

Top quark mass effects in Higgs physics

— Towards HJ production at NLO —



MAX-PLANCK-GESELLSCHAFT

Matthias Kerner

Radcor 2017

St. Gilgen — 28 September 2017

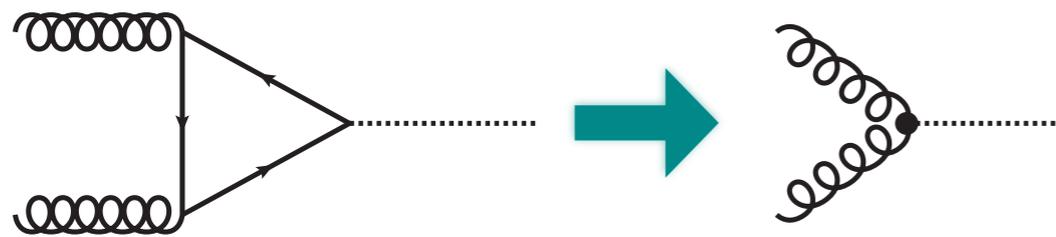


Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

In collaboration with
Stephen Jones, Gionata Luisoni

HEFT vs. full theory

Many calculations in Higgs physics done in the $m_t \rightarrow \infty$ limit (Higgs EFT)



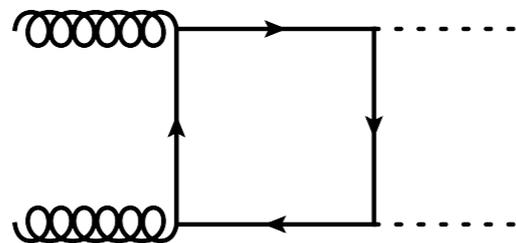
$$\mathcal{L}_{\text{int}} = -\frac{\lambda}{4} H G_a^{\mu\nu} G_{a,\mu\nu}$$

$$\text{with } \lambda = -\frac{\alpha_s}{3\pi v} + \mathcal{O}(\alpha_s^2)$$

HEFT valid if m_t is large compared to all other scales

H + X production: large phase space region with $\sqrt{s} > 2m_t$

e.g. Higgs Pair production

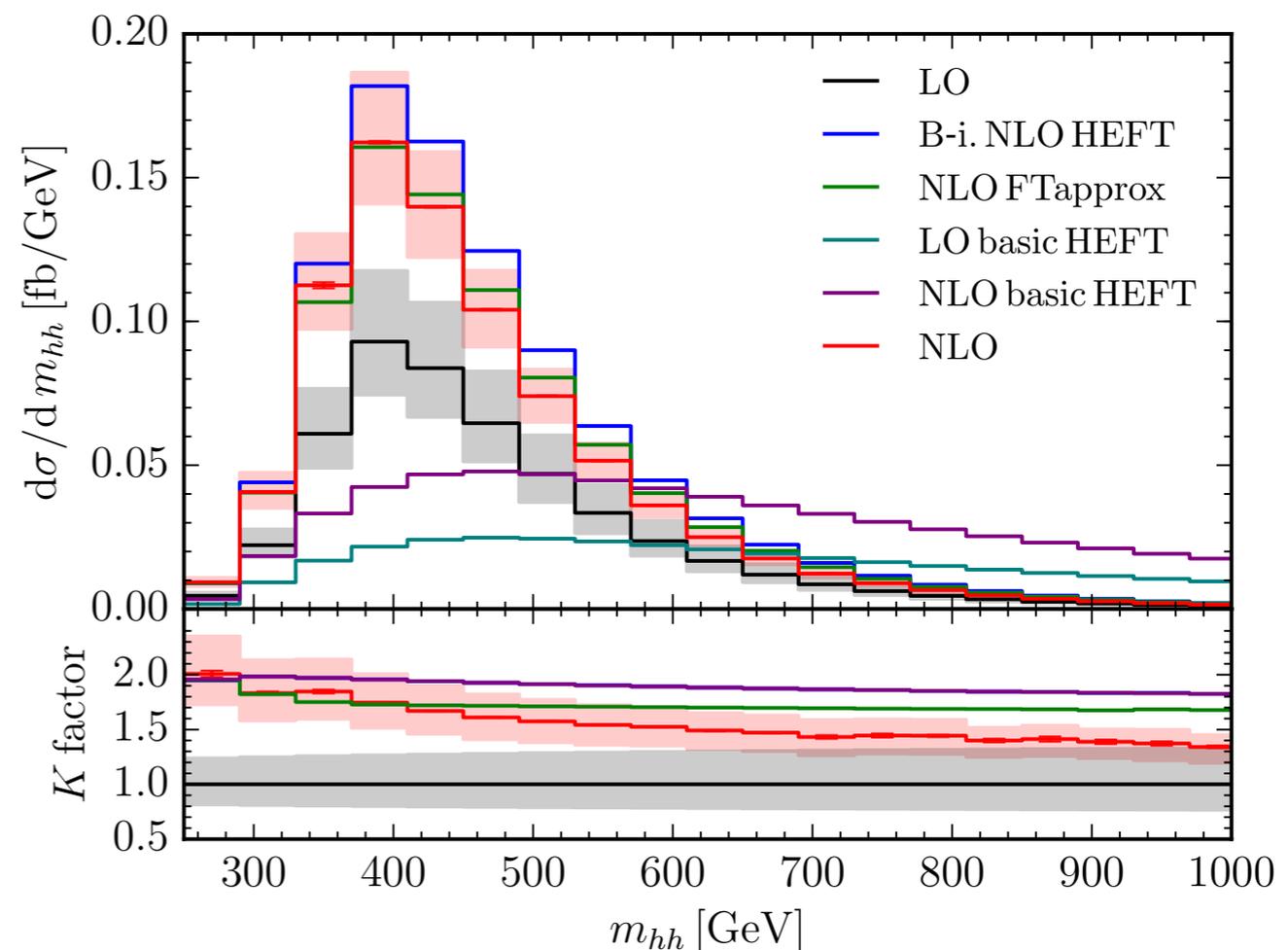


top mass effects at NLO [Borowka, Geiner, Heinrich, Jones, MK, Schlenk, Schubert, Zirke 16]

compared to NLO HEFT:

