\pi f_\pi / f_a)$

In a large part of parameter space axion can be dark matter

"Generic" feature of string compactifications (but expected decay constants?)

Small couplings mean discovery hard, but several ideas

Resonance effects: possible to measure its mass with relative accuracy

$$\delta m/m \sim 10^{-6}$$

Depending on experiment other couplings as well

Could we exploit such a high precision experiment?

What could we learn?

Possible to infer the UV completion of the axion and its cosmology?

Topics to study

- Precision physics at zero temperature
- Physics at finite temperature
- Cosmology and astrophysics

Lagrangian

UV Lagrangian:

$$\mathcal{L}_a = \frac{1}{2}(\partial a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu} + \frac{1}{4}\, a\, g^0_{a\gamma\gamma} F_{\mu\nu}\tilde{F}^{\mu\nu} + j^\mu_{a,0}\frac{\partial_\mu a}{2 f_a}$$

where $g^0_{a\gamma\gamma} = \dfrac{\alpha_{em}}{2\pi f_a}\dfrac{E}{N}$ Anomalous EM coupling

$j^\mu_{a,0} = c^0_q \bar{q}\gamma^\mu\gamma_5 q$ Model dependent axial current

Rotation

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \to e^{i\gamma_5 \frac{a}{2f_a} Q_a} \begin{pmatrix} u \\ d \end{pmatrix}, \qquad \mathrm{tr} Q_a = 1$$

Gives

$$\mathcal{L}_a = \frac{1}{2}(\partial a)^2 + \frac{1}{4} a\, g_{a\gamma\gamma} F_{\mu\nu} \tilde{F}^{\mu\nu} + j_a^\mu \frac{\partial_\mu a}{2 f_a} - \bar{q}_L M_a q_R + h.c.$$

Where

$$M_a = e^{i \frac{a}{2f_a} Q_a} M_q\, e^{i \frac{a}{2f_a} Q_a}, \quad M_q = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix},$$

All non-derivative couplings to QCD in quark mass matrix

Coupling to axial current only multiplicatively renormalised

Chiral Perturbation Theory

Axion: external source, non-derivative couplings via dressed mass matrix, derivative coupling an external axial vector current

Low energy correlators from chiral perturbation theory [Georgi, Kaplan, Randall]

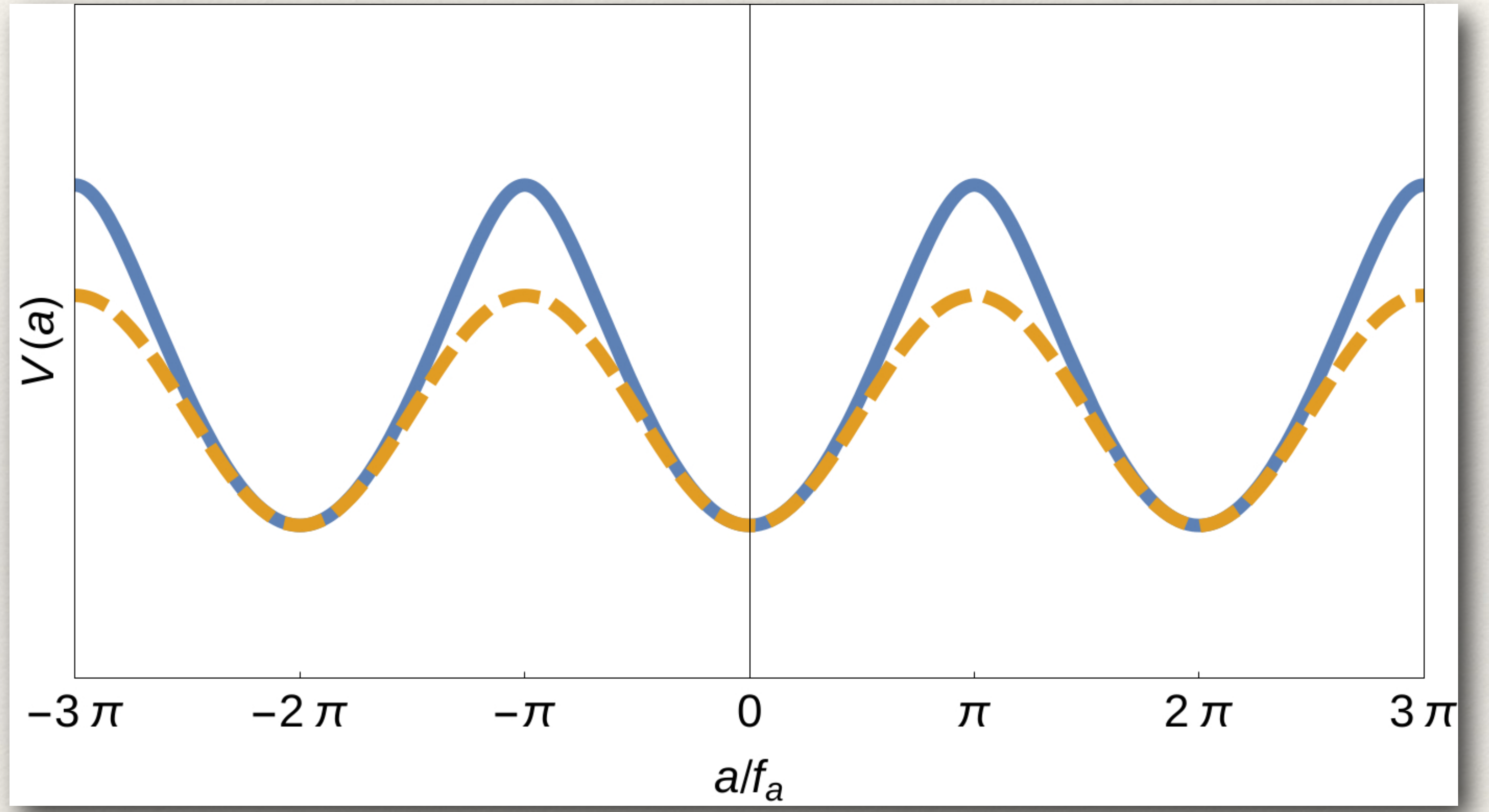
$$\mathcal{L}_{p^2} \supset 2B_0 \frac{f_\pi^2}{4} \langle U M_a^\dagger + M_a U^\dagger \rangle$$

where $U = e^{i\Pi/f_\pi}\,, \quad \Pi = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$

Free to shift axion into the first two generations only

Potential

$$V(a,\pi^0) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4m_u m_d}{(m_u+m_d)^2}\sin^2\left(\frac{a}{2f_a}\right)}$$



Not a cosine

$$m_a^2 = \frac{m_u m_d}{(m_u+m_d)^2}\frac{m_\pi^2 f_\pi^2}{f_a^2}$$

Mass at NLO

From CPT, loops and higher terms, e.g.

$$\mathcal{L}_{p^4} \supset \frac{l_7}{8} \left\langle \left(D^\mu U + D^\mu U^\dagger \right) 2B_0 \left(M_a + M_a^\dagger \right) \right\rangle$$

$$m_a^2 = \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f_\pi^2}{f_a^2} \left[1 + 2\frac{m_\pi^2}{f_\pi^2} \left(h_1^r - h_3^r - l_4^r + \frac{m_u^2 - 6 m_u m_d + m_d^2}{(m_u + m_d)^2} l_7^r \right) \right]$$

Turns out no logs at this order

Also true in 3-flavour

Determining low energy constants

$$l_7^r = \frac{m_u + m_d}{m_s} \frac{f_\pi^2}{8m_\pi^2} - 36L_7 - 12L_8^r + \frac{\log(m_\eta^2/\mu^2)+1}{64\pi^2} + \frac{3\log(m_K^2/\mu^2)}{128\pi^2} = 7(4)\cdot 10^{-3},$$

$$h_1^r - h_3^r - l_4^r = -8L_8^r + \frac{\log(m_\eta^2/\mu^2)}{96\pi^2} + \frac{\log(m_K^2/\mu^2)+1}{64\pi^2} = (4.8 \pm 1.4) \times 10^{-3}$$

$$z \equiv \frac{m_u^{\overline{\rm MS}}(2 \text{ GeV})}{m_d^{\overline{\rm MS}}(2 \text{ GeV})} = 0.48(3)$$

$$m_a = 5.70(6)(4)\ \mu\text{eV} \left(\frac{10^{12}\text{GeV}}{f_a}\right) = 5.70(7)\ \mu\text{eV} \left(\frac{10^{12}\text{GeV}}{f_a}\right)$$

First error from z, second from low energy constants

Potential

$$V(a)^{\rm NLO} = -m_\pi^2(\tfrac{a}{f_a}) f_\pi^2 \left\{1 - 2\frac{m_\pi^2}{f_\pi^2}\left[l_3^r + l_4^r - \frac{(m_d-m_u)^2}{(m_d+m_u)^2} l_7^r - \frac{3}{64\pi^2}\log\left(\frac{m_\pi^2}{\mu^2}\right)\right]\right.$$

$$\left. + \frac{m_\pi^2(\tfrac{a}{f_a})}{f_\pi^2}\left[h_1^r - h_3^r + l_3^r + \frac{4m_u^2 m_d^2}{(m_u+m_d)^4}\frac{m_\pi^8 \sin^2(\tfrac{a}{f_a})}{m_\pi^8(\tfrac{a}{f_a})} l_7^r - \frac{3}{64\pi^2}\left(\log\left(\frac{m_\pi^2(\tfrac{a}{f_a})}{\mu^2}\right) - \frac{1}{2}\right)\right]\right\}$$

