Two-loop bosonic electroweak calculation to the weak mixing angle of the Zbb Vertex

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Outline

Asymmetries measured at the Z pole

We study the process $e^+e^- \to (Z) \to b\bar{b}$

Pseudo-observables, unfolded at the Z peak

forward-backward asymmetry $A_{\mathrm{FB}}^{\mathrm{b\bar{b}},0} = \frac{3}{4} A_{\mathrm{e}} A_{\mathrm{b}}$

f-b left-right asymmetry $A_{\mathrm{FB,LR}}^{\mathrm{b\bar{b}},0} = \frac{3}{4} P_{\mathrm{e}} A_{\mathrm{b}}$, P_{e} is the electron polarization

$$A_{\mathrm{b}} = \frac{2\Re e \frac{v_b}{a_b}}{1 + \left(\Re e \frac{v_b}{a_b}\right)^2} = \frac{1 - 4|Q_b| \sin^2\theta_{\mathrm{eff}}^{\mathrm{b}}}{1 - 4|Q_b| \sin^2\theta_{\mathrm{eff}}^{\mathrm{b}} + 8Q_b^2 (\sin^2\theta_{\mathrm{eff}}^{\mathrm{b}})^2} \quad (1)$$

Definition of the effective weak mixing angle

$$\sin^2\theta_{\mathrm{eff}}^{\mathrm{b}} = \frac{1}{4|Q_b|} \left(1 - \Re e \frac{v_b}{a_b}\right) \quad (2)$$

v_b and a_b are effective vector coupling and axial-vector coupling of the $Zb\bar{b}$ vertex

Pole scheme, accuracy calculations

- The ill defined Z-propagator is replaced by the Breit-Wigner function

$$\frac{1}{s - M_Z^2 + i\varepsilon}\Big|_{s=M_Z^2} \rightarrow \frac{1}{s - \overline{M}_Z^2 + i\overline{M}_Z\overline{\Gamma}_Z}\Big|_{s=M_Z^2} \qquad (3)$$

- The width $\overline{\Gamma}_Z$ is now a prediction of the theory and mixes the diagrams
- In the pole scheme, near the Z pole, the amplitude is written as

$$\mathcal{A}^{e^+e^-\to b\bar{b}} = \frac{R}{s - s_0} + S + (s - s_0)S' + \ldots, \quad s_0 = \overline{M}_Z^2 - i\overline{M}_Z\overline{\Gamma}_Z \qquad (4)$$

- Due to the analyticity of the S-matrix, $R, S, S', \ldots, s_0$ are individually gauge-invariant and UV-finite. IR-finite, when soft and collinear real photon emission is added. [Willenbrock, Valencia,1991] [Sirlin,1991] [Stuart,1991] [Riemann, 1991, 1992] [H. Veltman,1994] [Passera, Sirlin, 1998] [Gambino, Grassi, 2000] [Bohm, Harshman, 2000].
- Experiment: $\mathcal{A} \propto \frac{1}{s - M_Z^2 + is\Gamma_Z/M_Z}$
  $\overline{M}_Z^2 = M_Z^2/(1 + \Gamma_Z^2/M_Z^2)$, $\overline{M}_Z \approx M_Z - 34$ MeV.
  $\overline{\Gamma}_Z^2 = \Gamma_Z^2/(1 + \Gamma_Z^2/M_Z^2)$, $\overline{\Gamma}_Z \approx \Gamma_Z - 1$ MeV [Bardin, Leike, Riemann, Sachwitz, 1988]

