RG analysis at higher orders in perturbativ QFTs in CMP

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[arXiv:1605.09423] & [arXiv:1703.08801] & [arXiv:1709.05057]

Overview

Motivation

- Obtain reliable & precise predictions
- In HEP the SM (QFT with $\mathcal{L}_{\mathrm{SM}}$) provides collider observables
- New physics $\mathcal{L}_{\text{“real”}} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_X$?
- New physics includes realization of $SUSY$? (MSSM, nMSSM, ...)
- Origin of $\mathcal{L}_{\mathrm{SM}}$?
- Origin of QFTs?

How to proceed?

- Build and run LHC to find new physics
- Calculate observables at higher order in PT
- Investigate on various QFTs (may not have HEP relevancy)

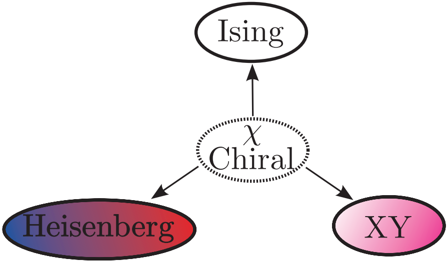
Work in collaboration with...

(from west to east)

- *Simon Fraser University (Burnaby)*
  Igor Herbut
- *University of Alberta (Edmonton)*
  Joseph Maciejko
  Chien-Hung Lin
- *University of Heidelberg*
  Michael Scherer ($\rightarrow$ Köln)
  Luminita Mihaila
  Bernhard Ihrig ($\rightarrow$ Köln)
- *DESY Zeuthen*
  Peter Marquard

Content

- Perturbativ RGE analysis at 4-loops around $D = 4 - \epsilon$
- Gross-Neveu-Yukawa Models



Models

- Emerge as LEET in condensed matter physics

⚠ Strongly coupled systems in $D = 2 + 1$

- Dirac systems with N ψ's coupled to a bosonic order parameter ϕ
- Lorenzian Symmetry
- Fermion Kinetic Term in $\mathcal{L}$ (no gap)

$$\mathcal{L}_\psi = \bar{\psi}(x)\not{\partial}\psi(x)\,.$$

$\not{\partial} = \gamma^\mu \cdot \partial_\mu$.
+Clifford algebra: $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$.
(Ex.Rep.: $\gamma^0 = \sigma_3, \gamma^1 = \sigma_1, \gamma^2 = \sigma_2$)

- Global Symmetry/ DoF[ϕ] $\rightarrow \{\mathrm{Ising}, \mathrm{Heisenberg}, \mathrm{XY}\}$

Models

- $\phi \in \mathbb{R}$ (1 DoF) & 'Global Z_2' (no cubics in pot.)$\rightarrow$ χ-Ising

$$\mathcal{L}_{\chi I} = \mathcal{L}_\psi + g\phi\bar{\psi}\psi + \tfrac{1}{2}\phi(m^2 - \partial_\mu^2)\phi + \lambda\phi^4 \,.$$

- $\phi \in \mathbb{C}$ (2 DoF) & Global U_1 $\rightarrow$ χ-XY

$$\mathcal{L}_{\chi XY} = \mathcal{L}_\psi + g\phi\bar{\psi}P_+\psi + g\phi^*\bar{\psi}P_-\psi + |\partial_\mu\phi|^2 + m^2|\phi|^2 + \lambda|\phi|^4 \,.$$

- $\vec{\phi} \in \mathbb{R}^3$ (3 DoF) & Global SU_2 $\rightarrow$ χ-Heisenberg

$$\mathcal{L}_{\chi H} = \mathcal{L}_\psi + g\,\bar{\psi}(\vec{\phi}\cdot\vec{\sigma})\psi + \tfrac{1}{2}\vec{\phi}\cdot\left(m^2 - \partial_\mu^2\right)\vec{\phi} + \lambda\left(\vec{\phi}\cdot\vec{\phi}\right)^2 \,.$$

✓ Renormalizability around $D = 4$

Renormalization

- All UV divergences absorbable via $\overline{\text{MS}}$ field and coupling redefinitions

$$\begin{aligned} \phi \to \phi^0 &= \sqrt{Z_\phi}\phi\,, & \psi \to \psi^0 &= \sqrt{Z_\psi}\psi\,, \\ \lambda \to \lambda^0 &= \mu^\epsilon Z_\lambda \lambda\,, & g \to g^0 &= \mu^{\epsilon/2} Z_g g\,. \end{aligned}$$