- total cross section reduced by 14%
- large corrections at high m_{hh}

→ talk by Stephen Jones



HJ@NLO — Motivation

HJ production at large p_T

- top mass effects expected to be large for high p_T
LO: -40% for $p_T > 250$ GeV
- high p_T resolves particle in loop
→ sensitivity to BSM effects

This talk: toward HJ production at NLO with full m_t dependence

only subset of 2-loop integrals known analytically

→ use numerical methods for evaluation of virtual amplitude

HJ Production — known results

1) LO (full m_t dependence)

[Ellis, Hinchliffe, Soldate, van der Bij 87]

[Baur, Glover 89]

2) NLO

$K \approx 1.6$

- HEFT

[de Florian, Grazzini, Kunszt 99; Glosser, Schmidt 02;
Ravindran, Smith, van Neerven 02]

- approximated m_t dependence

[Harlander, Neumann, Ozeren, Wiesemann 12]

[Neumann, Wiesemann 14] [Frederix, Frixione,

Vryonidou, Wiesemann 16] [Neumann, Williams 16]

[Caola, Forte, Marzani, Muselliand, Vita 16]

[Braaten, Zhang, Zhang 17]

- top-bottom interference → talk by Chris Wever

[(Lindert,) Melnikov, Tancredi, Wever 16, 17]

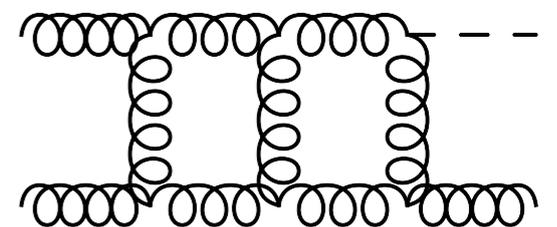
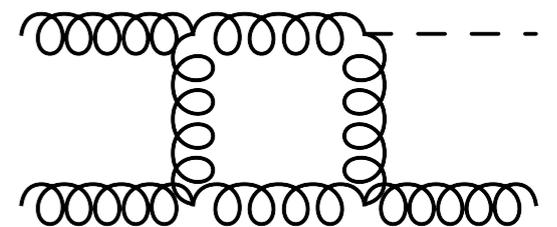
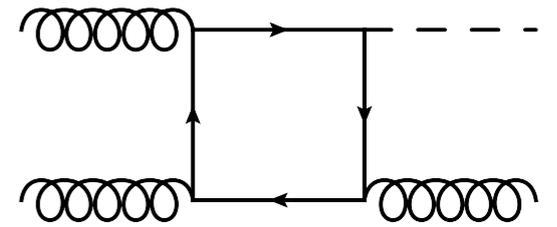
3) NNLO (HEFT)

$K \approx 1.2$

[Boughezal, Caola, Melnikov, Petriello, Schulze 13, 14]

[Chen, (Martinez,) Gehrmann, Glover, Jaquier 14, 16]

[Boughezal, Focke, Giele, Liu, Petriello 15]



Partonic Channels

	$p_T > 30 \text{ GeV}$	$p_T > 300 \text{ GeV}$
$gg \rightarrow Hg$	5836 fb (73%)	53.1 fb (62.5%)
$qg \rightarrow Hq, \bar{q}g \rightarrow H\bar{q}$	2096 fb (26%)	31.0 fb (36.5%)
$q\bar{q} \rightarrow Hg$	38 fb (0.5%)	0.8 fb (1%)

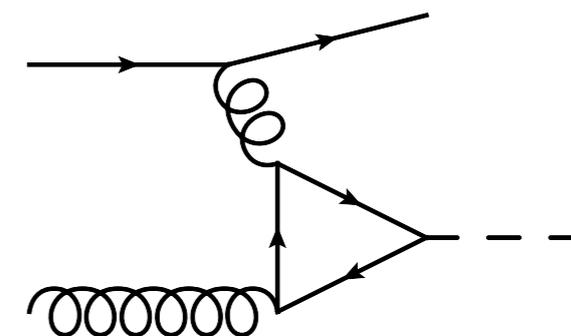
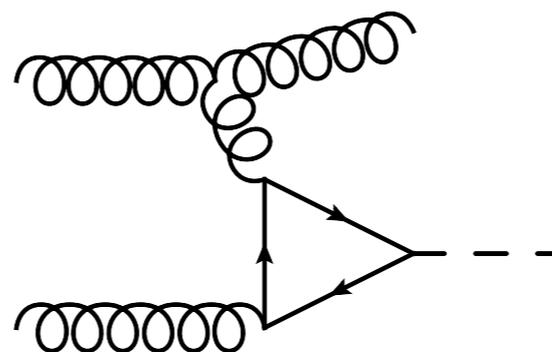
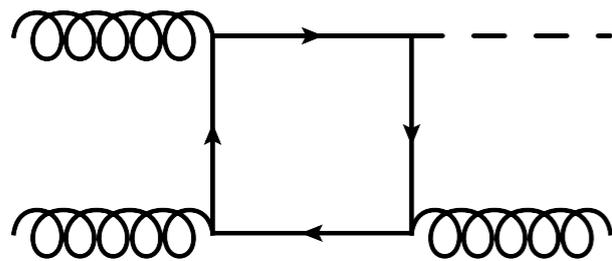
$$\sqrt{s} = 13 \text{ TeV}$$