Vertex form factor

- The $\sin^2\theta_{\rm eff}^{\rm b}$ is contained in the residue R in $\mathcal{A}^{[e^+e^-\to b\bar{b}]}$
- The Residue R of $\mathcal{A}^{[e^+e^-\to b\bar{b}]}$ factorizes into initial- and final state vertex form factors and Z-propagator corrections
- Calculation of $A_{\rm b}$ rests on the calculation of the vertex form factor

$$V_\mu^{Zb\bar{b}} = \gamma_\mu[\hat{v}_b(s) - \hat{a}_b(s)\gamma_5] \tag{5}$$

- The effective vector and axial-vector components can be projected via

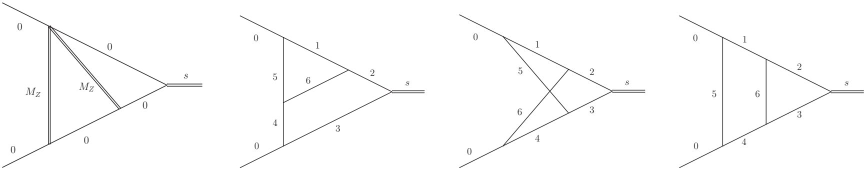
$$\hat{v}_b(s) = \frac{1}{2(2-D)s}\mathrm{Tr}[\gamma^\mu \not{p}_1 V_\mu^{Zb\bar{b}} \not{p}_2] \tag{6}$$

$$\hat{a}_b(s) = \frac{1}{2(2-D)s}\mathrm{Tr}[\gamma_5\gamma^\mu \not{p}_1 V_\mu^{Zb\bar{b}} \not{p}_2] \tag{7}$$

- $D = 4 - 2\epsilon$ is the space-time dimension
- $p_{1,2}$ are the momenta of the external b-quarks and $s = (p_1 + p_2)^2$
- The hat in $\hat{v}_b(s)$ and $\hat{a}_b(s)$ donates the $Z-\gamma$ mixing

Samples of Feynman integral topologies for the $Z\bar{b}b$ vertex

After projection only scalar integrals remain, but may contain non-trivial combinations of scalar products in the numerator.



(a) (b) (c) (d)

Some of most difficult cases:

- (b) $m_4 = M_Z$ and $m_1 = M_W, m_t$ and $m_5 = m_6 = m_t, M_W$
- (c) $m_1 = m_4 = M_Z$
- (d) $m_5 = M_Z$ and $m_6 = M_W, m_t$ and $m_2 = m_3 = m_t, M_W$

Two independent numerical calculations

- The integrals contain up to three dimensionless parameters

$$\left\{ \frac{M_H^2}{M_Z^2}, \frac{M_W^2}{M_Z^2}, \frac{m_t^2}{M_Z^2}, \frac{(M_Z + i\varepsilon)^2}{M_Z^2} \right\} \tag{8}$$

- Many of them contain ultraviolet and infrared singularities, even though the divergences cancel in the final result
- In general, it is not possible to compute all integrals analytically with available methods and tools, but instead one has to resort to numerical integration strategies
- The aim is to obtain eight significant digits, to be obtained with two completely independent calculations

Numerical Methods

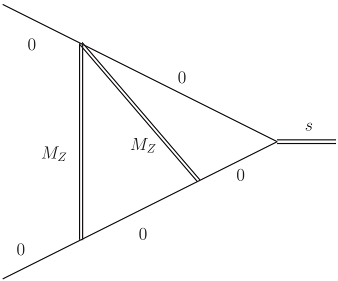
Sector decomposition

FIESTA 3 [A.V.Smirnov, 2014] and SecDec 3 [Borowka, et. al., 2015]

Mellin-Barnes integral approach

- With AMBRE 2 [Gluza, et. al., 2011] (AMBRE 3 [Dubovyk, et. al., 2015]) we derived Mellin-Barnes representations for planar (non-planar) topologies. One may use PlanarityTest [Bielas, et. al, 2013] for automatic identification.
- Expansion in terms of $\epsilon = (4 - D)/2$ is done with MB [Czakon, 2006], MBresolve [A. Smirnov, V. Smirnov, 2009], barnesroutines (D. Kosower).
- For the numerical treatment of massive Mellin-Barnes integrals with Minkowskian kinematics, the package MBnumerics [Dubovyk, Riemann, Usovitsch] is being developed since 2015.