- With

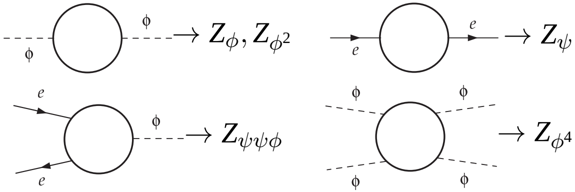
$$Z_{\phi^4} = Z_\lambda Z_\phi^2\,, \qquad Z_{\psi\psi\phi} = Z_g \sqrt{Z_\phi} Z_\psi\,, \qquad Z_{\phi^2} = Z_\phi Z_{m^2}\,,$$

renormalized Lagrangians read

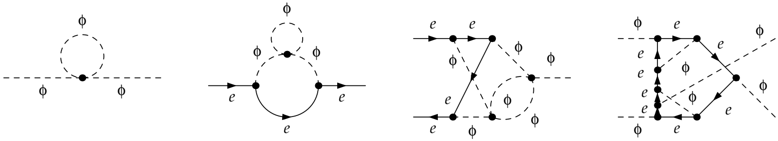
$$\begin{aligned} \mathcal{L}_{\chi I} &= Z_\psi \mathcal{L}_\psi + Z_{\psi\psi\phi} g \phi \bar\psi \psi + \tfrac{1}{2}\phi(Z_{\phi^2} m^2 - Z_\phi \partial_\mu^2)\phi + Z_{\phi^4}\lambda\phi^4\,. \\ \mathcal{L}_{\chi XY} &= Z_\psi \mathcal{L}_\psi + Z_{\psi\psi\phi} g(\phi\bar\psi P_+\psi + \phi^*\bar\psi P_-\psi) \\ &\quad + Z_\phi|\partial_\mu\phi|^2 + Z_{\phi^2} m^2|\phi|^2 + Z_{\phi^4}\lambda|\phi|^4\,. \\ \mathcal{L}_{\chi H} &= \mathcal{L}_\psi + Z_{\psi\psi\phi} g\,\bar\psi(\vec\phi\cdot\vec\sigma)\psi \\ &\quad + \tfrac{1}{2}\vec\phi\cdot\left(Z_{\phi^2}m^2 - Z_\phi\partial_\mu^2\right)\vec\phi + Z_{\phi^4}\lambda\left(\vec\phi\cdot\vec\phi\right)^2\,. \end{aligned}$$

How to obtain Zs?

- Using DREG get poles in small $\epsilon = 4 - d$ of 1-PI n-point functions at $L = 1, 2, 3, 4$ loops for “any” kinematics



- Example Diagrams (Ising)



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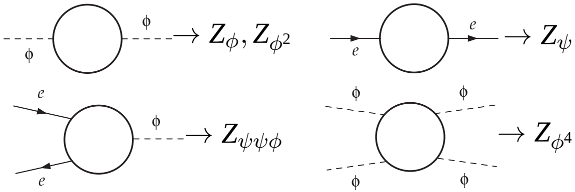

- Number of Diagrams

	χIsing & χHeisenberg				χXY			
Loops	1	2	3	4	1	2	3	4
Z_ψ	1	4	31	323	2	14	200	4014
Z_ϕ, Z_{ϕ^2}	2	6	36	358	2	9	112	2198
$Z_{\psi\psi\phi}$	1	11	145	2199	2	41	1002	28701
Z_{ϕ^4}	9	93	1476	26976	9	173	5029	147023

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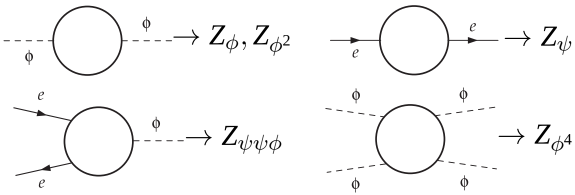
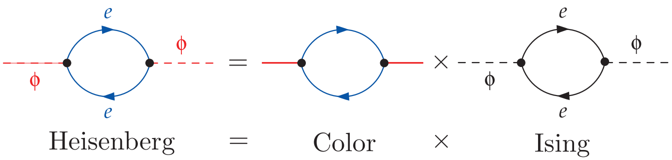


- χ*Heisenberg* diagrams factorize $\quad (T_F^a = \frac{1}{2}\sigma^a \in su_2\ )$



How to obtain Zs?