$$m_H = 125 \text{ GeV}$$

$$m_t = 173.05 \text{ GeV}$$

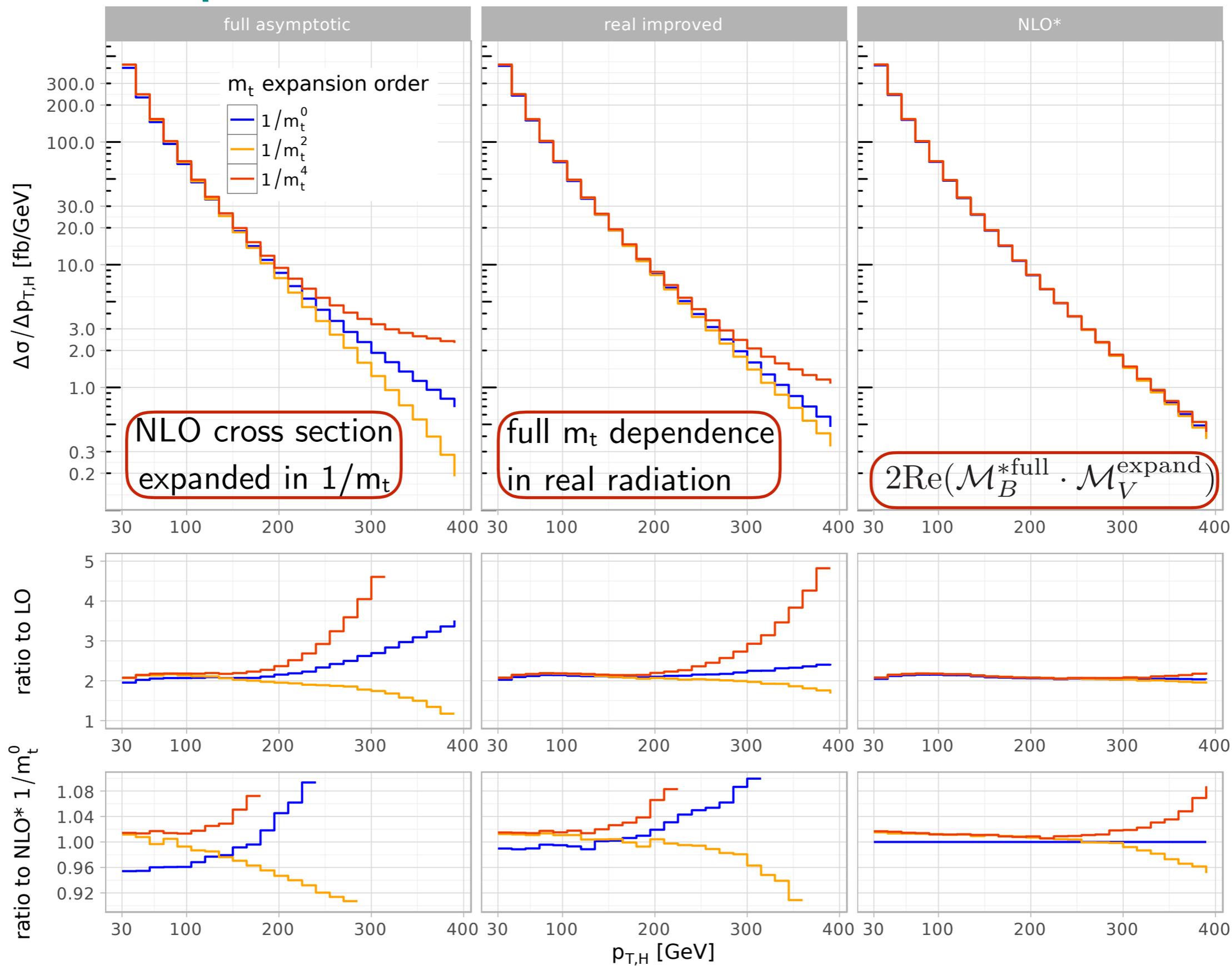
PDF4LHC15

$$\mu_R = \mu_F = m_H$$



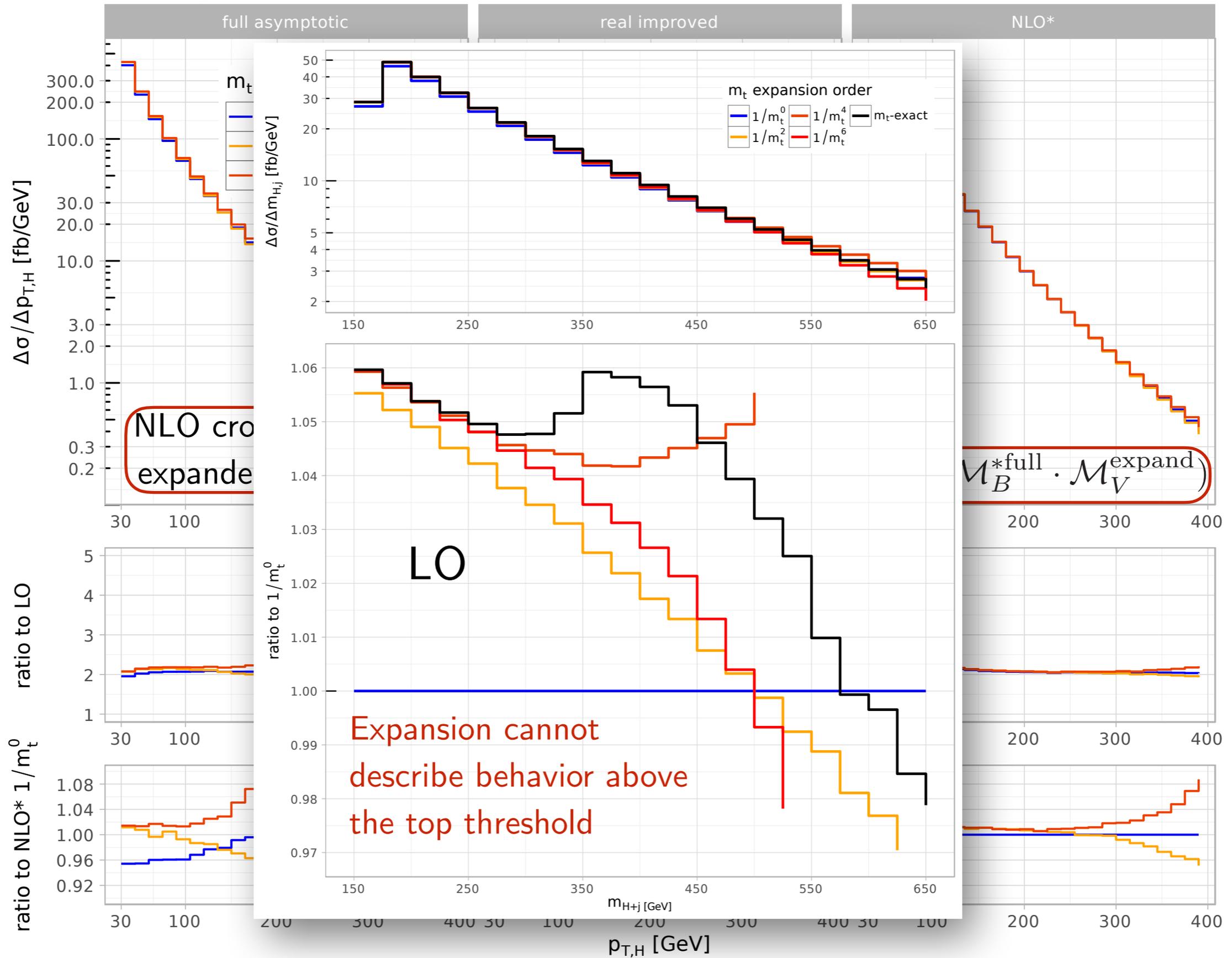
Approximated results

[Neumann, Williams 16]



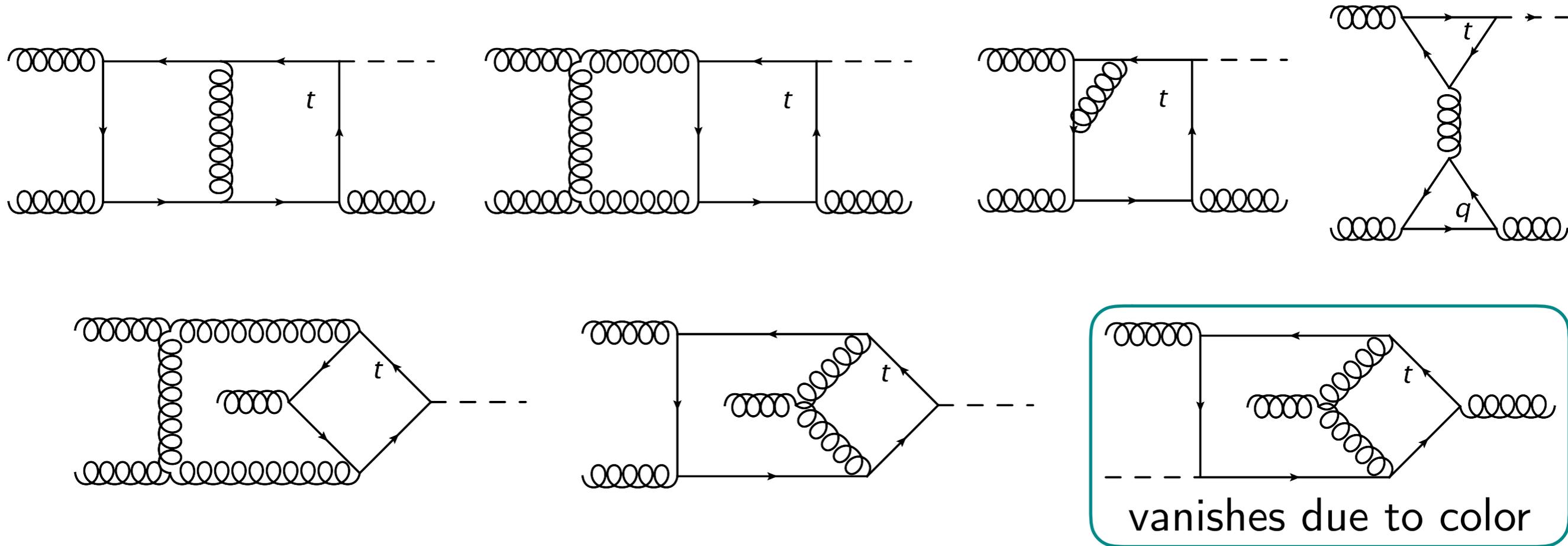
Approximated results

[Neumann, Williams 16]



Virtual Amplitude — Feynman Diagrams

