

Exercise 1

Evaluate

$$q_r \frac{\partial}{\partial q_r} T_1(n_1, n_2, n_3, n_4, n_5)$$

for various values of n_1, \dots, n_5 using the routine `t1.frm` (in MINCER/)

- 1) On paper, choose a routing for q through T_1 (⊖)
- 2) Find the operator $(1^\pm, 2^\pm, \dots)$ relation:

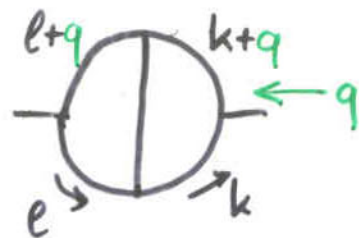
$$q_r \frac{\partial}{\partial q_r} T_1(\dots) = [\dots 1^+ \dots 2^- 4^+ \dots] T_1(\dots)$$

- 3) Implement this relation into `t1.frm`

$$L \text{ dia} = \dots I(n_1+1, n_2, \dots) + \dots I(n_1, n_2, n_3-1, \dots) + \dots$$

- 4) Could you derive the result without comp. algebra?
- 5) Choose a different routing for q
- 6) Try 3-loop.

Solution to Exercise 1 (i)



$$= \int dk dl \frac{1}{[(l+q)^2]^{n_1} [(k+q)^2]^{n_2} [k^2]^{n_3} [l^2]^{n_4} [(k-l)^2]^{n_5}} \equiv I$$

$$q_r \frac{\partial}{\partial q_r} \frac{1}{[(l+q)^2]^{n_1}} = -2n_1 \frac{1}{[(l+q)^2]^{n_1+1}} \{ q \cdot (l+q) \}$$

$$= - \frac{n_1}{[(l+q)^2]^{n_1+1}} \left\{ 2q^2 + \underbrace{(l+q)^2}_{1^-} - \underbrace{l^2}_{4^-} - q^2 \right\}$$

$$q_r \frac{\partial}{\partial q_r} \frac{1}{[(k+q)^2]^{n_2}} = - \frac{n_2}{[(k+q)^2]^{n_2+1}} \left\{ q^2 + \underbrace{(k+q)^2}_{2^-} - \underbrace{k^2}_{3^-} \right\}$$

$$\Rightarrow q_r \frac{\partial}{\partial q_r} I = \left[-n_1 (q^2 1^+ + 1 - 1^+ 4^-) - n_2 (q^2 2^+ + 1 - 2^+ 3^-) \right] I$$

Solution to Exercise 1 (2)

- mass dim of I : $\underbrace{2 \cdot D}_{\text{loop integr.}} - \underbrace{2 \cdot (n_1 + \dots + n_5)}_{\text{propagators}}$

only dimensional parameter: q^2

$$\Rightarrow I \sim (q^2)^{D - n_1 - \dots - n_5}$$

$$\Rightarrow q_\mu \frac{\partial}{\partial q_\mu} I = 2(D - n_1 - \dots - n_5) \cdot I$$

Exercise

Using AIR, calculate $T_1(2,1,1,1,1)$

by modifying `script-T1.map`
and `check-T1.map`

Compare the result to MINCER using `t1.frm`.

While AIR runs, watch the `calc.map` file.