- Chosing “massiv tadpole kinematics”
  - Use single IR regulator mass M in every denominator
  - Expand in small external momenta $|p_i|/M \ll 1$
- All integrals are 1-scale tadpole integrals
- Use infrared rearrangement [Chetyrkin,Misiak,Munz] to consistently subtract artificial terms $\sim M^2$
- Fully massiv 4-loop MI [Czakon’04]

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  - Fully massiv 4-loop MI [Czakon'04]
- Fully automatized setup
  - *QGRAF* [Nogueira]
  - *q2e* [Seidensticker]
  - *exp* [Seidensticker]
  - *FORM* [Vermaseren & Co]
  - *MATAD* [Steinhauser] OR *CRUSHER* [Marquard,Seidel]
  - *COLOR* [Ritbergen,Schellekens,Vermaseren'98]

  - modified *majoranas.pl* [Harlander]
  - *RecalcPrefac* [NZ]

β- & γ- Functions

- $\beta_y = \frac{dy}{d\ln\mu} \quad \beta_\lambda = \frac{d\lambda}{d\ln\mu} \qquad\qquad (\frac{g^2}{8\pi} \to y, \frac{\lambda}{8\pi} \to \lambda)$

$$\beta_y = -\epsilon y + \beta_y^{\text{(1L)}} + \beta_y^{\text{(2L)}} + \beta_y^{\text{(3L)}} + \beta_y^{\text{(4L)}},$$
$$\beta_\lambda = -\epsilon\lambda + \beta_\lambda^{\text{(1L)}} + \beta_\lambda^{\text{(2L)}} + \beta_\lambda^{\text{(3L)}} + \beta_\lambda^{\text{(4L)}},$$

- $\gamma_x = d\ln Z_x/d\ln\mu \qquad \forall x \in \{\psi, \phi, \phi^2\}$

$$\gamma_\psi = \gamma_\psi^{\text{(1L)}} + \gamma_\psi^{\text{(2L)}} + \gamma_\psi^{\text{(3L)}} + \gamma_\psi^{\text{(4L)}},$$
$$\gamma_\phi = \gamma_\phi^{\text{(1L)}} + \gamma_\phi^{\text{(2L)}} + \gamma_\phi^{\text{(3L)}} + \gamma_\phi^{\text{(4L)}},$$
$$\gamma_{\phi^2} = \gamma_{\phi^2}^{\text{(1L)}} + \gamma_{\phi^2}^{\text{(2L)}} + \gamma_{\phi^2}^{\text{(3L)}} + \gamma_{\phi^2}^{\text{(4L)}}.$$

β- & γ- Functions for χ*Ising*

Example Results @ 1 & 2 Loops:

$$\beta^{(1L)}_{y,\chi I} = (3+2N)y^2\,, \qquad \beta^{(1L)}_{\lambda,\chi I} = 36\lambda^2 + 4Ny\lambda - Ny^2\,,$$

$$\beta^{(2L)}_{y,\chi I} = 24y\lambda(\lambda - y) - (\frac{9}{8} + 6N)y^3\,, \beta^{(2L)}_{\lambda,\chi I} = 4Ny^3 + 7Ny^2\lambda - 72Ny\lambda^2 - 816\lambda^3\,,$$

$$\gamma^{(1L)}_{\psi,\chi I} = \frac{y}{2}\,, \qquad \gamma^{(1L)}_{\phi,\chi I} = 2Ny\,, \qquad \gamma^{(1L)}_{\phi^2,\chi I} = -12\lambda\,,$$

$$\gamma^{(2L)}_{\psi,\chi I} = -\frac{y^2}{16}(12N+1)\,, \gamma^{(2L)}_{\phi,\chi I} = 24\lambda^2 - \frac{5Ny^2}{2}\,, \gamma^{(2L)}_{\phi^2,\chi I} = 144\lambda^2 - 2Ny(y - 12\lambda)\,.$$