gluon channels:



quark channels:



Analytic results for planar integrals

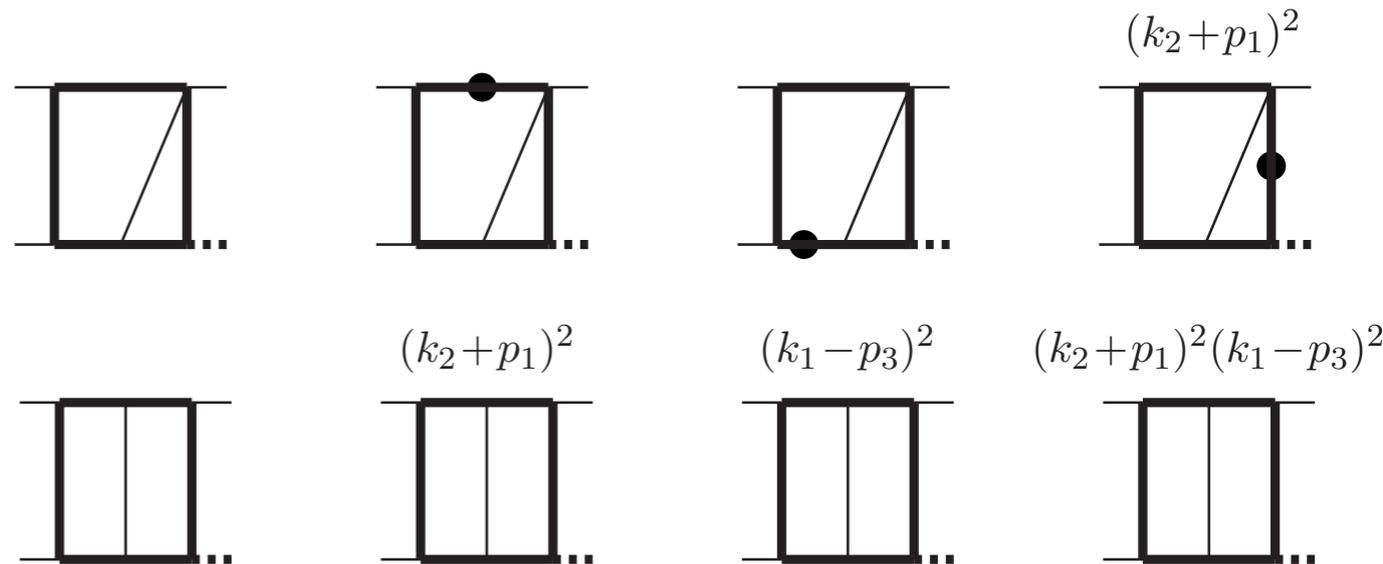
[Bonciani, Del Duca, Frellesvig, Henn, Moriello, Smirnov 16]

most planar integrals can be expressed in terms of

- log, Li_2 up to weight 2
- 1-fold integrals at weights 3,4
- alphabet with 3 variables, 49 letters, many square roots

