

Improvements of the sector-improved residue subtraction scheme

Arnd Behring, Michal Czakon, Rene Poncelet

Institute for Theoretical Particle Physics and Cosmology
RWTH Aachen University



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Outline

Introduction

New phase space construction

't Hooft Veltman scheme

C++ implementation of STRIPPER

Conclusions

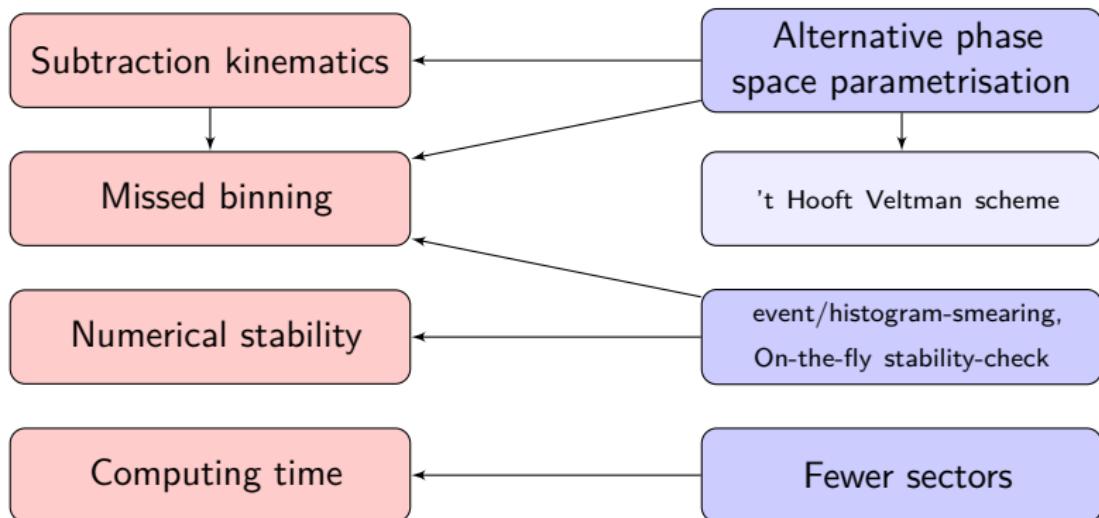
NNLO subtraction schemes

Handling real radiation contribution in NNLO calculations
cancellation of infrared divergences

Increasing number of available NNLO calculations with a variety of schemes

- **qT-slicing** [Catani,Grazzini, '07], [Ferrera,Grazzini,Tramontano, '11], [Catani,Cieri,DeFlorian,Ferrera,Grazzini,'12],
[Gehrmann,Grazzini,Kallweit,Maierhofer,Manteuffel,Rathlev,Torre,'14-'15], [Bonciani,Catani,Grazzini,Sargsyan,Torre,'14-'15]
- **N-jettiness slicing** [Gaunt,Stahlhofen,Tackmann,Walsh, '15], [Boughezal,Focke,Giele,Liu,Petriello,'15-'16],
[Boughezal,Campbell,Ellis,Focke,Giele,Liu,Petriello,'15]. [Campbell,Ellis,Williams,'16]
- **Antenna subtraction** [Gehrmann, GehrmannDeRidder,Glover,Heinrich,'05-'08], [Weinzierl,'08,'09],
[Currie,Gehrman,GehrmanDeRidder,Glover,Pires,'13-'17], [Bernreuther,Bogner,Dekkers,'11,'14],
[Abelof,(Dekkers),GehrmanDeRidder,'11-'15], [Abelof,GehrmanDeRidder,Maierhofer,Pozzorini,'14],
[Chen,Gehrman,Glover,Jaquier,'15]
- **Colorful subtraction** [DelDuca,Somogyi,Troscanyi,'05-'13], [DelDuca,Duhr,Somogyi,Tramontano,Troscanyi,'15]
- **Sector-improved residue subtraction (STRIPPER)** [Czakon,'10,'11],
[Czakon,Fiedler,Mitov,'13,'15], [Czakon,Heymes,'14] [Czakon,Fiedler,Heymes,Mitov,'16,'17],
[Boughezal,Caola,Melnikov,Petriello,Schulze,'13,'14], [Boughezal,Melnikov,Petriello,'11],
[Caola,Czernecki,Liang,Melnikov,Szafron,'14], [Brucherseifer,Caola,Melnikov,'13-'14], [Caola, Melnikov, Röntsch,'17]

How to improve the STRIPPER subtraction scheme?



Idea: Optimisation through minimisation

Formulation

Hadronic cross section:

$$\sigma_{h_1 h_2}(P_1, P_2) = \sum \int_0^1 dx_1 dx_2 f_{a/h_1}(x_1, \mu_F^2) f_{b/h_2}(x_2, \mu_F^2) \hat{\sigma}_{ab}(x_1 P_1, x_2 P_2; \alpha_S(\mu_R^2), \mu_R^2, \mu_F^2)$$

Partonic cross section:

$$\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0)} + \hat{\sigma}_{ab}^{(1)} + \hat{\sigma}_{ab}^{(2)} + \mathcal{O}(\alpha_S^3)$$

Contributions with different final state multiplicities and convolutions:

$$\hat{\sigma}_{ab}^{(2)} = \hat{\sigma}_{ab}^{\text{RR}} + \hat{\sigma}_{ab}^{\text{RV}} + \hat{\sigma}_{ab}^{\text{VV}} + \hat{\sigma}_{ab}^{\text{C2}} + \hat{\sigma}_{ab}^{\text{C1}}$$

$$\hat{\sigma}_{ab}^{\text{RR}} = \frac{1}{2\hat{s}} \int d\Phi_{n+2} \left\langle \mathcal{M}_{n+2}^{(0)} \middle| \mathcal{M}_{n+2}^{(0)} \right\rangle F_{n+2}$$

