Real Parton Emission and Automated Dipole Subtraction

K. Hasegawa
(Kouhei.Hasegawa@desy.de)
DESY Zeuthen

Collaboration with
S. Moch (DESY Zeuthen)
P. Uwer (Humboldt-Universität zu Berlin)

Reference: Nucl.Phys.Proc.Suppl.183:268-273,2008
arXiv:0807.3701 [hep-ph]
(Full paper will appear soon)

Computer Algebra and Particle Physics 2009, DESY Zeuthen, 1 April 2009

Contents

Part 1 : Theory and Package (45 min)

Part 2 : Exercise (45 min)

Part 1 : Theory and Package

1. Introduction

- Large Hadron Collider (LHC) at CERN

  - Energy Frontier : $\sqrt{S} \simeq 14\text{TeV}$

    ⟶ Direct production of Higgs and new particles beyond the Standard Model

  - Proton-Proton collision : $pp \rightarrow \mathrm{X}$

    ⟶ Events are triggered by the QCD interaction

  - We need estimate the Standard Model predictions to identify New Physics

$$\text{(New Physics)} = \text{(LHC signals)} - \text{(the SM predictions)}$$

  - The rate of QCD processes with high momentum transfer can be predicted by the perturbative expansion in the small strong coupling constant

For example, $\alpha_s(m_t) \simeq 0.1$

■ Perturbative QCD

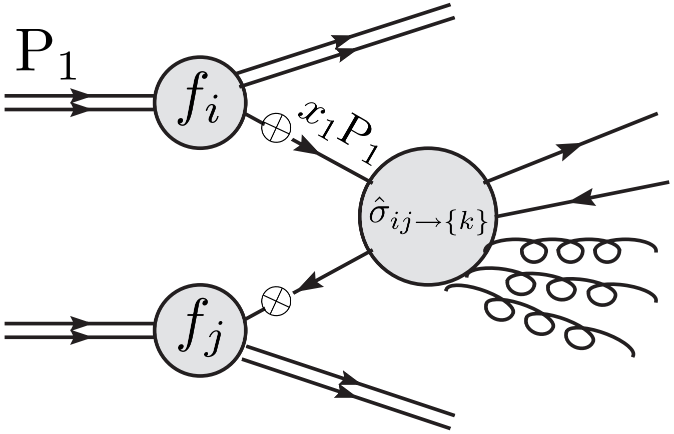
- Master Formula : Factorization of the hard scattering process

$$\sigma_{pp\to X} = \sum_{i,j,\{k\}} \int dx_1 dx_2 f_i(x_1) f_j(x_2) \hat{\sigma}_{ij\to\{k\}}(\alpha_s, Q) \otimes D_{\{k\}\to X}$$

Parton ditribution function
(Non-perturbative)

Subprocess partonic cross section
(Perturbative)

Jet algorithm
Parton shower
Hadronization model



- Perturbative expansion of the partonic cross section

$$\hat{\sigma}_{ij\to\{k\}} = \sigma_{\rm LO}(1 + \alpha_s C_1 + \alpha_s^2 C_2 + \cdots)$$

Leading order (LO)
Next-to-leading order (NLO)
Next-to-next-to-leading order (NNLO)

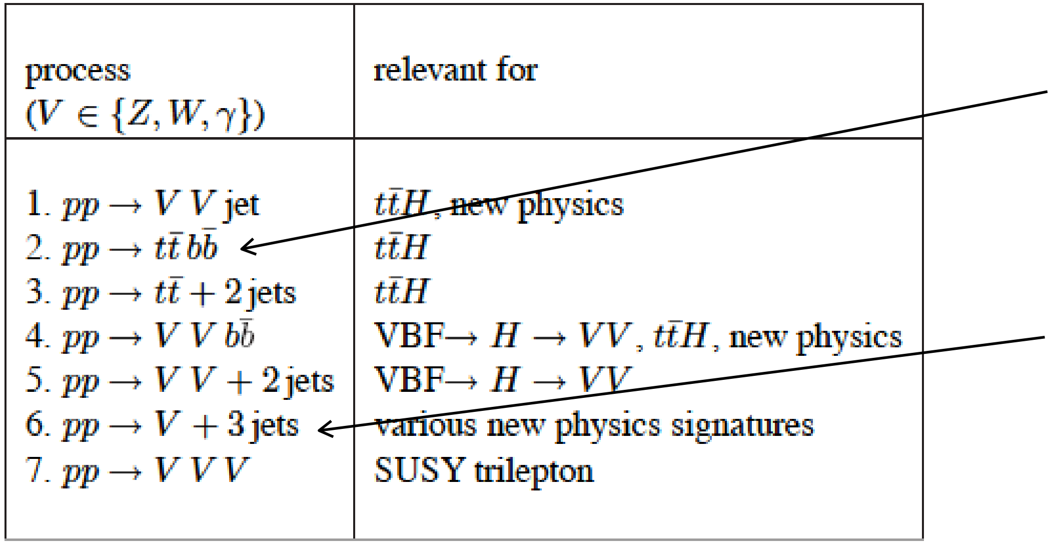
■ Leading order (LO)

- LO(Tree level) is well automated

Typical ones : Alpgen, CompHep, FeynArts, GRACE, HELAC/PHEGAS, MadGraph, ・・・

■ Next-to-leading order (NLO)

- LO has a large uncertainty from the renomalization/factorization scale dependences
- NLO is not yet fully automatized
- Process with multi-parton legs are difficult
- LHC priority NLO wish list in Les Houches 2005 (hep-ph/0604120)



process ($V \in \{Z, W, \gamma\}$)	relevant for
1. $pp \to V\,V$ jet	$t\bar{t}H$, new physics
2. $pp \to t\bar{t}\,b\bar{b}$	$t\bar{t}H$
3. $pp \to t\bar{t} + 2$ jets	$t\bar{t}H$
4. $pp \to V\,V\,b\bar{b}$	VBF→ $H \to VV$, $t\bar{t}H$, new physics
5. $pp \to V\,V + 2$ jets	VBF→ $H \to VV$
6. $pp \to V + 3$ jets	various new physics signatures
7. $pp \to V\,V\,V$	SUSY trilepton

$q\bar{q}$ mode
A. Bredenstein, A.Denner, S.Dittmaier, S.Pozzorini,
JHEP0808(2008)108

C.Berger, Z.Bern, L.Dixon, F.Cordero,
D.Forde, T.Gleisberg, H.Ita, D.Kosower, D.Maitre,
arXiv:0902.2760

-These predictions are urgently needed for the successful operation of LHC

-The computation of these radiative corrections is now a very active field

- QCD at NLO : $\sigma_{\rm NLO} = \sigma_{\rm real} + \sigma_{\rm virtual}$

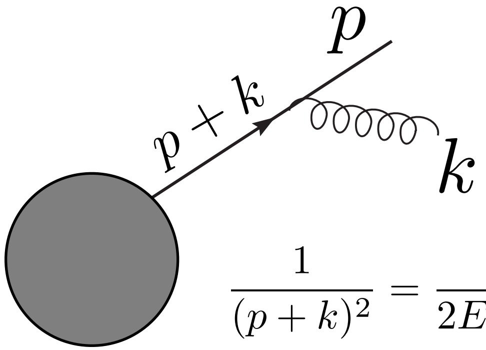
  - Why is the NLO of Multi-parton legs process so difficult ?

- Real correction

  - One additional gluon to LO

    Because it can not be resolved to LO event in some phase space region

  - Soft and collinear singularities



$$\frac{1}{(p+k)^2} = \frac{1}{2E_k E_p(1-\beta\cos\theta_{kp})} \rightarrow \infty$$

$\mathrm{E}_k \rightarrow 0$

: Soft divergence
(IR divergence)

Detector can not be aware of it

$m_q = 0$ and $\theta_{kq} \rightarrow 0$

: Collinear divergence
(mass divergence)

Detector can not resolve it to quark

  - Phase space integral of those singularities

	Cutoff		Dimensional regularization
Soft region:	$\int \frac{d^3k}{k}\frac{1}{k^2} \simeq \int_{\mu_k} \frac{dk}{k} \simeq \log\mu_k + \cdots$	$\longleftrightarrow$	$\int \frac{d^{D-1}k}{k}\frac{1}{k^2} \simeq \int_0 \frac{dk}{k^{1+\epsilon}} \simeq -\frac{1}{\epsilon} + \cdots$
Collinear region:	$\int_{-1}^{1} d\cos\theta \frac{1}{1-\cos\theta} \simeq \int_{\mu_\theta} \frac{d\theta}{\theta} \simeq \log\mu_\theta + \cdots$	$\longleftrightarrow$	$\int_0 \frac{d\theta}{\theta^{1+\epsilon}} \simeq -\frac{1}{\epsilon} + \cdots$

■ Virtual correction

- One loop diagram

Because the intermediate state can not be observed

- Include the ultraviolet (UV) and soft/collinear divergences

UV divergence

1-loop diagram $\supset \int \frac{d^4 l}{(2\pi)^4} \frac{(l^2)^m}{(l^2-\Delta)^n} = \frac{i(-1)^{n+m}}{16\pi^2}\left(\frac{1}{\Delta}\right)^{n-m-2} \frac{\Gamma(m+2)\Gamma(n-m-2)}{\Gamma(n)} \longrightarrow \infty$

$(n-m-2 \leq 0)$

Integral over shifted loop momenta

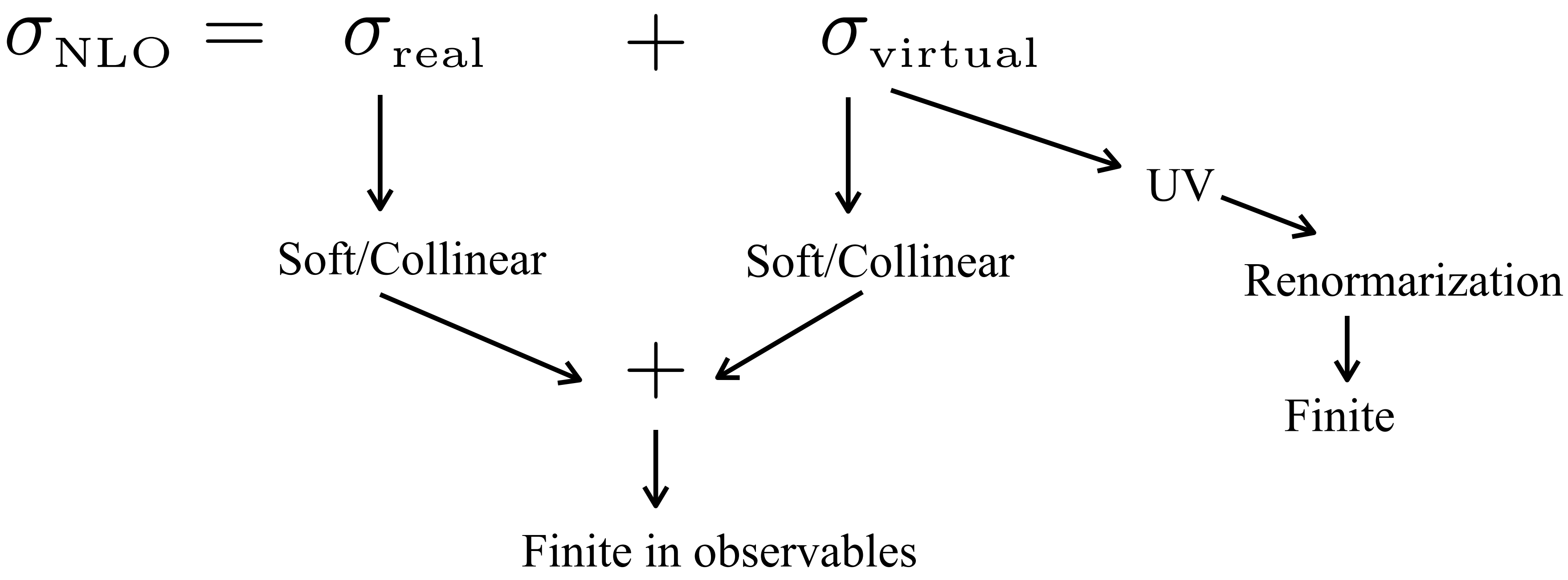
$\supset \int dx_1 dx_2 \cdots \left(\frac{1}{\Delta(x_1, x_2, \cdots)}\right)^{n-m-2}$ Soft/collinear divergence $\longrightarrow \infty$

Integral over Feynmann parameters

- First difficulty of NLO with multi-parton leg:
  Evaluation of 1 loop diagram with 5 legs (Pentagon), 6 legs (Hexagon) , and more

Still active field 30 years after the pioneer work of Passarino-Veltman

- QCD at NLO : Cancellation of soft/collinear singularities



$$\sigma_{\mathrm{NLO}} = \sigma_{\mathrm{real}} + \sigma_{\mathrm{virtual}}$$

Soft/Collinear

Soft/Collinear

UV

Renormarization

Finite

$+$

Finite in observables

:IR safe (more precisely, soft/collinear safe)

Kinoshita-Lee-Nauenberg theorem

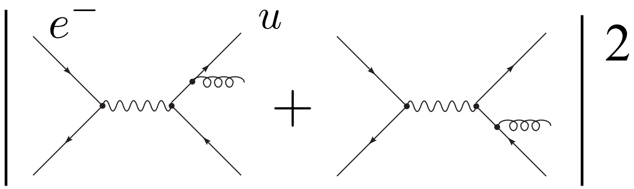
■ The simplest example : $e^-e^+ \to u\bar{u}$

- Typical procedure: Dimensional regularization

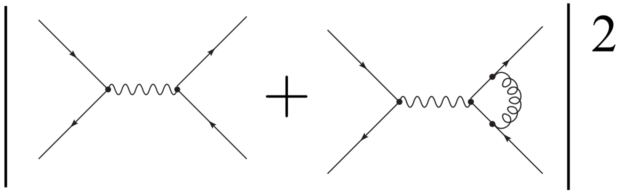
Calculate all quantity (phase space and matrix element) in dimension: $\mathrm{D} = 4 - 2\epsilon$

$$\sigma_{\mathrm{NLO}} = \sigma_{\mathrm{real}} + \sigma_{\mathrm{virtual}}$$

$$= \int d\Phi_3 |\mathrm{M}(e^-e^+ \to u\bar{u}g)|^2 \Big|_{\text{D-dim}} + \int d\Phi_2 |\mathrm{M}(e^-e^+ \to u\bar{u})|^2_{\text{1-loop}} \Big|_{\text{D-dim}}$$



+



Phase space $\int_0^1 \mathrm{d}x_1 \int_{1-x_1}^1 \mathrm{d}x_2 \frac{1}{[(1-x_1)(1-x_2)(1-x_3)]^\epsilon}$ $\int_0^1 dv \frac{1}{[v(1-v)]^\epsilon}$