2 sectors contain elliptic functions

can be expressed as 2- and 3-fold iterated integrals with elliptic kernel



→ results not available in closed form
no results for non-planar integrals

We calculate all integrals full numerically using SecDec

Virtual Amplitude — Form Factors

gluon channel: $\mathcal{M} = \epsilon_\mu(p_1)\epsilon_\nu(p_2)\epsilon_\tau(p_3)\mathcal{M}^{\mu\nu\tau}$

$$\begin{aligned} \mathcal{M}^{\mu\nu\tau}(s, t, u) = & F_{212}(sg^{\mu\nu} - 2p_2^\mu p_1^\nu)(up_1^\tau - tp_2^\tau)/(2t) \\ & + F_{332}(ug^{\nu\tau} - 2p_3^\nu p_2^\tau)(tp_2^\mu - sp_3^\mu)/(2s) \\ & + F_{311}(tg^{\tau\mu} - 2p_1^\tau p_3^\mu)(sp_3^\nu - up_1^\nu)/(2u) \\ & + F_{312}\left(g^{\mu\nu}(up_1^\tau - tp_2^\tau) + g^{\nu\tau}(tp_2^\mu - sp_3^\mu) + g^{\tau\mu}(sp_3^\nu - up_1^\nu) \right. \\ & \left. + 2p_3^\mu p_1^\nu p_2^\tau - 2p_2^\mu p_3^\nu p_1^\tau\right)/2 \end{aligned}$$

Thanks: Glover, Frellesvig

Form factors are gauge invariant and cyclic:

$$F_{311}(u, s, t) = F_{332}(t, u, s) = F_{212}(s, t, u)$$

$$F_{312}(u, s, t) = F_{312}(t, u, s) = F_{312}(s, t, u)$$

quark channels:

$$\begin{aligned} \mathcal{M} = & F_q \left(\bar{u}(q_{\bar{q}})\not{p}_g v(p_q)p_q \cdot \varepsilon - \bar{u}(q_{\bar{q}})\not{\varepsilon} v(p_q)p_q \cdot p_g \right) \\ & + F_{\bar{q}} \left(\bar{u}(q_{\bar{q}})\not{p}_g v(p_q)p_{\bar{q}} \cdot \varepsilon - \bar{u}(q_{\bar{q}})\not{\varepsilon} v(p_q)p_{\bar{q}} \cdot p_g \right) \end{aligned}$$

Gehrmann, Glover,
Jaquier, Koukoutsakis

Integral Reduction

Full Integration-by-Parts reduction [Chetyrkin, Tkachov 81; Laporta 01]

achieved using Reduze [von Manteuffel, Studerus 12]

with modifications to

- change order of solving the system of equations,
 sorting the equations by number of unreduced integrals
- specify list of required integrals
 → consider only equations containing these integrals

unreduced amplitude: 3767 integrals

up to 3 inverse propagators for 7-propagator integrals

up to 4 inverse propagators for factorizing 6-propagator integrals

reduced amplitude: 458 integrals

up to 6 master integrals per sector

choose quasi-finite basis of master integrals [von Manteuffel, Panzer, Schabinger 14]

→ requires integrals in shifted dimension [Tarasov 96; Lee 10]

→ requires reduction of integrals with 2 inverse propagators and 2 dots

Integral Reduction

Reduction done using two different setups:

1) fix mass ratio

$$\frac{m_h^2}{m_t^2} = \frac{12}{23} \rightarrow \begin{aligned} m_h &= 125 \text{ GeV} \\ m_t &= 173.055 \text{ GeV} \end{aligned}$$

2) keep full mass dependence

Total size of Reduze
reduction directory:

250 GB

1100 GB

Size of reduced amplitude:
symbolic d-dependent coefficients
after simplifications with
Fermat and Mathematica

780 MB

?

Size of c++ code:
for coefficients
after expansion in ε

340 MB

?

including m_b, Γ_t possible:

X

✓

Loop Integrals — Sector Decomposition

Numerical evaluation of loop integrals with **SecDec**

[Borowka, Heinrich, Jahn, Jones, MK, Schlenk, Zirke]

- Sector decomposition [Binoth, Heinrich '00]

factorizes overlapping singularities

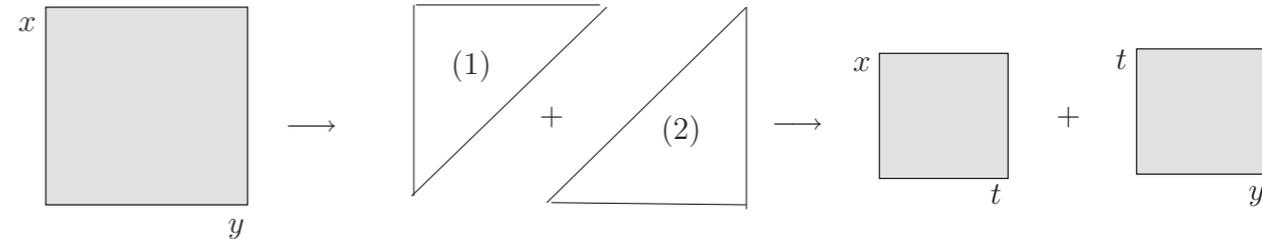
- Subtraction of poles & expansion in ϵ

- Contour deformation [Soper 00; Binoth et al. 05, Nagy, Soper 06, Borowka et al. 12]

analytic continuation from Euclidean to physical region

→ finite integrals at each order in ϵ

→ numerical integration possible



SecDec 3 used for amplitude evaluation

interface to amplitude based on calculation of HH production

[Borowka, Greiner, Heinrich, Jones, MK, Schlenk, Schubert, Zirke]