$\hat{\sigma}_{ab}^{\text{C1}} = (\text{single convolution}) F_{n+1}$

$$\hat{\sigma}_{ab}^{\text{RV}} = \frac{1}{2\hat{s}} \int d\Phi_{n+1} 2\text{Re} \left\langle \mathcal{M}_{n+1}^{(0)} \middle| \mathcal{M}_{n+1}^{(1)} \right\rangle F_{n+1}$$

$\hat{\sigma}_{ab}^{\text{C2}} = (\text{double convolution}) F_n$

$$\hat{\sigma}_{ab}^{\text{VV}} = \frac{1}{2\hat{s}} \int d\Phi_n \left(2\text{Re} \left\langle \mathcal{M}_n^{(0)} \middle| \mathcal{M}_n^{(2)} \right\rangle + \left\langle \mathcal{M}_n^{(1)} \middle| \mathcal{M}_n^{(1)} \right\rangle \right) F_n$$

Sector decomposition

Several layers of decomposition

Selector functions

$$1 = \sum_{i,j} \left[\sum_k S_{ij,k} + \sum_{k,l} S_{i,k;j,l} \right]$$

Factorization of double soft limits

$$\theta(u_1^0 - u_2^0) + \theta(u_2^0 - u_1^0)$$

Sector parametrisation

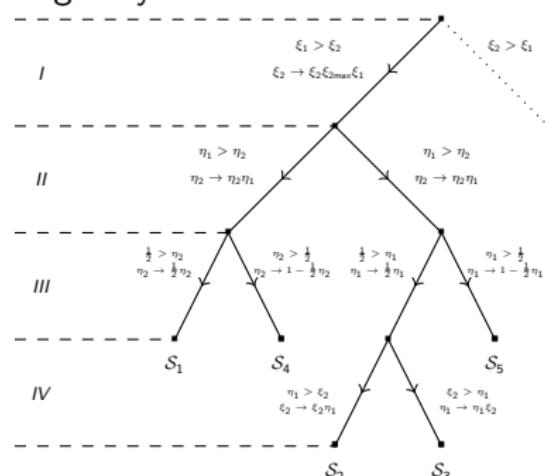
Parametrisation of u_i with respect to the reference parton r :

Angles: $\hat{\eta}_i = \frac{1}{2}(1 - \cos \theta_{ir}) \in [0, 1]$

Energies: $\hat{\xi}_i = \frac{\mu_i^0}{\mu_{\max}^0} \in [0, 1]$

Triple collinear factorisation

originally: 5 sub-sectors



Sector decomposition

Several layers of decomposition

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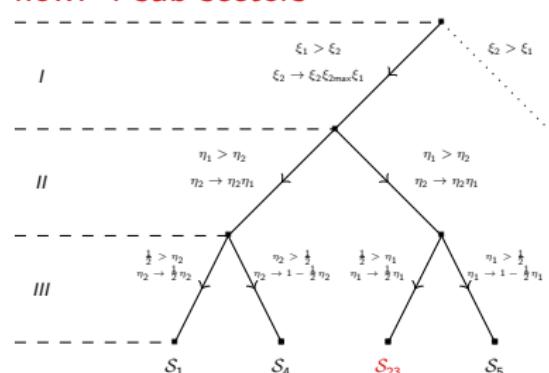
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Triple collinear factorisation

now: 4 sub-sectors



[Caola, Melnikov, Röntsch '17]

→ yesterday's talk by Raoul Röntsch

New phase space construction: Idea

Goal

Phase space construction with a minimal # of subtraction kinematics

Old construction

- Start with unresolved partons
 - Fill remaining phase space with Born configuration
- Non-minimal # kinematic configurations
(e.g. single soft and collinear limits yield different configurations)

New construction

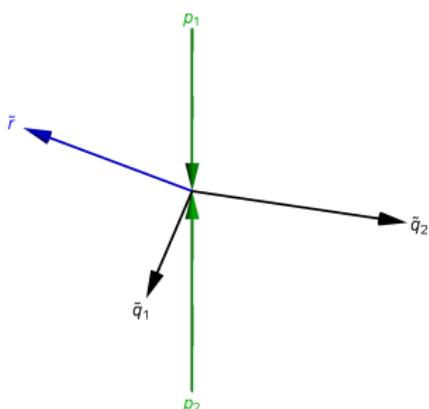
- Start with Born configuration
- Add unresolved partons (u_i)
- Cleverly adjust Born configuration to accommodate the u_i

New phase space construction

Mapping from $n+2$ to Born configuration: $\{P, r_j, u_k\} \rightarrow \{\tilde{P}, \tilde{r}_j\}$
 Modification of [Frixione,Webber'02]/[Frixione,Nason,Oleari'07]

Requirements:

- Keep direction of reference r fixed
- Invertible for fixed u_i :
 $\{\tilde{P}, \tilde{r}_j, u_k\} \rightarrow \{P, r_j, u_k\}$
- Preserve $q^2 = \tilde{q}^2$, $\tilde{q} = \tilde{P} - \sum_{j=1}^{n_{fr}} \tilde{r}_j$



Main steps:

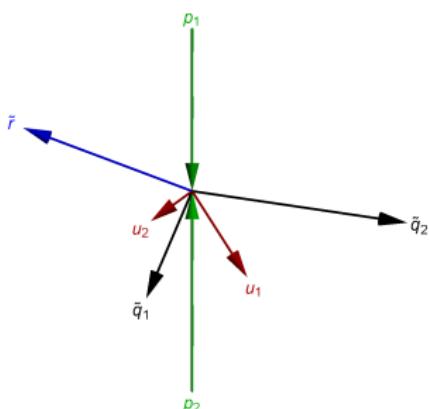
- Generate Born phase space configuration
- Generate unresolved partons u_i
- Rescale reference momentum
- Boost non-reference momenta of the Born configuration