Mellin-Barnes Coefficients in the ϵ expansion



- Known analytic result for Master integrals in [Aglietti, Bonciani,2004]
- We will now study this integral with one irreducible numerator

$$\mathcal{I}(z_1, z_2) = \frac{-(-\frac{s}{M_Z^2})^{1+z_1}\Gamma[-z_1]\Gamma[1+z_1]^2\Gamma[1+z_1-z_2]\Gamma[-z_2]\Gamma[1+z_2]^3\Gamma[-z_1+z_2]}{2\Gamma[3+z_1]\Gamma[1-z_1+z_2]\Gamma[2+z_1+z_2]} \tag{9}$$

- Mellin-Barnes integration variables $z_i = x_i + i\, t_i$, where the x_i are fixed and $t_i \in (-\infty, +\infty)$
- Here $x_1 = -\frac{2}{3}$, $x_2 = -\frac{1}{3}$

Numerical integration in MB.m and MBnumerics.m

$$z_i^{\mathrm{L}} = x_i + i\,\ln\left(\frac{t_i}{1-t_i}\right),\ t_i \in (0,1),\ \text{Jacobians: } J_i^{\mathrm{L}} = \frac{i}{t_i(1-t_i)} \quad (10)$$

$$I^{\mathrm{MB.m}} = \frac{1}{(2\pi i)^2}\int_0^1\int_0^1 J_1^{\mathrm{L}} J_2^{\mathrm{L}}\,\mathcal{I}\left(z_1^{\mathrm{L}}, z_2^{\mathrm{L}}\right)\mathrm{d}t_1\mathrm{d}t_2 \quad (11)$$

$$z_i^{\mathrm{T}} = x_i + n_i + \frac{(i+\theta)}{\tan(-\pi\, t_i)},\ t_i \in (0,1),\ \text{Jacobians: } J_i^{\mathrm{T}} = \frac{(i+\theta)\pi}{\sin^2(\pi\, t_i)} \quad (12)$$

$$I^{\mathrm{MBnumerics.m}}(n_1, n_2) = \frac{1}{(2\pi i)^2}\int_0^1\int_0^1 J_1^{\mathrm{T}} J_2^{\mathrm{T}}\,\hat{\mathcal{I}}\left(z_1^{\mathrm{T}}, z_2^{\mathrm{T}}\right)\mathrm{d}t_1\mathrm{d}t_2 \quad (13)$$

- $n_i \in \mathrm{Integers}$, shifts [Anastasiou, Daleo, 2006]
- $\theta \in \mathrm{Reals}$, rotations [Freitas, Huang, 2010]
- tangent mapping imposes $\mathcal{I}[\prod_i \Gamma_i] \to \hat{\mathcal{I}}[e^{\sum_i \ln(\Gamma_i)}]$
- $I^{\mathrm{MBnumerics.m}}(n_1, n_2)$ is now a discrete function in n_i

Asymptotic behavior in generalized spherical coordinates for $r \to \infty$

Euclidean kinematics

$$\Re e \left(\lim_{r \to \infty} \mathcal{I} \right) \approx \frac{e^{-\beta\, r}}{r^{\alpha}}, \ \beta > 0 \text{ and } \alpha \text{ arbitrary} \tag{14}$$

for any angular direction

Minkowskian kinematics (physical momenta)

$$\Re e \left(\lim_{r \to \infty} \mathcal{I} \right) \approx \frac{1}{r^{\alpha}}, \ \alpha \text{ arbitrary} \tag{15}$$

- logarithmic mapping always has an infinity at the boundary
- For $\alpha \geq 2$ tangent mapping has no infinities at the boundaries
- $\alpha < 2$ in either mapping the integrand is not absolutely convergent

Shifts and rotations

Contour rotations θ. The transformations $z_i = x_i + (i + \theta)\, t_i$ do not cross poles and may introduce $\beta > 0$ for any angular direction in Minkowskian kinematics

Contour shifts n_i. Treat the Mellin-Barnes integrals as discrete functions: $z_i = x_i + n_i + i\, t_i$.

- May improve convergence by shifting: $\Re e(\lim_{r \to \infty} \mathcal{I}) \approx \frac{1}{r^\alpha}\ \alpha \geq 2$.
- Add up all crossed poles (integrals with one dimension less)
- May reduce the order of magnitude of the shifted integral
- The shifted integral and its poor knowledge becomes numerically less important
- In effect, the procedure consists of a summing over a finite number of residues with a controlled remainder.