New version of SecDec: **pySecDec** → talk by Stephan Jahn

- implementation in python and Form

- modular structure

- generates libraries that can be directly linked to amplitude code

Loop Integrals — Amplitude Structure

rewrite loop integrals with r propagators and s inverse propagators as

$$I_{r,s}(s, t, m_h^2, m_t^2) = (M^2)^{-L\epsilon} (M^2)^{2L-r+s} I_{r,s} \left(\frac{s}{M^2}, \frac{t}{M^2}, \frac{m_h^2}{M^2}, \frac{m_t^2}{M^2} \right)$$

arbitrary scale

and write renormalized form factors as

$$F^{\text{virt}} = a^{3/2} \left(F^{(1)} + a \left(\frac{n_g}{2} \delta Z_A + \frac{3}{2} \delta Z_a \right) F^{(1)} + a \delta m_t^2 \mathcal{F}^{\text{ct},(1)} + a F^{(2)} + \mathcal{O}(a^2) \right)$$

$$F^{(1)} = \left(\frac{\mu_R^2}{M^2} \right)^\epsilon \left[b_0^{(1)} + b_1^{(1)} \epsilon + b_2^{(1)} \epsilon^2 + \mathcal{O}(\epsilon^3) \right], \quad \text{(1-loop)}$$

$$F^{\text{ct},(1)} = \left(\frac{\mu_R^2}{M^2} \right)^\epsilon \left[c_0^{(1)} + c_1^{(1)} \epsilon + \mathcal{O}(\epsilon^2) \right], \quad \text{(mass counter-term)}$$

$$F^{(2)} = \left(\frac{\mu_R^2}{M^2} \right)^{2\epsilon} \left[\frac{b_{-2}^{(2)}}{\epsilon^2} + \frac{b_{-1}^{(2)}}{\epsilon} + b_0^{(2)} + \mathcal{O}(\epsilon) \right], \quad \text{(2-loop)}$$

→ scale variations do not require re-computation of $b_i^{(n)}, c_i^{(n)}$

Loop Integrals — Numerical Integration

after sector decomposition and expansion in ϵ :
amplitude written in terms of 22 675 finite integrals

- all integrals evaluated using Quasi-Monte-Carlo integration

- generating vector

- constructed component-by-component [Nuyens 07]
- minimizing worst-case error
- for fixed lattice sizes

- $\mathcal{O}(n^{-1})$ scaling of integration error

- dynamically set n for each integral, minimizing

$$T = \sum_{\text{integral } i} t_i + \lambda \left(\sigma^2 - \sum_i \sigma_i^2 \right) \quad \sigma_i = c_i \cdot t_i^{-e}$$

σ_i = error estimate (including coefficients in amplitude)
 λ = Lagrange multiplier σ = precision goal

- parallelization on gpu

- avoid reevaluation of integrals for different orders in ϵ and form factors

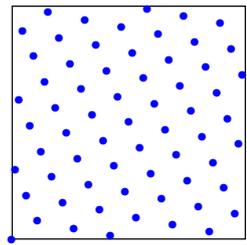
$$F^a = \sum_i \left[\left(\sum_j C_{i,j}^a \epsilon^j \right) \cdot \left(\sum_k I_{i,k} \epsilon^k \right) \right] = \frac{C_{1,-2}^a I_{1,0} + C_{1,-1}^a I_{1,-1} + \dots}{\epsilon^2} + \frac{C_{1,-1}^a I_{1,0} + \dots}{\epsilon^1} + \dots$$

compute only once

QMC rank-1 lattice rule

$$I = \int d\vec{x} f(\vec{x}) \approx I_k = \frac{1}{n} \sum_{i=1}^n f(\vec{x}_{i,k})$$

$$\vec{x}_{i,k} = \left\{ \frac{i \cdot \vec{g}}{n} + \vec{\Delta}_k \right\}$$

$\{ \dots \}$ = fractional part 

\vec{g} = generating vector

$\vec{\Delta}_k$ = randomized shift

m different estimates $I_1 \dots I_m$

→ error estimate

Review: Dick, Kuo, Sloan

Phase-Space integration & Real Radiation

Phase-space integration of virtual corrections:

- generate unweighted events based on differential LO cross section
 - nearly perfect importance sampling for evaluating total cross section
- include additional p_T -dependent reweighing factor
 - enhances number of events in tail of distribution, reducing their weight

Real radiation matrix elements generated with GoSam [Cullen et. al.]

Implemented in Powheg-Box framework [Alioli, Hamilton, Nason, Oleari, Re, Zanderighi]
allows to calculate the cross section

- at fixed order
- including a parton shower

Current Status

Real radiation

- fully implemented
- checked
- results available, but more statistics required

Virtual corrections

- full code generated & compiled
- pole cancellation checked
- verified permutation symmetry of form factors
- ToDo:
 - compare to HEFT result
 - test convergence of finite part in various phase-space regions
 - evaluate cross section

pole cancellation (at low m_{hj} , p_T)

	gluon channels	quark channels
ϵ^{-2}	5-8 digits	8 digits
ϵ^{-1}	3-5 digits	7 digits

Comparison of HJ and HH

	HJ production	HH production
#Form factors	4+2	2
Full reduction	✓	only planar
(quasi-) finite basis	✓	only planar
#Master integrals including crossings	458	327
#Master integrals neglecting crossings	120	215
#Integrals after sector decomposition and expansion in ϵ	22675	11244
Code size coefficients	~340 MB	~80 MB
Code size integrals	~330 MB	~580 MB
Compile time coefficients	~ 2 weeks	few days
Compile time integrals	~4 hours	~1-2 days
Time for linking the program	~3-4 days	few hours

Challenges (far) ahead

future plans to extend the calculation

- effects of top width
- bottom quark mass effects
 - large mass ratios
 - numerical integration might be challenging
- combine with parton shower



requires reduction with full m_h and m_t dependence
→ more complicated coefficients

HJ production at NLO

- top quark mass effects expected to be large at high p_T
- implementation of NLO correction with full m_t dependence
 - 2-loop amplitude calculated numerically
 - code generation finished
 - some more testing required

Thank you for your attention!