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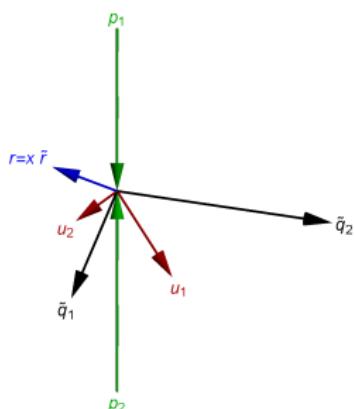
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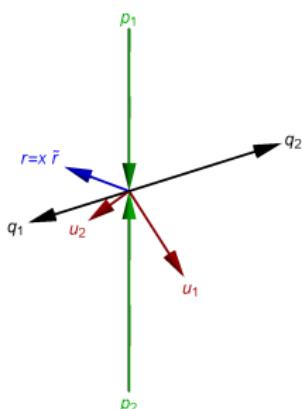
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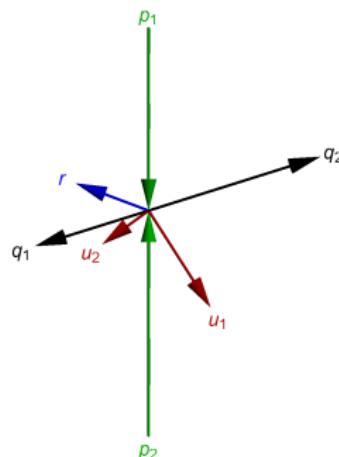


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Behaviour in singular limits

Collinear limit of u_2
(sector 1, $\eta_2 \rightarrow 0$)



Soft limit of u_2
(sector 1, $\xi_2 \rightarrow 0$)

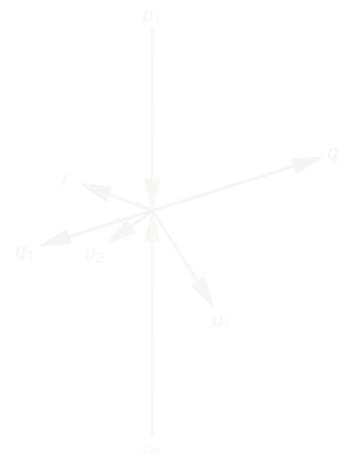
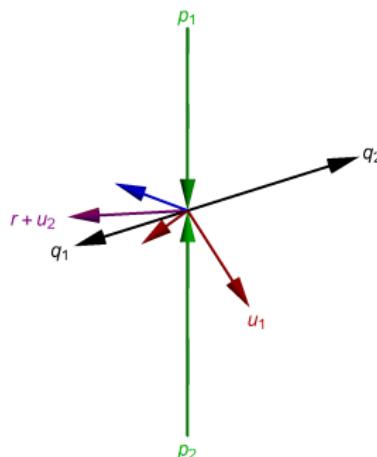


→ Both singular limits approach the same kinematic configuration

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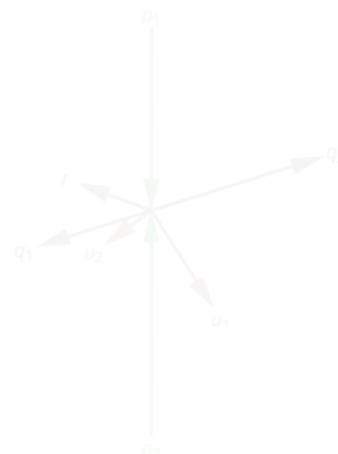
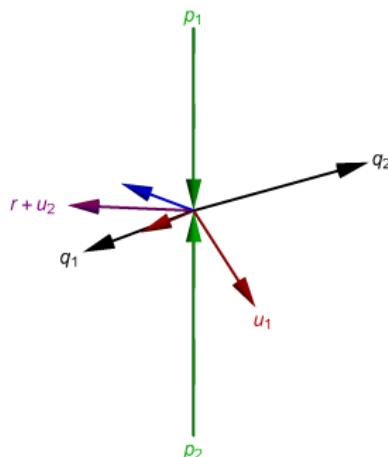


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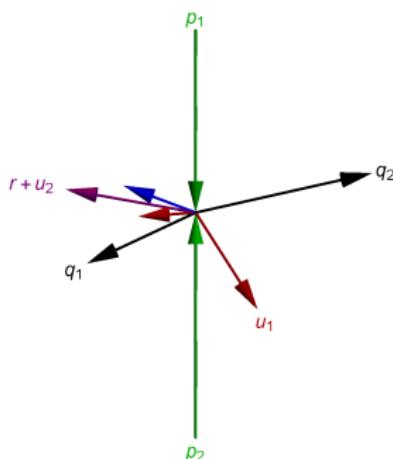
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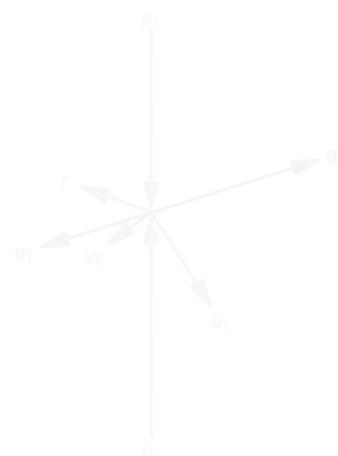
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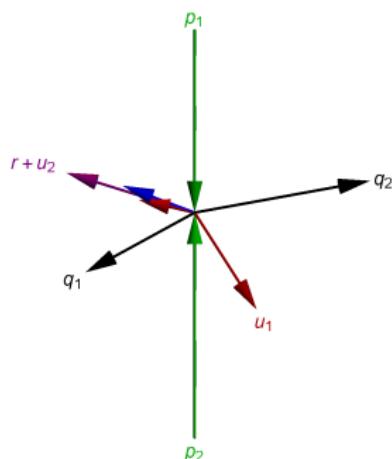
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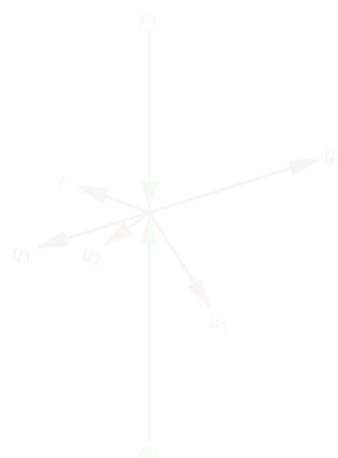
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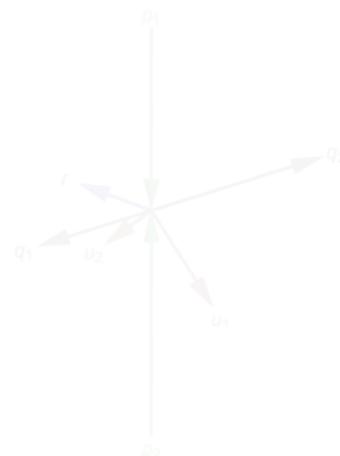
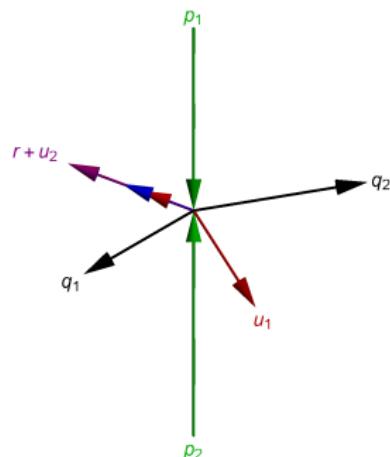