✓ 2-loop results [Rosenstein,Kovner,Yu‘93]

β- & γ- Functions for $\chi Ising$

Example Result @ 4 Loops:

$$
\begin{aligned}
\beta^{(4L)}_{\lambda,\chi\mathrm{I}} =& 41472\left(-39\zeta_3-60\zeta_5+\frac{\pi^4}{10}-\frac{3499}{96}\right)\lambda^5+\frac{1}{240}\lambda N y^4\left(-60\zeta_3\left(912N^2-4156N-4677\right)\right.\\
&\left.+1200\zeta_5(157-168N)-4\pi^4(450N+41)+25(4N(337N+3461)+5847)\right)\\
&+\frac{Ny^5\left(480\zeta_3(12N(14N-15)+277)+2400\zeta_5(128N+65)+8\pi^4(64N-77)+160N(1289-386N)-67095\right)}{1920}\\
&+\frac{1}{80}\lambda^2 N y^3\left(835200\zeta_5+1920\zeta_3(3N(4N-61)+19)+72\pi^4(24N+31)-40N(288N+15649)+1057825\right)\\
&+\frac{4}{5}\lambda^3 N y^2\left(-86400\zeta_5+540\zeta_3(4N-69)+7890N-288\pi^4-72605\right)+\frac{36}{5}\left(-17280\zeta_3+96\pi^4-6775\right)\lambda^4 N y\,.
\end{aligned}
$$

- ✓ $y=0\rightarrow Z_2\phi^4$ text book results [Kleinert,Schulte-Frohlinde'01]
- ! 6-loop $O(N)\phi^4$ [Batkovich,Chetyrkin,Kompaniets'16]
- ✓ large N limit for critical exponents[Derkachov,Gracey,Kivel,Stepanenko,Vasiliev][Gracey'17]

β- & γ- Functions for χXY

$N = 1/2, y \to 2h^2, \lambda \to \lambda^2/4$ [S.Thomas'05@KITP]

$$\overline{h}^2 = h^2/(4\pi)^2, \overline{\lambda}^2 = \lambda^2/(4\pi)^2$$

$$\begin{aligned}
\beta_{h^2} &= -\epsilon\overline{h}^2 + 12\overline{h}^4 \\
&\quad +8\overline{h}^2\overline{\lambda}^4 - 64\overline{h}^4\overline{\lambda}^2 + 8\overline{h}^6 \\
&\quad -40\overline{h}^2\overline{\lambda}^6 + 300\overline{h}^4\overline{\lambda}^4 + 1632\overline{h}^6\overline{\lambda}^2 - [1652 - 576\zeta_3]\overline{h}^8, \\
\beta_{\lambda^2} &= -\epsilon\overline{\lambda}^2 + 20\overline{\lambda}^4 + 8\overline{h}^2\overline{\lambda}^2 - 16\overline{h}^4 \\
&\quad -80\overline{h}^2\overline{\lambda}^4 + 16\overline{h}^4\overline{\lambda}^2 + 256\overline{h}^6 - 240\overline{\lambda}^6 \\
&\quad +904\overline{h}^2\overline{\lambda}^6 + [3832 + 3264\zeta_3]\overline{h}^4\overline{\lambda}^4 - [8664 + 2688\zeta_3]\overline{h}^6\overline{\lambda}^2 \\
&\quad -[768 + 3072\zeta_3]\overline{h}^8 + [4936 + 3072\zeta_3]\overline{\lambda}^8, \\
\gamma_\phi &= 4\overline{h}^2 - 24\overline{h}^4 + 8\overline{\lambda}^4 \\
&\quad -40\overline{\lambda}^6 - 60\overline{h}^2\overline{\lambda}^4 + 160\overline{h}^4\overline{\lambda}^2 + [20 + 192\zeta_3]\overline{h}^6, \\
\gamma_\psi &= 4\overline{h}^2 - 16\overline{h}^4 \\
&\quad -44\overline{h}^2\overline{\lambda}^4 + 128\overline{h}^4\overline{\lambda}^2 + [192\zeta_3 - 4]\overline{h}^6.
\end{aligned}$$