$|\mathrm{M}|^2$ $\frac{x_1^2 + x_2^2 - \epsilon x_3^2}{(1-x_1)(1-x_2)}$ $|\mathrm{M}_{\mathrm{LO}}| \cdot \mathrm{Re}\Big[\int dxdy \frac{\Gamma(\epsilon)}{(-xyq^2)^\epsilon}\Big((1-\epsilon)^2 - \epsilon\frac{(1-x)(1-y)-\epsilon xy}{xy}\Big)\Big]$

$$= \sigma_{\mathrm{LO}}^{(\epsilon)} \frac{\alpha_s}{2\pi} C_{\mathrm{F}} \left(\frac{4\pi}{q^2}\right)^\epsilon \frac{4\Gamma(1-\epsilon)^2}{\Gamma(3-3\epsilon)} \left[\left(\underline{\frac{1}{\epsilon^2} - \frac{3}{\epsilon}} + \frac{5}{2} + O(\epsilon)\right) + \left(\underline{-\frac{1}{\epsilon^2} + \frac{3}{\epsilon}} - \frac{7}{4} + O(\epsilon)\right)\right]$$

Cancelation of $1/\epsilon$ poles

$$= \sigma_{\mathrm{LO}} \cdot \frac{\alpha_s}{\pi}$$:Finite results

- This method is not practical for the multi-parton leg processes
  - -The complexity and the long expression
  - -The phase space integral of n and (n+1) particles in D-dimension

■ Dipole subtraction

- A general and practical procedure to treat soft/collinear divergences at QCD NLO

S.Catani and M.H.Seymour, Nucl.Phys.B485(1997)291
S.Catani, S.Dittmaier, M.H.Seymour, Z.Trocsanyi, Nucl.Phys.B627(2002)189

1. Construct the counter terms which cancel all soft/collinear divergences
2. Subtract it from $\sigma_{\rm real}$ and add it to $\sigma_{\rm virtual}$

$$\sigma_{\rm NLO} = \sigma_{\rm real} + \sigma_{\rm virtual}$$

$$= \left(\sigma_{\rm real} - \sigma_{\rm a}\right) + \left(\sigma_{\rm virtual} + \sigma_{\rm a}\right)$$

$$= \int {\rm d}\Phi_{m+1}\left[|{\rm M}_{\rm real}|^2 - \sum_i {\rm D}_i\right]\Bigg|_{\rm D=4} + \int {\rm d}\Phi_m\left[|{\rm M}_{\rm 1\text{-}loop}|^2 + \int {\rm d}\Phi_1 \sum_i {\rm D}_i\right]\Bigg|_{\rm D=4}$$

Finite

$$\int_0 dk\left(\frac{1}{k} - \frac{1}{k}\right) < \infty$$

Finite

- Real correction does not need any reguralization
  Calculation (phase space and matrix element) is in 4-dimension

- Dipole term is systematically constructed based on
  the factorization of soft/collinear singularities
  $\longrightarrow$ reduction to Born level

$$\mathrm{D}_i \simeq \frac{1}{s_i}\mathrm{V}_i \cdot |\mathrm{M}_i|_{\rm Born}$$

Singular part

$\uparrow$

dipole splitting function
(Universal)

- Integration of dipole term is analitically done once for all

■ Multi-parton leg processes

- Dipole subtraction makes it possible
- It requires many dipole terms and repeats the same kinds of calculation at huge times
  (Order 50 dipoles)
- The algorithm is a combinatorial way

⇨ The automatization is required and it is possible

■ Our aim

1. Automatize the dipole subtraction
2. Apply it to the QCD backgrounds and the relevant signals in LHC

-There is recent work in the same direction

- T. Gleisberg and F. Krauss, Eur.Phys.J.C53(2008)501, arXiv0709.2881
- M.H. Seymour and C. Tevlin, arXiv0803.2231
- R. Frederix and T. Gehrmann and N. Greiner, JHEP0809:122, arXiv0808.2128

- We present today our package of an automated dipole subtraction:

AutoDipole Version 1.0beta

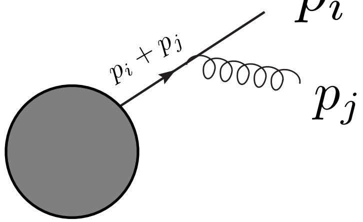
- This version includes the subtracted real emission part : $|\mathrm{M}|^2_{\mathrm{real}} - \sum_i \mathrm{D}_i$

⇨ In this talk we treat with only tree level diagrams

2. Dipole Subtraction

- Soft limit

- Factorization of the amplitude in the soft limit is an universal way



$$\mathrm{M}_{m+1}(p_i, p_j) = \epsilon_\mu^{a\star} \bar{u}(p_i) i g_s t^a \gamma^\mu \frac{i(\not{p}_i + \not{p}_j)}{(p_i + p_j)^2} \mathrm{M}_m(p_i + p_j)$$

Eikonal approximation

$$|\vec{p}_j| \ll |\vec{p}_i|$$

$$\simeq \epsilon_\mu^{a\star} g_s \frac{p_i^\mu}{p_i \cdot p_j} \bar{u}(p_i)_\alpha (t^a)_{\alpha\beta} \mathrm{M}_m(p_i)_\beta$$

- No spin correlation
- Color correlation

$$p_j = \lambda q_j \qquad \lambda \to 0$$

$$< 1, \cdots, i, \cdots, j, \cdots, m+1 || 1, \cdots, i, \cdots, j, \cdots m+1 >_{m+1}$$

$$\longrightarrow \qquad -\frac{1}{\lambda^2} 4\pi\alpha_s < 1, \cdots, i, \cdots, m+1 | [J^\mu]^\dagger J_\mu | 1, \cdots, i, \cdots m+1 >_m$$

Eikonal current: $J^\mu = \sum_i \mathrm{T}_i \frac{p_i^\mu}{p_i \cdot q_j}$

Color operator

$$\left[\begin{array}{l} \mathrm{T}_i | i, u_\alpha >= (t^{a_i})_{\alpha\beta} | i, u_\beta > \\ \mathrm{T}_i | i, \bar{u}_\alpha >= -(t^{a_i})_{\beta\alpha} | i, \bar{u}_\beta > \\ \mathrm{T}_i | i, g_a >= i f_{a c_i b} | i, g_b > \end{array} \right.$$

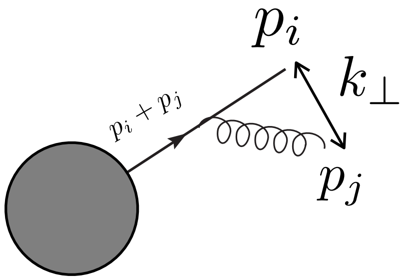
$$\longrightarrow \quad -\frac{1}{\lambda^2} 8\pi\alpha_s \sum_i \frac{1}{p_i \cdot q_j} \sum_{k(\neq i)} < 1, \cdots, i, \cdots, m+1 | \frac{p_i \cdot p_k}{(p_i + p_k) \cdot q_j} \mathrm{T}_i \cdot \mathrm{T}_k | \underline{1, \cdots, i, \cdots m+1 >}_m$$

Reduced Born

$$\equiv \mathrm{S}_{(j)} \equiv \sum_{i,k(\neq j)} \mathrm{S}_{i(j),k}$$

Collinear limit

- Factorization of the amplitude in the collinear limit is an universal way



$$p_i^\mu = zp^\mu + k_\perp^\mu - \frac{k_\perp^2}{z}\frac{n^\mu}{2p\cdot n} \qquad p_j^\mu = (1-z)p^\mu - k_\perp^\mu - \frac{k_\perp^2}{1-z}\frac{n^\mu}{2p\cdot n} \qquad 2p_i\cdot p_j = -\frac{k_\perp^2}{z(1-z)}$$

$$k_\perp \to 0$$

$$\mathrm{M}_{m+1}(p_i,p_j) \to g\frac{1}{\sqrt{p_i\cdot p_j}}f(z)\bar{u}(p_i+p_j)_\alpha t^a_{\alpha\beta}\mathrm{M}_m(p_i+p_j)_\beta$$

- Self-square of the matrix element has the leading singularity

$$< 1,\cdots,i,\cdots,j,\cdots,m+1||1,\cdots,i,\cdots,j,\cdots m+1 >_{m+1}$$

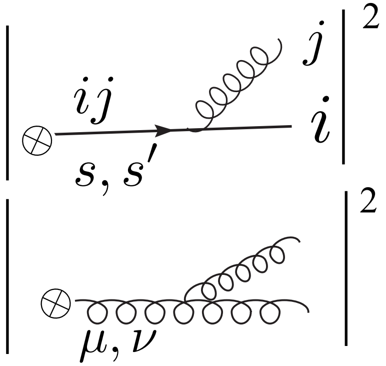
$$\longrightarrow \quad \frac{1}{p_i\cdot p_j}4\pi\alpha_s < 1,\cdots,ij,\cdots,m+1|\hat{P}_{(ij),i}(z,k_\perp)|1,\cdots,ij,\cdots m+1 >_m \;\equiv \mathrm{C}_{ij}$$

-Altarelli-Parisi splitting function: $\hat{P}_{(ij),i}(z,k_\perp)$

Square of the splitting amplitudes

$$< s|\hat{P}_{qq}(z,k_\perp)|s' >= \delta_{ss'}C_F\left[\frac{1+z^2}{1-z}\right]$$

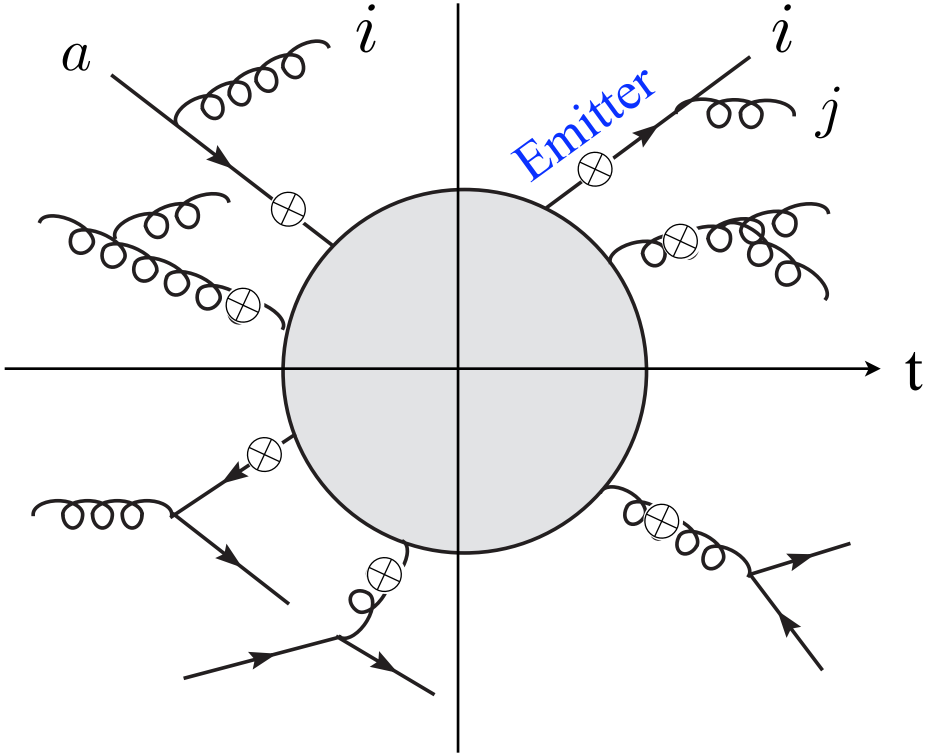
$$< \mu|\hat{P}_{gg}(z,k_\perp)|\nu >= 2C_A\left[-\underset{\underset{\sum\limits_{\lambda=L,R}\epsilon_\mu^{(\lambda)}\epsilon_\nu^{(\lambda)\star}}{\uparrow}}{g_{\mu\nu}}\left(\frac{z}{1-z}+\frac{1-z}{z}\right)-2z(1-z)\underset{\underset{\text{Different helicity}}{\uparrow}}{\frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2}}\right]$$



- Gluon spin correlation
- No color correlation

■ Construction of dipole terms

1. Choose emitter pair



Choose all possible leg-pair which matches one of the seven patterns

(a, i) or (i, j)

Initial parton=a,b
Final parton=i,j,k

2. Choose spectator

Choose a different leg from emitter pair

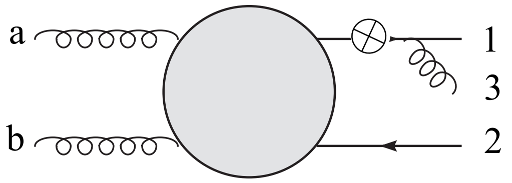
Spectator : $k \neq i, j$ $\quad b \neq a$

spectator emitter pair	k	b
(i, j)	$\mathrm{D}_{ij,k}$ $(k \neq i, j)$	$\mathrm{D}_{ij}^{\;b}$
(a, i)	$\mathrm{D}^{ai}_{\;k}$ $(k \neq i)$	$\mathrm{D}^{ai,b}$ $(b \neq a)$

3. Use dipole formulae

$$D_{ij,k}(p_1,\cdots,p_{m+1}) = -\frac{1}{2p_i\cdot p_j} < 1,\cdots,\tilde{ij},\cdots,\tilde{k},\cdots,m{+}1|\frac{T_k\cdot T_{ij}}{T_{ij}^2}V_{ij,k}|1,\cdots,\tilde{ij},\cdots,\tilde{k},\cdots,m{+}1 >_m$$

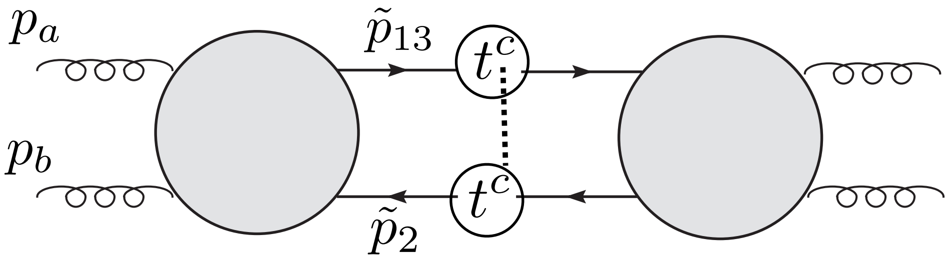
Example : $g(a)g(b) \to u(1)\bar{u}(2)g(3)$



$$D_{13,2}(p_1,p_2,p_3,p_a,p_b) = -\frac{1}{2p_1\cdot p_3}\langle gg\to\tilde{u}\tilde{\bar{u}}|\frac{T_{\bar{u}}\cdot T_{ug}}{T_{ug}^2}V_{13,2}|gg\to\tilde{u}\tilde{\bar{u}}\rangle_2$$

- Dipole splitting function : $V_{13,2}(z,y) = \delta_{ss'}8\pi\alpha C_F\left[\frac{2}{1-z_i(1-y_{ij,k})}-(1+z_i)\right]$