Numerical effects

Point in the kinematics: $s = 3 + i10^{-16}$, $M_Z^2 = 1$

$$
\begin{aligned}
I^{\text{MB.m}} &= 0.069619\mathbf{0}689628302 + 1.705511\mathbf{8}49228807\, i \\
I^{\text{tangent}} &= 0.0696190691\mathbf{5}06288 + 1.705511853\mathbf{8}46761\, i \\
I^{\text{logarithm}}_{\text{rotation}} &= 0.069619069154\mathbf{5}005 + 1.7055118538396\mathbf{7}5\, i \\
I^{\text{tangent}}_{\text{rotation}} &= 0.0696190691545\mathbf{01}4 + 1.7055118538396\mathbf{7}3\, i \\
I^{\text{MBnumerics.m}} &= 0.0696190691545\mathbf{01}4 + 1.7055118538396\mathbf{7}1\, i
\end{aligned} \tag{16}
$$

- Best optimization in MB.m is used
- 500 000 evaluation points are used
- All calculations are evaluated with CUHRE (CUBA) [Hahn, 2015]
- Since $\frac{1}{t^\alpha}$, $\alpha > 2$, tangent mapping gives improvement
- After contour rotation the mapping is not mandatory
- MBnumerics.m takes control over shifts, rotations, mappings

Effective weak mixing angle $\sin^2\theta_{\mathrm{eff}}^{\mathrm{b}}$

- The standard model prediction for the effective weak mixing angle can be written as

$$\sin^2\theta_{\mathrm{eff}}^{\mathrm{b}} = \left(1 - \frac{M_W^2}{M_Z^2}\right)(1 + \Delta\kappa_{\mathrm{b}}) \tag{17}$$

- The bosonic electroweak two-loop corrections amount to

$$\Delta\kappa_{\mathrm{b}}^{(\alpha^2,\mathrm{b}os)} = -0.9855 \times 10^{-4} \tag{18}$$

Collection of radiative corrections

Order	Value $[10^{-4}]$	Order	Value $[10^{-4}]$
α	468.945	$\alpha_t^2\alpha_s$	1.362
$\alpha\alpha_s$	-42.655	α_t^3	0.123
$\alpha_t\alpha_s^2$	-7.074	α_{ferm}^2	3.866
$\alpha_t\alpha_s^3$	-1.196	α_{bos}^2	-0.986

Table : *Comparison of different orders of radiative corrections to* $\Delta\kappa_{\text{b}}$

- Input Parameters: M_Z, Γ_Z, M_W, Γ_W, M_H, m_t, α_s and $\Delta\alpha$
- $\Delta\alpha$ is the shift of the electromagnetic fine structure constant due to light fermion loops between the scales $q^2 = 0$ and M_Z^2
- one-loop contributions [Akhundov, Bardin, Riemann, 1986] [Beenakker, Hollik, 1988]
- two-loop fermionic contributions [Awramik, Czakon, Freitas, Kniehl, 2009]
- two-loop bosonic contributions [Dubovyk, Freitas, Riemann, Usovitsch, 2016]

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$\mathcal{O}(\alpha\alpha_s)$ QCD corrections:

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Partial higher order corrections of order $\mathcal{O}(\alpha_t\alpha_s^2)$:

[1] L. Avdeev, J. Fleischer, S. Mikhailov, O. Tarasov, $O(\alpha\alpha_s^2)$ correction to the electroweak ρ parameter, Phys. Lett. B336 (1994) 560–566, Erratum: Phys. Lett. B349 (1995) 597. arXiv:hep-ph/9406363, doi:10.1016/0370-2693(94)90573-8.