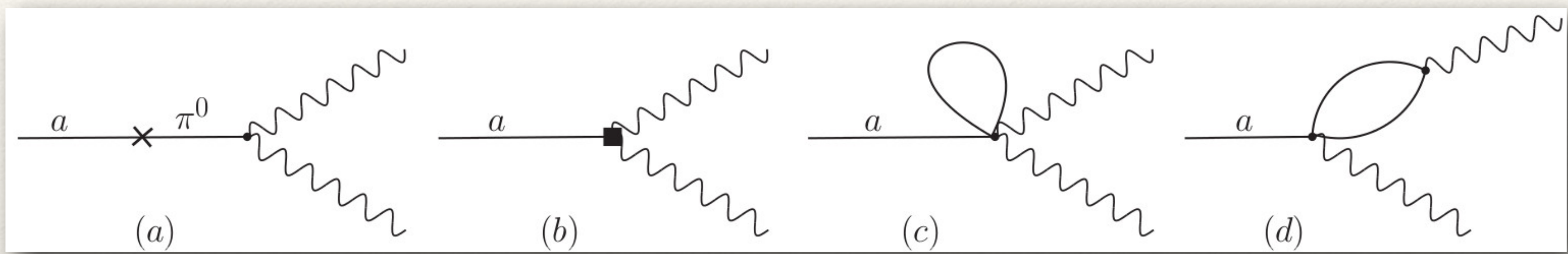
Similar calculation: 1-loop + NLO Lagrangian + rewriting physical constants

Easy to extract domain wall tension to sub-% level

$$\sigma = 2 f_a \int_0^\pi d\theta\, \sqrt{2[V(\theta) - V(0)]}$$

$$= 8.97(5)\, m_a f_a^2$$

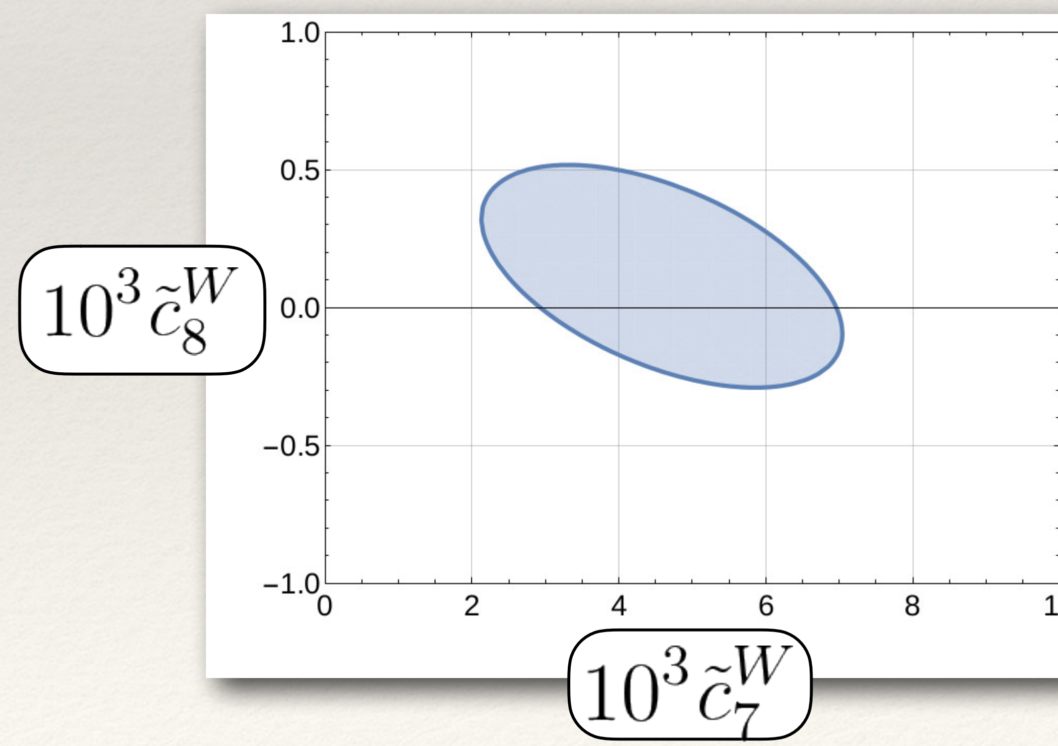
Coupling to Photons

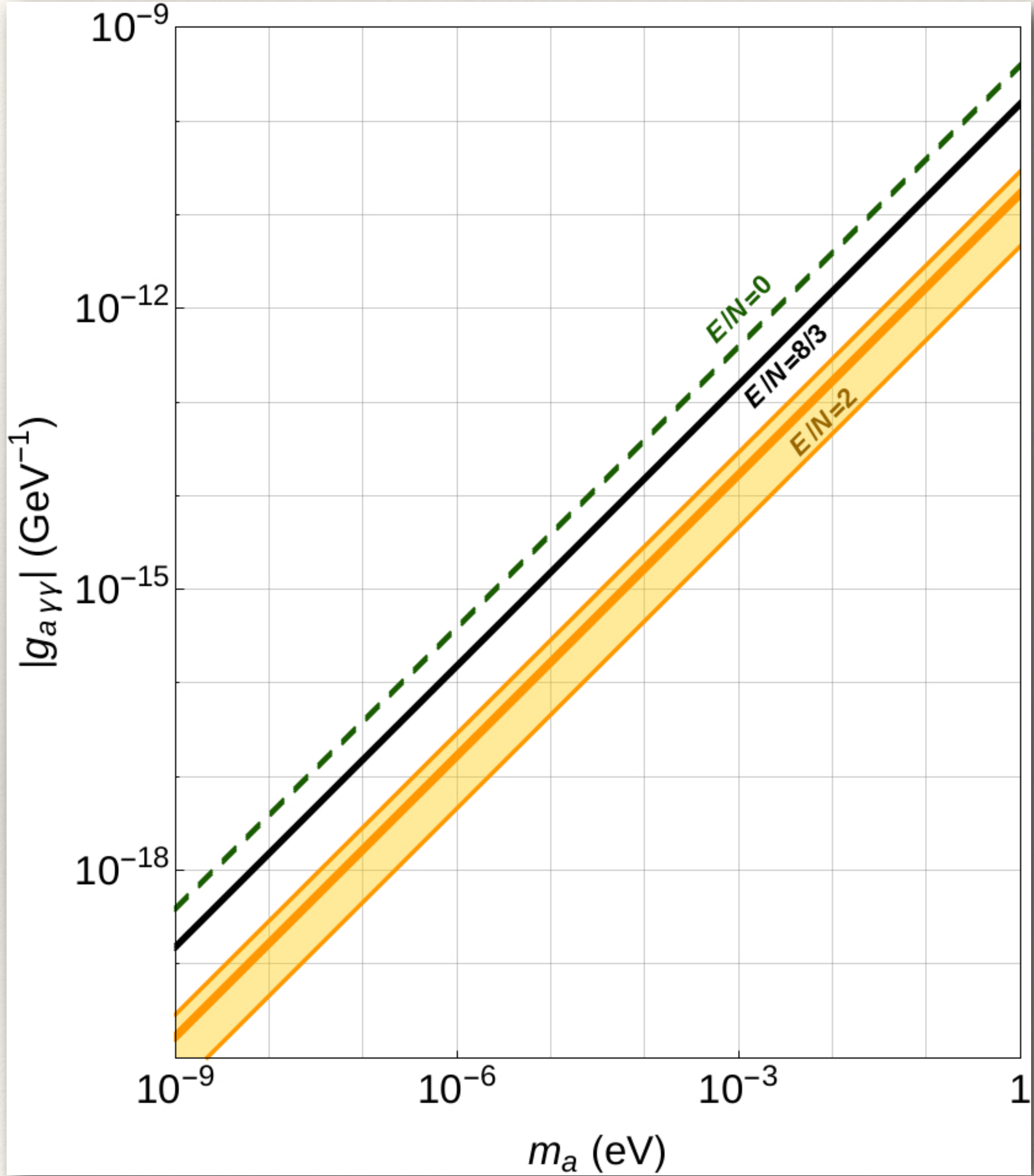


$$g_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a}\left\{\frac{E}{N} - \frac{2}{3}\frac{4m_d + m_u}{m_d + m_u} + \frac{m_\pi^2}{f_\pi^2}\frac{8m_u m_d}{(m_u+m_d)^2}\left[\frac{8}{9}\left(5\tilde{c}_3^W + \tilde{c}_7^W + 2\tilde{c}_8^W\right) - \frac{m_d - m_u}{m_d + m_u} l_7^r\right]\right\}$$

New low energy constants to determine

Harder to get these precisely





$$g_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a}\left[\frac{E}{N} - 1.92(4)\right]$$

Possible to get near cancellation for particular UV models

Couplings to Matter

Safest thing to do is use an effective theory well below QCD mass gap,

Non-relativistic nucleons $$\frac{\partial_\mu a}{2f_a} c_N \bar{N} \gamma^\mu \gamma_5 N$$