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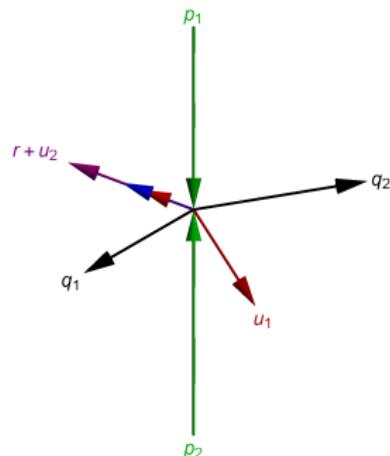
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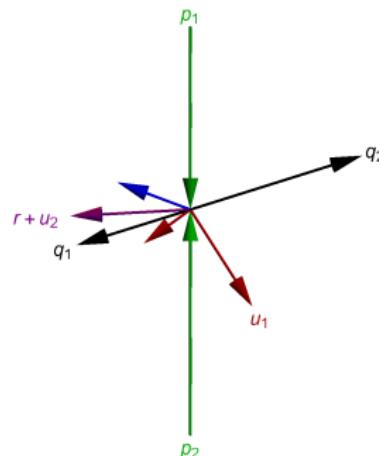
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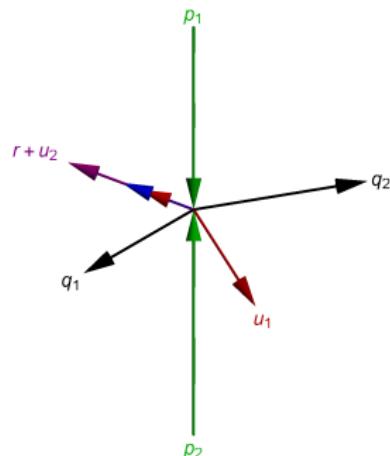
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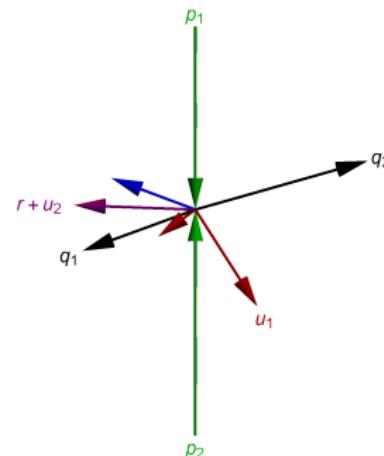
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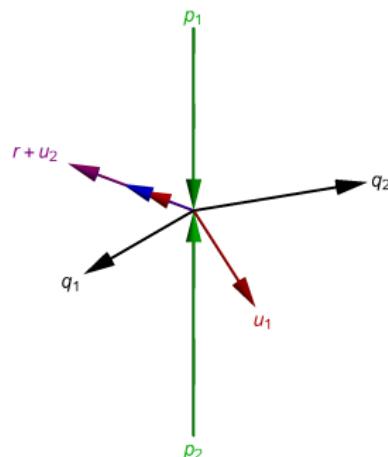
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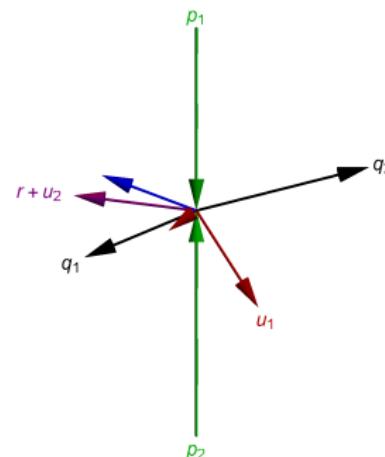
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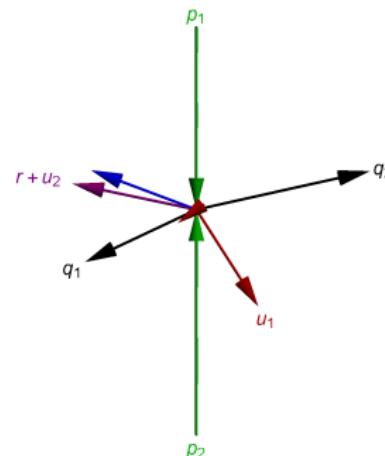
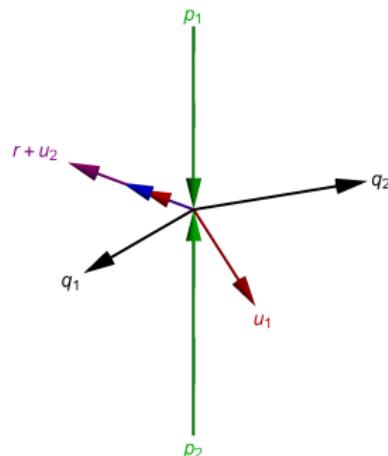