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Simple fitting formula

$$\Delta\kappa_{\rm b}^{(\alpha^2,{\rm bos})} = k_0 + k_1\, c_{\rm H} + k_2\, c_{\rm t} + k_3\, c_{\rm t}^2 + k_4\, c_{\rm H}\, c_{\rm t} + k_5\, c_{\rm W} \tag{19}$$

$$c_{\rm H} = \log\left(\frac{M_{\rm H}}{M_{\rm Z}} \times \frac{91.1876{\rm GeV}}{125.1{\rm GeV}}\right)$$

$$c_{\rm t} = \left(\frac{m_{\rm t}}{M_{\rm Z}} \times \frac{91.1876{\rm GeV}}{173.2{\rm GeV}}\right)^2 - 1 \tag{20}$$

$$c_{\rm W} = \left(\frac{M_{\rm W}}{M_{\rm Z}} \times \frac{91.1876{\rm GeV}}{80.385{\rm GeV}}\right)^2 - 1$$

$$k_0 = -0.98605 \times 10^{-4}, \quad k_1 = 0.3342 \times 10^{-4}, \quad k_2 = 1.3882 \times 10^{-4},$$
$$k_3 = -1.7497 \times 10^{-4}, \quad k_4 = -0.4934 \times 10^{-4}, \quad k_5 = -9.930 \times 10^{-4} \tag{21}$$

The deviations to the full calculation amount to average (maximal) 5×10^{-8} (1.2×10^{-7}), in the input parameter ranges.

Currently most precise prediction for $\sin^2\theta^{\rm b}_{\rm eff}$

$$\begin{aligned}\sin^2\theta^{\rm b}_{\rm eff} =& s_0 + d_1\,L_H + d_2\,L_H^2 + d_3\,\Delta_\alpha + d_4\,\Delta_t + d_5\,\Delta_t^2 \\ &+ d_6\,\Delta_t\,L_H + d_7\,\Delta_{\alpha_{\rm s}} + d_8\,\Delta_t\,\Delta_{\alpha_{\rm s}} + d_9\,\Delta_Z\end{aligned} \tag{22}$$

$$\begin{aligned}&L_H = \log\left(\frac{M_{\rm H}}{125.7{\rm Ge}V}\right), \quad \Delta_t = \left(\frac{m_{\rm t}}{173.2{\rm Ge}V}\right)^2 - 1, \quad \Delta_Z = \frac{M_{\rm Z}}{91.1876{\rm Ge}V} - 1, \\ &\Delta_\alpha = \frac{\Delta\alpha}{0.0059} - 1, \qquad \Delta_{\alpha_{\rm s}} = \frac{\alpha_{\rm s}}{0.1184} - 1.\end{aligned} \tag{23}$$

$$\begin{aligned}s_0 = 0.232704,\quad & d_1 = 4.723\times10^{-4}, & d_2 = 1.97\times10^{-4}, \quad & d_3 = 2.07\times10^{-2}, \\ & d_4 = -9.733\times10^{-4}, & d_5 = 3.93\times10^{-4}, \quad & d_6 = -1.38\times10^{-4}, \\ & d_7 = 2.42\times10^{-4}, & d_8 = -8.10\times10^{-4}, \quad & d_9 = -0.664.\end{aligned} \tag{24}$$

- $M_{\rm W}$ is calculated from the Fermi constant G_μ [Awramik, et al., 2004]
- The deviations to the full calculation amount to average (maximal) 2×10^{-7} (1.3×10^{-6}), in the input parameter ranges.

Conclusions and Outlook

- We calculate the asymmetry parameter A_{b} which can be related to the asymmetric pseudo-observables
- The calculation is done in the pole scheme
- The main challenge was the calculation of massive two-loop vertex diagrams
- New automatized tools AMBRE 3 and MBnumerics for the evaluation of the Mellin-Barnes integrals in Minkowskian kinematics together with sector decomposition programs SecDec 3 and Fiesta 3 are sufficient to calculate all integrals
- No reduction of integrals to masters
- Final calculation at two-loop order to the electroweak effective weak mixing angle $\sin^2\theta^{\mathrm{b}}_{\mathrm{eff}}$ is presented as a simple fitting formula
- Next - and last - steps: Γ_{Zbb}, $\Gamma_{Z_{\mathrm{tot}}}$