- Color linked Born squared (CLBS): $\langle gg\to\tilde{u}\tilde{\bar{u}}|T_{\bar{u}}\cdot T_{ug}|gg\to\tilde{u}\tilde{\bar{u}}\rangle_2$



$$T_X^a = \begin{cases} t^a & (X = \text{quark}) \\ f^a & (X = \text{gluon}) \end{cases}$$

- Reduced momenta satisfy the energy-momentum conservation and on-shell condition

$$p_a^\mu + p_b^\mu = \tilde{p}_{13}^\mu + \tilde{p}_2^\mu \qquad \tilde{p}_{13}^2 = \tilde{p}_2^2 = 0$$

⇨ Make it possible to reduce into the physical born amplitude, which can be calculated by the well automated LO softwares

---

$$\tilde{p}_{ij}^\mu = p_i^\mu + p_j^\mu - \frac{y_{ij,k}}{1-y_{ij,k}}p_k^\mu \qquad \tilde{p}_k^\mu = \frac{1}{1-y_{ij,k}}p_k^\mu \qquad z_i = \frac{p_i\cdot p_k}{p_j\cdot p_k + p_i\cdot p_k} \qquad y_{ij,k} = \frac{p_i\cdot p_j}{p_i\cdot p_j + p_j\cdot p_k + p_k\cdot p_i}$$

Soft/collinear limits of dipole terms

$$|\mathrm{M}|^2_{\mathrm{real}} - \sum_i \mathrm{D}_i$$:IR safe (Soft/Collinear safe)

$$\begin{tikzcd}
\bigcup & & \\
\mathrm{V}_{ij,k} \arrow[r] \arrow[dr] & \frac{1}{\lambda}16\pi\alpha_s\mathrm{T}^2_{ij}\frac{p_i \cdot p_k}{q_j \cdot (p_i + p_k)} & \text{(Soft limit)} \\
\otimes & 8\pi\alpha_s\mathrm{P}_{ij} & \text{(Collinear limit)}
\end{tikzcd}$$

$$< 1, \cdots, ij, \cdots, m+1 | \mathrm{T}_{ij} \cdot \mathrm{T}_k | 1, \cdots, ij, \cdots m+1 >_m$$

- To reproduce the color factors of collinear limits, the identity is used

$$\sum_{k=1}^{m+1} < 1, \cdots, ij, \cdots, m+1 | \mathrm{T}_{ij} \cdot \mathrm{T}_k | 1, \cdots, ij, \cdots m+1 >_m$$

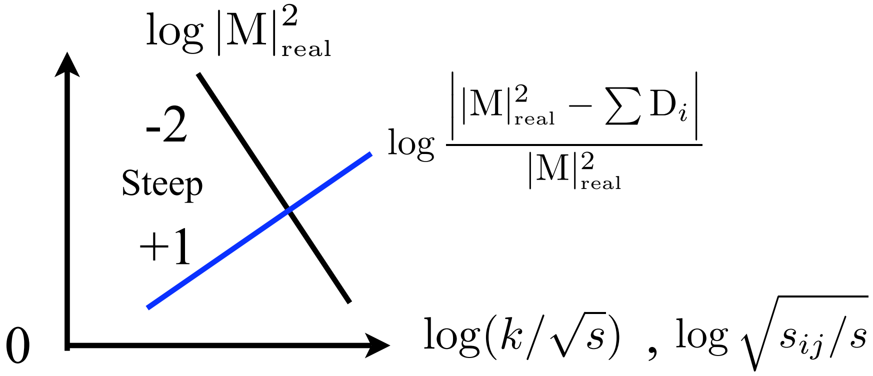
$$= - < 1, \cdots, ij, \cdots, m+1 | \mathrm{T}_{ij} \cdot \mathrm{T}_{ij} | 1, \cdots, ij, \cdots m+1 >_m$$

All soft/collinear singularities are cancelled by the dipole terms

- Limiting behavior

  - We can predict the limiting behavior

$$|\mathrm{M}|^2_{\mathrm{real}} - \sum_i \mathrm{D}_i = \begin{cases} \frac{1}{k^2}(a_0 + a_1 k + a_2 k^2 + \cdots) - \frac{1}{k^2} a_0 = \frac{1}{k}(a_1 + a_2 k + \cdots) & \text{Soft} \; (k \longrightarrow 0) \\ \frac{1}{s_{ij}}(b_0 + b_1\sqrt{s_{ij}} + b_2 s_{ij} + \cdots) - \frac{1}{s_{ij}} b_0 = \frac{1}{\sqrt{s_{ij}}}(b_1 + b_2\sqrt{s_{ij}} + \cdots) & \text{Collinear} \; (\theta_{ij} \longrightarrow 0) \end{cases}$$



- Final formula

  - Initial partons

    - Gluon emission from initial partons produces the collinear singularity which is not cancelled by the virtual correction ⟶ Those singularities should be factorized into PDF

    - For the purpose, the collinear subtraction term is introduced

$$\sigma_c(a, \text{Non-parton} \to \{k\}) = -\frac{\alpha_s}{2\pi}\frac{1}{\Gamma(1-\epsilon)}\sum_b \int_0^1 dz \left[-\frac{1}{\epsilon}\left(\frac{4\pi\mu_{\mathrm{R}}^2}{\mu_{\mathrm{F}}^2}\right)^{2\epsilon} P^{ab}(z)\right]\sigma_{\text{Born}}(b, \text{Non-parton} \to \{k\})$$

  - Define jet observable

$$F_J^{(m)}(p_1, \cdots, p_m) \quad \text{:Jet defining function}$$

Soft limit: $F_J^{(m+1)}(p_1, \cdots, p_j = \lambda q_j, \cdots, p_{m+1}) \to F_J^{(m)}(p_1, \cdots, p_{m+1}) \qquad (\lambda \to 0)$

Collinear limit: $F_J^{(m+1)}(p_1, \cdots, p_i, \cdots, p_j, \cdots, p_{m+1}) \to F_J^{(m)}(p_1, \cdots, p, \cdots, p_{m+1})$

$(p_i \to zp \text{ and } p_j \to (1-z)p)$

$$\sigma_{\text{NLO}} = \sigma_{\text{real}} + \sigma_{\text{virtual}} + \sigma_c$$

$$= \underline{(\sigma_{\text{real}} - \sigma_a)} + (\sigma_{\text{virtual}} + \sigma_{\text{c}} + \sigma_a)$$

$$= \underline{\int d\Phi_{m+1} \boxed{\left(|\mathrm{M}|^2_{\text{real}} F^{(m+1)} - \sum_i \mathrm{D}_i F_i^{(m)}\right)}} + \int d\Phi_m \left[|\mathrm{M}|^2_{\text{1-loop}} + < \mathrm{I}(\epsilon) >_m\right] F^{(m)} + \int_0^1 dx \int d\Phi_m \left[< (\mathrm{K}(x) + \mathrm{P}(x)) >_m\right] F^{(m)}$$

Our packege includes only this real emission part

3. Automatization

- Package: AutoDipole   Version 1.0beta

K. Hasegawa, S.Moch, and P.Uwer
Nucl.Phys.Proc.Suppl.183:268-273,2008 (arXiv:0807.3701 [hep-ph])

- Automated Dipole subtraction

- Mathematica code and an interface with MadGraph

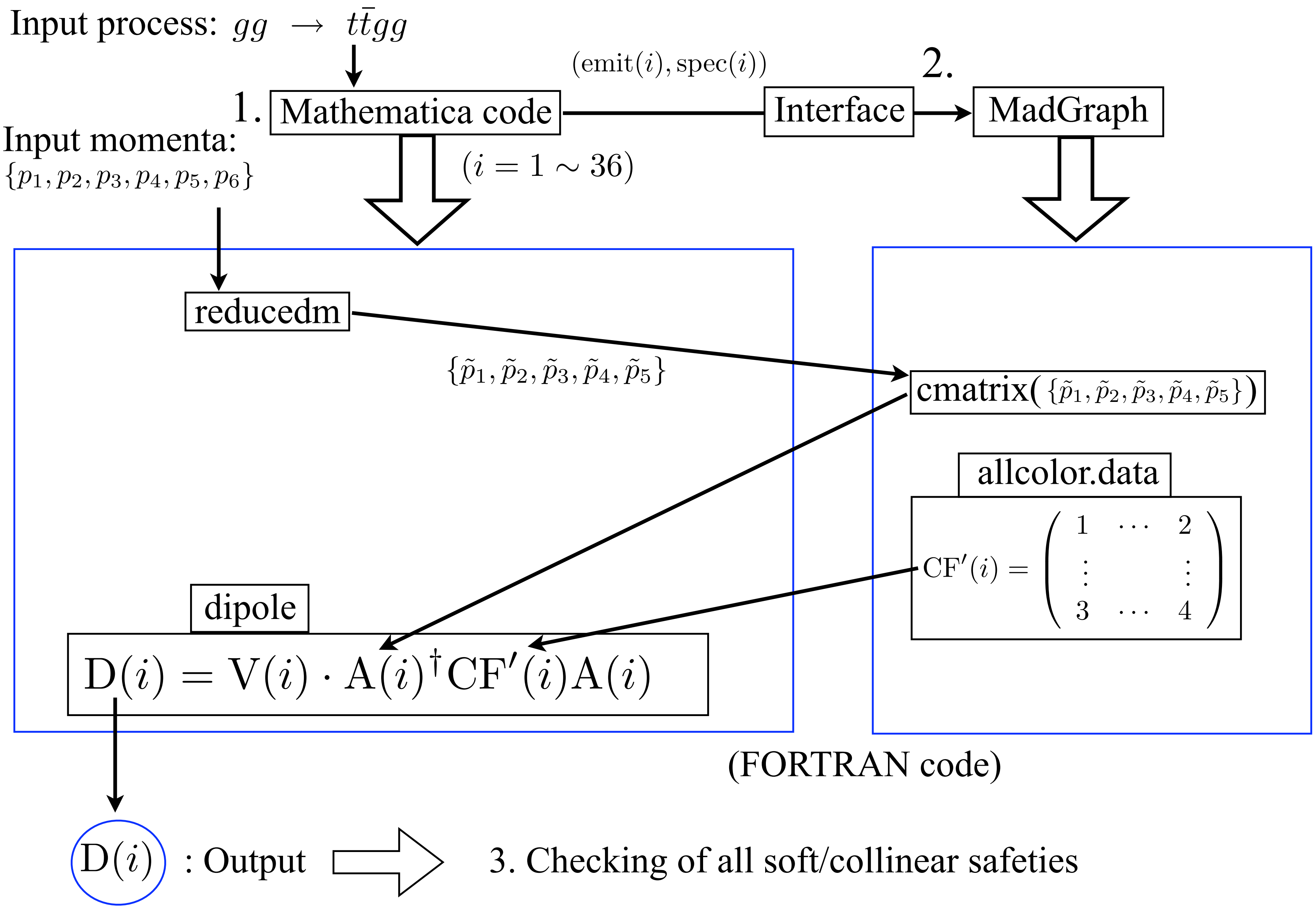
- The version 1.0beta includes the subtracted real emission part : $|\mathrm{M}|^2_{\mathrm{real}} - \sum_i \mathrm{D}_i$

- Generate the Fortran routines to calculates the subtracted real emission

- Check all soft/collinear safeties

- The version 1.0beta is now available
The complete packege is publicly available soon

■ Scheme to calculate $|\mathrm{M}|^2 - \sum_i \mathrm{D}_i$ in AutoDipole



Input process: $gg \to t\bar{t}gg$

1. Mathematica code

$(\mathrm{emit}(i), \mathrm{spec}(i))$

Interface

2. MadGraph

Input momenta: $\{p_1, p_2, p_3, p_4, p_5, p_6\}$

$(i = 1 \sim 36)$

reducedm

$\{\tilde{p}_1, \tilde{p}_2, \tilde{p}_3, \tilde{p}_4, \tilde{p}_5\}$

cmatrix($\{\tilde{p}_1, \tilde{p}_2, \tilde{p}_3, \tilde{p}_4, \tilde{p}_5\}$)

allcolor.data

$$\mathrm{CF}'(i) = \begin{pmatrix} 1 & \cdots & 2 \\ \vdots & & \vdots \\ 3 & \cdots & 4 \end{pmatrix}$$

dipole

$$\mathrm{D}(i) = \mathrm{V}(i) \cdot \mathrm{A}(i)^{\dagger} \mathrm{CF}'(i) \mathrm{A}(i)$$

(FORTRAN code)

$\mathrm{D}(i)$: Output

3. Checking of all soft/collinear safeties

1. Mathematica code

Input: $gg \to u\bar{u}g$ (Process at NLO real correction)

↓

Creation of dipole terms (Write down all D_i except for CLBS)

