Limit on Focusing of Electron Beams

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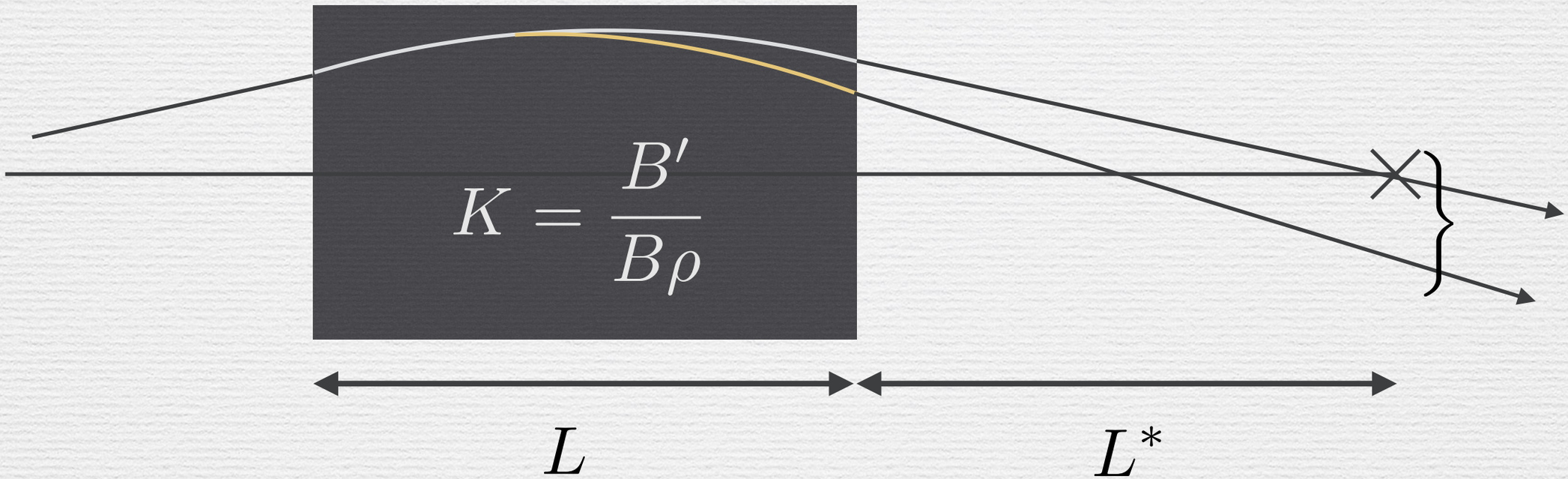

$$\sigma_y^{*2} = \beta_y^* \varepsilon_y + \frac{110}{3\sqrt{6\pi}} r_e \lambda_e \left(\frac{\gamma^2 \varepsilon_y}{\beta_y^*} \right)^{5/2} F\left(\sqrt{K}L, \sqrt{K}L^*\right)$$

$$F\left(\sqrt{K}L, \sqrt{K}L^*\right) \equiv \int_0^{\sqrt{K}L} d\phi \left| \sin\phi + \sqrt{K}L^* \cos\phi \right|^3 \times \left[\int_0^{\phi} d\phi' \left(\sin\phi' + \sqrt{K}L^* \cos\phi' \right)^2 \right]^2$$