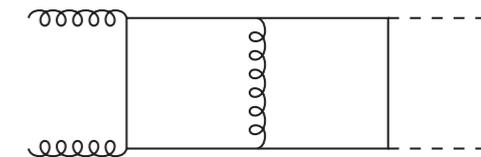
Backup

HH Amplitude Evaluation — Example

$$\sqrt{s} = 327.25 \text{ GeV}, \sqrt{-t} = 170.05 \text{ GeV}, M^2 = s/4$$

contributing integrals:

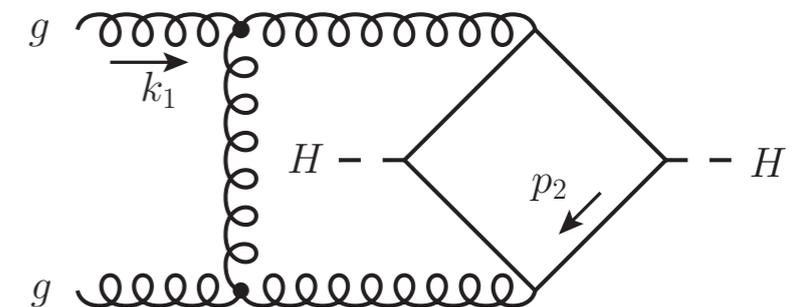
integral	value	error	time [s]
...			
F1_011111110_ord0	(0.484, 4.96e-05)	(4.40e-05, 4.23e-05)	11.8459
...			
N3_111111100_k1p2k2p2_ord0	(0.0929, -0.224)	(6.32e-05, 5.93e-05)	235.412
N3_111111100_1_ord0	(-0.0282, 0.179)	(8.01e-05, 9.18e-05)	265.896
N3_111111100_k1p2k1p2_ord0	(0.0245, 0.0888)	(5.06e-05, 5.31e-05)	282.794
N3_111111100_k1p2_ord0	(-0.00692, -0.108)	(3.05e-05, 3.05e-05)	433.342



≈ 700
integrals

$$I(s, t, m_t^2, m_h^2) = - \left(\frac{\mu^2}{M^2} \right)^{2\epsilon} \Gamma(3 + 2\epsilon) M^{-4} \left(\frac{A_{-2}}{\epsilon^2} + \frac{A_{-1}}{\epsilon^1} + A_0 + \mathcal{O}(\epsilon) \right)$$

sector decomposition



sector	integral value	error	time [s]	#points
5	(-1.34e-03, 2.00e-07)	(2.38e-07, 2.69e-07)	0.255	1310420
6	(-1.58e-03, -9.23e-05)	(7.44e-07, 5.34e-07)	0.266	1310420
...				
41	(0.179, -0.856)	(1.10e-05, 1.22e-05)	29.484	79952820
42	(0.359, -1.308)	(1.40e-06, 1.58e-06)	80.24	211436900
44	(0.0752, -1.185)	(5.44e-07, 6.76e-07)	99.301	282904860

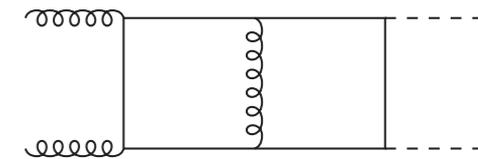
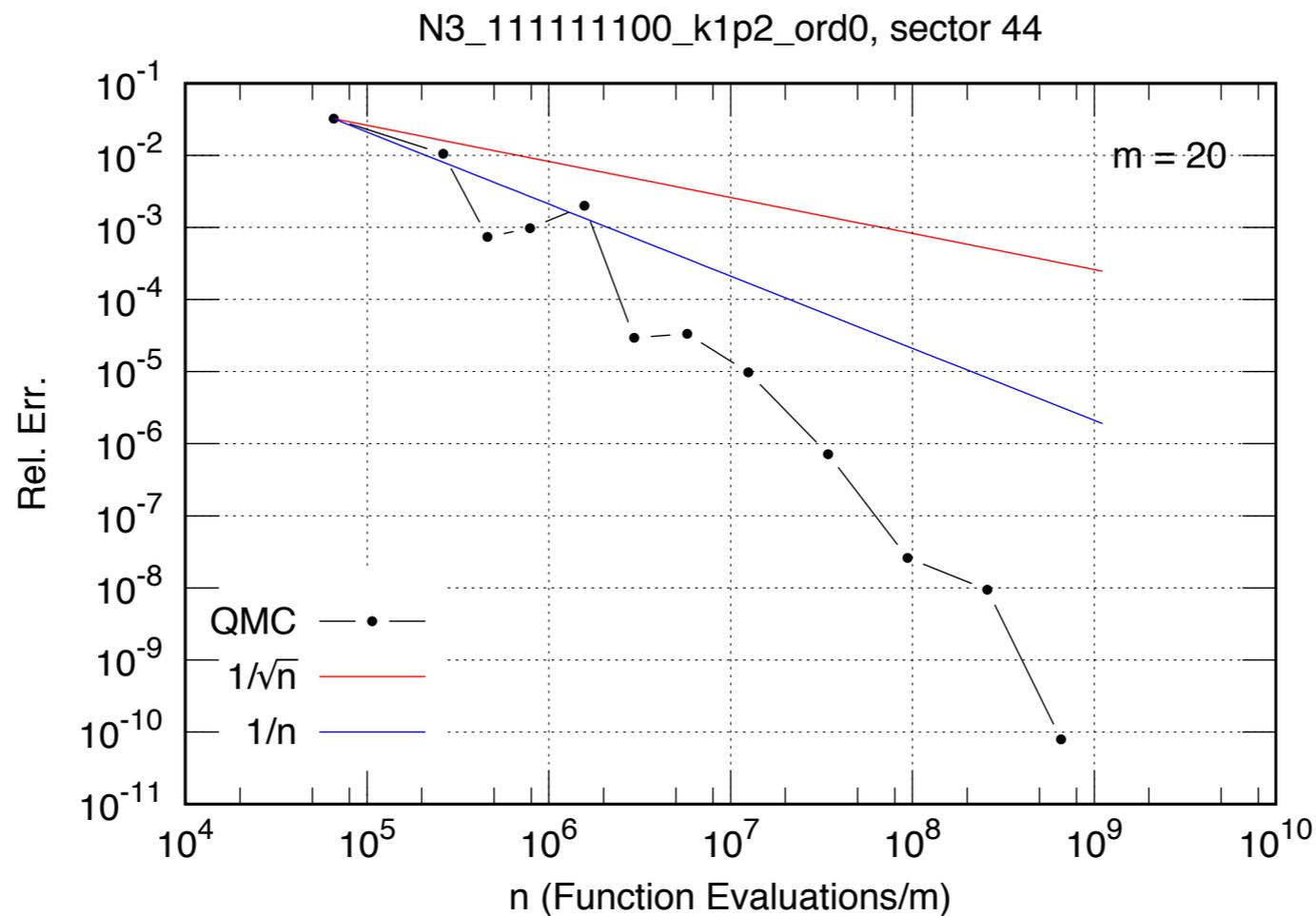
slide:
MK, L&L 2016