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when $\overline{\lambda} = \overline{h}$: $\overline{h}^2 = h^2/(4\pi)^2$

$$\begin{aligned}
\beta_{h^2} &= -\epsilon\overline{h}^2 + 12\overline{h}^4 - 48\overline{h}^6 + 48\overline{h}^8(5 + 12\zeta_3) + \dots, \\
\beta_{\lambda^2} &= -\epsilon\overline{h}^2 + 12\overline{h}^4 - 48\overline{h}^6 + 48\overline{h}^8(5 + 12\zeta_3) + \dots, \\
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&\quad -192\overline{h}^{10}(9 + 60\zeta_3 - 18\zeta_4 + 80\zeta_5)\,, \\
\gamma_\phi &= 4\overline{h}^2 - 16\overline{h}^4 + 16\overline{h}^6(5 + 12\zeta_3) \\
&\quad -64\overline{h}^8(9 + 60\zeta_3 - 18\zeta_4 + 80\zeta_5)\,, \\
\Rightarrow \beta_{h^2} &\stackrel{?}{=} (-\epsilon + 3\gamma_\phi)\overline{h}^2\,.
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✓ $h^2 = \lambda^2$ limit reproduces Wess-Zumino model [Avdeev,Goroshni‘82]

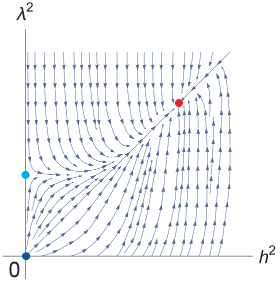
RG Fixed Points @ 3 Loops χXY

- Searching for IR fixed points ($\mu \to 0$) at $d = 3$:

$$\left(\bar{h}_*^2, \bar{\lambda}_*^2\right) = (0,0)\,, \qquad \textit{(unstable)}$$

$$\left(\bar{h}_*^2, \bar{\lambda}_*^2\right) = \left(0, \frac{\epsilon}{20} + \frac{3\epsilon^2}{100} - \frac{384\zeta_3 - 103}{20000}\epsilon^3\right), \qquad \textit{(unstable)}$$

$$\bar{h}_*^2 = \bar{\lambda}_*^2 = \frac{\epsilon}{12} + \frac{\epsilon^2}{36} - \frac{4\zeta_3 - 1}{144}\epsilon^3. \qquad \textit{(stable)}$$



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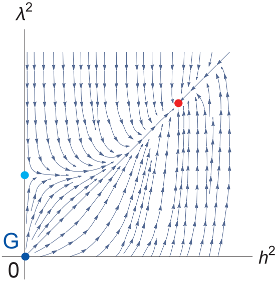

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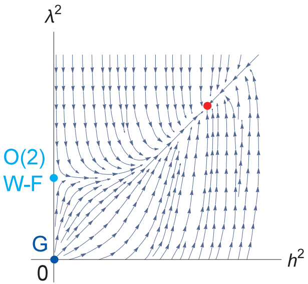

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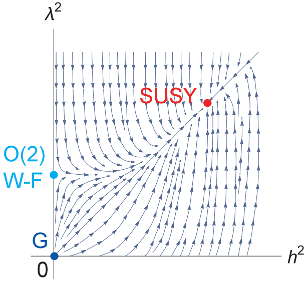

Emergent $\mathcal{N} = 2$ SUSY in χXY

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$$\mathcal{L} = \int d^2\theta d^2\bar{\theta}\, \Phi^\dagger \Phi + \left(\int d^2\theta \frac{h}{3} \Phi^3 + \text{h.c.} \right) ,$$

$$\begin{aligned} \Phi(y) =& \phi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y)\,, \\ y^\mu =& x^\mu - i\theta\gamma^\mu\bar{\theta}\,. \end{aligned}$$

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- SUSY non-renormalization theorems [Strassler'03@TASI]

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- SUSY non-renormalization theorems [Strassler'03@TASI]
- At the SUSY fixed point we check:

$$\gamma_\phi^* = \gamma_\psi^* = \frac{\epsilon}{3} + \mathcal{O}(\epsilon^5).$$

Determination of Critical Exponents

Phenomenology of Quantum Phasetransitions

$x = (X - X_c)/X_c$

$$\xi \sim |x|^{-\nu}(1 + C|x|^{\omega} + \ldots)$$

- correlation length around Quantum Critical Point (QCP) ξ
- correlation length exponent ν
- subleading/stability exponent ω