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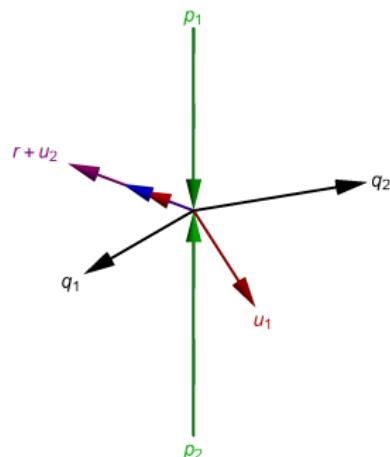
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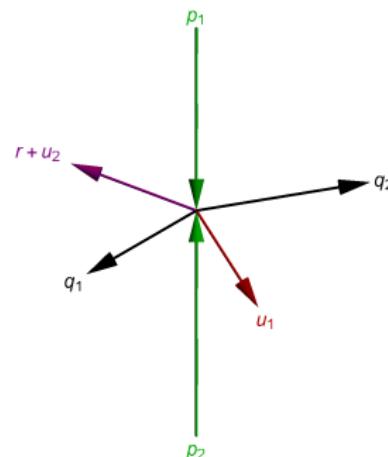
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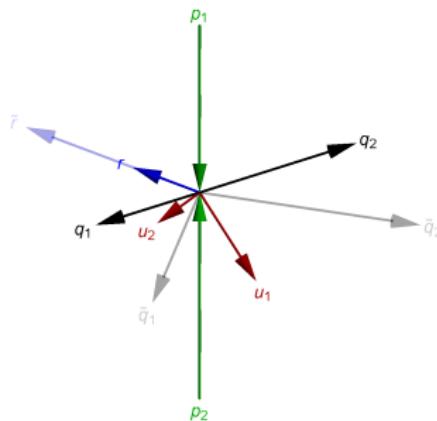
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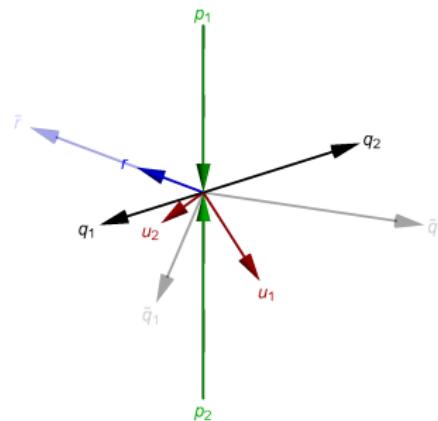
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Triple collinear limit of u_1 & u_2
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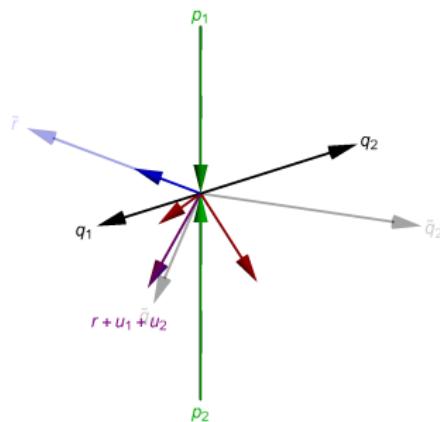
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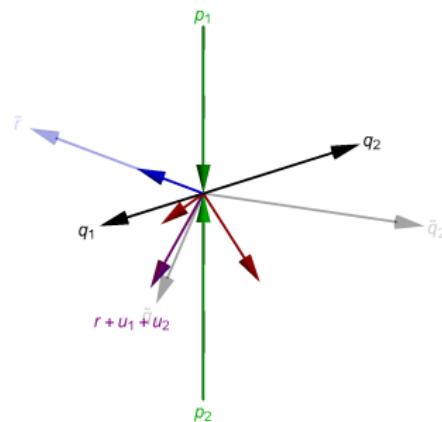
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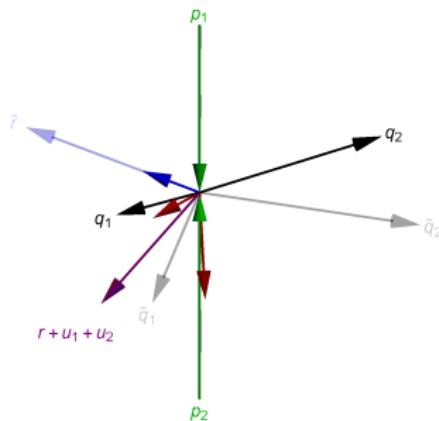
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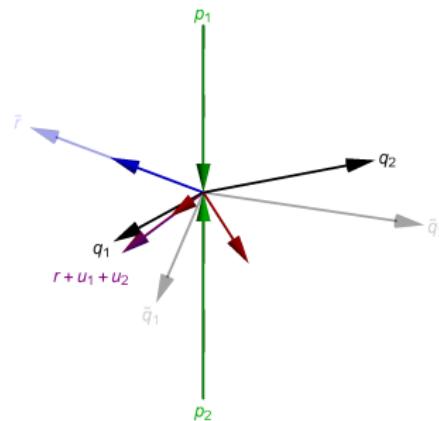