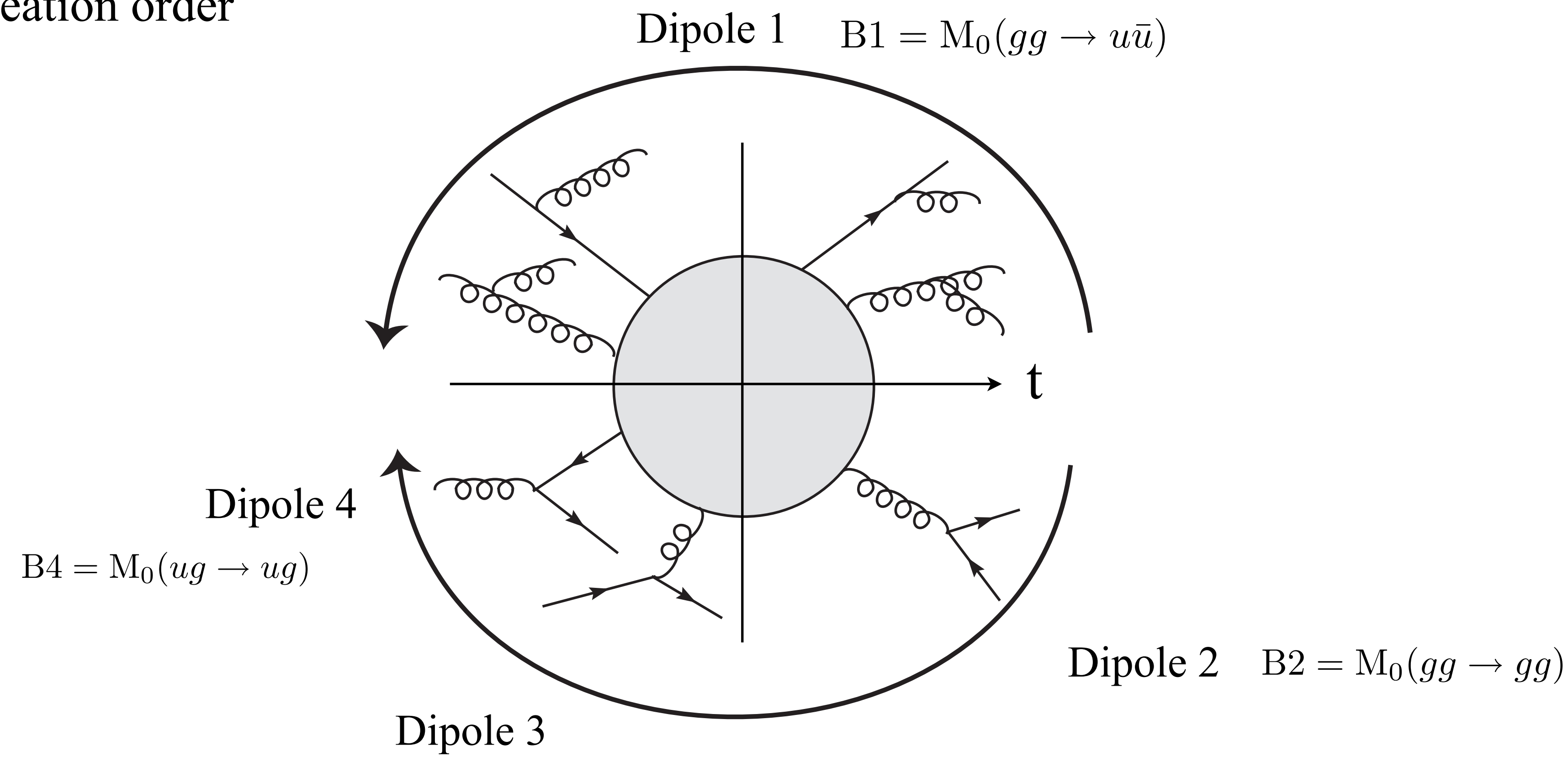
↓

Show the contents of all dipoles and all soft/collinear limits

↓

Write Fortran files: dipole.f reducedm.f and interface for MadGraph

- Creation order



- Output

Process: $gg \to u\bar{u}g$

- Contents of dipole terms

```
Number of dipoles

[Dipole1] : 12

 B1 : 12

{Splitting (1):(i,j)=(f,g)}:  6 (0)

                [1.(ij,k)=(fg,k): Dij,k]  2 (0)

                [2.(ij,a)=(fg,a): Dij^a]  4 (0)

{Splitting (2):(i,j)=(g,g)}:  0 (0)

                [3.(ij,k)=(gg,k): Dij,k]  0 (0)

                [4.(ij,a)=(gg,a): Dij^a]  0 (0)

{Splitting (3):(a,i)=(f,g)}:  0 (0)

                [5.(ai,k)=(fg,k): D^ai,k]  0 (0)

                [6.(ai,b)=(fg,b): D^ai,b]  0 (0)

{Splitting (4):(a,i)=(g,g)}:  6 (0)

                [7.(ai,k)=(gg,k): D^ai,k]  4 (0)

                [8.(ai,b)=(gg,b): D^ai,b]  2 (0)

--------------------

[Dipole2] : 3

{Splitting (5):(i,j)=(f,fbar)}

  (u-ubar splitting) B2u : 3

                [9-u.(ij,k)=(u ubar,k): Dij,k]  1 (0)

                [10-u.(ij,a)=(u ubar,a): Dij^a]  2 (0)

  (d-dbar splitting) B2d : 0

                [9-d.(ij,k)=(u ubar,k): Dij,k]  0 (0)

                [10-d.(ij,a)=(u ubar,a): Dij^a]  0 (0)

  (b-bbar splitting) B2b : 0

                [9-b.(ij,k)=(u ubar,k): Dij,k]  0 (0)

                [10-b.(ij,a)=(u ubar,a): Dij^a]  0 (0)

  (t-tbar splitting) B2t : 0

                [9-t.(ij,k)=(u ubar,k): Dij,k]  0 (0)

                [10-t.(ij,a)=(u ubar,a): Dij^a]  0 (0)

--------------------

[Dipole4] : 12
```

```
{Splitting (7):(a,i)=(g,f) or (g,fbar)}

 ((a,i)=(g,u) splitting)  B4u : 6

             [13-u.(ai,k)=(gu,k): D^ai,k]  4 (0)

             [14-u.(ai,b)=(gu,b): D^aib]  2 (0)

 ((a,i)=(g,ubar) splitting)   B4ubar : 6

             [13-ubar.(ai,k)=(g ubar,k): D^ai,k]  4 (0)

             [14-ubar.(ai,b)=(g ubar,b): D^aib]  2 (0)

 ((a,i)=(g,d) splitting) B4d : 0

             [13-d.(ai,k)=(gd,k): D^ai,k]  0 (0)

             [14-d.(ai,b)=(gd,b): D^aib]  0 (0)

 ((a,i)=(g,dbar) splitting) B4dbar : 0

             [13-dbar.(ai,k)=(g ubar,k): D^ai,k]  0 (0)

             [14-dbar.(ai,b)=(g ubar,b): D^aib]  0 (0)

-------------------

[Total] : 27

 (Massive dipoles : 0)

-------------------

END

************************************************************************

The collinear and soft limits and the corresponding dipoles

NLO: {{g, p[1]}, {g, p[2]}} --> {{u, p[3]}, {ubar, p[4]}, {g, p[5]}}

-------------------

Collinear pairs      Corresponding dipoles

1. {3, 5}             {1, 3, 4}

2. {4, 5}             {2, 5, 6}

3. {1, 5}             {7, 8, 11}

4. {2, 5}             {9, 10, 12}

5. {3, 4}             {13, 14, 15}

6. {1, 3}             {16, 17, 20}

7. {2, 3}             {18, 19, 21}

8. {1, 4}             {22, 23, 26}

9. {2, 4}             {24, 25, 27}

-------------------

Soft gluon   Collinear assemble    Corresponding dipoles

1. {5}       {1, 2, 3, 4}         {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

              -------------------

              END
```

- All soft/collinear limits

■ 2. MadGraph with our interface

- MadGraph

T. Stelzer and W.F. Long, Phys.Commun.81(1994) 357, hep-ph/9401258
Johan Alwall et al, JHEP 0709:028,2007, arXiv:0706.2334

  - An automated general LO in the Standard Model, MSSM, and some others models
  - Write down the Fortran codes to calculate the matrix element squared
  - Numerical evaluation of the helicity amplitude
  - In order to calculate the helicity amplitude, HELAS library is used

K. Hagiwara, H. Murayama, I. Watanabe, Nucl.Phys.B367(1991)257

  - Color decomposition

$$\mathrm{M} = \sum_a C_a J_a \qquad J_1 = A_1 - A_3 + \cdots$$: Joint amplitude

- Our interface

  - Normal Born squared

$< 1, \cdots, m || 1, \cdots, m >_m$

→ Color linked Born squared

$< 1, \cdots, m | \mathrm{T}_i \cdot \mathrm{T}_k | 1, \cdots, m >_m$

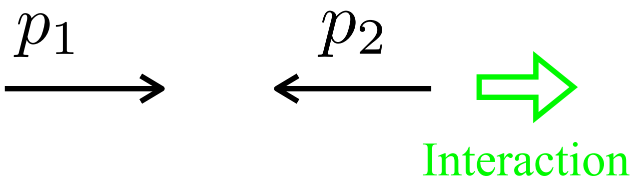
→ Different helicity Born squared

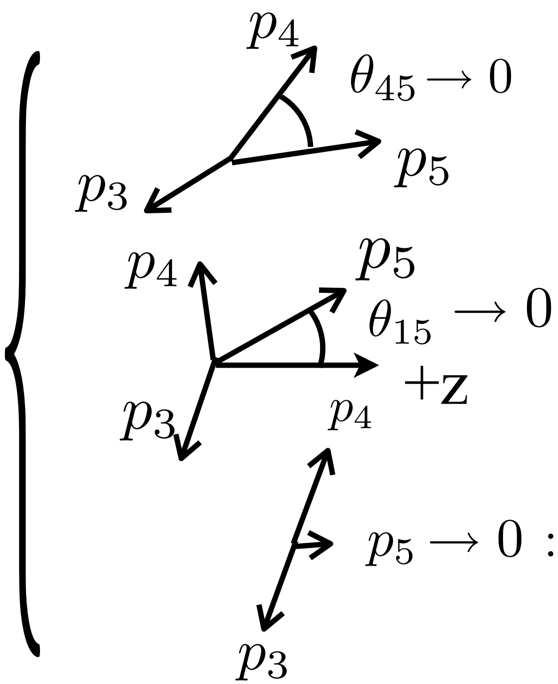
$< 1, \cdots, (i, \lambda), \cdots, m || 1, \cdots, (i, \lambda'), \cdots, m >_m$

■ 3. Check the soft/collinear limits

- Configurate all soft/collinear limits

$g(1)g(2) \to u(3)\bar{u}(4)g(5)$



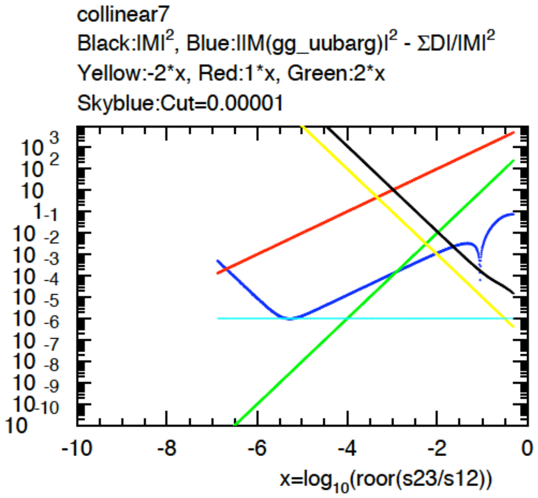


$\theta_{45} \to 0$: Colllinear limit of Final-Final spliting

$\theta_{15} \to 0$: Colllinear limit of Initial-Final spliting

$p_5 \to 0$: Soft limit

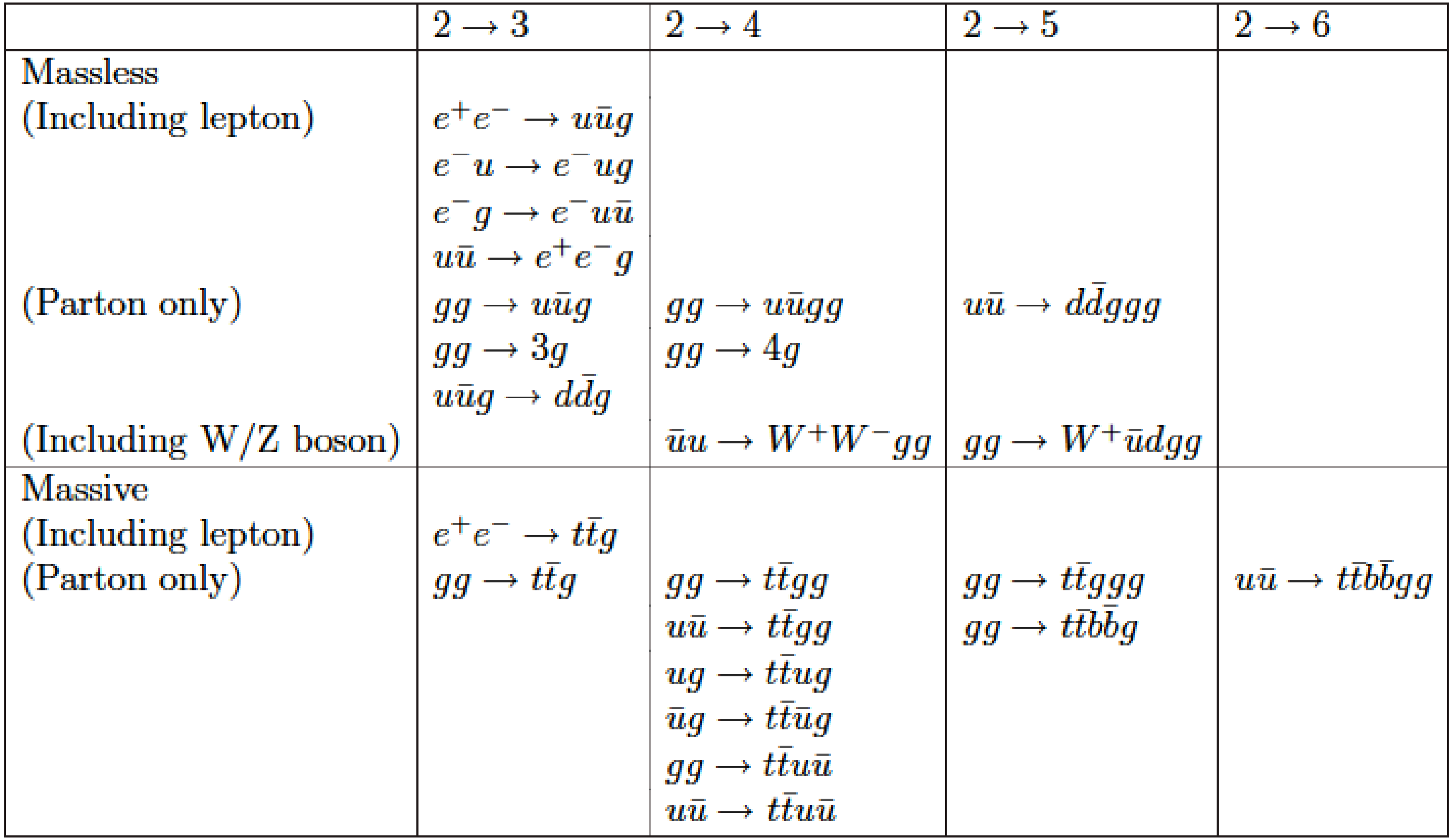
- Check the cancellation of the singularities



- After the soft/collinear safeties are confirmed, an user may go to the MonteCarlo phase space integral with PDF and a jet algorithm

Status of AutoDipole

- We checked the cancellation of all soft/collinear singularities in the following real emission processes



	$2 \to 3$	$2 \to 4$	$2 \to 5$	$2 \to 6$
Massless				
(Including lepton)	$e^+e^- \to u\bar{u}g$ $e^-u \to e^-ug$ $e^-g \to e^-u\bar{u}$ $u\bar{u} \to e^+e^-g$			
(Parton only)	$gg \to u\bar{u}g$ $gg \to 3g$ $u\bar{u}g \to d\bar{d}g$	$gg \to u\bar{u}gg$ $gg \to 4g$	$u\bar{u} \to d\bar{d}ggg$	
(Including W/Z boson)		$\bar{u}u \to W^+W^-gg$	$gg \to W^+\bar{u}dgg$	
Massive				
(Including lepton)	$e^+e^- \to t\bar{t}g$			
(Parton only)	$gg \to t\bar{t}g$	$gg \to t\bar{t}gg$ $u\bar{u} \to t\bar{t}gg$ $ug \to t\bar{t}ug$ $\bar{u}g \to t\bar{t}\bar{u}g$ $gg \to t\bar{t}u\bar{u}$ $u\bar{u} \to t\bar{t}u\bar{u}$	$gg \to t\bar{t}ggg$ $gg \to t\bar{t}b\bar{b}g$	$u\bar{u} \to t\bar{t}b\bar{b}gg$