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Which phase transition?

$$\mathcal{L}_{\chi XY} = \bar{\psi}\partial\!\!\!/\psi + |\partial_\mu \phi|^2 + m^2|\phi|^2 + \lambda^2|\phi|^4 + h(\phi^* \psi^T i\sigma_2 \psi + h.c.)\,.$$

- Semimetal phase Global $U(1)$ symmetry for $m^2 > 0$

$$\psi \to e^{i\theta}\psi\,, \qquad \phi \to e^{2i\theta}\phi\,, \qquad \langle 0|\phi|0\rangle = \phi_0 = 0\,.$$

- Phasetransition at QCP for $m^2 = 0$
- Superconducting phase
  Spontaneously broken global $U(1)$ symmetry for $m^2 < 0$

$$\langle 0|\phi|0\rangle = \phi_0 \neq 0\,.$$

Critical Exponents χXY

- Stability exponent ($\omega \gtrless 0$ IR stable/unstable $*$)

$$\omega = \left.\frac{\mathrm{d}\beta_{h^2}(\lambda^2 = h^2)}{\mathrm{d}h^2}\right|_{h=h_*} = \epsilon - \frac{1}{3}\epsilon^2 + \left(\frac{1}{18} + \frac{2}{3}\zeta_3\right)\epsilon^3 + a\epsilon^4 + \mathcal{O}(\epsilon^5)\,.$$

Critical Exponents χXY

- Stability exponent ($\omega \gtrless 0$ IR stable/unstable $*$)

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- Correlation length exponent

$$\nu^{-1} = 2 + \gamma^*_{m^2}\,, \qquad \gamma_{m^2} = \frac{\mathrm{d}\ln Z_{m^2}}{\mathrm{d}\ln\mu}\,, \qquad Z_{m^2} m^2 = m_0^2\,,$$

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- OR: Using "Grassmannian continuation" of the SUSY Lagrangian

$$\gamma^*_{m^2} = -\left.\frac{\mathrm{d}\beta_{h^2}(\lambda^2 = h^2)}{\mathrm{d}h^2}\right|_{h=h_*}\,.$$

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- Asymptotic Series!?
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- Try Padé approximations

Emergent $\mathcal{N} = 1$ SUSY in $\chi Ising$

- For $N = 1/4$: 1 Majorana $\leftrightarrow$ 1 real Scalar SUSY in $D = 3$

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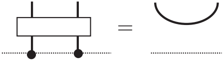
$D = 3$ (using Pauli-matrices)

$$\mathrm{tr}\{\gamma^\mu \gamma^\nu \gamma^\rho\} \sim \varepsilon^{\mu\nu\rho} \,.$$

Clifford for Pedestrians

$$\frac{1}{2}\{\gamma^\mu, \gamma^\nu\} = g^{\mu\nu}\mathbb{I}$$



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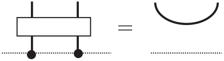
- $\frac{1}{2}\{\gamma^\mu, \gamma^\nu\} = g^{\mu\nu}\mathbb{I}$ 

- Irreducible Bilinears in D Dimensions

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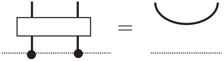
- $\frac{1}{2}\{\gamma^\mu, \gamma^\nu\} = g^{\mu\nu}\mathbb{I}$ 

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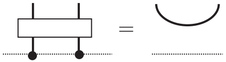

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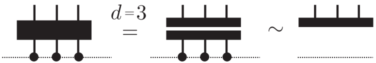
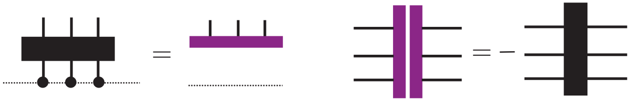
➡ In $D = 3$ (ε_{ijk} has 1 lin. indp. component!)