Need matrix elements $s^\mu \Delta q \equiv \langle p | \bar{q} \gamma^\mu \gamma_5 q | p \rangle_Q$: use neutron decay and lattice

$$c_p^{\mathrm{KSVZ}} = -0.48(3)\,,$$
$$c_n^{\mathrm{KSVZ}} = 0.03(3)\,,$$

$$c_p^{\mathrm{DFSZ}} = -0.622 + 0.434 \sin^2\beta \pm 0.024\,,$$
$$c_n^{\mathrm{DFSZ}} = 0.249 - 0.415 \sin^2\beta \pm 0.024\,.$$

Finite Temperature

As T increases QCD gets weaker, and axion mass decreases

Eventually the precision of calculations will improve,

Important for axion relic abundance (also interesting in its own right)

Low Temperature

Compute temperature dependence with CPT

But one loop correction is only from local NLO couplings, so

$$\frac{m_a^2(T)}{m_a^2} = \frac{\chi_{top}(T)}{\chi_{top}} \overset{\text{NLO}}{=} \frac{m_\pi^2(T) f_\pi^2(T)}{m_\pi^2 f_\pi^2} = \frac{\langle \bar{q} q \rangle_T}{\langle \bar{q} q \rangle}$$

$$= 1 - \frac{3}{2} \frac{T^2}{f_\pi^2} J_1\left[\frac{m_\pi^2}{T^2}\right] , \qquad J_1[\xi] = \frac{1}{\pi^2} \frac{\partial}{\partial \xi} \int_0^\infty dq\, q^2 \log\left(1 - e^{-\sqrt{q^2+\xi}}\right) .$$

Effects of heavy states suppressed by $e^{m/T}$

Ratio m/T_c not huge, and many new states appear, so breaks down at crossover

High Temperatures

At high enough temperatures, instanton calculation (Gross, Pisarski, Yaffe)

$$f_a^2 m_a^2(T) \simeq 2 \int d\rho\, n(\rho, 0) e^{-\frac{2\pi^2}{g_s^2} m_{D1}^2 \rho^2 + \ldots}$$

Where $n(\rho, 0) \propto m_u m_d e^{-8\pi^2/g_s^2}$ is zero temperature instanton density

Integral is over instanton size ρ

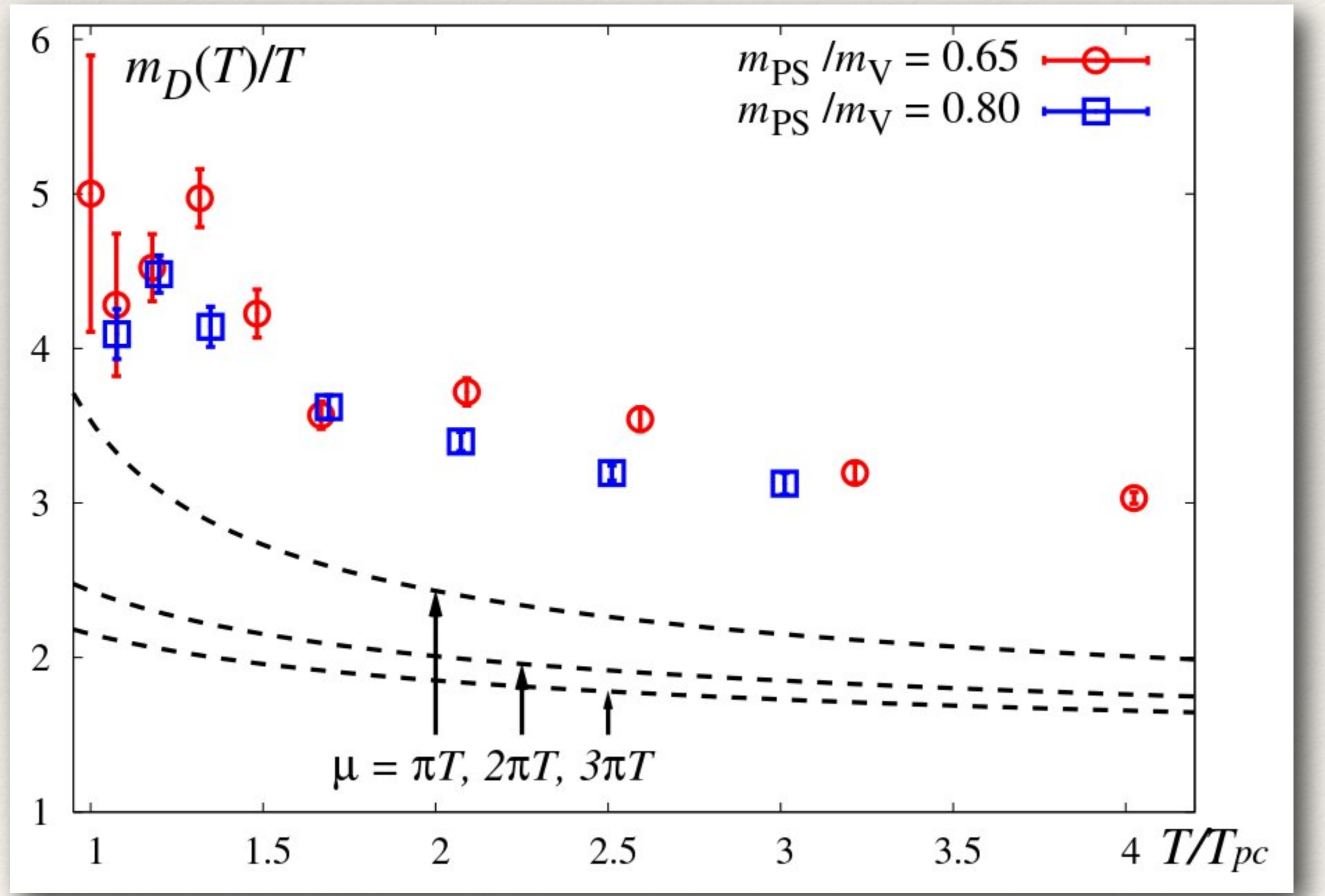
Cut off by screening from leading order Debye mass

$$m_{D1}^2 = g_s^2 T^2 (1 + n_f/6)$$

Finite T QCD convergence…

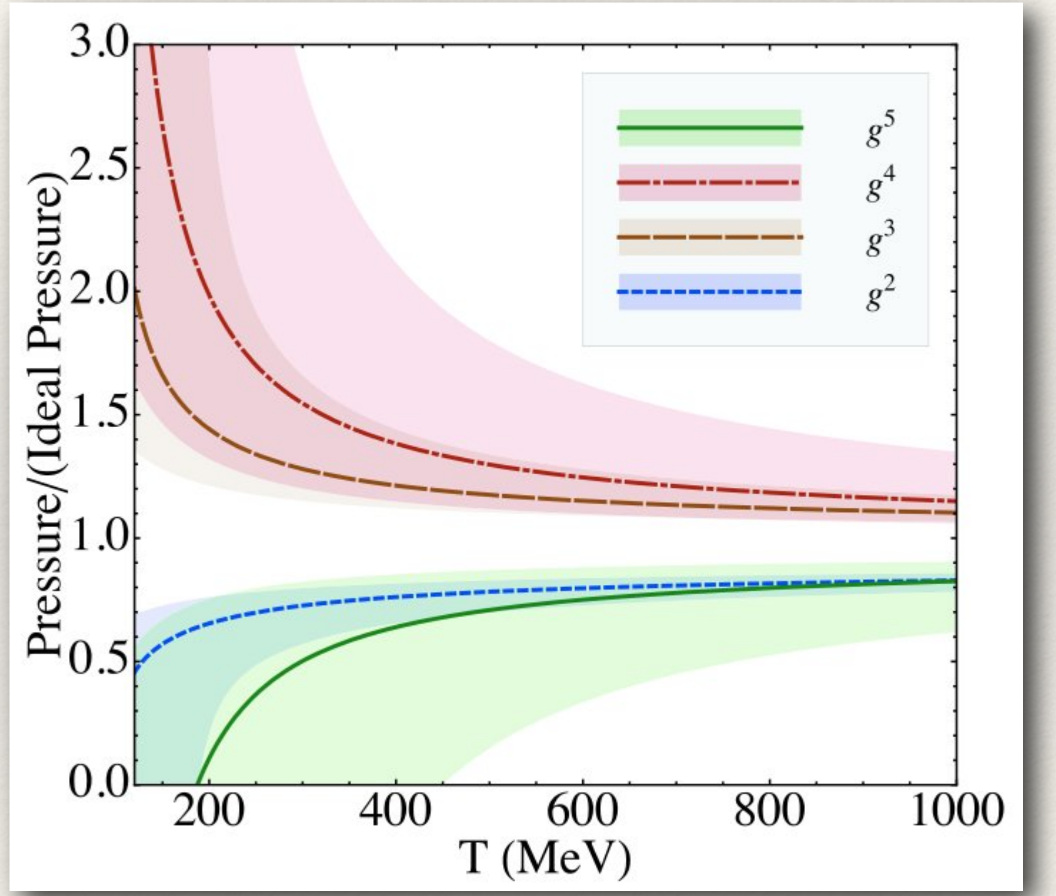
E.g. $$m_D = m_{D0} + \frac{N}{4\pi} g^2 T \log \frac{m_{D0}}{g^2 T} + c_N g^2 T + \mathcal{O}\left(g^3 T\right)$$