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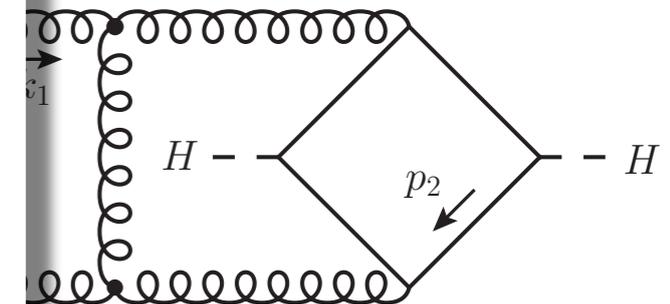
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...			
N3_111111100_k1p2			
N3_111111100_1_orc			
N3_111111100_k1p2			
N3_111111100_k1p2			
...			
sector	in		
5	(-1.34e-0		
6	(-1.58e-0		
...			
41	(0.179, -0.856)	(1.10e-05, 1.22e-05)	29.484
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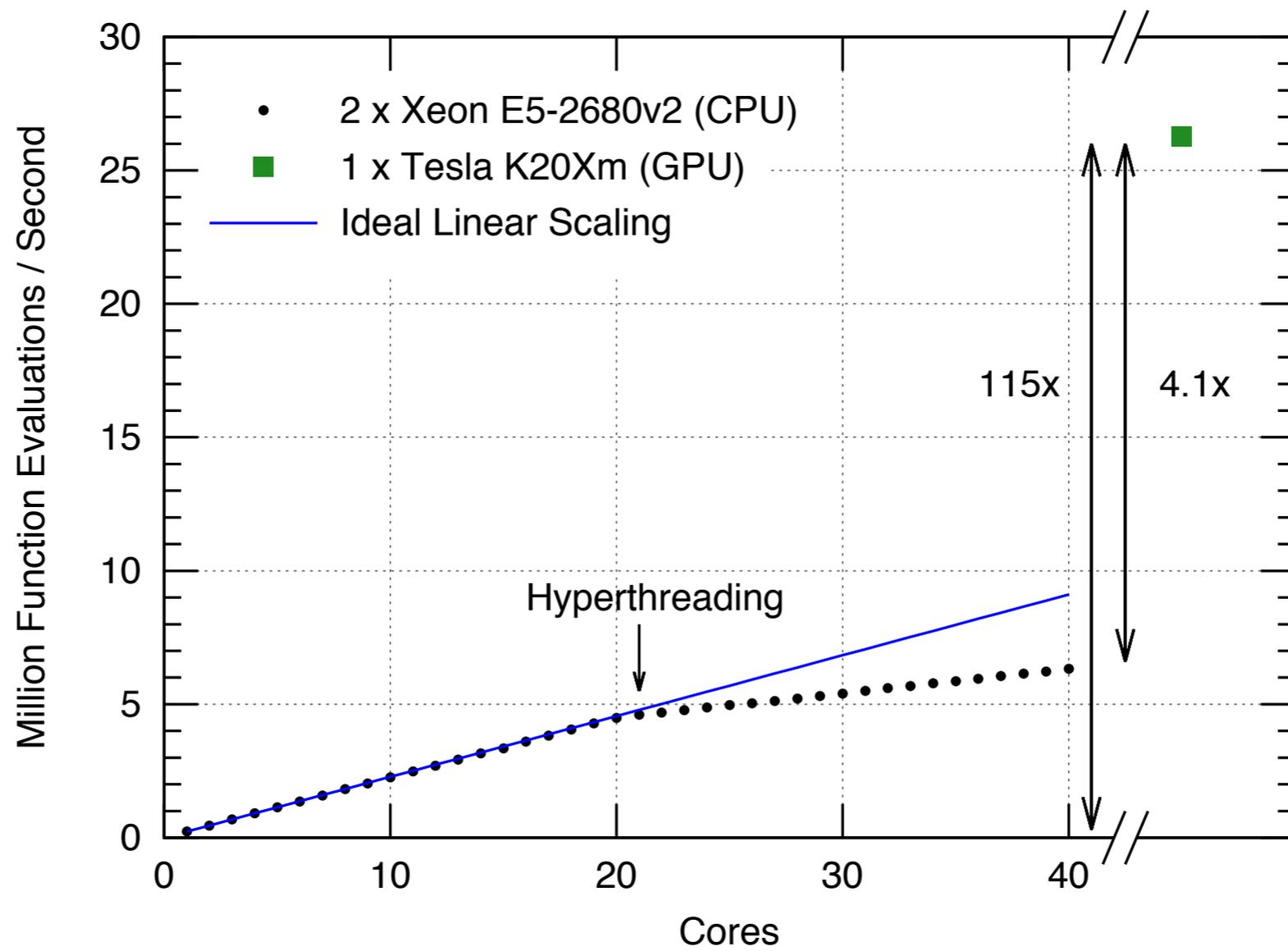


≈ 700
integrals

412
896
794
342



slide:
MK, L&L 2016



plot:
Stephen Jones