Saving $\mathcal{N}=1$ SUSY "with Trump"

- Modify DREG & demand in D dimensions:



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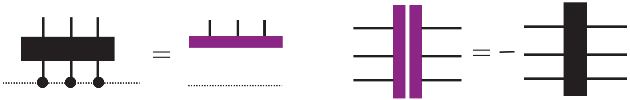

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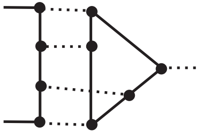

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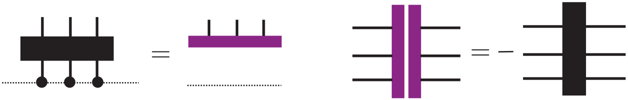

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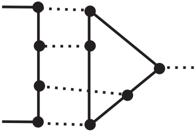

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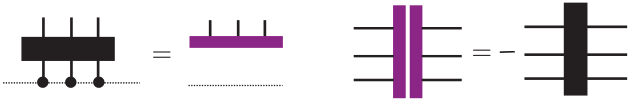

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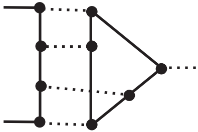


- ✓ Contribution can be obtained via a majorana fermion calculation using an explicit spin $SU(2)$ algebra
- ✓ SUSY relations restored

$$\beta_y^{(4L)*} = \beta_\lambda^{(4L)*} \to \lambda^* = y^* \,,$$
$$\nu^{-1} = \frac{D-\gamma^*}{2} \,.$$

Comparing Results χXY

$N = 1/2$	$1/\nu$	γ_ϕ^*	γ_ψ^*	ω
this work, $P_{[2/2]}$	1.128	1/3	1/3	0.872
this work, $P_{[3/1]}$	1.130	1/3	1/3	0.870
conf. bootstrap[1]	1.090	1/3	1/3	0.910
$N = 2$	$1/\nu$	γ_ϕ^*	γ_ψ^*	ω
this work, $P_{[2/2]}$	0.840	0.810	0.117	0.796
this work, $P_{[3/1]}$	0.841	0.788	0.108	0.780
functional RG[2]	0.862	0.88	0.062	0.878
Monte Carlo[3]	1.06(5)	0.71(3)		

[1] [Bobev,El-Showk,Mazac,Paulos'15]

[2] [Classen, Herbut, Scherer'17]

[3] [Li,Jiang,Jian,Yao,'17]

Comparing Results $\chi Ising$

$N = 1/4$	$1/\nu$	γ_ϕ^*	γ_ψ^*	ω
this work, $P_{[2/2]}$	1.415	0.171	0.171	0.843
this work, $P_{[3/1]}$	1.415	0.170	0.170	0.838
FRG[4] (Regulator 1)	1.385	0.174	0.174	0.765
FRG[4] (Regulator 2)	1.395	0.167	0.167	0.782
conf. bootstrap[5]		0.164	0.164	

[4] [Gies,Hellwig,Wipf,Zanusso'17]

[5] [Iliesiu,Kos,Poland,Pufu,Simmons-Duffin,Yacoby'16]

Comparing Results $\chi Ising$

$N=2$	$1/\nu$	γ_ϕ^*	γ_ψ^*	ω
this work, $P_{[2/2]}$	0.931	0.7079	0.0539	0.794
this work, $P_{[3/1]}$	0.945	0.6906	0.0506	0.777
$(2+\epsilon)$, (ϵ^4, Padé)[6]	0.931	0.745	0.082	
FRG[7]	0.994(2)	0.7765	0.0276	
conformal bootstrap[8]	0.88	0.742	0.044	
Monte Carlo[9]	1.20(1)	0.62(1)	0.38(1)	

6 [Gracey,Luthe,Schroder'16]

7 [Knorr'16]

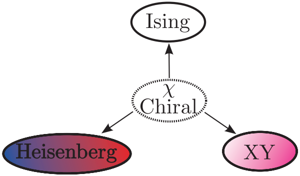
8 [Iliesiu,Kos,Poland,Pufu,Simmons-Duffin'17]

9 [Chandrasekharan,Li'13]

Summary & Outlook

Summary

- 4-Loop βs & γs for Gross-Neveu-Yukawa Models



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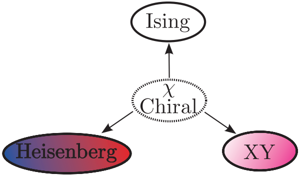

- Kinematics $\leftrightarrow$ Symmetry
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Outlook

- More sophisticated/beyond Padé analysis (large order behavior?)

To backup slides $\rightarrow$

Where to find SUSY?