~gT



WHOT-QCD Collaboration

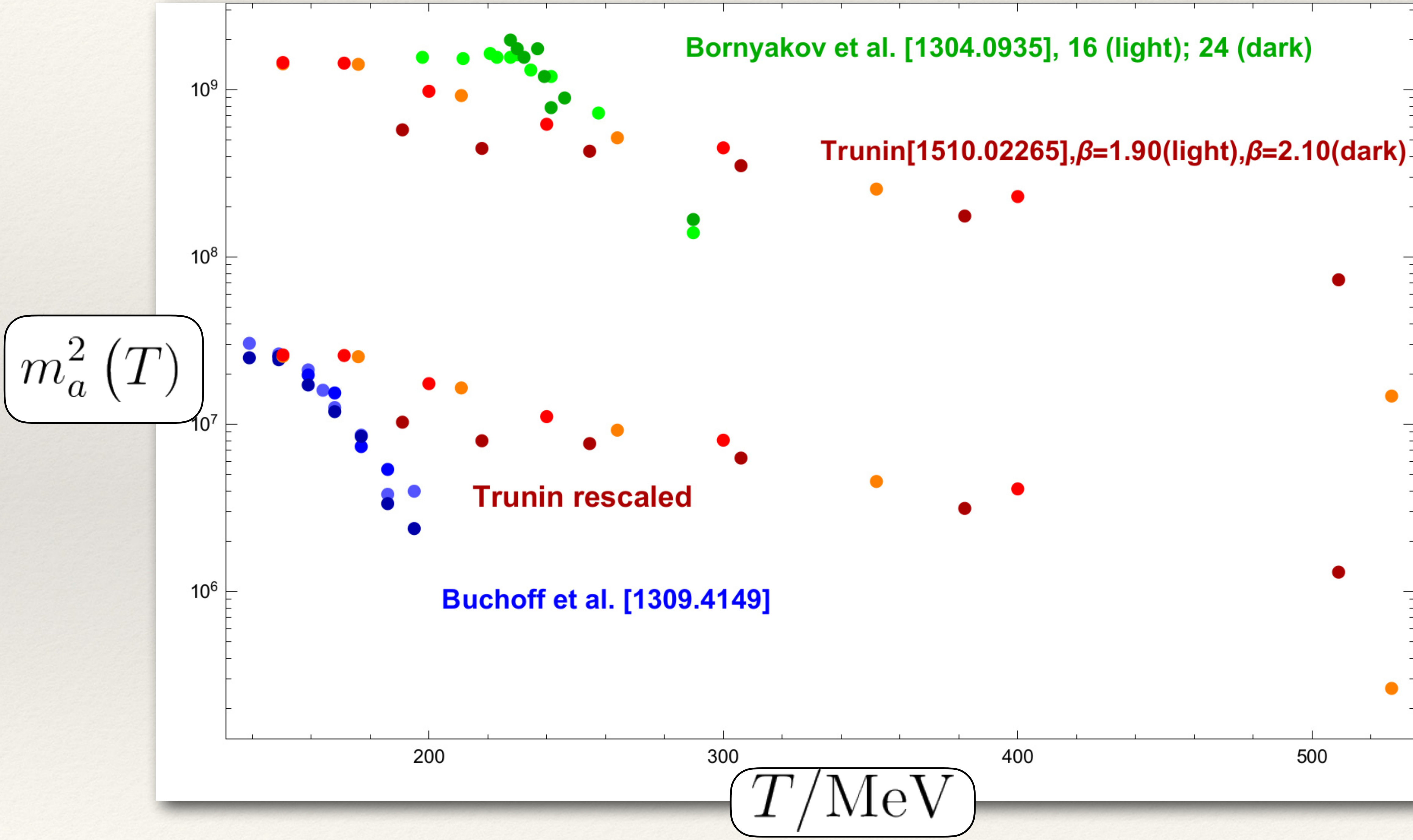
Debye mass



Andersen et al 1103.2528

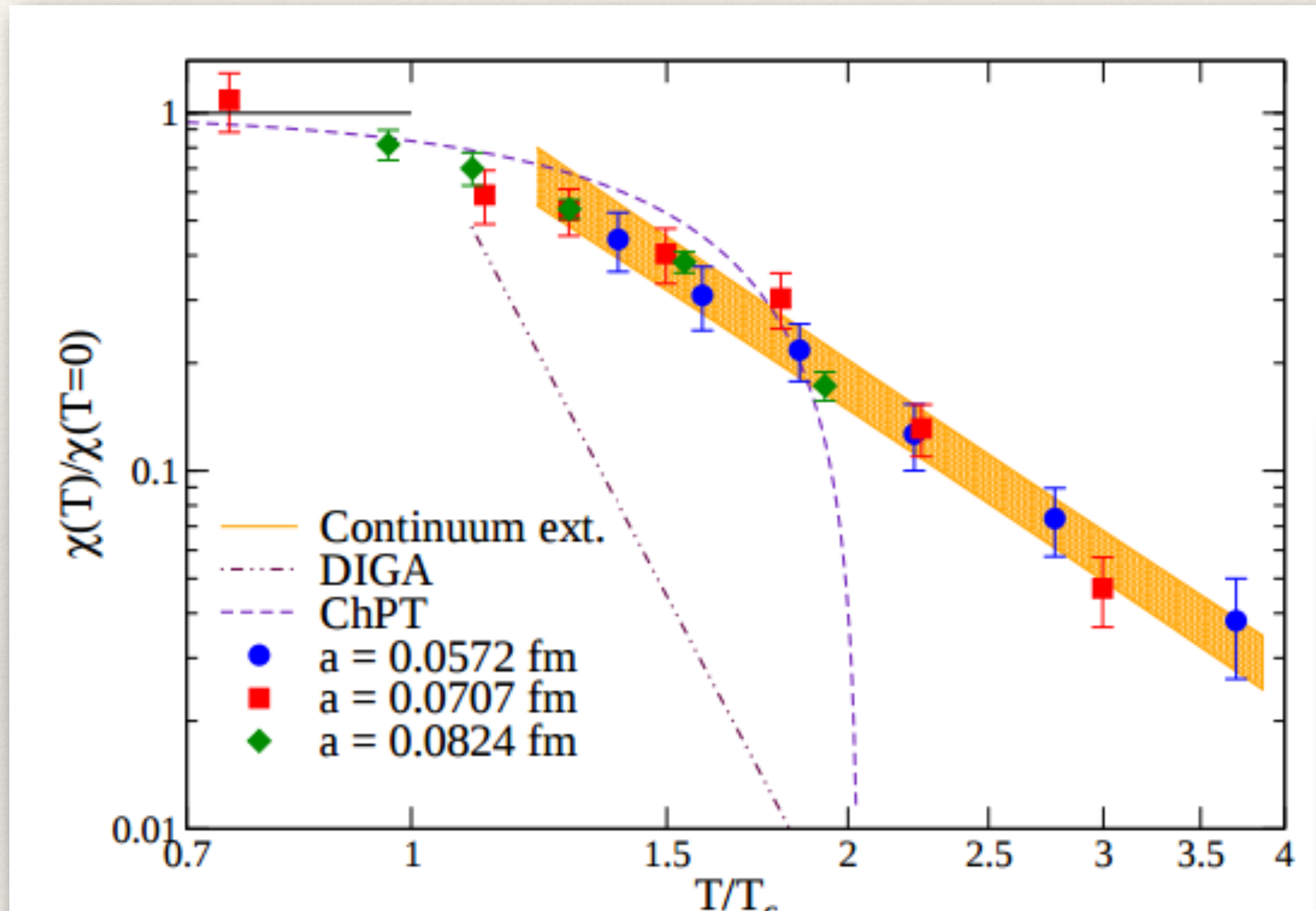
Pressure

Lattice

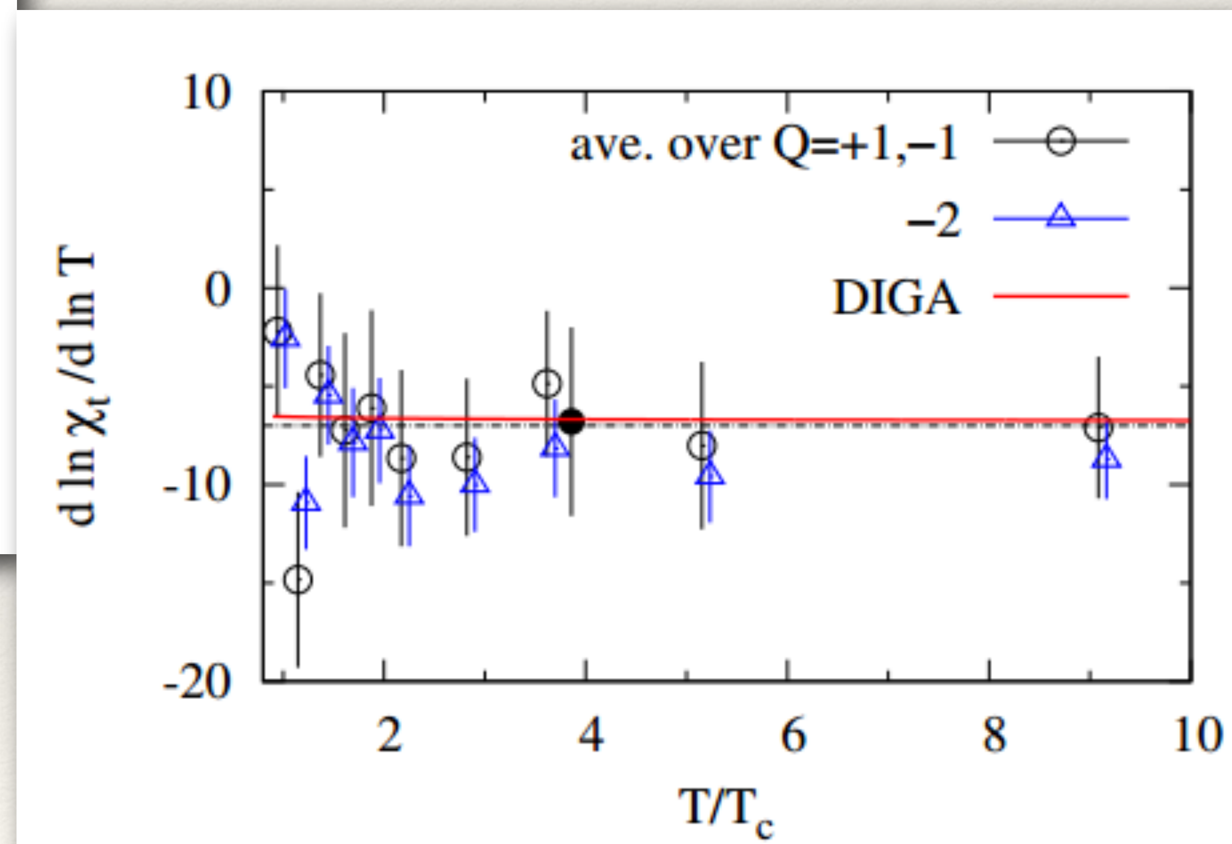


Older data: totally unclear

More recent data



Bonati et al. '15



Frison et al '16

Insight from gauge configurations?

Future work:

Can we understand the physics by looking at the gauge configurations?

Instantons with a different size than predicted, or something else entirely?

Some preliminary attempts in Bornyakov et al. 1304.0935

Axion Dark Matter

Evolution