Status of AutoDipole - continued

- Agreements with the independent results

- $gg \to t\bar{t}gg$ $u\bar{u} \to t\bar{t}gg$ $ug \to t\bar{t}ug$ $\bar{u}g \to t\bar{t}\bar{u}g$ $gg \to t\bar{t}u\bar{u}$ $u\bar{u} \to t\bar{t}gg$

  - The modes of NLO real emission process to $pp \to t\bar{t} + 1\mathrm{jet}$
  - All dipoles completely agree with the results in

    S. Dittmaier, P. Uwer and S. Weinzierl, arXiv:0810.0452 and
    Phys.Rev.Lett.98(2007)262002,hep-ph/0703120

- $u\bar{u} \to W^+W^-gg$

  - One mode of NLO real emission process to $pp \to W^+W^- + 1\mathrm{jet}$
  - All 10 dipoles completely agree with the results in

    S. Dittmaier, S. Kallweit, P. Uwer, Phys.Rev.Lett.100(2008)062003,
    arXiv:0710.1577 [hep-ph]

4. Outlook

■ Summary

- Dipole subtraction is a general and practical procedure in NLO QCD
- We automated it in the package: AutoDipole (Version 1.0beta)
    - Mathematica code and an interface with MadGraph
- We apply it to some QCD backgrounds in LHC
    - We checked all soft/collinear safeties in many processes

      Complex ones: $gg \to t\bar{t}ggg$ $gg \to t\bar{t}b\bar{b}g$ $gg \to W^+\bar{u}dgg$
    - We obtained the complete agreement about all dipoles with the independent results in the processes,

      $gg \to t\bar{t}gg$ $\bar{u}d \to W^+W^-gg$
- The beta version is now available

■ Plan

- The complete packege is publicly available soon
- Compute new and complete NLO QCD predictions for important background at LHC
- Automate the creation of the integrated dipole

Part 2 : Exercise

1. Use of AutoDipole
2. Exercise 1: $e^- e^+ \to u\bar{u}g$
3. Exercise 2: $gg \ \to \ t\bar{t}g$
4. Outlook

1. Use of AutoDipole

- Download package and install

  - Go to directory: usr1/tmp

```
cd /usr1/tmp/
```

  - Download the package from the CAPP09 Website

```
https://https://indico.desy.de/conferenceOtherViews.py?view=standard&confId=1573
```

  - Decompress it

```
tar xvf AutoDipole_V1.0beta.tar
```

  - Install MadGraph

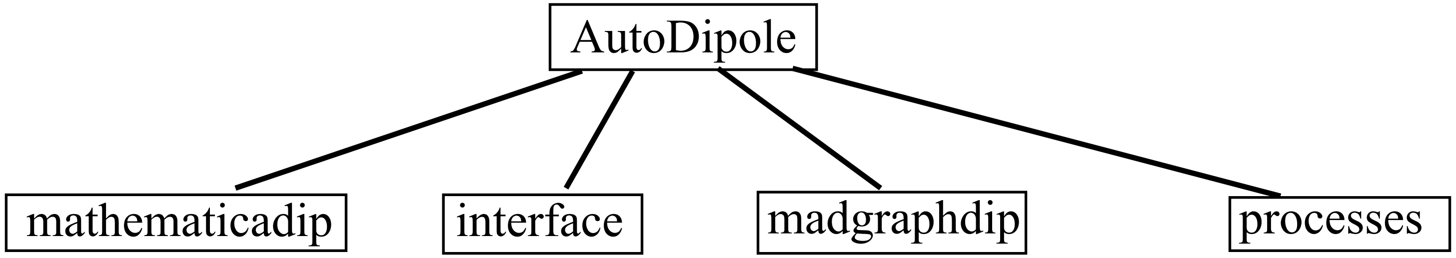
```
cd ./AutoDipole_V1.0beta/madgraphdip/MG_ME_SA_V4.2.6/MadGraphII/
```

```
make
```

  - Go back to the home directory

```
cd ../../
```

■ Directory structure



AutoDipole

mathematicadip — Mathematica code

interface — For MadGraph

madgraphdip — MadGraph (Stand Alone)

processes — Created processes

■ Execution

0. Set up : ./mathematicadip/parameter.m
   ./madgraphdip/MadGraph/ interactions.dat param_card.dat
1. Input to Mathematica code and run : ./mathematicadip/ Realprocess[{g, g}, {t, tbar, g}]
2. Run of MadGraph with interface : ./processes/ ./createdir.csh
3. Checkings : ./processes/ make checkIR

2. Exercise 1: $e^- e^+ \to u\bar{u}g$

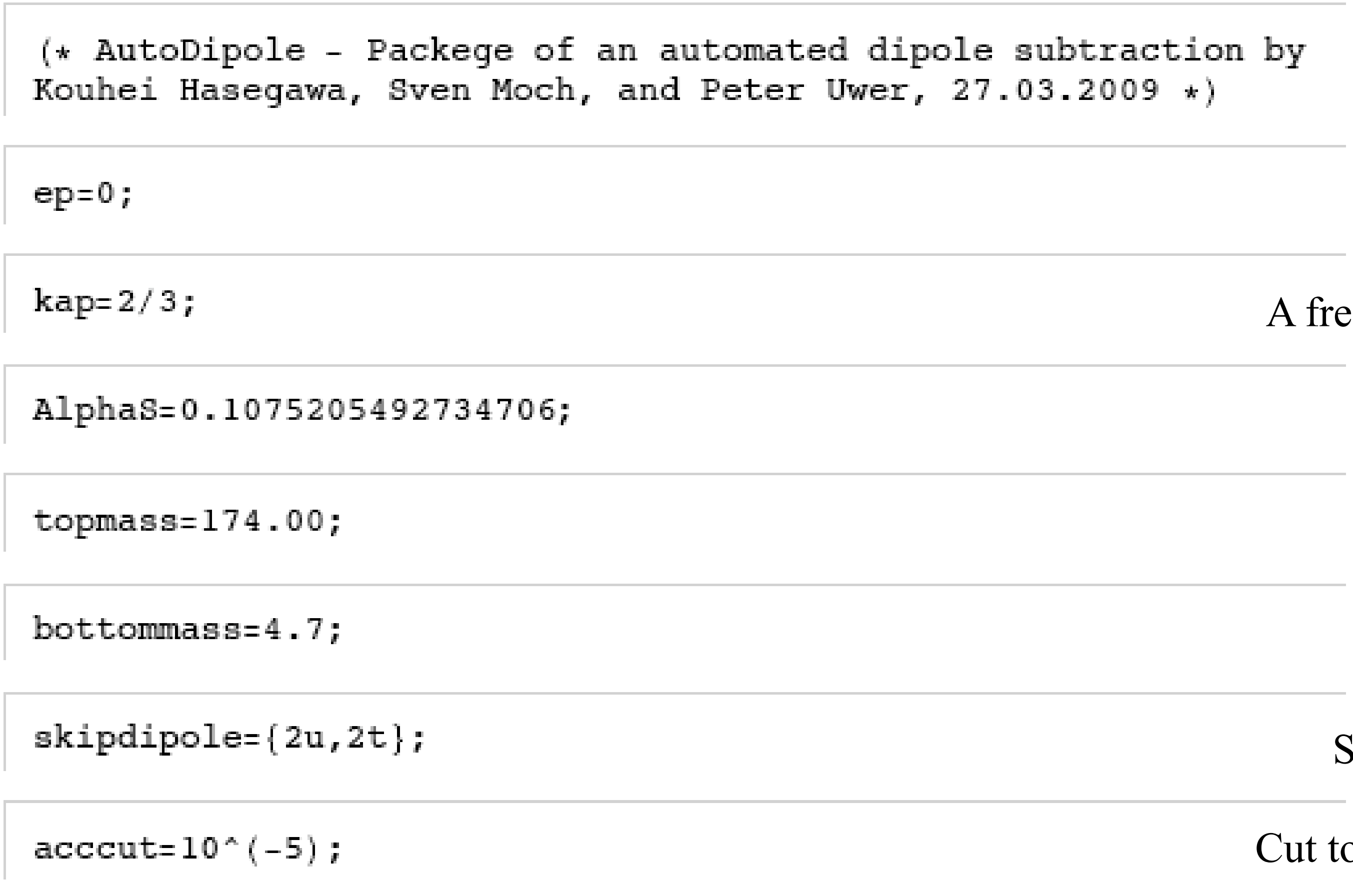
We are in mathematicadip/

- 0. Set up

  - Set up file for Mathematica code :

```
cd mathematicadip/
```

```
mathematica parameter.m&
```



```
(* AutoDipole - Packege of an automated dipole subtraction by
Kouhei Hasegawa, Sven Moch, and Peter Uwer, 27.03.2009 *)
```

```
ep=0;
```

$D = 4 - 2\epsilon$

```
kap=2/3;
```

A freedom for non-singular part

```
AlphaS=0.1075205492734706;
```

α_s

```
topmass=174.00;
```

m_{top}

```
bottommass=4.7;
```

m_{bottom}

```
skipdipole={2u,2t};
```

Splitting which is skipped

```
acccut=10^(-5);
```

Cut to define soft/collinear safeties

- Set up file for MadGraph : /madgraphdip/MG_ME_SA_V4.2.6/Models/sm/ interactions.dat
param_card.dat

- They are in MadGraph

interactions.dat

Quark-quark-gluon vertex

```
#
# QCD interactions
#

d d g GG QCD
u u g GG QCD
s s g GG QCD
c c g GG QCD
b b g GG QCD
t t g GG QCD

g g g G  QCD

g g g g G G QCD QCD

#
# QED interactions
#

#d d a GAD QED
u u a GAU QED
#s s a GAD QED
#c c a GAU QED
#b b a GAD QED
#t t a GAU QED

e-  e-  a GAL QED
#mu- mu- a GAL QED
#ta- ta- a GAL QED
      .
      .
      .
```

param_card.dat

```
Block SMINPUTS      # Standard Model inputs
    1       1.32506980E+02   # alpha_em(MZ)(-1) SM MSbar
    2       1.16639000E-05   # G_Fermi
#    3       1.18000000E-01   # alpha_s(MZ) SM MSbar
    3       0.1075205492734706E+00
    4       9.11880000E+01   # Z mass (as input parameter)
Block MGYUKAWA     # Yukawa masses m/v=y/sqrt(2)
#    PDG         YMASS
    5       4.20000000E+00   # mbottom for the Yukawa  y_b
    4       1.42000000E+00   # mcharm  for the Yukawa  y_c
    6       1.64500000E+02   # mtop    for the Yukawa  y_t
   15       1.77700000E+00   # mtau    for the Yukawa  y_ta
Block MGCKM     # CKM elements for MadGraph
    1   1     9.75000000E-01   # Vud for Cabibbo matrix
Block MASS      #  Mass spectrum (kinematic masses)
#       PDG       Mass
        5     4.70000000E+00   # bottom   pole mass
#        6     1.74300000E+02   # top      pole mass
        6     1.74000000E+02   # top
       15     1.77700000E+00   # tau      mass
       23     9.11880000E+01   # Z        mass
       24     8.04190000E+01   # W        mass
       25     1.20000000E+02   # H        mass
                    .
                    .
                    .
```

α_s

m_{bottom}

m_{top}

- The values should be consistent with ones in ./mathematicadip/parameter.m
- We use the default ones and do not have to do anything here