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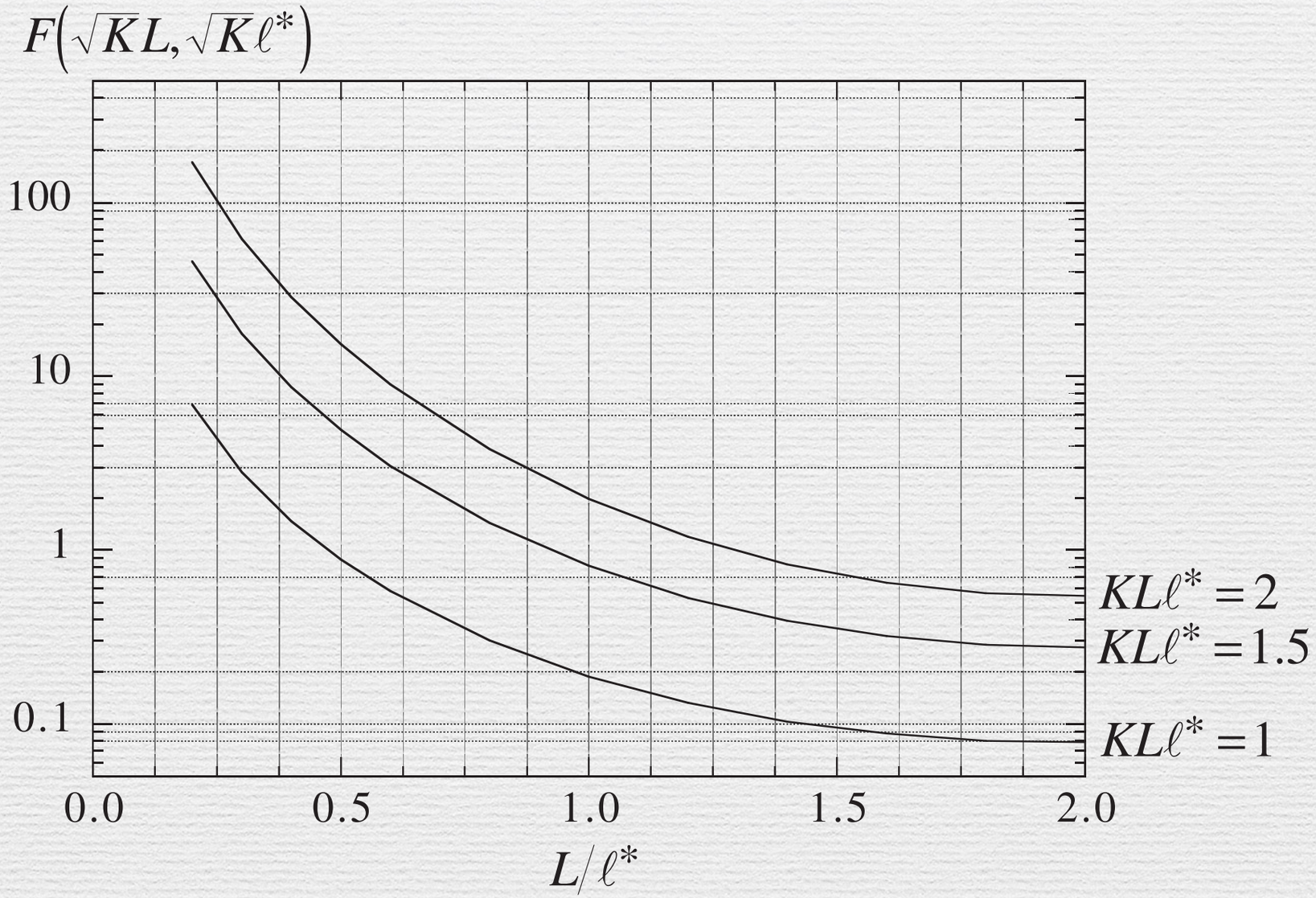

Figure 1: Function $F(\sqrt{K}L, \sqrt{K}\ell^*)$.

The minimum spot size

$$\sigma^*_{y,\min} = \sqrt{\frac{7}{5}} \left(\frac{\gamma\varepsilon_y}{\lambda_e}\right)^{5/7} \lambda_e \left[\frac{275}{3\sqrt{6\pi}} \alpha F\left(\sqrt{K}L, \sqrt{K}L^*\right)\right]^{1/7}$$

$$\approx 1.0 \times \left(\frac{\gamma\varepsilon_y}{\lambda_e}\right)^{5/7} \lambda_e \qquad (@\ KLL^* = 2, L/L^* = 1)$$

with $$\beta^*_{y,\min} = \gamma\lambda_e \left(\frac{\gamma\varepsilon_y}{\lambda_e}\right)^{3/7} \left[\frac{275}{3\sqrt{6\pi}} \alpha F\left(\sqrt{K}L, \sqrt{K}L^*\right)\right]^{2/7}$$

$$\gamma\varepsilon_y = \frac{\Delta y \Delta p_y}{mc} \geq \frac{\hbar}{2mc} = \frac{\lambda_e}{2}$$