$$\ddot{a} + 3H\dot{a} + m_a^2(T)\, f_a \sin\left(\frac{a}{f_a}\right) = 0\,.$$

At high temperatures axion field fixed:

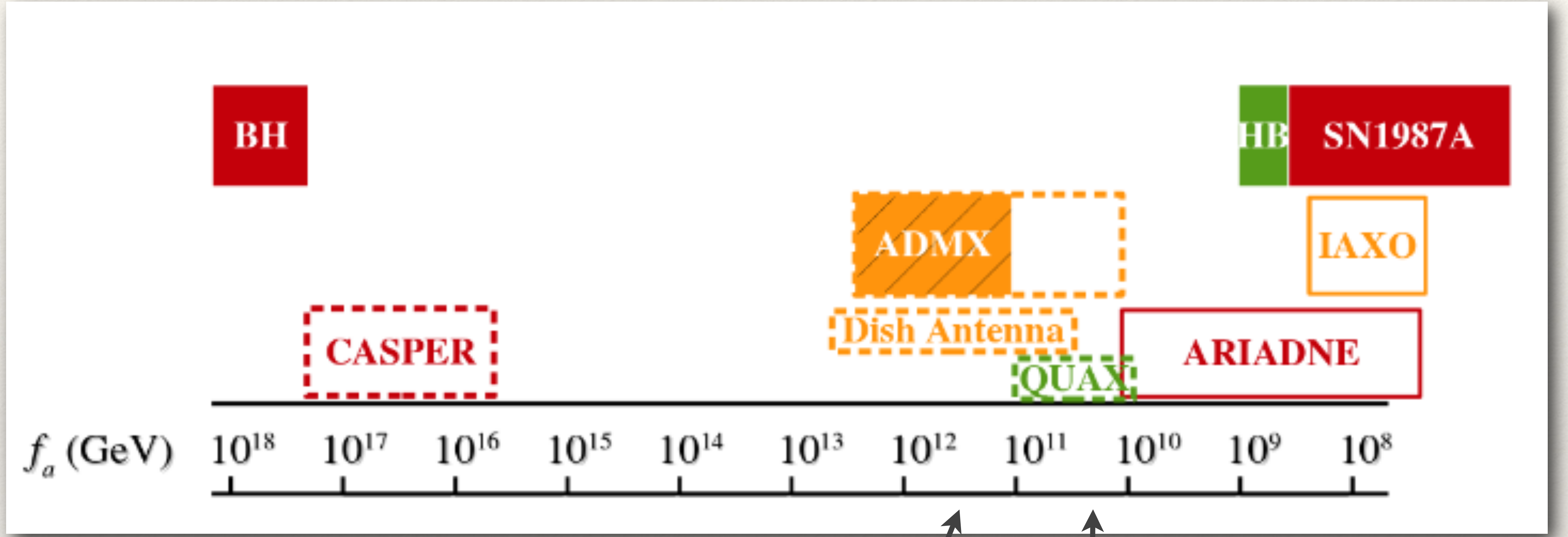
- At constant value over observable universe if PQ breaking before inflation
- Randomly in $a \in f_a\,[0, 2\pi]$ if PQ broken after reheating

Axion starts oscillating when $m_a(T) \sim 3H$

Quickly reaches solution where comoving number density is an adiabatic invariant

Coherent oscillations of the axion act as cold dark matter

Misalignment

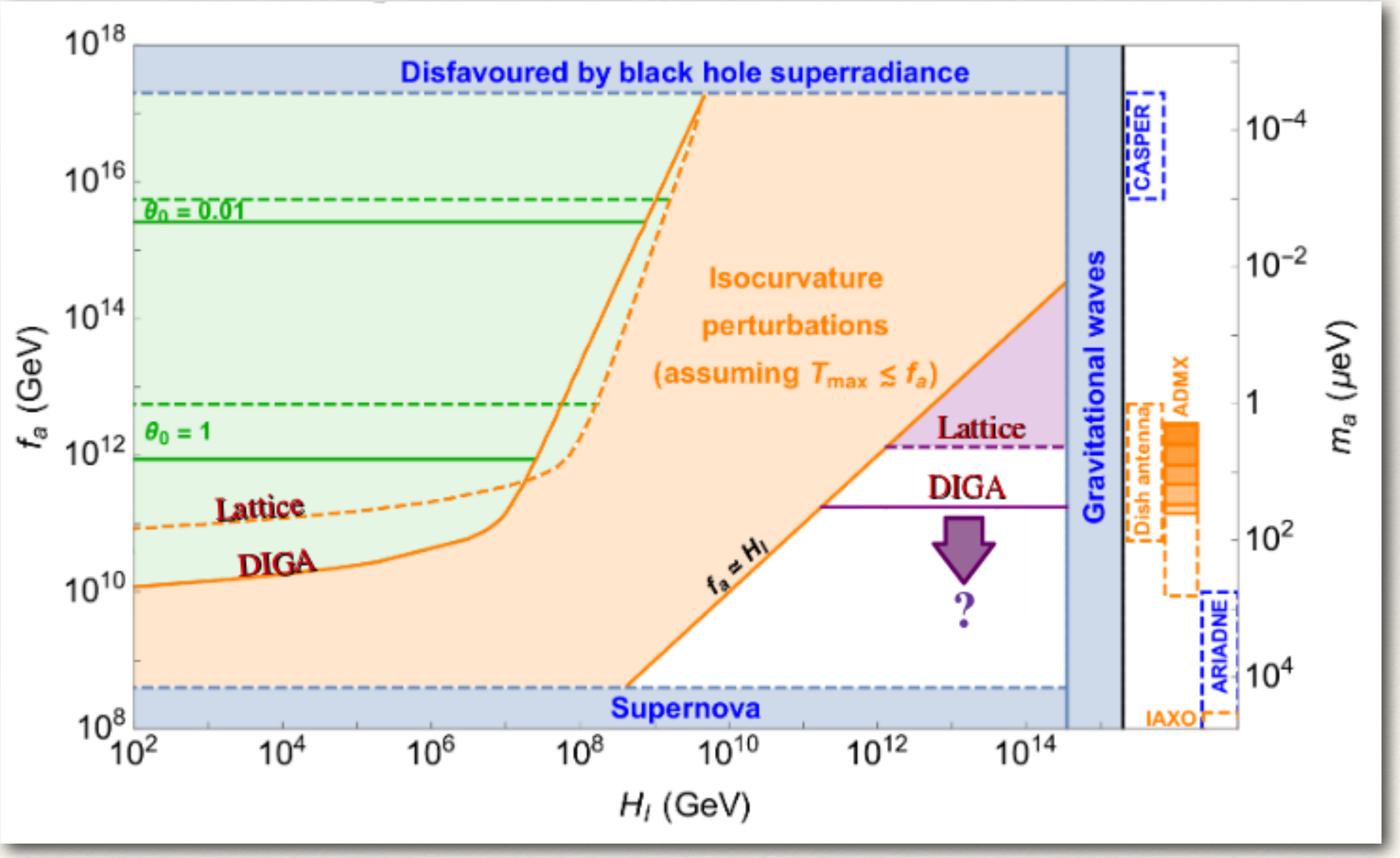


PQ breaking after inflation:

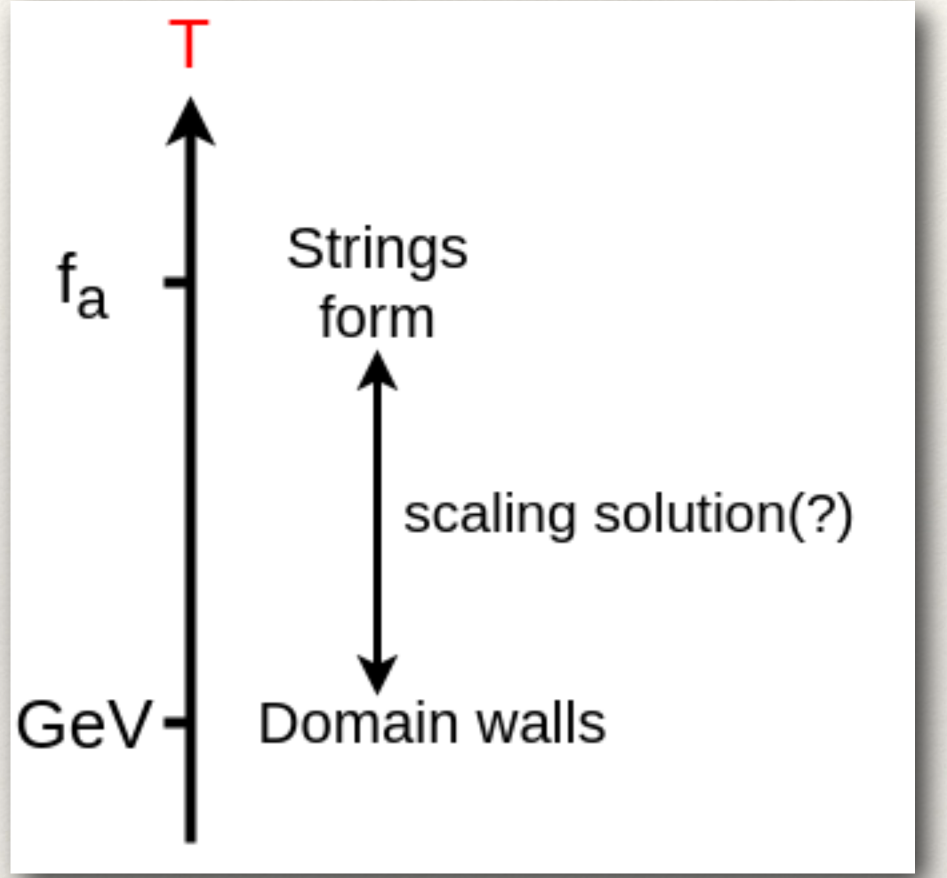
Lattice

Instanton

Cosmological Parameter Space



Strings and Domain walls



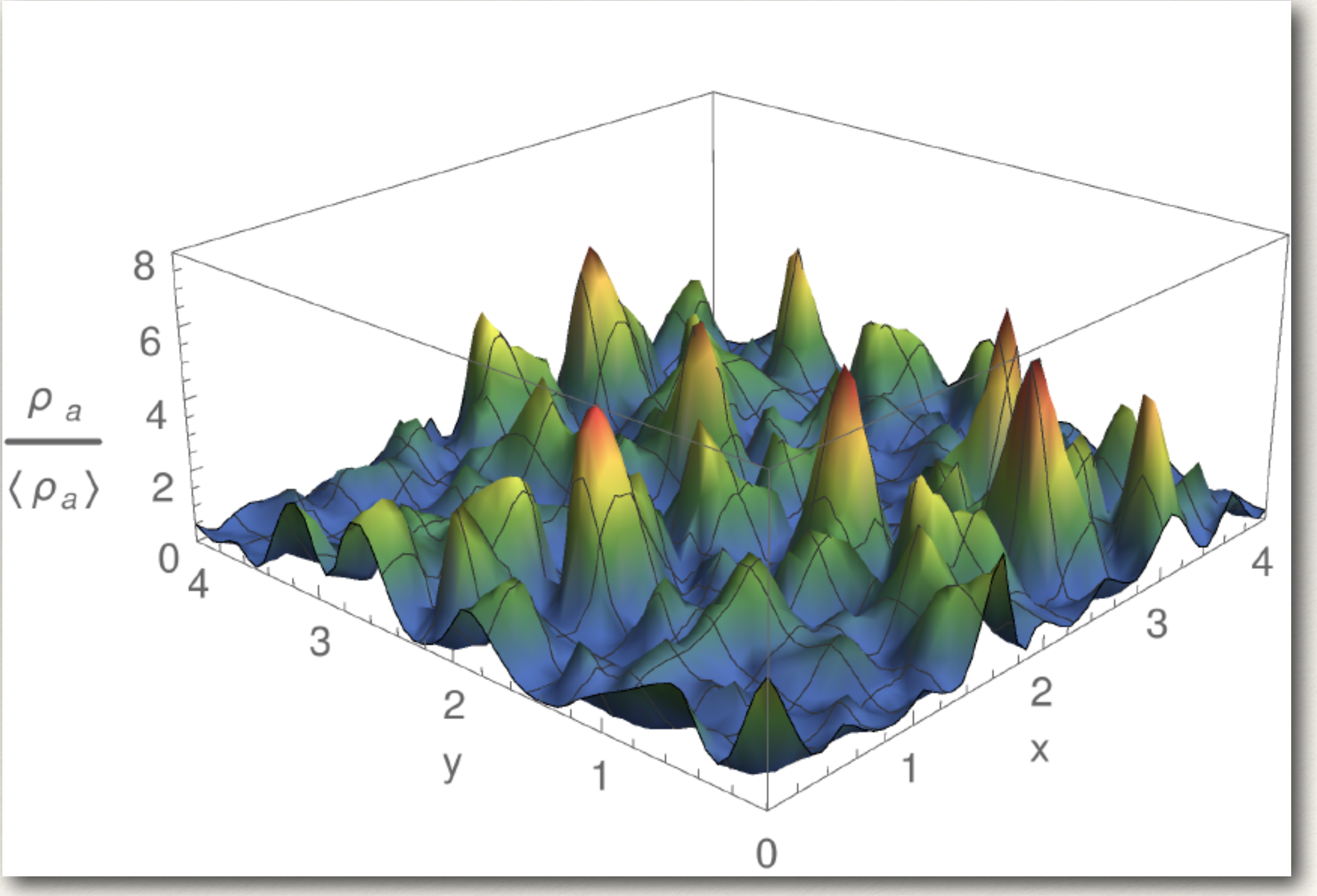
$$T = \frac{E}{L} \simeq \pi \int_{\sim 1/m_s}^{\ell} r dr \, \frac{f_a^2}{r^2} = \pi f_a^2 \ln(m_s \ell)$$

e.g. Shellard, Moore, etc.

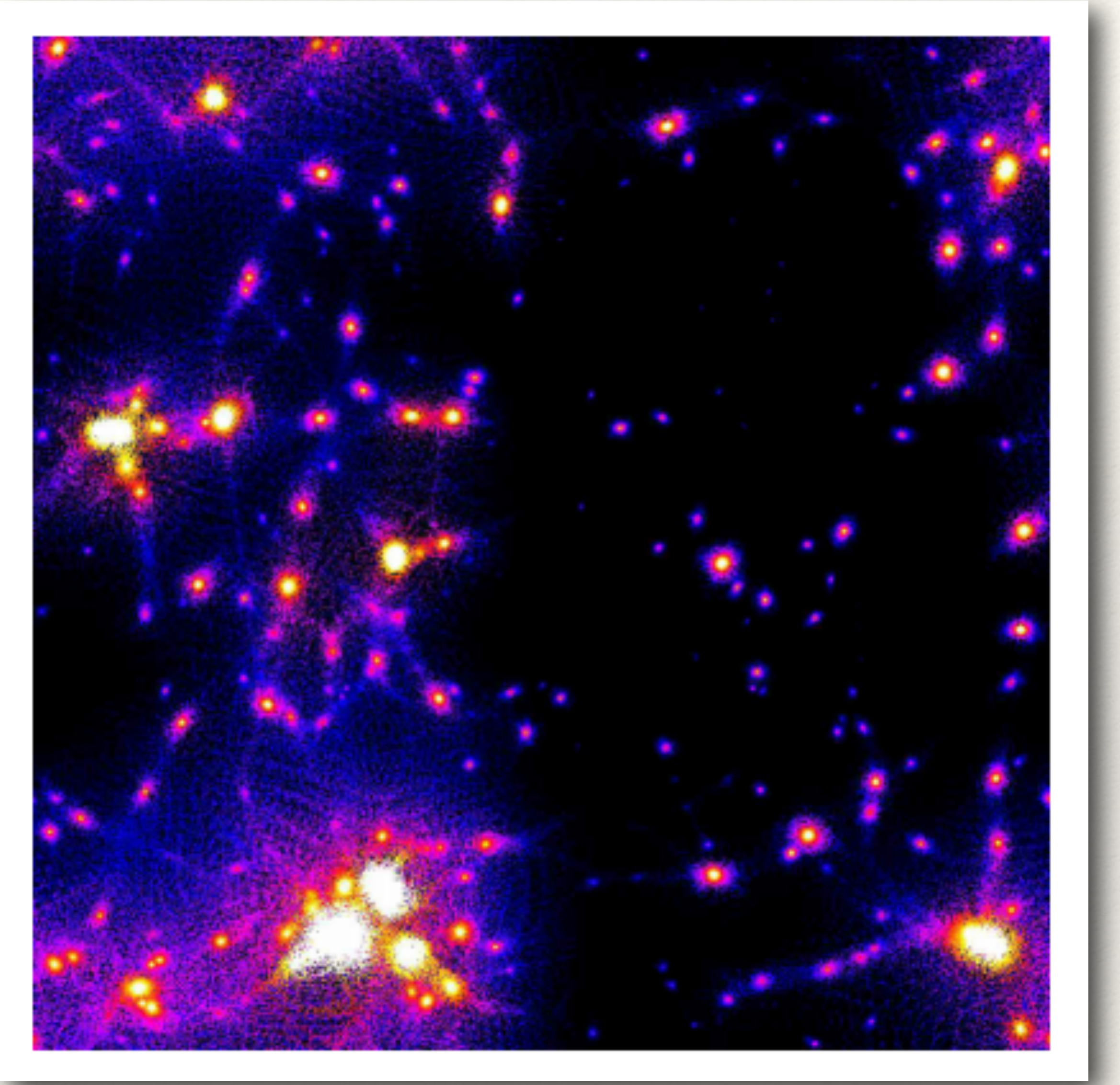
Current best numerics: Kawasaki et al: 1412.0789, all at one scale