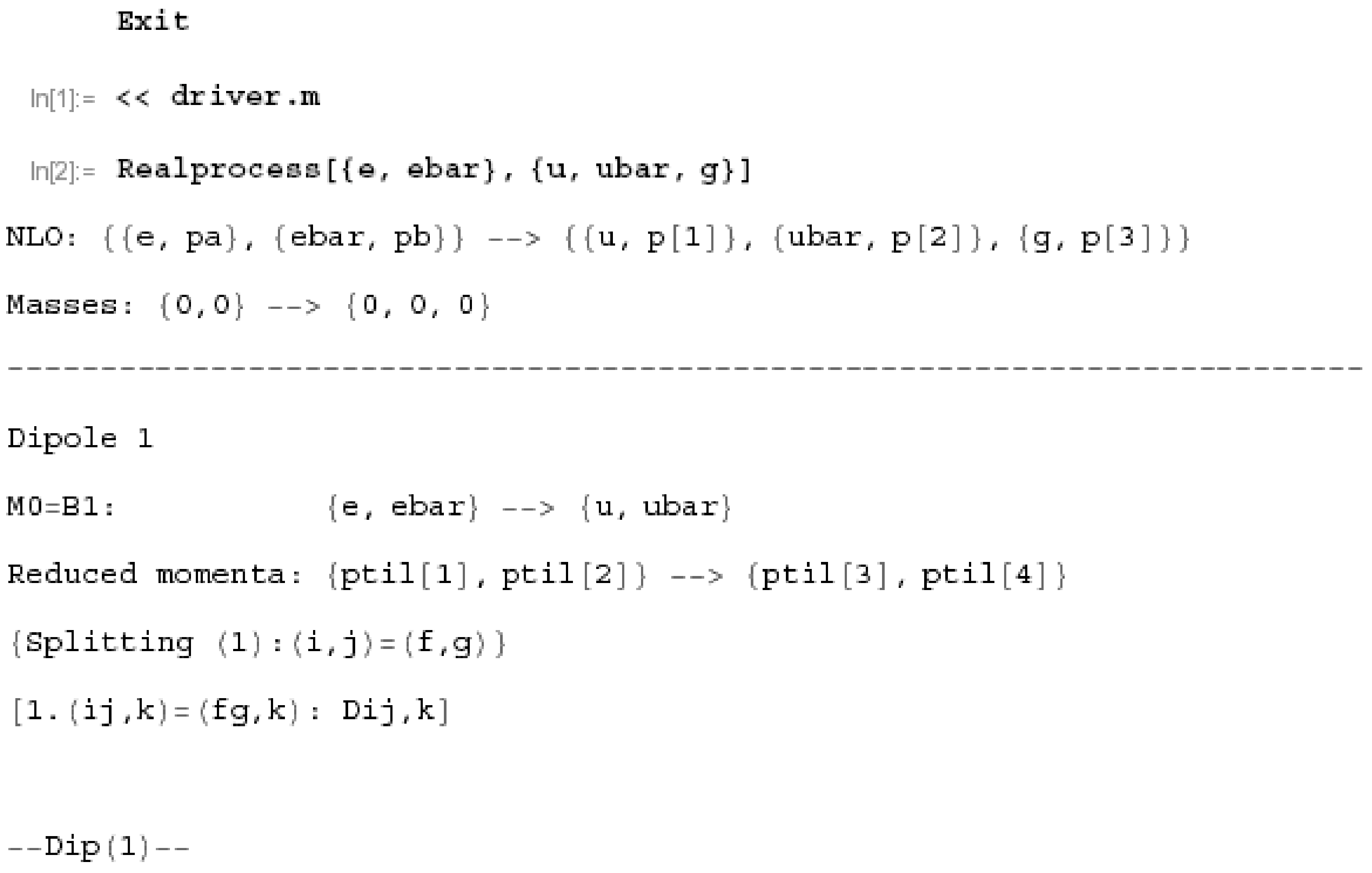
■ 1. Input to Mathematica code and run

We are in mathematicadip/

`mathematica exedip.nb&` - Open file exedip.nb

`<< driver.m` - Includes package

`Realprocess[{e, ebar}, {u, ubar, g}]` - Input real emission process and run

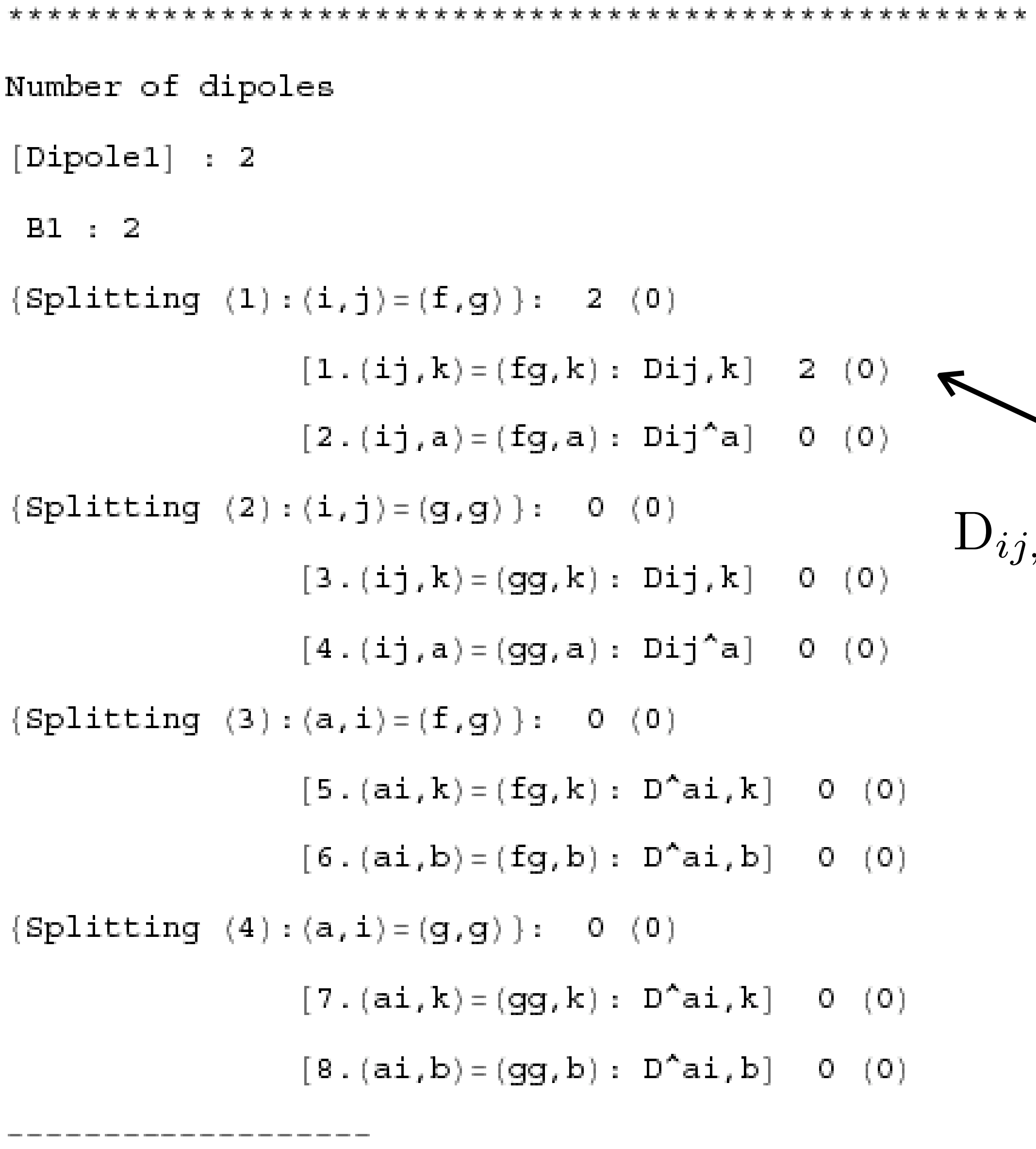


```
     Exit

In[1]:= << driver.m

In[2]:= Realprocess[{e, ebar}, {u, ubar, g}]

NLO: {{e, pa}, {ebar, pb}} --> {{u, p[1]}, {ubar, p[2]}, {g, p[3]}}

Masses: {0,0} --> {0, 0, 0}

-------------------------------------------------------------------------

Dipole 1

M0=B1:              {e, ebar} --> {u, ubar}

Reduced momenta: {ptil[1], ptil[2]} --> {ptil[3], ptil[4]}

{Splitting (1):(i,j)=(f,g)}

[1.(ij,k)=(fg,k): Dij,k]


--Dip(1)--
```

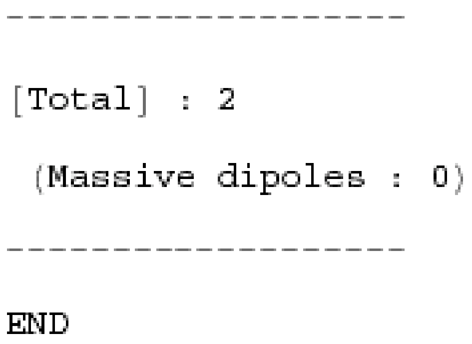
■ 1. Input to Mathematica code and run - continued

We are in processes/

- At the end of Output: Contents of dipole are shown



```
*****************************************************

Number of dipoles

[Dipole1] : 2

 B1 : 2

{Splitting (1):(i,j)=(f,g)}:  2 (0)

              [1.(ij,k)=(fg,k): Dij,k]  2 (0)

              [2.(ij,a)=(fg,a): Dij^a]  0 (0)

{Splitting (2):(i,j)=(g,g)}:  0 (0)

              [3.(ij,k)=(gg,k): Dij,k]  0 (0)

              [4.(ij,a)=(gg,a): Dij^a]  0 (0)

{Splitting (3):(a,i)=(f,g)}:  0 (0)

              [5.(ai,k)=(fg,k): D^ai,k]  0 (0)

              [6.(ai,b)=(fg,b): D^ai,b]  0 (0)

{Splitting (4):(a,i)=(g,g)}:  0 (0)

              [7.(ai,k)=(gg,k): D^ai,k]  0 (0)

              [8.(ai,b)=(gg,b): D^ai,b]  0 (0)

------------------
```



```
-------------------

[Total] : 2

 (Massive dipoles : 0)

-------------------

END
```

Type of dipole

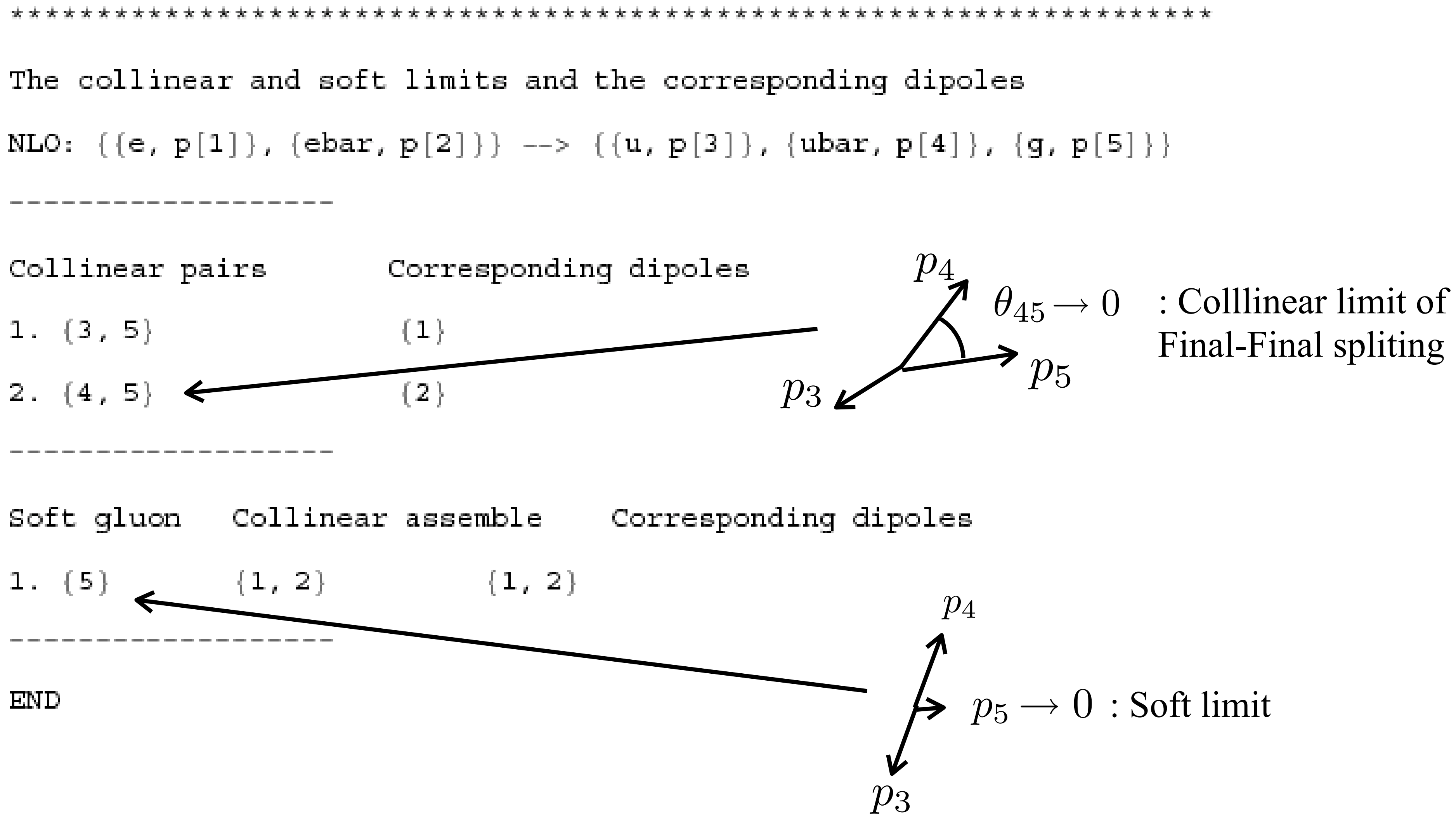
$$D_{ij,k} = D_{\text{quark gluon, something}}$$

in final state

■ 1. Input to Mathematica code and run - continued

We are in mathematicadip/

- At the end of Output: All soft/collinear limits and the corresponding dipoles are also shown



```
**********************************************************************

The collinear and soft limits and the corresponding dipoles

NLO: {{e, p[1]}, {ebar, p[2]}} --> {{u, p[3]}, {ubar, p[4]}, {g, p[5]}}

-------------------

Collinear pairs        Corresponding dipoles

1. {3, 5}               {1}

2. {4, 5}               {2}

-------------------

Soft gluon   Collinear assemble    Corresponding dipoles

1. {5}        {1, 2}             {1, 2}

-------------------

END
```

$\theta_{45} \rightarrow 0$: Colllinear limit of Final-Final spliting

$p_5 \rightarrow 0$: Soft limit

2. Run of MadGraph with interface

We are in processes/

```
cd ../processes/
```

```
./createdir.csh
```

- Directory : Proc_e-e+_uuxg is produced

This includes the closed Fortran routines to calculate $|\mathrm{M}|^2_{\mathrm{real}} - \sum_i \mathrm{D}_i$

3. Checkings

- Go to directory: Proc_e-e+_uuxg

```
cd Proc_e-e+_uuxg
```

- Check the values of the sum of all dipolles on the 10 phase space points

```
make
```

```
./check
```

$\sum \mathrm{D}_i$ $\quad$ $\sum \mathrm{D}_i/|\mathrm{M}|^2_{\mathrm{real}}$ $\quad$ $\left(|\mathrm{M}|^2_{\mathrm{real}} - \sum \mathrm{D}_i\right)/|\mathrm{M}|^2_{\mathrm{real}}$

```
             |M|^2               SumDipole               Ratio                   Accuracy
  1   0.300494427478150E-05   0.306483766095212E-05   0.101993161293315E+01   -0.199316129331459E-01
  2   0.824571736656559E-05   0.831794218053564E-05   0.100875906980063E+01   -0.875906980063354E-02
  3   0.108020837472608E-05   0.113477335925199E-05   0.105051338778941E+01   -0.505133877894137E-01
  4   0.398997680814314E-06   0.456062859950099E-06   0.114302133039801E+01   -0.143021330398015E+00
  5   0.781169201607950E-06   0.814699986512918E-06   0.104292384394565E+01   -0.429238439456497E-01
  6   0.348167959120634E-04   0.342689909364597E-04   0.984266071553875E+00    0.157339284461249E-01
  7   0.148082090268567E-05   0.156101669648346E-05   0.105415630860717E+01   -0.541563086071683E-01
  8   0.459595997449995E-06   0.531616229332302E-06   0.115670334877131E+01   -0.156703348771315E+00
  9   0.722990538823266E-06   0.804546570364715E-06   0.111280373277663E+01   -0.112803732776628E+00
 10   0.326611878319585E-04   0.329691186754487E-04   0.100942803565732E+01   -0.942803565732195E-02
```

- Check all soft/collinear limits

We are in processes/Proc_e-e+_uuxg

```
make checkIR
```

```
./checkIR
```

⟶ Output: resIRcheck

```
more resIRcheck
```

```
Cut condition: (|M|^2-SumD)/|M|^2 <  0.100000000000000E-04

 ----------Collinear limits----------
Log10(Root(S_ij/S)) < Cut(i) (i=1,2)
   1   -0.244942287339502E+01
   2   -0.348223120739498E+01
 Maximum value of Cut(i)
   -0.244942287339502E+01
 Corresponding S_ij/S
    0.126227579153667E-04
 -----------------------------------


 -----------Soft limits--------------
Log10(2*E_soft/root(s)) < Softcut(i) (i=1,1)
   1   -0.319070974314666E+01
 Maximum value of Softcut(i)
   -0.319070974314666E+01
 Corresponding (2*E_soft/root(s))^2
    0.415509075188428E-06
 -----------------------------------


 Infrared safeties of all collinear and soft limits are confirmed for
    S_ij/S > 0.126227579153667E-04
```

← This cut value is set as a parameter ‘acccut’ at parameter.m

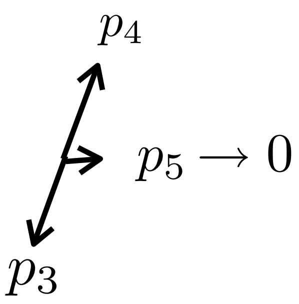
← Confirmation of all soft/collinear safeties