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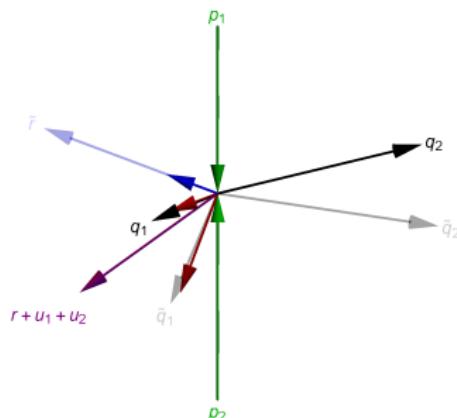
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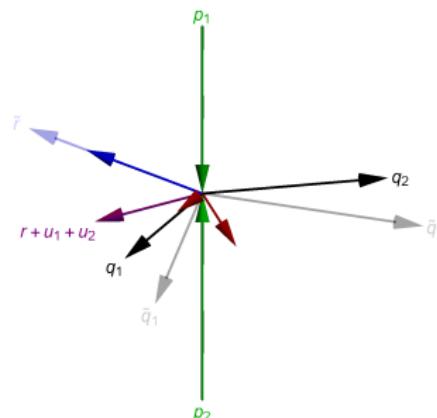
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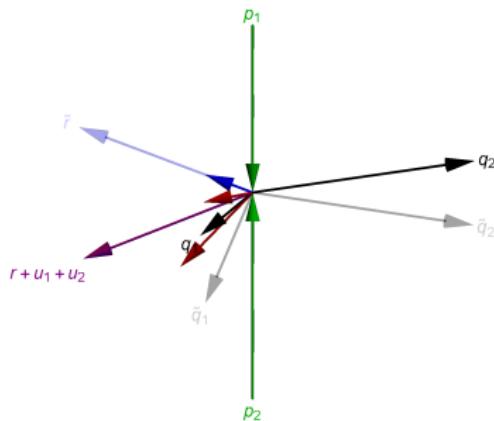
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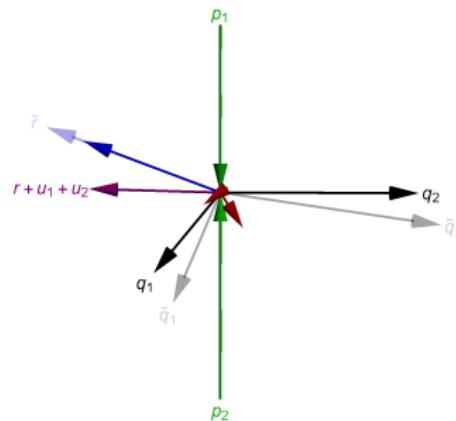
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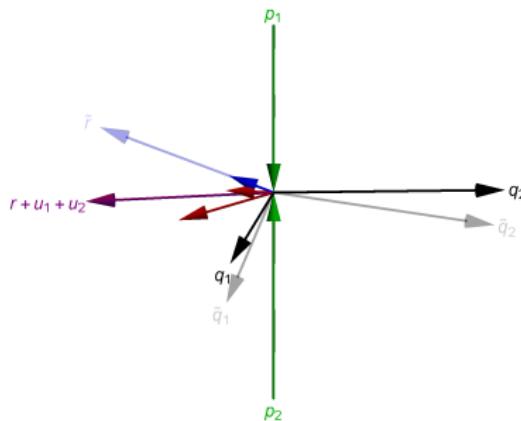
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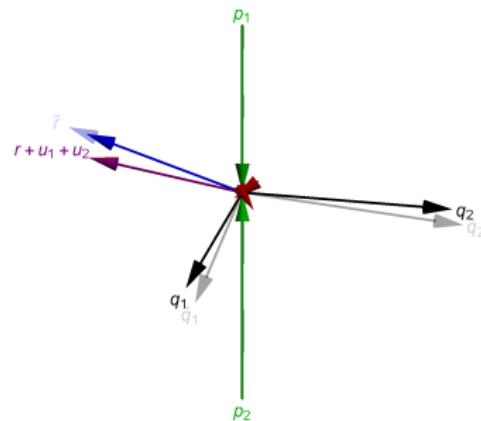
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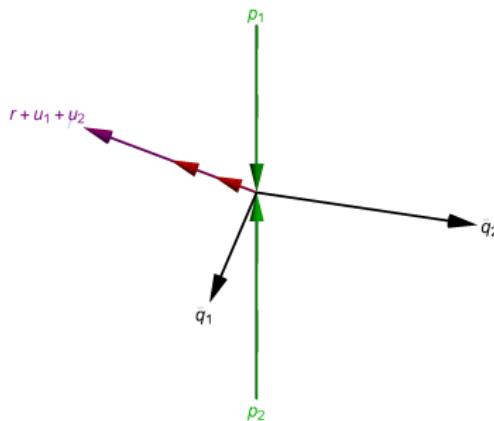
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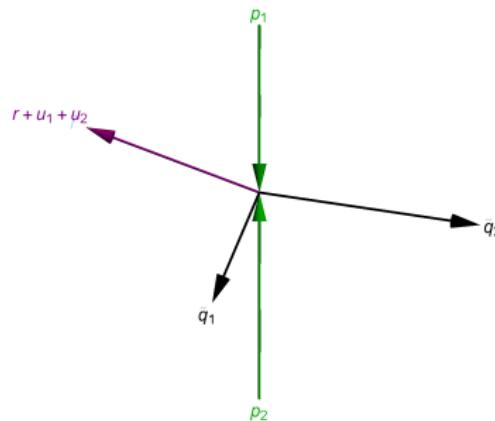
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Consequences