Miniclusters

Don't yet know the effect of strings and domain walls, but can do something at least



Late time evolution



$$\rho \simeq 140\Phi^3 (\Phi + 1)\, \rho_c$$

$$\Phi = \delta\rho/\rho \simeq 6$$

$$M_m \simeq 3 \times 10^{-11} M_\odot \left(\frac{\mathrm{GeV}}{T_0}\right)^3$$

$$R_m \simeq \frac{10^{11}\,\mathrm{m}}{\Phi\,(1+\Phi)^{1/3}} \left(\frac{\mathrm{GeV}}{T_0}\right)$$

Zurek et al
astro-ph/0607341

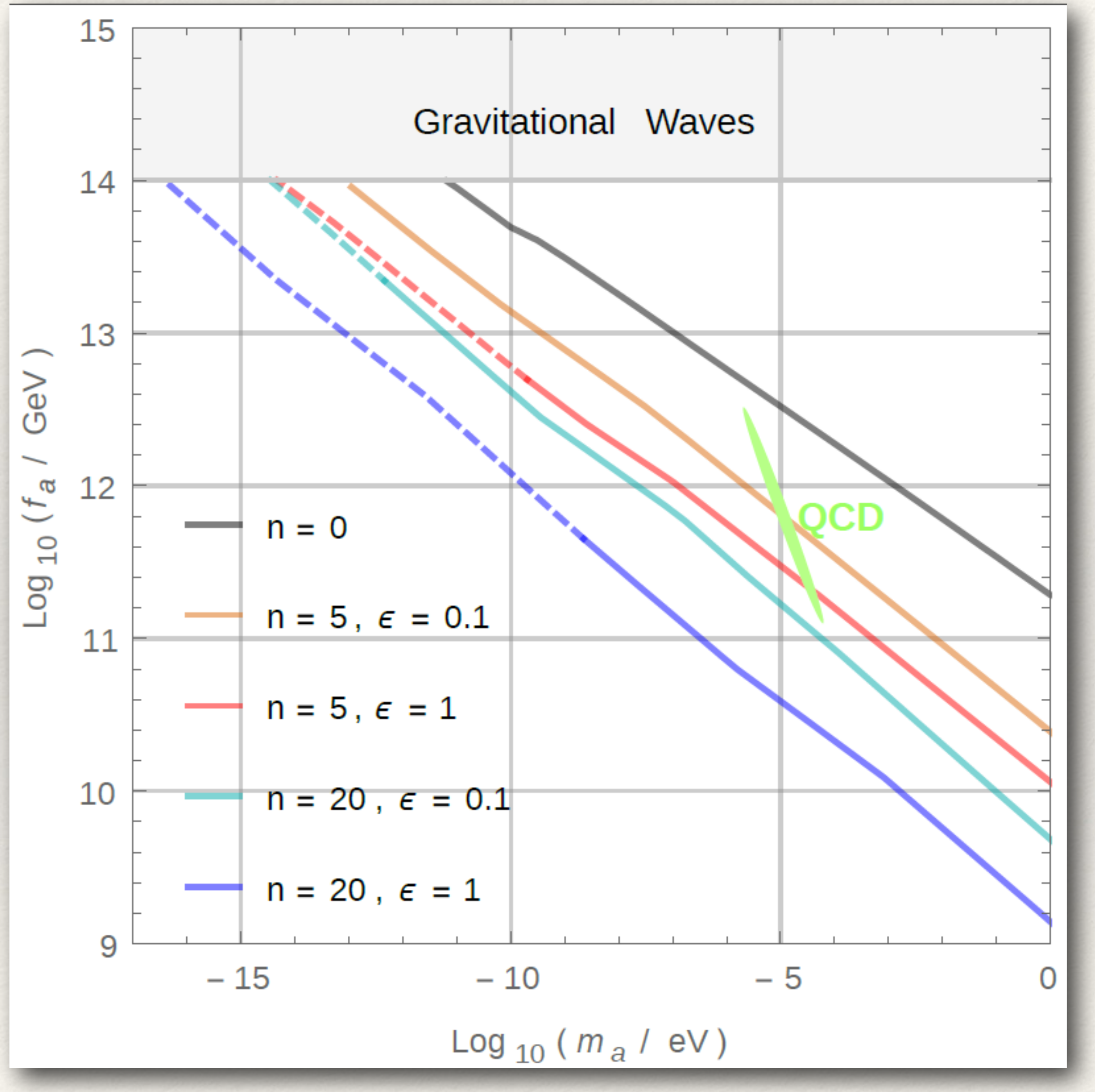
can further contraction to bose stars occur?

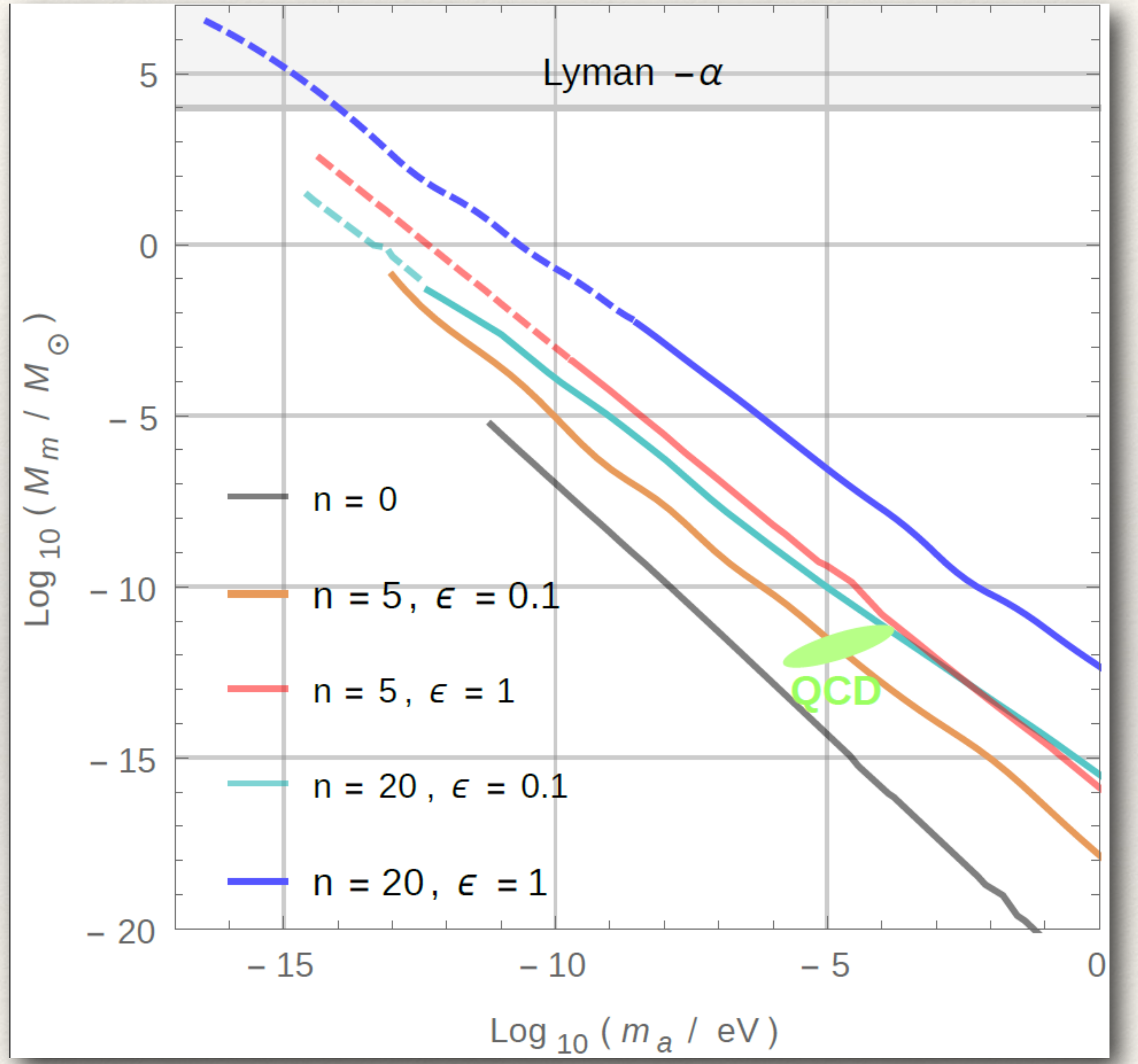
Summary

- Zero and low temperature properties to % level accuracy
- Finite temperature physics poorly known (future lattice studies can help)
- Significant effect on dark matter and cosmology
- Effect on string and domain wall contributions to axion relic density?