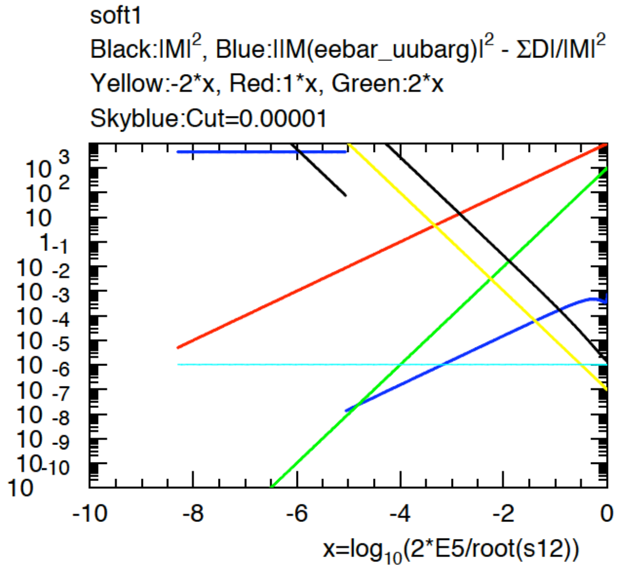
We are in processes/Proc_e-e+_uuxg

- Plots on all soft/collinear limits

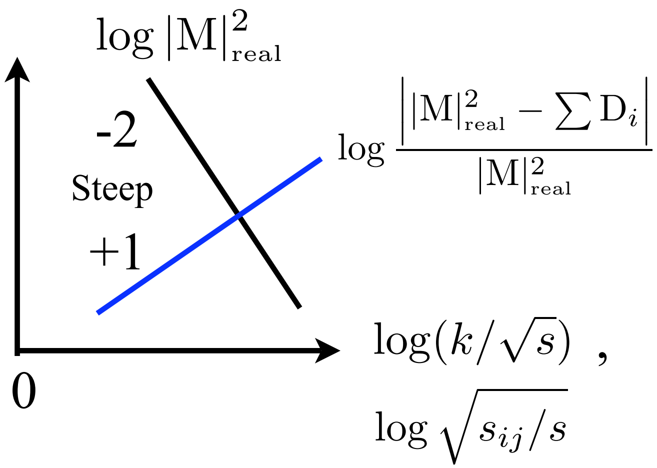
```
./plotall
```

- Soft limit

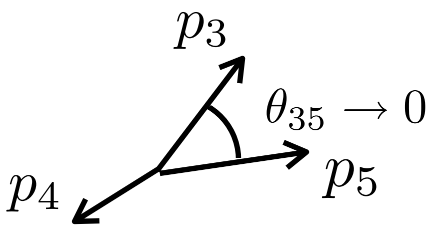


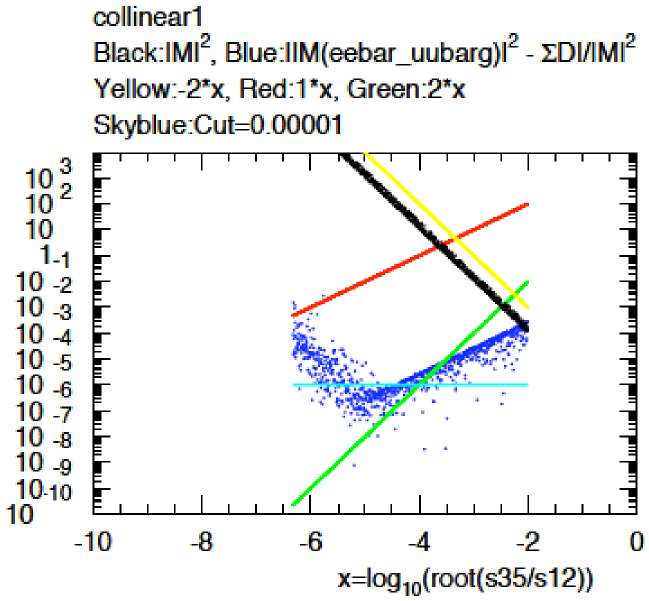


Prediction of the steep



- Collinear limit





$\theta_{45} \to 0$ limit is also similar to this case

We can easily confirm the soft/collinear safeties by seeing these plots, especially the steep

■ Available fields and notation in the present version 1.0beta

Notation for input: Realprocess[{e, ebar}, {u, ubar, g}]

- Parton	
- Quark	
$(u, \bar{u})$	(u,ubar)
$(d, \bar{d})$	(d,dbar)
$(b, \bar{b})$	(b,bbar)
$(t, \bar{t})$	(t,tbar)
- Gluon g	g
- Non-Parton	
- Lepton	
(e^-, e^+)	(e,ebar)
- Gauge boson	
γ	gamma
(W^+, W^-)	(Wp,Wm)
Z	Z

- It is straightforward to include more fields like

other quarks and leptons, Higgs boson, and super partners

- Available interactions are same with ones in MadGraph

3. Exercise 2: $gg \to t\bar{t}g$

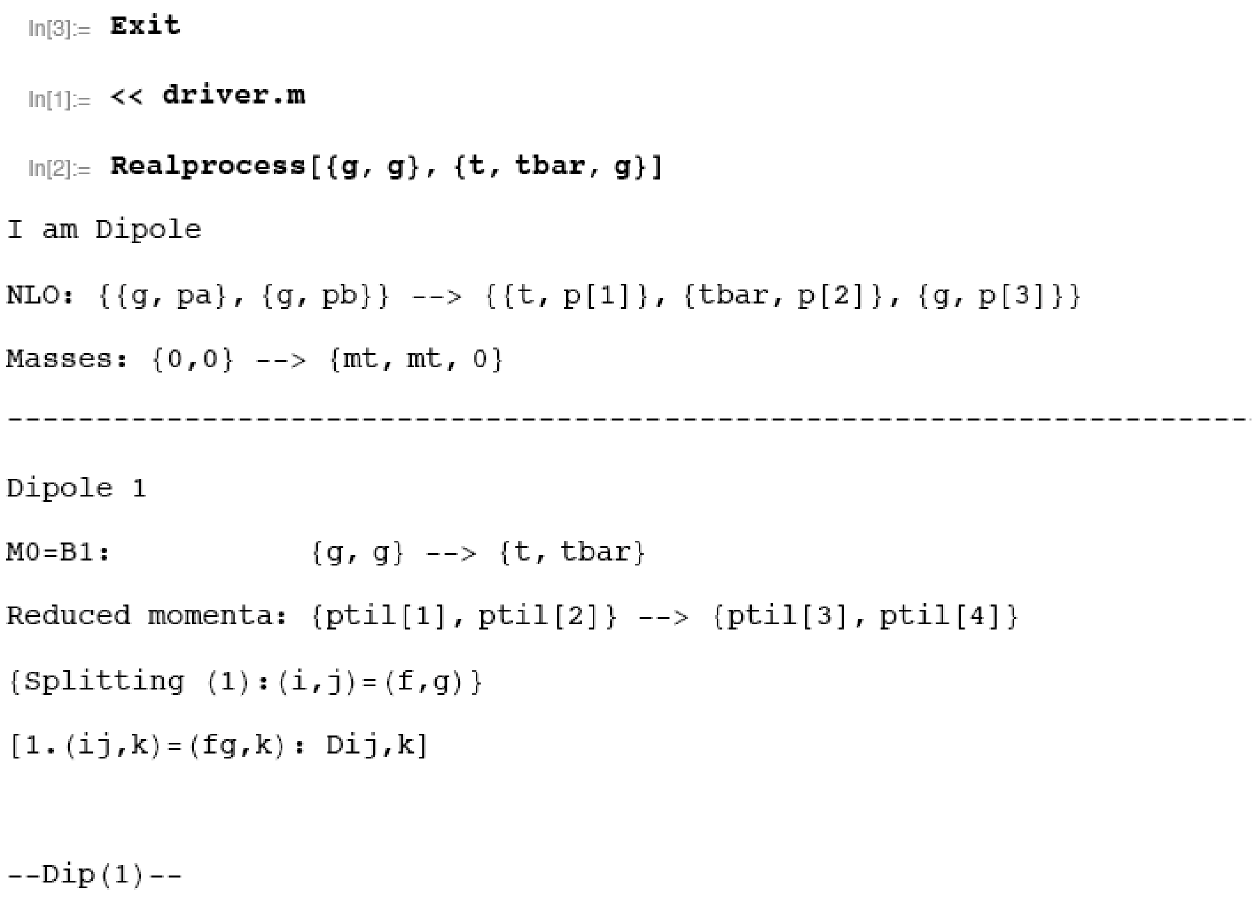
We are in mathematicadip/

- 0. Set up

  - Same with Exercise 1 skipdipole={2u,2t}; ⟶ t-tbar splitting is skipped

- 1. Input to Mathematica code and run

  Realprocess[{g, g}, {t, tbar, g}] - Input real emission process



```
In[3]:= Exit

In[1]:= << driver.m

In[2]:= Realprocess[{g, g}, {t, tbar, g}]

I am Dipole

NLO: {{g, pa}, {g, pb}} --> {{t, p[1]}, {tbar, p[2]}, {g, p[3]}}

Masses: {0,0} --> {mt, mt, 0}

----------------------------------------------------------------------

Dipole 1

M0=B1:            {g, g} --> {t, tbar}

Reduced momenta: {ptil[1], ptil[2]} --> {ptil[3], ptil[4]}

{Splitting (1):(i,j)=(f,g)}

[1.(ij,k)=(fg,k): Dij,k]


--Dip(1)--
```

■ 1. Input to Mathematica code and run - continued

We are in mathematicadip/

- At the end of Output: Contents of dipole are shown

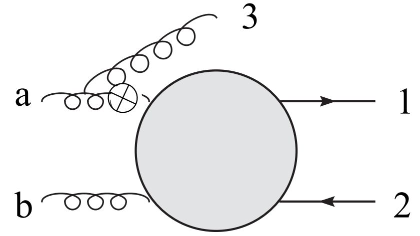
```
****************************************************

Number of dipoles

[Dipole1] : 12

 B1 : 12

{Splitting (1):(i,j)=(f,g)}:  6 (6)

               [1.(ij,k)=(fg,k): Dij,k]  2 (2)

               [2.(ij,a)=(fg,a): Dij^a]  4 (4)

{Splitting (2):(i,j)=(g,g)}:  0 (0)

               [3.(ij,k)=(gg,k): Dij,k]  0 (0)

               [4.(ij,a)=(gg,a): Dij^a]  0 (0)

{Splitting (3):(a,i)=(f,g)}:  0 (0)

               [5.(ai,k)=(fg,k): D^ai,k]  0 (0)

               [6.(ai,b)=(fg,b): D^ai,b]  0 (0)

{Splitting (4):(a,i)=(g,g)}:  6 (4)

               [7.(ai,k)=(gg,k): D^ai,k]  4 (4)

               [8.(ai,b)=(gg,b): D^ai,b]  2 (0)

-------------------
```

```
-----------------

[Total] : 12

 (Massive dipoles : 10)

-----------------

END
```

(ij, k)=(quark gluon, something in final state)

(ij, a)=(quark gluon, something in initial state)

- Gluon radiation from the initial gluon



(ai, k)=(gluon gluon, something in final state)

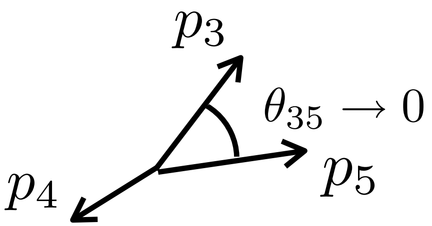
(ai, b)=(gluon gluon, something in initial state)

■ 1. Input to Mathematica code and run - continued

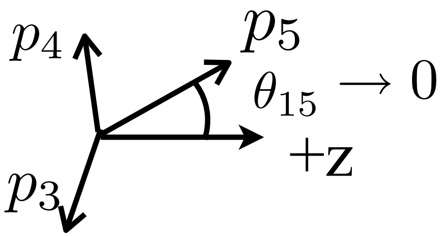
We are in mathematicadip/

-All soft/collinear limits and the corresponding dipoles are also shown

```
***************************************************************************

The collinear and soft limits and the corresponding dipoles

NLO: {{g, p[1]}, {g, p[2]}} --> {{t, p[3]}, {tbar, p[4]}, {g, p[5]}}

-------------------

Collinear pairs       Corresponding dipoles

1. {3, 5}              {1, 3, 4}

2. {4, 5}              {2, 5, 6}

3. {1, 5}              {7, 8, 11}

4. {2, 5}              {9, 10, 12}

Collinear pairs

1.

2.

include a massive quark

-------------------

Soft gluon   Collinear assemble    Corresponding dipoles

1. {5}       {1, 2, 3, 4}          {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

-------------------

END
```

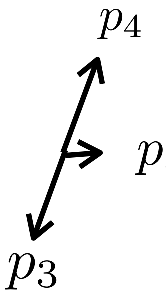


: Colllinear limit of Final-Final spliting



: Colllinear limit of Initial-Final spliting

Collinear limits 1 and 2 do not include the collinear divegences



$p_5 \rightarrow 0$: Soft limit

We are in processes/

- 2. Run of MadGraph with interface

```
cd ../processes/
```

```
./createdir.csh
```

- Directory : Proc_gg_ttxg is produced

- 3. Checkings

```
cd Proc_gg_ttxg
```

- Check the values of the sum of all dipolles on the 10 phase space points

```
make
```

```
./check
```

$\sum \mathrm{D}_i$ $\quad$ $\sum \mathrm{D}_i/|\mathrm{M}|^2_{\mathrm{real}}$ $\quad$ $\left(|\mathrm{M}|^2_{\mathrm{real}} - \sum \mathrm{D}_i\right)/|\mathrm{M}|^2_{\mathrm{real}}$

```
       IMI^2                  SumDipole              Ratio                  Accuracy
1      0.400893569363976E-03  0.405385268735139E-03  0.101120421906066E+01  -0.112042190606586E-01
2      0.554612468603335E-03  0.687236683409599E-03  0.123912952252993E+01  -0.239129522529935E+00
3      0.231759860037041E-03  0.308531272164432E-03  0.133125413570374E+01  -0.331254135703743E+00
4      0.262017095449925E-03  0.457611638297119E-03  0.174649534798990E+01  -0.746495347989895E+00
5      0.117434085443178E-03  0.161142425242084E-03  0.137219466251181E+01  -0.372194662511809E+00
6      0.267551495703035E-02  0.267541549586891E-02  0.999962825413785E+00   0.371745862150187E-04
7      0.927338018137340E-03  0.113228428952350E-02  0.122100492741344E+01  -0.221004927413437E+00
8      0.277838316144724E-03  0.509438726318734E-03  0.183357980780941E+01  -0.833579807809412E+00
9      0.353722424746050E-03  0.706243188852582E-03  0.199660281464943E+01  -0.996602814649425E+00
10     0.875738991423606E-03  0.882235650188843E-03  0.100741848750468E+01  -0.741848750467973E-02
```