Features

- Minimal number of subtraction kinematics
- Only one DU configuration
 - pole cancellation for each Born phase space point
- Expected improved convergence of invariant mass distributions, since $\tilde{q}^2 = q^2$

Unintentional features

- Construction in lab frame
- Original construction of 't Hooft Veltman corrections
[Czakon,Heymes'14] is spoiled

't Hooft Veltman scheme

Treat resolved particles in 4 dimensions (momenta and polarisations)

- Avoid unnecessary ϵ -orders of the matrix elements
- Avoid growth of dimensionality of phase space integrals

Make resolved phase space 4-dim. using measurement function, e.g.

$$F_n \rightarrow F_n \mathcal{N}^{-(n-1)\epsilon} \prod_{i=1}^{n-1} \delta^{(-2\epsilon)}(q_i)$$

Finite parts:

$$\sigma_F^{RR}$$

$$\sigma_F^{RV}$$

$$\sigma_F^{VV}$$

Finite remainder
parts:

$$\sigma_{FR}^{RV}, \sigma_{FR}^{VV}, \sigma_{FR}^{C2}$$

Single (SU) and
double (DU) un-
resolved parts:

$$\sigma_{SU}^{RR}, \sigma_{SU}^{RV}, \sigma_{SU}^{C1}$$

$$\sigma_{DU}^{RR}, \sigma_{DU}^{RV}, \sigma_{DU}^{C1}, \sigma_{DU}^{VV}, \sigma_{DU}^{C2}$$

$$\sigma_{SU}^{RR}, \sigma_{SU}^{RV}, \sigma_{SU}^{C1}$$

$$\sigma_{DU}^{RR}, \sigma_{DU}^{RV}, \sigma_{DU}^{C1}, \sigma_{DU}^{VV}, \sigma_{DU}^{C2}$$

't Hooft Veltman scheme

Goal: Make SU and DU separately finite

Idea: Move “divergent parts” of SU to DU before applying 'tHV scheme

- SU contribution: $\sigma_{SU} = \sigma_{SU}^{RR} + \sigma_{SU}^{RV} + \sigma_{SU}^{C1}$ with

$$\sigma_{SU}^c = \int d\Phi_{n+1} (I_{n+1}^c F_{n+1} + I_n^c F_n)$$

- We know: NLO cross section is finite
→ F_{n+1} part of SU is finite: Poles cancel between RR, RV and C1 (with NLO measurement function)
- With NNLO measurement function: Additional poles arise
→ SU no longer finite by itself
- Non-cancelling ϵ poles are generated by terms with F_n
→ can be moved to DU

→ **Task:** Identify non-cancelling parts of SU

't Hooft Veltman scheme

Task: Identify non-cancelling parts of SU

- Use parametrised measurement functions

$$F_{n+1}^\alpha = F_{n+1} \theta \left(\min_{i,j} \eta_{ij} - \alpha \right) \theta \left(\min_i \frac{u_i^0}{E_{\text{norm}}} - \alpha \right)$$

- Construct:

$$\sigma_{\text{SU}}^c - \mathcal{I}_c^\alpha = \int d\Phi_{n+1} \left(I_{n+1} F_{n+1} + I_n F_n - [I_{n+1}]_{1/\epsilon^2, 1/\epsilon} F_{n+1}^\alpha \right)$$

- Rearrangements allow to extract the non-cancelling part:

$$N^c(\alpha) = \int d\Phi_{n+1} [I_n]_{1/\epsilon^2, 1/\epsilon} F_n \theta_\alpha$$

- Analytically extract divergences ($\ln^k \alpha$) and cancel them exactly
- Take limit $\alpha \rightarrow 0$ to remove dependence on α
- Subtract from σ_{SU} and add to σ_{DU}
 - separately finite SU and DU contributions
 - ready for application of 'tHV scheme

Looks like slicing, but it is slicing *only* for divergences
 → no actual slicing parameter in the result

C++ implementation of STRIPPER

Features of the implementation

- General subtraction framework
 - Provides a general set of subtraction terms
 - Tree-level amplitudes are calculated automatically using a Fortran library [van Hameren '09] [Bury, van Hameren '15]
 - User has to provide the 1- and 2-loop amplitudes
- Separate evaluation of coefficients of scales and PDFs
→ Cheaper calculations with several scales and PDFs
- FastNLO interface
 - Allows to produce tables for fast fits
 - FastNLO tables for $t\bar{t}$ differential distributions released this spring [Czakon, Heymes, Mitov '17]

Example: Driver program

```
// define initial state and (multiple) PDFs
InitialState initial("p","p",Ecms,Emin);
initial.include(LHAPDFsetName);

// define (multiple) scales
ScalesList scales(FixedScales(mt,mt,"muR = mt, muF = mt"));
scales.include(DynamicalScalesHT4(1.,1.));

// set up observables to be calculated
Measurement measurement;
measurement.include(TransverseMomentum({"t"}),
                    {{Histogram::bins(40,0.,2000.)}});

// initialise MC generator and specify contribution to calculate
Generator generator(incoming,scales,measurement);
generator.include({{"g","g"}, {"t","t~","g","g"}}, 2,2,0,0, false);

// run integration with 10^6 points
generator.run(1000000);

// write results
ofstream xml("ttbar.xml");
generator.measurement().print(xml);
xml.close();
```

Conclusions

- Minimization of the STRIPPER scheme
- Fewer subsectors in triple collinear sectors
- Alternative phase space parametrisation
- New formulation of 't Hooft Veltman scheme
- Implementation of STRIPPER as a C++ library
- Currently performing tests for a variety of processes:
 $pp \rightarrow t\bar{t}$, $e^+e^- \rightarrow 2/3j$, t decay, DIS, Drell-Yan, Higgs decay, dijets

Backup

- Phase space → 19
- Factorisation and subtraction terms → 20
- SU contribution → 21
- SU finiteness → 22

Phase space

Common starting point for all phase spaces:

$$d\Phi_n = dQ^2 \left[\prod_{j=1}^{n_{fr}} \mu_0(r_j) \prod_{k=1}^{n_u} \mu_0(u_k) \delta_+ \left(\left(P - \sum_{j=1}^{n_{fr}} r_j - \sum_{k=1}^{n_u} u_k \right)^2 - Q^2 \right) \right] \\ \prod_{i=1}^{n_q} \mu_{m_i}(q_i) (2\pi)^d \delta^{(d)} \left(\sum_{i=1}^{n_q} q_i - q \right)$$

with

$$\text{with } \mu_m(k) \equiv \frac{d^d k}{(2\pi)^d} 2\pi \delta(k^2 - m^2) \theta(k^0),$$

n : # final state particles, n_{fr} : # final state references, n_u : # additional partons