  Work in progress, but hard
- Relatively dense objects often form, but the late time dynamics are uncertain

Allowed ALP parameter space?



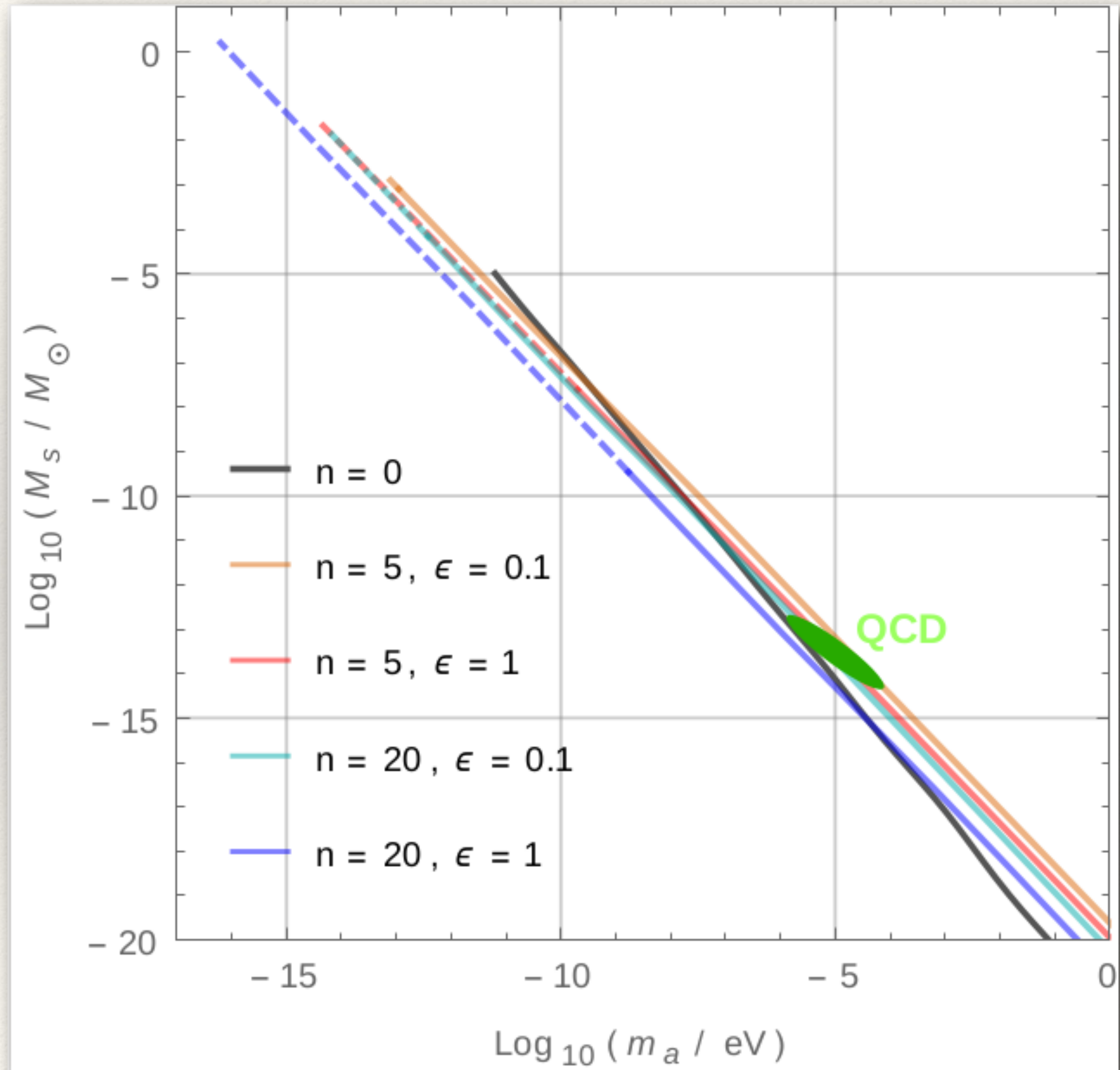


$$m_a = \frac{\epsilon^2 \Lambda^2}{f_a}$$

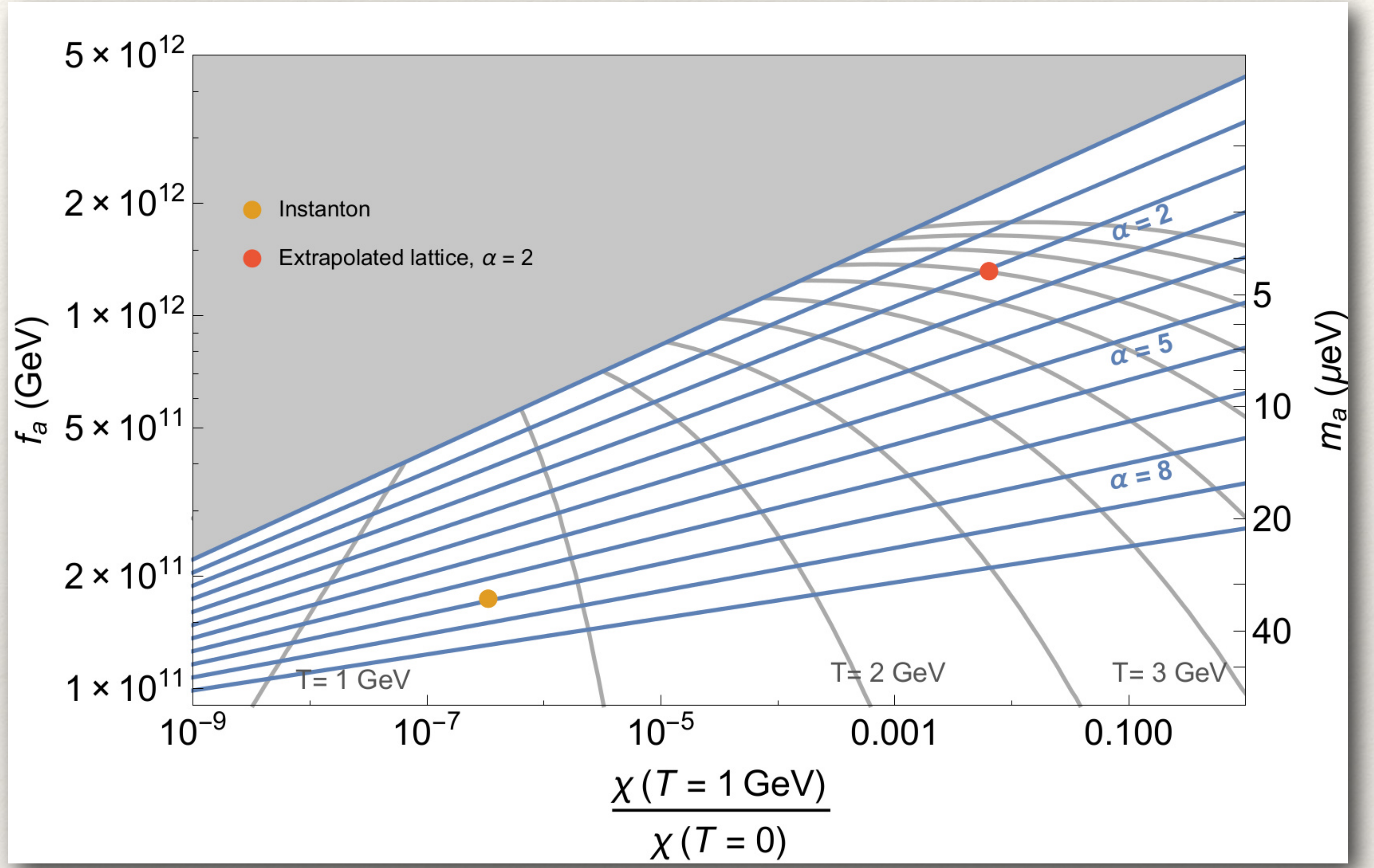
$$m_a^2(T) = m_a^2 \left(\frac{\Lambda}{T}\right)^n$$

ALP stars

- Maximum star size in ALP models:
- Typically light



Uncertainty on required f_a



Starts oscillating near 1 GeV, value of power not important

If axion mass has dropped a large amount already at 1 GeV, high precision

Oscillates at much higher temperature

Behaviour at T above 1 GeV (i.e. power in approximation) very important