■ 3. Checkings - continued

We are in processes/Proc_gg_ttxg

- Check all soft/collinear limits

```
make checkIR
```

```
./checkIR
```

⟶ Output: resIRcheck

```
more resIRcheck
```

```
Cut condition: (IMI^2-SumD)/IMI^2 <  0.100000000000000E-04

 ----------Collinear limits----------
Log10(Root(S_ij/S)) < Cut(i) (i=1,4)
    1   -0.100000000000000E+03
    2   -0.100000000000000E+03
    3   -0.195941167550387E+01
    4   -0.261212258826533E+01
 Maximum value of Cut(i)
    -0.195941167550387E+01
 Corresponding S_ij/S
     0.120552618763782E-03
 ------------------------------------

 -----------Soft limits--------------
Log10(2*E_soft/root(s)) < Softcut(i) (i=1,1)
    1   -0.130917931411337E+01
 Maximum value of Softcut(i)
    -0.130917931411337E+01
 Corresponding (2*E_soft/root(s))^2
     0.240791621767016E-02
 ------------------------------------

 Infrared safeties of all collinear and soft limits are not confirmed
```

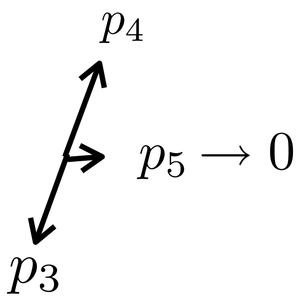
← Confirmation of all soft/collinear safeties

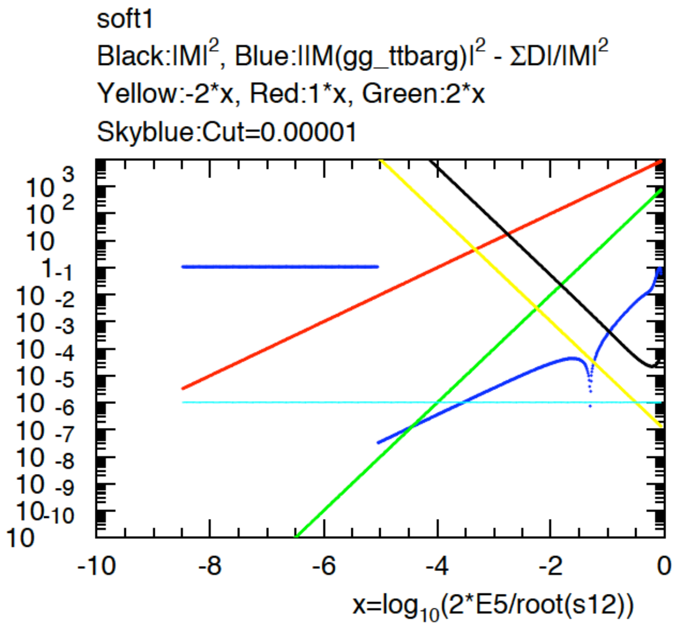
- Plots on all soft/collinear limits

We are in processes/Proc_gg_ttxg

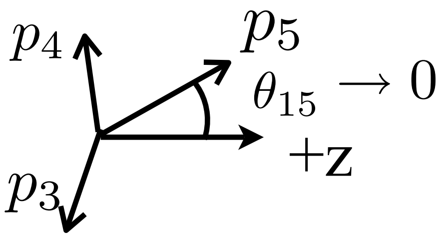
```
./plotall
```

- Soft limit

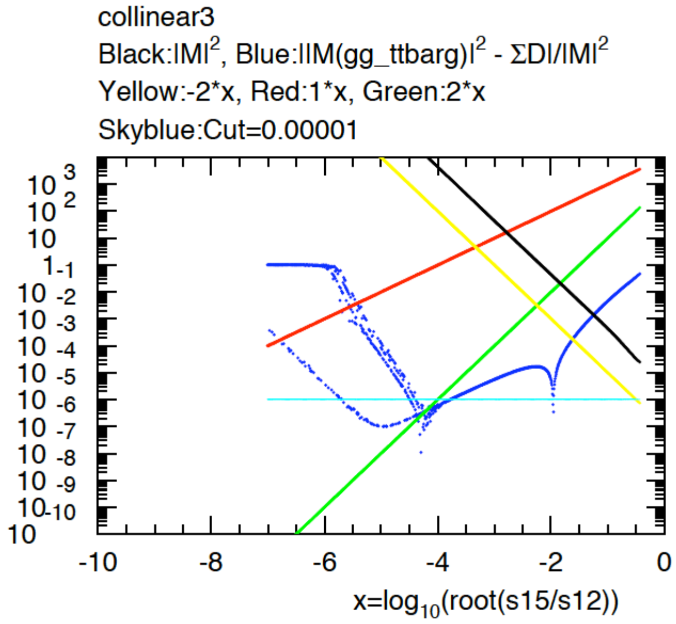




- Collinear limit



-Initial-Final spliting



We can confirm the soft/collinear safeties

4. Outlook

■ Summary

- AutoDipole Version1.0beta : An automated dipole subtraction for $|\mathrm{M}|^2_{\mathrm{real}} - \sum_i \mathrm{D}_i$
  - Mathematica code and an interface with MadGraph

- Use

  0. Set up
  1. Input to Mathematica code and run
  2. Run of MadGraph with interface
  3. Checking of all soft/collinear safeties

- We had the exercises for the processes : $e^- e^+ \to u\bar{u}g$ and $gg \to t\bar{t}g$
  - We got the all dipoles and confirmed all soft/collinear safeties

■ Further works (if you like)

- Try more complex processes like $gg \to u\bar{u}gg$
- Try the phase space integral and obtain parton and hadron level cross section

■ Future of AutoDipole

- The complete packege is publicly available soon
- Compute new and complete NLO QCD predictions for important background at LHC
- Automate the creation of the integrated dipole

Extra Slide

■ Unification of soft and collinear limits

$$|\mathrm{M}|^2_{\mathrm{real}} \xrightarrow[\text{[All soft/collinear limits]}]{} \sum_{\substack{i,j,k \\ (i \neq j \neq k)}} \mathrm{S}_{i(j),k} + \sum_{\substack{i,j \\ (i \neq j)}} \mathrm{C}_{ij}$$

$$= \sum_{\substack{i,j,k \\ (i \neq j \neq k)}} (\mathrm{S}_{i(j),k} + \mathrm{C}'_{ij,k}) = \sum_{\substack{i,j,k \\ (i \neq j \neq k)}} \mathrm{U}_{ij,k}$$

- The color consevation can split C_{ij} into $\mathrm{C}'_{ij,k}$

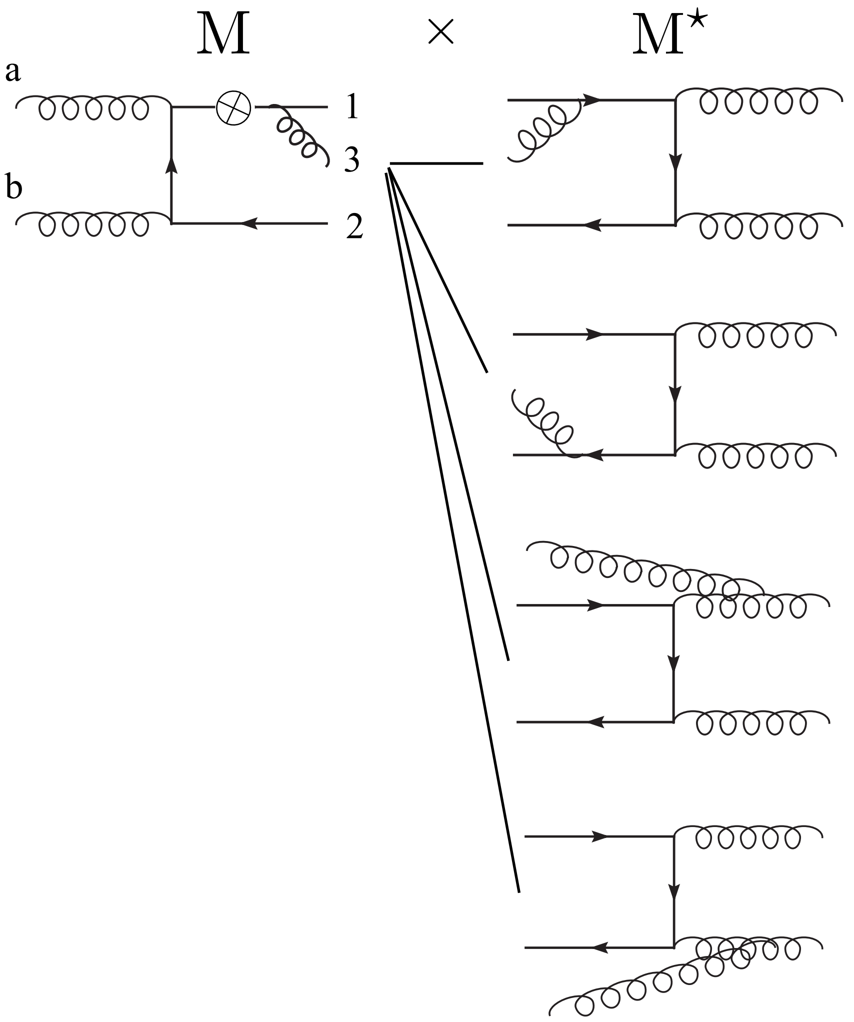
$$< 1, \cdots, ij, \cdots, m+1|\mathrm{T}_{ij} \overset{\overset{\text{Casimir: } C_F, C_A}{\downarrow}}{\cdot} \mathrm{T}_{ij}|1, \cdots, ij, \cdots m+1 >_m = -\sum_{\substack{k=1 \\ (k \neq ij)}}^{m+1} < 1, \cdots, ij, \cdots, m+1|\mathrm{T}_{ij} \cdot \mathrm{T}_k|1, \cdots, ij, \cdots m+1 >_m$$

$$\longrightarrow \quad \mathrm{C}_{ij} = \sum_k \mathrm{C}'_{ij,k}$$

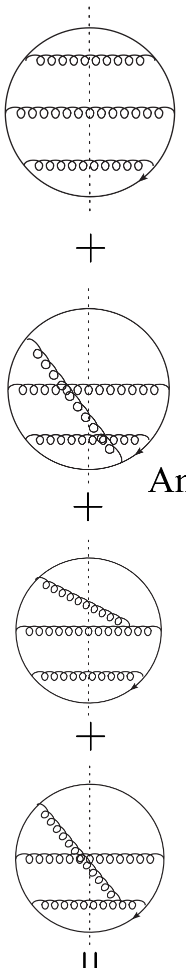
-We can construct IR safe matrix element squared as

$$|\mathrm{M}|^2_{\mathrm{real}} - \sum_{\substack{i,j,k \\ (i \neq j \neq k)}} \mathrm{U}_{ij,k}$$

-Example: NLO $gg \to u\bar{u}g$ $(i,j) = (1,3)$



Color factor



$\Delta_{13,1}$ $\mathrm{Tr}[t^a t^b t^c t^c t^b t^a]$

$+$

$\Delta_{13,2}$ $\mathrm{Tr}[t^a t^b t^c t^a t^c t^b]$ $\times(-1)$

Anti-quark

$+$

$\Delta_{13,b}$

$+$

$\Delta_{13,a}$

$\|$

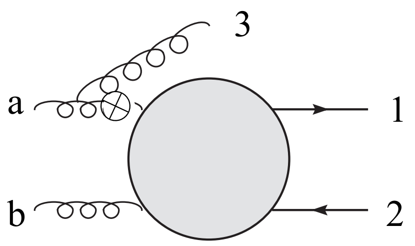
0

(Color conservation)

- We can rewrite the diagonal color factor as

$$\Delta_{13,1}\mathrm{X} = -(\Delta_{13,2} + \Delta_{13,a} + \Delta_{13,b})\mathrm{X}$$

- Emitter = gluon case



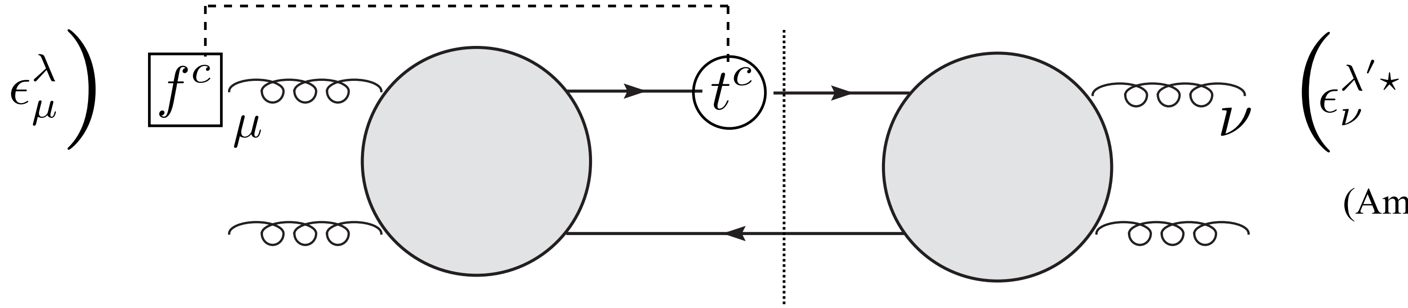
$$D^{a3}_{1}(p_1,p_2,p_3,p_a,p_b) = -\frac{1}{2p_a\cdot p_3}\,\frac{1}{x_{31,a}}\Big\langle \underset{(\mu)}{\tilde{g}g} \to \tilde{u}\bar{u}\Big|\frac{\mathrm{T}_u\cdot \mathrm{T}_{gg}}{\mathrm{T}^2_{gg}}\mathrm{V}^{a3}\underline{(\mu,\nu)}\Big|\underset{(\nu)}{\tilde{g}g} \to \tilde{u}\bar{u}\Big\rangle_2$$

splitting function:

$$\mathrm{V}^{ai}_{k}(x,u)^{\mu\nu} = 16\pi\alpha_s C_A\Big[-g^{\mu\nu}\Big(\frac{1}{1-x_{ik,a}+u_i}-1+x_{ik,a}(1-x_{ik,a})\Big)+\frac{1-x_{ik,a}}{x_{ik,a}}\,\frac{u_i(1-u_i)}{p_i\cdot p_k}\Big(\frac{p_i^{\mu}}{u_i}-\frac{p_k^{\mu}}{1-u_i}\Big)\Big(\frac{p_i^{\nu}}{u_i}-\frac{p_k^{\nu}}{1-u_i}\Big)\Big]$$

$$\times$$

Color linked Born squared (CLBS)



(Amputate polarization vector)

Different helicity squared (DHS)