Factorization and subtraction terms

SU phase space

$$\int \int_0^1 d\eta d\xi \eta^{a_1 - b_1 \epsilon} \xi^{a_2 - b_2 \epsilon}$$

Factorized singular limits

$$\int d\Phi_n \prod dx_i \underbrace{x_i^{-1-b_i \epsilon}}_{\text{singular}} \tilde{\mu}(\{x_i\})$$

$$\underbrace{\prod x_i^{a_i+1} \langle \mathcal{M}_{n+2} | \mathcal{M}_{n+2} \rangle}_{\text{regular}} F_{n+2}$$

DU phase space

$$\int \int \int \int_0^1 d\eta_1 d\xi_1 d\eta_2 d\xi_2 \eta_1^{a_1 - b_1 \epsilon} \xi_1^{a_2 - b_2 \epsilon} \eta_2^{a_3 - b_3 \epsilon} \xi_2^{a_4 - b_4 \epsilon}$$

Regularisation

$$x^{-1-b\epsilon} = \underbrace{\frac{-1}{b\epsilon}}_{\text{pole term}} + \underbrace{[x^{-1-b\epsilon}]}_{\text{reg. + sub.}} +$$

$$\int_0^1 dx [x^{-1-b\epsilon}]_+ f(x) = \int_0^1 \frac{f(x) - f(0)}{x^{1+b\epsilon}}$$

The single unresolved (SU) contribution

- SU contribution

$$\sigma_{SU} = \sigma_{SU}^{RR} + \sigma_{SU}^{RV} + \sigma_{SU}^{C1} \quad \text{with}$$

$$\sigma_{SU}^c = \int d\Phi_{n+1} (I_{n+1}^c F_{n+1} + I_n^c F_n)$$

- NLO measurement function ($\alpha \neq 0$)

$$\int d\Phi_{n+1} (I_{n+1}^{RR} + I_{n+1}^{RV} + I_{n+1}^{C1}) F_{n+1}^\alpha = \text{finite in 4 dim.}$$

- All divergences cancel in d -dimensions

$$\sum_c \int d\Phi_{n+1} \left[\frac{I_{n+1}^{c,(-2)}}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)}}{\epsilon} \right] F_{n+1}^\alpha \equiv \sum_c \mathcal{I}^c = 0$$

SU finiteness for $\alpha = 0$

$$\begin{aligned}
 \sigma_{SU} &= \sigma_{SU}^{RR} + \sigma_{SU}^{RV} + \sigma_{SU}^{C1} \underbrace{-\mathcal{I}^{RR} - \mathcal{I}^{RV} - \mathcal{I}^{C1}}_{=0} \\
 \sigma_{SU}^c - \mathcal{I}^c &= \int d\Phi_{n+1} \left\{ \left[\frac{I_{n+1}^{c,(-2)}}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)}}{\epsilon} + I_{n+1}^{c,(0)} \right] F_{n+1} + \left[\frac{I_n^{c,(-2)}}{\epsilon^2} + \frac{I_n^{c,(-1)}}{\epsilon} + I_n^{c,(0)} \right] F_n \right\} \\
 &\quad - \int d\Phi_{n+1} \left[\frac{I_{n+1}^{c,(-2)}}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)}}{\epsilon} \right] F_{n+1} \theta_\alpha(\{\alpha_i\}) \\
 &= \int d\Phi_{n+1} \left[\frac{I_{n+1}^{c,(-2)} F_{n+1} + I_n^{c,(-2)} F_n}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)} F_{n+1} + I_n^{c,(-1)} F_n}{\epsilon} \right] (1 - \theta_\alpha(\{\alpha_i\})) \\
 &\quad + \int d\Phi_{n+1} \left[I_{n+1}^{c,(0)} F_{n+1} + I_n^{c,(0)} F_n \right] + \int d\Phi_{n+1} \left[\frac{I_n^{c,(-2)}}{\epsilon^2} + \frac{I_n^{c,(-1)}}{\epsilon} \right] F_n \theta_\alpha(\{\alpha_i\}) \\
 &=: \underbrace{Z^c(\alpha)}_{\text{integrable, zero volume for } \alpha \rightarrow 0} + \underbrace{C^c}_{\text{no divergences}} + \underbrace{N^c(\alpha)}_{\text{only } F_n \rightarrow \text{DU}}
 \end{aligned}$$

The function $N^c(\alpha)$

Looks like slicing, but it is slicing *only* for divergences
 → no actual slicing parameter in result

Power-log-expansion

$$N^c(\alpha) = \sum_{k=0}^{\ell_{\max}} \ln^k(\alpha) N_k^c(\alpha)$$

- all $N_k^c(\alpha)$ regular in α
- start expression independent of $\alpha \Rightarrow$ all logs cancel
- only $N_0^c(0)$ relevant

Putting parts together

$$\sigma_{SU} - \sum_c N_0^c(0) \text{ and } \sigma_{DU} + \sum_c N_0^c(0)$$

are finite in 4 dimension



SU contribution

$$\sigma_{SU} - \sum_c N_0^c = \sum_c C^c$$

original expression σ_{SU} in 4-dim without poles, no further ϵ pole cancellation

Calculation of $N_0^c(0)$

For each sector/contribution:

1. extraction of $d\Phi_{n+1}$ from $d\Phi_{n+2}|_{SU \text{ pole}}$ (only for RR contribution)

$$d\Phi_{n+2}|_{SU \text{ pole}} = \left(\underbrace{d\Phi_n d^d \mu(u_1)}_{d\Phi_{n+1}} d^d \mu(u_2) \right) \Big|_{u_2 \text{ col/soft}}$$

2. expansion in ϵ up to ϵ^{-1} (except $d\Phi_{n+1}$): $d^d \Phi_{n+1} \left(\frac{\dots}{\epsilon^2} + \frac{\dots}{\epsilon} \right)$
3. Identifying $\ln^k(\alpha)$'s from x_i integrations over θ function

$$\theta_\alpha(\hat{\eta}, u^0) = \theta(\hat{\eta} - \alpha) \theta(\hat{\xi} u_{max}/E_{norm} - \alpha)$$

→ discard them

4. perform integration over θ -functions of non-cancelling and non-vanishing (in $\alpha \rightarrow 0$ limit) terms