- @ LHC

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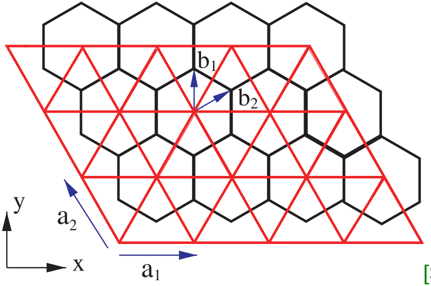
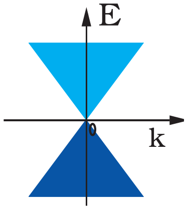
- On the surface of topological insulator
  @ the semimetal-superconductor QCP





[S.-S.Lee‘07]

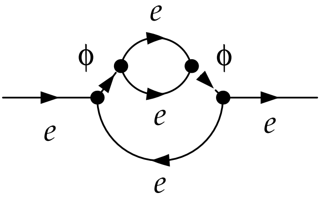
χXY in smart

$$\mathcal{L}_{\chi XY} = \bar{\psi}\partial\!\!\!/\psi + |\partial_\mu \phi|^2 + m^2|\phi|^2 + \lambda^2|\phi|^4 + h(\phi^*\psi^T i\sigma_2\psi + h.c.)\,.$$

vs.

$$\mathcal{L}_{\chi XY} = \bar{\psi}\partial\!\!\!/\psi + |\partial_\mu \phi|^2 + m^2|\phi|^2 + \lambda^2|\phi|^4 + g\phi\bar{\psi}P_+\psi + g\phi^*\bar{\psi}P_-\psi\,.$$

	χXY v1				χXY v2			
Loops	1	2	3	4	1	2	3	4
Z_ψ	1	3	16	116	2	14	200	4014
Z_ϕ, Z_{ϕ^2}	2	4	22	148	2	9	112	2198
$Z_{\psi\psi\phi}$	0	2	25	296	2	41	1002	28701
Z_{ϕ^4}	4	35	369	4388	9	173	5029	147023



Renormalizability

Is the QFT with

$$\mathcal{L} = \bar{\psi}_\alpha(\not{\partial} + g[T_R^a]^\alpha_\beta \phi_a)\psi^\beta - \frac{1}{2}\phi_a \partial_\mu^2 \phi_a + \lambda(\phi_a\phi_a)^2\,,$$

a renormalizeable theory in $d = 4$ when we assume that T_R obey a finite dimensional simple Lie algebra?

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Lie algebra	Betti numbers
$A_r \sim SU(r+1)$	$2, 3, \ldots, r+1$
$B_r \sim SO(2r+1)$	$2, 4, 6, \ldots, 2r$
$C_r \sim Sp(2r)$	$2, 4, 6, \ldots, 2r$
$D_r \sim SO(2r)$	$2, 4, 6, \ldots, 2r-2, r$
G_2	$2, 6$
F_4	$2, 6, 8, 12$
E_6	$2, 5, 6, 8, 9, 12$
E_7	$2, 6, 8, 10, 12, 14, 18$
E_8	$2, 8, 12, 14, 18, 20, 24, 30$

Superspace

$$
\begin{aligned}
d^2\theta &\equiv -\frac{1}{4} d\theta^\alpha d\theta^\beta \varepsilon_{\alpha\beta}, \\
d^2\bar{\theta} &\equiv -\frac{1}{4} d\bar{\theta}^\alpha d\bar{\theta}^\beta \varepsilon_{\alpha\beta}, \\
\theta^2 &\equiv \theta^\alpha \theta_\alpha = \theta^\alpha \varepsilon_{\alpha\beta} \theta^\beta,
\end{aligned}
$$

$$\int d^2\theta d^2\bar{\theta}\, \Phi^\dagger \Phi = i\bar{\psi}\not{\partial}\psi + |\partial_\mu \phi|^2 + |F|^2. \qquad \textit{kinetic terms}$$

$$\int d^2\theta\, \frac{h}{3} \Phi^3 = h\phi^2 F + h\phi\psi^T i\sigma_2 \psi. \qquad \textit{super potential terms}$$

Auxiliary fields are eliminated using EQM: $F = -h\phi^{*2}$, $F^* = -h\phi^2$.